

Elayn Martín-Gay

Prealgebra



Eighth Edition



Prealgebra

This page intentionally left blank

Prealgebra

Eighth Edition

Elayn Martín-Gay

University of New Orleans



330 Hudson Street, NY NY 10013

Director, Portfolio Management: *Michael Hirsch*
Courseware Portfolio Manager: *Mary Beckwith*
Courseware Portfolio Management Assistant: *Alison Oehman*
Managing Producer: *Karen Wernholm*
Content Producer: *Patty Bergin*
Media Producer: *Audra Walsh*
Manager, Courseware QA: *Mary Durnwald*
Senior Content Developer, Math: *Eric Gregg*
Product Marketing Manager: *Alicia Frankel*
Field Marketing Manager: *Jennifer Crum and Lauren Schur*
Product Marketing Assistant: *Hanna Lafferty*
Senior Author Support/Technology Specialist: *Joe Vetere*
Manager, Rights and Permissions: *Gina Cheselka*
Manufacturing Buyer: *Carol Melville, LSC Communications*
Text Design: *Tamara Newnam*
Composition and Production Coordination: *Integra*
Illustrations: *Scientific Illustrators*
Senior Designer: *Barbara T. Atkinson*
Cover Design: *Tamara Newman*
Cover Image: *Tamara Newman*

Copyright © 2019, 2015, 2011 by Pearson Education, Inc. All Rights Reserved.
Printed in the United States of America. This publication is protected by copyright,
and permission should be obtained from the publisher prior to any prohibited
reproduction, storage in a retrieval system, or transmission in any form or by
any means, electronic, mechanical, photocopying, recording, or otherwise. For
information regarding permissions, request forms and the appropriate contacts
within the Pearson Education Global Rights & Permissions department, please visit
www.pearsoned.com/permissions/.

Attributions of third party content appear on page P1, which constitutes an extension
of this copyright page.

PEARSON, ALWAYS LEARNING, and MYLAB MATH are exclusive trademarks
owned by Pearson Education, Inc. or its affiliates in the U.S. and/or other countries.

Unless otherwise indicated herein, any third-party trademarks that may appear in this
work are the property of their respective owners and any references to third-party
trademarks, logos or other trade dress are for demonstrative or descriptive purposes
only. Such references are not intended to imply any sponsorship, endorsement, au-
thorization, or promotion of Pearson's products by the owners of such marks, or any
relationship between the owner and Pearson Education, Inc. or its affiliates, authors,
licensees or distributors.

Library of Congress Cataloging-in-Publication Data

Names: Martin-Gay, K. Elayn, author.
Title: Prealgebra / Elayn Martin-Gay.
Description: 8th edition. | New York : Pearson, [2019] | Includes index. |
Revised edition published, 8th edition, Boston, 2015.
Identifiers: LCCN 2017034234 | ISBN 9780134707648 (se : alk. paper) | ISBN
9780134708829 (aie : alk. paper) | ISBN 9780134708706 (ebook)
Subjects: LCSH: Arithmetic--Textbooks.
Classification: LCC QA107.2 .M37 2019 | DDC 513/.1--dc23
LC record available at <https://lcn.loc.gov/2017034234>



This book is dedicated to students everywhere—
and we should all be students. After all, is there anyone among
us who truly knows too much? Take that hint and continue
to learn something new every day of your life.

Best wishes from a fellow student:
Elayn Martin-Gay

This page intentionally left blank

Contents

Preface xiii

Applications Index xxiii

1 **The Whole Numbers** 1

- 1.1** Study Skill Tips for Success in Mathematics 2
- 1.2** Place Value, Names for Numbers, and Reading Tables 8
- 1.3** Adding and Subtracting Whole Numbers, and Perimeter 17
- 1.4** Rounding and Estimating 32
- 1.5** Multiplying Whole Numbers and Area 40
- 1.6** Dividing Whole Numbers 52
 - Integrated Review—Operations on Whole Numbers** 66
- 1.7** Exponents and Order of Operations 68
- 1.8** Introduction to Variables, Algebraic Expressions, and Equations 75
 - Group Activity 84
 - Vocabulary Check 85
 - Chapter Highlights 85
 - Chapter Review 89
 - Getting Ready for the Test 95
 - Chapter Test 96

2 **Integers and Introduction to Solving Equations** 98

- 2.1** Introduction to Integers 99
- 2.2** Adding Integers 108
- 2.3** Subtracting Integers 116
- 2.4** Multiplying and Dividing Integers 124
 - Integrated Review—Integers** 133
- 2.5** Order of Operations 135
- 2.6** Solving Equations: The Addition and Multiplication Properties 142
 - Group Activity 151
 - Vocabulary Check 152
 - Chapter Highlights 152
 - Chapter Review 154
 - Getting Ready for the Test 159
 - Chapter Test 160
 - Cumulative Review 162

3 Solving Equations and Problem Solving 164

- 3.1** Simplifying Algebraic Expressions **165**
- 3.2** Solving Equations: Review of the Addition and Multiplication Properties **175**
Integrated Review—Expressions and Equations 184
- 3.3** Solving Linear Equations in One Variable **186**
- 3.4** Linear Equations in One Variable and Problem Solving **193**
 - Group Activity **202**
 - Vocabulary Check **202**
 - Chapter Highlights **203**
 - Chapter Review **205**
 - Getting Ready for the Test **209**
 - Chapter Test **210**
 - Cumulative Review **212**

4 Fractions and Mixed Numbers 214

- 4.1** Introduction to Fractions and Mixed Numbers **215**
- 4.2** Factors and Simplest Form **229**
- 4.3** Multiplying and Dividing Fractions **242**
- 4.4** Adding and Subtracting Like Fractions, Least Common Denominator, and Equivalent Fractions **254**
- 4.5** Adding and Subtracting Unlike Fractions **268**
Integrated Review—Summary on Fractions and Operations on Fractions 280
- 4.6** Complex Fractions and Review of Order of Operations **282**
- 4.7** Operations on Mixed Numbers **290**
- 4.8** Solving Equations Containing Fractions **307**
 - Group Activity **316**
 - Vocabulary Check **316**
 - Chapter Highlights **317**
 - Chapter Review **321**
 - Getting Ready for the Test **326**
 - Chapter Test **327**
 - Cumulative Review **329**

5 Decimals 331

- 5.1** Introduction to Decimals **332**
- 5.2** Adding and Subtracting Decimals **344**
- 5.3** Multiplying Decimals and Circumference of a Circle **357**
- 5.4** Dividing Decimals **366**
Integrated Review—Operations on Decimals 376
- 5.5** Fractions, Decimals, and Order of Operations **378**
- 5.6** Solving Equations Containing Decimals **387**
- 5.7** Decimal Applications: Mean, Median, and Mode **392**

Group Activity	400
Vocabulary Check	401
Chapter Highlights	401
Chapter Review	404
Getting Ready for the Test	410
Chapter Test	411
Cumulative Review	413

6 **Ratio, Proportion, and Triangle Applications** 416

6.1	Ratios and Rates	417
6.2	Proportions	427
	Integrated Review—Ratio, Rate, and Proportion	435
6.3	Proportions and Problem Solving	437
6.4	Square Roots and the Pythagorean Theorem	445
6.5	Congruent and Similar Triangles	453
	Group Activity	461
	Vocabulary Check	462
	Chapter Highlights	462
	Chapter Review	465
	Getting Ready for the Test	469
	Chapter Test	470
	Cumulative Review	472

7 **Percent** 474

7.1	Percents, Decimals, and Fractions	475
7.2	Solving Percent Problems with Equations	486
7.3	Solving Percent Problems with Proportions	493
	Integrated Review—Percent and Percent Problems	501
7.4	Applications of Percent	503
7.5	Percent and Problem Solving: Sales Tax, Commission, and Discount	515
7.6	Percent and Problem Solving: Interest	521
	Group Activity	527
	Vocabulary Check	528
	Chapter Highlights	528
	Chapter Review	531
	Getting Ready for the Test	534
	Chapter Test	535
	Cumulative Review	537

8 **Graphing and Introduction to Statistics and Probability** 539

8.1	Pictographs, Bar Graphs, Histograms, Line Graphs, and Introduction to Statistics	540
8.2	Circle Graphs	556
8.3	The Rectangular Coordinate System and Paired Data	564

Integrated Review—Reading Graphs 574**8.4** Graphing Linear Equations in Two Variables **577****8.5** Counting and Introduction to Probability **588**Group Activity **595**Vocabulary Check **596**Chapter Highlights **596**Chapter Review **599**Getting Ready for the Test **607**Chapter Test **609**Cumulative Review **615****9 Geometry and Measurement 617****9.1** Lines and Angles **618****9.2** Perimeter **629****9.3** Area, Volume, and Surface Area **639****Integrated Review—Geometry Concepts 655****9.4** Linear Measurement **656****9.5** Weight and Mass **669****9.6** Capacity **679****9.7** Temperature and Conversions Between the U.S. and Metric Systems **687**Group Activity **696**Vocabulary Check **697**Chapter Highlights **697**Chapter Review **701**Getting Ready for the Test **707**Chapter Test **708**Cumulative Review **710****10 Exponents and Polynomials 713****10.1** Adding and Subtracting Polynomials **714****10.2** Multiplication Properties of Exponents **724****Integrated Review—Operations on Polynomials 729****10.3** Multiplying Polynomials **730****10.4** Introduction to Factoring Polynomials **736**Group Activity **741**Vocabulary Check **741**Chapter Highlights **742**Chapter Review **743**Getting Ready for the Test **746**Chapter Test **747**Cumulative Review **748**

Appendices

Appendix A Tables 751

A.1 Tables of Geometric Figures 751

A.2 Table of Percents, Decimals, and Fraction Equivalents 753

A.3 Table on Finding Common Percents of a Number 754

A.4 Table of Squares and Square Roots 755

Appendix B Quotient Rule and Negative Exponents 756

Appendix C Scientific Notation 761

Appendix D Geometric Formulas 765

Student Resources 766

Study Skills Builders 767

Bigger Picture—Study Guide Outline 778

Practice Final Exam 780

Answers to Selected Exercises A1

Solutions to Selected Exercises A26

Subject Index I-1

Photo Credits P1

This page intentionally left blank

Preface

Prealgebra, Eighth Edition, was written to help students make the transition from arithmetic to algebra. To help students accomplish this, my goals for this text are:

- Most importantly, to write an organized, student-friendly text that is keyed to objectives and contains many worked-out examples.
- To introduce algebraic concepts early and repeat them often as I cover traditional arithmetic topics, thus laying the groundwork for the next algebra course your students will take.
- To show students the relevancy of mathematics in everyday life and in the workplace by emphasizing and integrating the following throughout this text: real-life and real-data applications, data interpretation, conceptual understanding, problem solving, writing, cooperative learning, number sense, estimation, critical thinking and geometric concepts.

The many factors that contributed to the success of the previous editions have been retained. In preparing the Eighth Edition, I considered comments and suggestions of colleagues, students, and many users of the prior edition throughout the country.

What's New in the Eighth Edition?

- **The Martin-Gay Program** has been revised and enhanced with a new design in the text and MyLab Math to actively encourage students to use the text, video program, and Video Organizer as an integrated learning system.
- **New Getting Ready for the Test** can be found before each Chapter Test. These exercises can increase student success by helping students prepare for their Chapter Test. The purpose of these exercises is to check students' conceptual understanding of the topics in the chapter as well as common student errors. It is suggested that students complete and check these exercises before taking a practice Chapter Test. All Getting Ready for the Test exercises are either Multiple Choice or Matching, and all answers can be found in the answer section of this text.

Video Solutions of all exercises can be found in MyLab Math. These video solutions contain brief explanations and reminders of material in the chapter. Where applicable, incorrect choices contain explanations.

Getting Ready for the Test exercise numbers marked in blue indicate that the question is available in **Learning Catalytics**. 

- **New Learning Catalytics** is an interactive student response tool that uses students' smartphones, tablets, or laptops to engage them in more sophisticated tasks and thinking. Generate class discussion, guide your lecture, and promote peer-to-peer learning with real-time analytics. Accessible through MyLab Math, instructors can use Learning Catalytics to:
 - Pose a variety of open-ended questions that help your students develop critical thinking skills.
 - Monitor responses to find out where students are struggling.
 - Use real-time data to adjust your instructional strategy and try other ways of engaging your students during class.
 - Manage student interactions by automatically grouping students for discussion, teamwork, and peer-to-peer learning.

- Pearson-created questions for developmental math topics are available to allow you to take advantage of this exciting technology. Additionally, “Getting Ready for the Test” exercises (marked in blue) are available in Learning Catalytics. Search the question library for “MGPre” and the chapter number, for example, MGPre7 would be the questions from Chapter 7.
- **Revised and updated Key Concept Activity Lab Workbook** includes Extension Exercises, Exploration Activities, Conceptual Exercises, and Group Activities. These activities are a great way to engage students in conceptual projects and exploration as well as group work. This workbook is available in MyLab Math, or can be packaged with a text or MyLab code.
- **Exercise Sets** have been carefully examined and revised. Special focus was placed on making sure that even- and odd-numbered exercises are carefully paired and that real-life applications are updated.
- **The Martin-Gay MyLab Math** course has been updated and revised to provide more exercise coverage, including assignable Video Check questions and an expanded video program. There are Lecture Videos for every section, which students can also access at the specific objective level; Student Success Tips videos; and an increased number of video clips at the exercise level to help students while doing homework in MyLab Math. Suggested homework assignments have been premade for assignment at the instructor’s discretion.

Key Continuing Resources and Pedagogical Features

- **Vocabulary, Readiness & Video Check Questions** continue to be available in the text and for assignment in MyLab Math. The **Readiness** exercises center on a student’s understanding of a concept that is necessary in order to continue to the exercise set. The **Video Check questions** are included in every section for every learning objective. These exercises are a great way to assess whether students have viewed and understood the key concepts presented in the videos. Answers to all Video Check questions are available in an answer section at the back of the text.
- **Interactive Lecture Series in MyLab Math**, featuring author Elayn Martin-Gay, provides students with active learning at their own pace. The videos offer the following resources and more:
 - A complete lecture for each section of the text** highlights key examples and exercises from the text. Pop-ups reinforce key terms, definitions, and concepts.
 - An interface with menu navigation features** allows students to quickly find and focus on the examples and exercises they need to review.
 - Interactive Concept Check** exercises measure students’ understanding of key concepts and common trouble spots.
 - Student Success Tips Videos** are 3-5 minute videos designed to be daily reminders to students to continue practicing and maintaining good organizational and study habits. They include student success tips for general college success, tips specific to success in math courses, and content-specific tips to avoid common mathematical mistakes.
- **The Interactive Lecture Series** also includes the following resources for test prep:
 - New Getting Ready for the Chapter Test Videos**
 - The Chapter Test Prep Videos** help students during their most teachable moment—when they are preparing for a test. This innovation provides step-by-step solutions for the exercises found in each Chapter Test. For the Eighth Edition, the chapter test prep videos are also available on YouTube™. The videos are captioned in English and Spanish.
 - The Practice Final Exam Videos** help students prepare for an end-of-course final. Students can watch full video solutions to each exercise in the Practice Final Exam at the end of this text.

- **The Video Organizer** helps students take notes and work practice exercises while watching the Interactive Lecture Series videos in their MyLab Math course. All content in the Video Organizer is presented in the same order as it is presented in the videos, making it easy for students to create a course notebook and build good study habits.
 - Covers all of the video examples in order.
 - Provides prompts with ample space for students to write down key definitions and properties.
 - Includes Play and Pause button icons to prompt students to follow along with the author for some exercises while they try others on their own.

The Video Organizer is available in a loose-leaf, notebook-ready format. It is also available for download in MyLab Math.

Key Pedagogical Features

The following key features have been retained and/or updated for the Eighth Edition of the text:

- **Problem-Solving Process** This is formally introduced in Chapter 3 with a four-step process that is integrated throughout the text. The four steps are **Understand, Translate, Solve, and Interpret**. The repeated use of these steps in a variety of examples shows their wide applicability. Reinforcing the steps can increase students' comfort level and confidence in tackling problems.
- **Exercise Sets Revised and Updated** The exercise sets have been carefully examined and extensively revised. Special focus was placed on making sure that even- and odd-numbered exercises are paired and that real-life applications were updated.
- **Examples** Detailed, step-by-step examples were added, deleted, replaced, or updated as needed. Many examples reflect real life. Additional instructional support is provided in the annotated examples.
- **Practice Exercises** Throughout the text, each worked-out example has a parallel Practice exercise. These invite students to be actively involved in the learning process. Students should try each Practice exercise after finishing the corresponding example. Learning by doing will help students grasp ideas before moving on to other concepts. Answers to the Practice exercises are provided at the bottom of each page.
- **Helpful Hints** Helpful Hints contain practical advice on applying mathematical concepts. Strategically placed where students are most likely to need immediate reinforcement, Helpful Hints help students avoid common trouble areas and mistakes.
- **Concept Checks** This feature allows students to gauge their grasp of an idea as it is being presented in the text. Concept Checks stress conceptual understanding at the point-of-use and help suppress misconceived notions before they start. Answers appear at the bottom of the page. Exercises related to Concept Checks are included in the exercise sets.
- **Mixed Practice Exercises** In the section exercise sets, these exercises require students to determine the problem type and strategy needed to solve it just as they would need to do on a test.
- **Integrated Reviews** This unique, mid-chapter exercise set helps students assimilate new skills and concepts that they have learned separately over several sections. These reviews provide yet another opportunity for students to work with "mixed" exercises as they master the topics.
- **Vocabulary Check** This feature provides an opportunity for students to become more familiar with the use of mathematical terms as they strengthen their verbal skills. These appear at the end of each chapter before the Chapter Highlights. Vocabulary, Readiness & Video exercises provide practice at the section level.

- **Chapter Highlights** Found at the end of every chapter, these contain key definitions and concepts with examples to help students understand and retain what they have learned and help them organize their notes and study for tests.
- **Chapter Review** The end of every chapter contains a comprehensive review of topics introduced in the chapter. The Chapter Review offers exercises keyed to every section in the chapter, as well as Mixed Review exercises that are not keyed to sections.
- **Chapter Test and Chapter Test Prep Videos** The Chapter Test is structured to include those problems that involve common student errors. The **Chapter Test Prep Videos** gives students instant access to a step-by-step video solution of each exercise in the Chapter Test.
- **Cumulative Review** This review follows every chapter in the text (except Chapter 1). Each odd-numbered exercise contained in the Cumulative Review is an earlier worked example in the text that is referenced in the back of the book along with the answer.
- **Writing Exercises**  These exercises occur in almost every exercise set and require students to provide a written response to explain concepts or justify their thinking.
- **Applications** Real-world and real-data applications have been thoroughly updated, and many new applications are included. These exercises occur in almost every exercise set and show the relevance of mathematics and help students gradually and continuously develop their problem-solving skills.
- **Review Exercises** These exercises occur in each exercise set (except in Chapter 1) and are keyed to earlier sections. They review concepts learned earlier in the text that will be needed in the next section or chapter.
- **Exercise Set Resource Icons** Located at the opening of each exercise set, these icons remind students of the resources available for extra practice and support:

MyLab Math

See Student Resources descriptions on page xvii for details on the individual resources available.

Exercise Icons These icons facilitate the assignment of specialized exercises and let students know what resources can support them.

-  Video icon: exercise worked in the Interactive Lecture Series found in MyLab Math.
-  Triangle icon: identifies exercises involving geometric concepts.
-  Pencil icon: indicates a written response is needed.
-  Calculator icon: optional exercises intended to be solved using a scientific or graphing calculator.

Group Activities Found at the end of each chapter, these activities are for individual or group completion, and are usually hands-on or data-based activities that extend the concepts found in the chapter, allowing students to make decisions and interpretations and to think and write about algebra.

Optional: Calculator Exploration Boxes and Calculator Exercises The optional Calculator Explorations provide keystrokes and exercises at appropriate places to give students an opportunity to become familiar with these tools. Section exercises that are best completed by using a calculator are identified by  for ease of assignment.

Student and Instructor Resources

STUDENT RESOURCES

Video Organizer

Designed to help students take notes and work practice exercises while watching the Interactive Lecture Series videos.

- Covers all of the video examples in order.
- Provides prompts with ample space for students to write down key definitions and rules.
- Includes “Play” and “Pause” button icons to prompt students to follow along with the author for some exercises while they try others on their own.
- Includes Student Success Tips Outline and Questions

Available in loose-leaf, notebook-ready format and in MyLab Math.

Key Concept Activity Lab Workbook

Includes Extension Exercises, Exploration Activities, Conceptual Exercises, and Group Activities. This workbook is available in MyLab Math, or can be packaged in printed form with a text or MyLab Math code.

Student Solutions Manual

Provides completely worked-out solutions to the odd-numbered section exercises; all exercises in the Integrated Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews.

INSTRUCTOR RESOURCES

Annotated Instructor’s Edition

Contains all the content found in the student edition, plus the following:

- Answers to even and odd exercises on the same text page
- Teaching Tips throughout the text placed at key points

Instructor’s Resource Manual with Tests and Mini-Lectures

This resource includes:

- Mini-lectures for each text section
- Additional practice worksheets for each section
- Several forms of tests per chapter—free response and multiple choice
- Answers to all items

Instructor’s Solutions Manual TestGen®

(These resources are available for download from MyLab Math or from the Instructor’s Resource Center on pearson.com.)

Instructor-to-Instructor Videos—available in the Instructor Resources section of the MyLab Math course.

Online Resources
MyLab Math (access code required)

MathXL® (access code required)

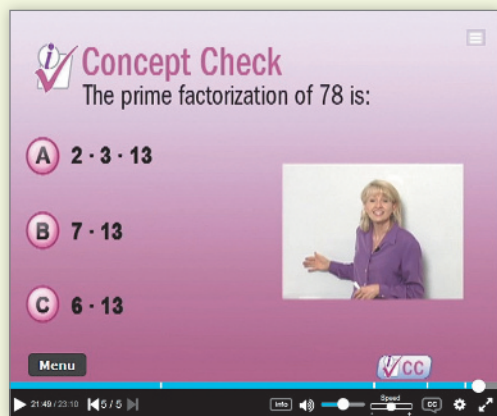
Resources for Success

Get the Most Out of MyLab Math for *Prealgebra*, Eighth Edition by Elayn Martin-Gay

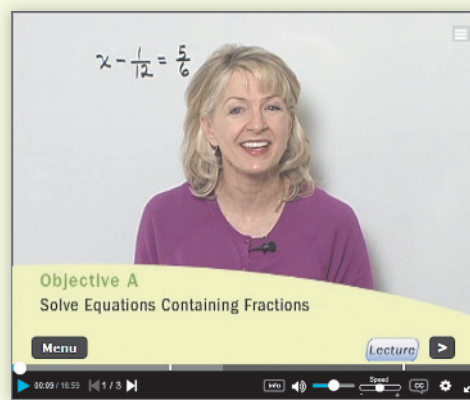
Elayn Martin-Gay believes that every student can succeed, and each MyLab course that accompanies her texts is infused with her student-centric approach. The seamless integration of Elayn's award-winning content with the #1 choice in digital learning for developmental math gives students a completely consistent experience from print to MyLab.

A Comprehensive and Dynamic Video Program

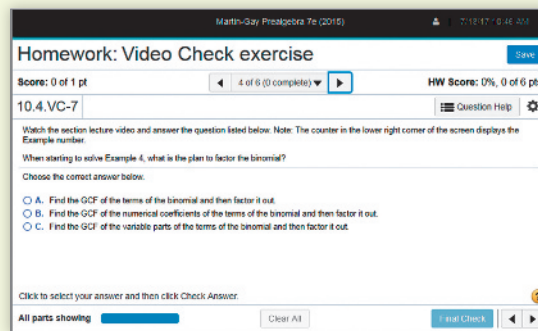
The **Martin-Gay video program** is 100% presented by Elayn Martin-Gay to ensure consistency with the text. The video program includes full section lectures and shorter objective level videos, and an intuitive navigation menu and pop-ups that reinforce key definitions.



All videos can be assigned as a **media assignment** in the Assignment Manager, to ensure that students are getting the most out of their MyLab resources. Additionally, **Video Check questions** ensure that students have viewed and understood the key concepts from the section lecture videos.



Within the section lecture videos, **Interactive Concept Checks** measure a student's understanding of key concepts and common trouble spots. Concept Checks ask students to try a question on their own within the video, after which Elayn Martin-Gay explains why they were correct or incorrect.

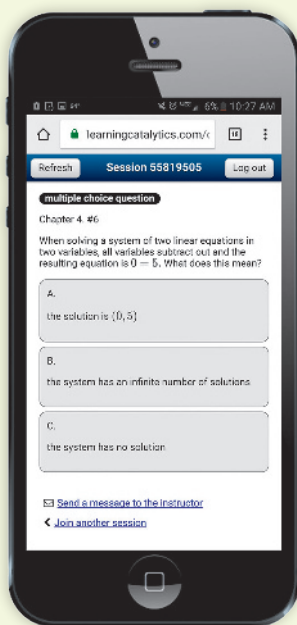
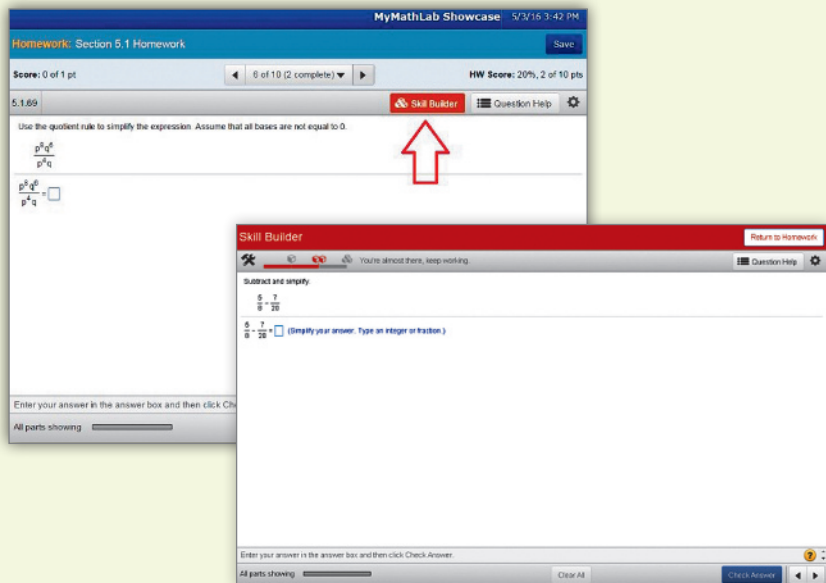


Additional hallmark Martin-Gay video types include Student Success Tip videos and Chapter Test Prep videos. **Student Success Tip videos** are in short segments designed to be daily reminders to stay organized and to study. **Chapter Test Prep videos**, a Martin-Gay innovation, help students during their most teachable moment—when they are preparing for a test—with step-by-step solutions for the exercises in the Chapter Test.

New Tools Improve Preparedness and Personalize Learning

New! Getting Ready for the Test video solutions cover every Getting Ready for the Test exercise. These come at the end of each chapter to give students an opportunity to assess if they understand the big picture concepts of the chapter, and help them focus on avoiding common errors.

New! Skill Builder exercises offer just-in-time additional adaptive practice. The adaptive engine tracks student performance and delivers questions to each individual that adapt to his or her level of understanding. This new feature allows instructors to assign fewer questions for homework, allowing students to complete as many or as few questions needed.



New Ways to Engage Students

New! Learning Catalytics

Martin-Gay-specific questions are pre-built and available through MyLab Math. Learning Catalytics is an interactive student response tool that uses students' smartphones, tablets, or laptops to engage them in more sophisticated tasks and thinking. **Getting Ready for the Test** exercises marked in blue in the text are pre-built in Learning Catalytics to use in class. These questions can be found in Learning Catalytics by searching for "MGPre".

New! Vocab and Readiness questions in MyLab Math have been expanded to 100% coverage, and are now available with a new **Drag and Drop functionality!** Drag and Drop exercises allow students to manually select elements of the question, such as expressions, words, graphs, or images, and place them into a designated target area.

Easier Start-Up for Instructors

Enhanced Sample Assignments make course set-up easier by giving instructors a starting point for each section. Each assignment has been carefully curated for this specific text, and includes a thoughtful mix of question types.

Acknowledgments

There are many people who helped me develop this text, and I will attempt to thank some of them here. Cindy Trimble was *invaluable* for contributing to the overall accuracy of the text. Gina Linko and Patty Bergin provided guidance throughout the production process.

A very special thank you goes to my editor, Mary Beckwith, for being there 24/7/365, as my students say. And, my thanks to the staff at Pearson for all their support: Barbara Atkinson, Alicia Frankel, Michael Hirsch, Chris Hoag, Paul Corey, Michelle Renda, Jenny Crum and Lauren Schur among many others.

I would like to thank the following reviewers for their input and suggestions that have affected this and previous editions:

Lisa Angelo, <i>Bucks Community College</i>	Teresa Hasenauer, <i>Indian River College</i>
Victoria Baker, <i>Nicholls State College</i>	Julia Hassett, <i>Oakton Community College</i>
Teri Barnes, <i>McLennan Community College</i>	Jeff Koleno, <i>Lorain County Community College</i>
Laurel Berry, <i>Bryant & Stratton</i>	Judy Langer, <i>Westchester Community College</i>
Thomas Blackburn, <i>Northeastern Illinois University</i>	Sandy Lofstock, <i>St. Petersburg College</i>
Gail Burkett, <i>Palm Beach Community College</i>	Stan Mattoon, <i>Merced College</i>
Anita Collins, <i>Mesa Community College</i>	Dr. Kris Mudunuri, <i>Long Beach City College</i>
Lois Colpo, <i>Harrisburg Area Community College</i>	Carol Murphy, <i>San Diego Miramar College</i>
Fay Dang, <i>Joliet Junior College</i>	Greg Nguyen, <i>Fullerton College</i>
Robert Diaz, <i>Fullerton College</i>	Jean Olsen, <i>Pikes Peak Community College</i>
Tamie Dickson, <i>Reading Area Community College</i>	Darlene Ornelas, <i>Fullerton College</i>
Latonya Ellis, <i>Gulf Coast Community College</i>	Warren Powell, <i>Tyler Junior College</i>
Sonia Ford, <i>Midland College</i>	Jeanette Shea, <i>Central Texas College</i>
Cheryl Gibby, <i>Cypress College</i>	Katerina Vishnyakova, <i>Collin County Community College</i>
Kathryn Gunderson, <i>Three Rivers Community College</i>	Corey Wadlington, <i>West Kentucky Community and Technical College</i>
Elizabeth Hamman, <i>Cypress College</i>	Edward Wagner, <i>Central Texas College</i>
Craig Hardesty, <i>Hillsborough Community College</i>	Jenny Wilson, <i>Tyler Junior College</i>
Lloyd Harris, <i>Gulf Coast Community College</i>	

I would also like to thank the following dedicated group of instructors who participated in our focus groups, Martin-Gay Summits, and our design review for the series. Their feedback and insights have helped to strengthen this edition of the text. These instructors include:

Billie Anderson, <i>Tyler Junior College</i>	Cheryl Cantwell, <i>Seminole Community College</i>
Cedric Atkins, <i>Mott Community College</i>	Jackie Cohen, <i>Augusta State College</i>
Lois Beardon, <i>Schoolcraft College</i>	Julie Dewan, <i>Mohawk Valley Community College</i>
Laurel Berry, <i>Bryant & Stratton</i>	Janice Ervin, <i>Central Piedmont Community College</i>
John Beyers, <i>University of Maryland</i>	Richard Fielding, <i>Southwestern College</i>
Bob Brown, <i>Community College of Baltimore County–Essex</i>	Cindy Gaddis, <i>Tyler Junior College</i>
Lisa Brown, <i>Community College of Baltimore County–Essex</i>	Nita Graham, <i>St. Louis Community College</i>
NeKeith Brown, <i>Richland College</i>	Pauline Hall, <i>Iowa State College</i>
Gail Burkett, <i>Palm Beach Community College</i>	Pat Hussey, <i>Triton College</i>

Dorothy Johnson, *Lorain County Community College*
 Sonya Johnson, *Central Piedmont Community College*
 Irene Jones, *Fullerton College*
 Paul Jones, *University of Cincinnati*
 Kathy Kopelousous, *Lewis and Clark Community College*
 Nancy Lange, *Inver Hills Community College*
 Judy Langer, *Westchester Community College*
 Lisa Lindloff, *McLinn Community College*
 Sandy Lofstock, *St. Petersburg College*
 Kathy Lovelle, *Westchester Community College*
 Jean McArthur, *Joliet Junior College*
 Kevin McCandless, *Evergreen Valley College*
 Daniel Miller, *Niagra County Community College*
 Marica Molle, *Metropolitan Community College*
 Carol Murphy, *San Diego Miramar College*

Greg Nguyen, *Fullerton College*
 Eric Oilila, *Jackson Community College*
 Linda Padilla, *Joliet Junior College*
 Davidson Pierre, *State College of Florida*
 Marilyn Platt, *Gaston College*
 Ena Salter, *Manatee Community College*
 Carole Shapero, *Oakton Community College*
 Janet Sibol, *Hillsborough Community College*
 Anne Smallen, *Mohawk Valley Community College*
 Barbara Stoner, *Reading Area Community College*
 Jennifer Strehler, *Oakton Community College*
 Ellen Stutes, *Louisiana State University Elinice*
 Tanomo Taguchi, *Fullerton College*
 MaryAnn Tuerk, *Elsin Community College*
 Walter Wang, *Baruch College*
 Leigh Ann Wheeler, *Greenville Technical Community College*
 Valerie Wright, *Central Piedmont Community College*

A special thank you to those students who participated in our design review: Katherine Browne, Mike Bulfin, Nancy Canipe, Ashley Carpenter, Jeff Chojnachi, Roxanne Davis, Mike Dieter, Amy Dombrowski, Kay Herring, Todd Jaycox, Kaleena Levan, Matt Montgomery, Tony Plese, Abigail Polkinghorn, Harley Price, Eli Robinson, Avery Rosen, Robyn Schott, Cynthia Thomas, and Sherry Ward.

Elayn Martin-Gay

Personal Acknowledgements

I would like to personally thank my extended family. Although this list has grown throughout the years, it still warrants mentioning in my texts as each of these family members has contributed to my work in one way or another – from suggesting application exercises with data and updating/upgrading my computer to understanding that I usually work on “Vacations.” I am deeply grateful to them all:

Clayton, Bryan (in heaven), Eric, Celeste, and Tové Gay; Leo and Barbara Miller; Mark and Madison Martin and Carrie Howard; Stuart and Earline Martin; Karen Martin Callac Pasch (in heaven); Michael, Christopher, Matthew, Nicole, and Jessica Callac; Dan Kirk; Keith, Mandy, Erin, and Clayton McQueen, Bailey Martin, Ethan, Avery, and Mia Barnes; Melissa and Belle Landrum.

About the Author

Elayn Martin-Gay has taught mathematics at the University of New Orleans for more than 25 years. Her numerous teaching awards include the local University Alumni Association’s Award for Excellence in Teaching, and Outstanding Developmental Educator at University of New Orleans, presented by the Louisiana Association of Developmental Educators.

Prior to writing textbooks, Elayn Martin-Gay developed an acclaimed series of lecture videos to support developmental mathematics students in their quest for success. These highly successful videos originally served as the foundation material for her texts. Today, the videos are specific to each book in the Martin-Gay series.

The author has also created Chapter Test Prep Videos to help students during their most “teachable moment”—as they prepare for a test—along with Instructor-to-Instructor videos that provide teaching tips, hints, and suggestions for each developmental mathematics course, including basic mathematics, prealgebra, beginning algebra, and intermediate algebra.

Elayn is the author of 12 published textbooks as well as multimedia, interactive mathematics, all specializing in developmental mathematics courses. She has also published series in Algebra 1, Algebra 2, and Geometry. She has participated as an author across the broadest range of educational materials: textbooks, videos, tutorial software, and courseware. This provides an opportunity of various combinations for an integrated teaching and learning package offering great consistency for the student.

Applications Index

Advertising and marketing

applications of bottle of lotion, 442
Internet advertising, 201, 483, 729
money spent on TV advertising, 65

Agriculture

acres of wheat planted/harvested in U.S., 549, 555
apple types grown in Washington State, 561
average farm size, 513
average price per pound for chickens, 373
bushels of oranges, 607
circumference of round barn, 637
corn production, 37, 596–597
crop losses, 536
diameter of giant sequoia tree, 693
fencing needed for enclosure, 173
fertilizer application per area, 439, 441
heaviest zucchini grown, 667
height of bamboo, 666
height of tree, 451, 459
height of tree from shadow length, 749
number of pecan trees in orchard, 748
pesticide needed for garden, 466
selling price of fertilizer, 520

Animals

amount of grain eaten by cow in a year, 50
amount spent on pet food, 442
average weight of mature whales, 422
bat wingbeats per second, 424
bill length of pelican, 657
butterfly migration conversion into miles, 693
condor population changes, 131
distance bees chase fleeing human, 277
diving speeds of birds, 198
endangered and threatened species, 24–25, 84, 541
female veterinarians in U.S., 509
height of termite nests in meters, 693
hummingbird wingbeats per second, 424
legal lobster size, 295
life expectancies, 199
lobster classification by weights, 316
medicine dosage for dog, 471
mosquito control substance amounts, 442
mourning dove population changes, 131
number of households owning reptiles, 509
number of neon tetras in tank, 460
sheep population, 30
speed of cockroach, 342
speed of sloth in tree, 277

Astronomy and space

antenna tracking distance in miles, 693
average temperature of Earth's surface, 695
circumference of Meteor Crater, 637
commercial space launches, 551
days for Neptune to orbit Sun, 37
degrees apart of sites of Deep Space Network, 627
degrees around Earth at equator, 627
deviation in mirrors of Hubble Space Telescope, 484
diameter of largest crater on near side of Moon, 693
distance from Earth to Moon, 356
distance from Earth to Sun, 366
distance from Pluto to Sun, 412
eclipse durations, 305
energy produced by Sun, 763
length of day on Mars, 342
orbit day lengths of planets around Sun, 342

planetary radius, 24
planets with days longer than Earth's, 225
radio wave distances, 366
radius of Earth at equator, 364
radius of Saturn, 162
size and weight conversions of space antenna, 693
speed of Sojourner, 470
surface temperature of planets, 103, 122
temperature of Earth's core, 695
viewing power of telescope, 763
volume of planetarium domes, 654
weight of person on Earth, 63

Automotive

amount of gasoline in tank, 686
amount of gasoline needed to fill tank, 686
car price increases, 511
car sales volume by size, 30
decrease in auto theft in U.S., 480
Ford vehicle sales, 513
gasoline mileage, 374, 425, 426, 441, 616, 781
gas-to-oil mixture for engine, 686
HEVs and BEVs sold in 12 months, 587
hybrid car sales, 539
increase in licensed drivers in U.S., 505–506
interior space of cars, 30
licensed drivers in U.S., 505–506
monthly cost of owning and operating autos, 348–349
number of cars manufactured in U.S. and Germany, 200
number of not-blue cars on lot, 321
number of not-white cars on lot, 322
number of registered vehicles in U.S., 505
passenger car and light truck sales, 587
percentage of people driving blue cars, 711
percent increase in number of registered vehicles in U.S., 505
perimeter of stop sign, 635
price per liter of gasoline, 686
speed of car and truck, 200
top speed of dragsters, 200
truck sales in U.S., 484
world motor vehicle production, 484

Aviation

air cargo and mail tonnage estimates, 35
cruising speed of Boeing 747, 693
flight time remaining before landing, 273–274, 473
length of Dreamliner, 470

Business

amount of money received from 3-D movies, 251
annual inflation rate, 610
billable hours, 511
brand value estimates, 37
car price increases, 511
commission rate, 517, 519, 532
commissions, 516, 519, 520, 532
consumer spending for each category, 328
cost of each item, 97, 353, 409
costs of manufacturing, 721
credit card late fees, 354
CVS store numbers by state, 30
decrease in number of employees, 512
defective products, 424, 426, 508, 511
discount, 517–518, 520, 521, 532, 533, 534, 536, 781
employee work shift length, 239
fraction of Gap Corporation brand stores, 228
fraction of goods types sold online, 266–267
fraction of Hallmark employees in Kansas City, 239

fraction of states in U.S. with Ritz-Carlton hotels, 239
fundraiser amounts collected, 609
Gap Inc. stores owned, 30
growth of wearable devices, 721
hourly pay rate, 532
hours worked per week, 600
Internet use by world regions, 89, 90
inventory of art dealer's shop, 226
losses, 105, 131
money change from purchase prices, 352
monthly production, 509
net income, 115
number of cars manufactured in U.S. and Germany, 200
number of Wal-Mart Sam's Clubs, 595
online spending per month, 563
page print area, 460
pay after taxes, 510
pay before taxes, 364
percent increase in price, 711
percent increase of employed nurses, 504
perimeter of large buildings, 23
price difference estimates, 38, 97
price of each item, 213
price rounding, 342
purchase price, 515–516, 518, 519
purchase price not covered by trade-in, 239
ratios of defective to good product, 426
revenue and profit from iPod covers sales, 741
revenue from downloaded singles, 343
revenue from music downloading sales, 513
salary increase, 511
sale of bike and accessories, 200
sale of tractor and plow, 200
sale price, 29, 517–518, 520, 532, 781
sale price of cruise, 252
sales decrease, 507
sales tax, 515–516, 518, 519, 521, 532, 533, 536, 616, 712
selling price of house, 520
state energy consumption, 208
tipping amounts, 521
total cost estimates, 38, 51, 91, 94, 97, 365
total monthly sales, 516
total price, 519, 521, 712
toy expenditures per child, 67
trade balances, 123
traveler spending per day, 183
unit price, 421, 425, 426, 436, 465, 468, 470, 538, 616
unit rate of work, 425
units assembled in certain length of time, 425
values of global brands, 199
waste dumping charge, 532
weight of coal delivered to steel mill, 677
wholesale and markup cost, 201

Chemistry and physics

Avogadro's number, 763
brine solution mixture in ice cream maker, 444
copper amount in alloy, 536
decibel levels, 30
distance light travels in 10,000 years, 764
excess amount of water in mixture, 686
gas/oil ratio for mower and chainsaw, 444
melting points of elements, 131
oxygen supply provided by lawns, 442
pesticide amounts for mosquito control, 442
rope lengths for experiment, 64
temperature conversions to Celsius, 696, 700, 712
volume of saline solution in containers, 705

Demographics

age distribution of U.S. population, 611
 ages of householders, 552
 areas/monuments maintained by Park Service, 241, 253, 279, 722
 average heights of humans in various countries, 349
 California population estimate, 253
 corrections officers in U.S., 512
 crime decrease, 532, 536
 daily average waste disposed per person, 554
 deaths from boating accidents, 513
 distribution of blood types, 239, 324, 384
 fastest growing occupations, 485, 527
 fraction of employees being men/women, 225
 fraction of Habitat for Humanity affiliates in U.S., 228
 fraction of states containing Indian reservations, 226
 households without landlines, 483
 indoor cinema sites in U.S., 505
 jobs with highest numerical increase, 574
 largest cities in world, 550
 largest population of Native Americans, 200
 number of female runners in race, 211
 number of first jobs in restaurant industry, 443
 number of girls on school bus, 253
 number of people in human chain, 40
 number of registered nurses, 505
 nursing home population, 443
 population density, 342
 population differences, 91
 population increase, 511, 536
 population of Japan, 512
 population of states, 509
 population of U.S., 37
 population projections, 29, 39, 51, 424
 primary languages spoken, 540
 tall buildings in world completed, 602
 tourist numbers projections, 200, 556, 750
 world population by continent, 483

Education

application increase for math scholarships, 506–507, 711
 area of book page, 651
 area of classroom no smoking signs, 704
 college costs spent on books, 508
 college expenses, 60, 91
 college freshmen majors, 241, 251
 degrees awarded in one year, 511
 enrollment decrease, 509
 enrollment increase estimates at community college, 39
 fraction of class being sophomores, 225
 fraction of students freshmen/not freshmen, 225
 fraction of students making an A on test, 239
 fraction of students with math/science as favorite subjects, 278
 fraction of two-year colleges, 253
 freshmen enrolled in prealgebra, 532
 grade point averages, 394–395, 398, 408, 412
 Head Start enrollment increase estimates, 39
 heights of students, 611
 living arrangements of students, 560
 math problems completed over time, 538
 minimum floor space students require, 441
 number of children in day care, 508
 number of freshmen in school, 504–505, 749
 number of library books for students, 424
 number of pages read by college classes, 31
 number of students at a university, 504
 number of students in class, 63
 number of teachers, 512
 nursing school applications accepted, 504
 persons age 25 completing 4 or more years of college, 600

student test scores, 544, 554, 575, 611–612
 test score averages, 61
 test score estimates, 38
 textbook costs, 50
 times for completing mazes by students, 393
 tuition increase, 511
 typing speed, 512
 unit rate of students per school, 426

Electronics and computers

area of faces of smartphones, 364
 conversion of CD diameter from centimeters to inches, 688
 decrease in numbers of cable TV systems, 505
 diameter of hard drives, 423
 growth of wearable devices, 721–722
 height of stack of CDs, 705
 increase in cell sites in U.S., 505
 Internet use by world regions, 89, 90
 Internet use in U.S., 385
 length of CD track in miles, 694
 megabytes held by DVDs/CDs, 46, 50, 329
 number of characters in line of print, 50
 number of download cards given to friends, 60, 163
 number of pixels on a screen, 50
 perimeter of smartphone, 353
 price of each item, 196, 213
 prices of Xbox and games, 199
 printer paper usage rate, 442
 purchases of HDTV by viewers, 484
 rate of growth of cell phone usage, 722
 ratio of diameters of floppy disks, 423
 revenue from downloaded singles, 343
 smartphone user numbers, 37
 text messages sent/received per day, 353
 thickness of MacBook, 342
 virtual reality device usage increase, 512
 volume and surface area of Space Cube computer, 653
 width of diskette in millimeters, 693

Entertainment and recreation

admission total cost, 46, 51
 amount of money received from 3-D movies, 251
 area of movie screen, 173
 best-selling albums in U.S., 551
 blue-ray DVD sales changes, 377
 card game scores, 115, 127
 circumference of Ferris wheels, 364
 deaths from boating accidents, 513
 diameter of DVD/LaserDiscs, 423
 digital movie screens in/not in U.S. and Canada, 280
 DVD sales changes, 377
 group price for climbing wall, 511
 height of climbing wall, 511
 indoor cinema sites in U.S., 505
 lottery tickets paid per lottery winners, 424
 lottery win amounts per person, 63
 lottery winnings paid over 20 years, 424
 miles hiked on trail, 252
 miles of Appalachian Trail in New York State, 503
 money earned by top-grossing concert tours, 424
 money earned by top-rated movies, 343, 354
 number of analog/digital movie screens in U.S., 131
 number of digital 3-D movie screens in U.S./Canada, 509
 number of DVDs, 208
 number of moviegoers in U.S. and Canada, 251
 number of roller coasters in amusement parks, 248
 perimeter of Monopoly board, 30
 perimeter of puzzle, 30

rating ratios of films, 435
 roller coaster heights and depths, 156
 sail measurements for boat, 467
 sale price of cruise, 252
 ski run ratings, 509
 ticket costs, 163
 times for completing mazes, 393

Finance

amount paid back on investment, 523–524
 annual inflation rate, 610
 compound interest, 523, 533, 536, 616
 credit card balance transfer charges, 375
 credit card late fees, 354
 currency exchange rates, 365
 fraction of states without efile, 240
 monthly investment payments, 616
 number of electronically filed income tax returns, 192
 simple interest, 522, 523–524, 533, 536, 750
 stock market losses, 130
 stock share changes, 353
 top six national debts, 764
 total investment amount, 523

Food and nutrition

actual weight of cocoa in boxes, 678
 actual weight of ham in 3 cartons, 678
 actual weight of pineapple in cartons, 678
 amount of beverage each person gets, 686
 amount of beverage poured into container, 686
 amount of beverage remaining in bottle, 686
 amount of Jell-O in each dish, 686
 amount of rice after combining two portions, 677
 amount of soup in 3 containers, 686
 annual food sales in U.S., 551
 area of top of pizza, 635
 average amount of Coca-Cola drunk, 423
 average price per pound for chickens, 373
 beverage sales, 533
 bottled water consumption, 535
 brine solution mixture in ice cream maker, 444
 caffeine content of items, 542
 calories from fat, 251, 423, 510
 calories in food items, 49, 51, 97, 442, 512
 candy thermometer conversion to Fahrenheit, 695
 chocolate consumption of selected countries, 355
 cholesterol in food item, 443
 color distribution of M&Ms, 561
 fast food eating habits, 443
 fiber content of items, 551
 food cooperative dividends, 508
 food preferences, 531
 grams of fat in food items, 49, 51
 grams of fat in hamburger, 324
 iced tea remaining, 705
 lobster classification by weights, 316
 milk beverages consumed, 575
 number of hamburgers from total pounds, 303
 number of McDonald's restaurants in U.S., 509
 number of rotten apples in shipment, 749
 number of servings obtained from pancake mix, 442
 number of Subway restaurants, 595
 oven setting in Fahrenheit degrees, 705
 pounds of sugar required for recipe, 443
 preferences for Coke and Pepsi, 441
 preservatives added to dried fruit, 678
 ratios of defective to good tomatoes, 426
 recipe ingredient amounts, 445, 468, 705, 711
 saturated fat in food items, 364
 special prices for pizzas, 653
 tea bag production daily at plant, 51
 volume of waffle ice cream cone, 652
 weight of 4 boxes of hamburger, 678

weight of candy ordered for party, 704
 weight of food on pallet, 50
 weight of oatmeal in a carton, 678
 weight of oatmeal in carton, 678
 weight of one serving of sunflower seeds, 677
 weight of partial package of hamburger, 678
 weight of several cans of 7-Up, 677
 weight of several cartons of bottles, 677

Geography and geology

area of Colorado, 748
 area of Utah, 653
 countries with small land areas, 422
 dam distribution by continent, 562
 depth of cave, 329
 depth of Grand Canyon, 666
 depth of ocean trench, 104, 115, 122, 666
 diving depths, 105, 114, 130, 156, 161
 elevation differences between points, 122, 158, 161
 elevation of deepest point in lake, 161
 elevations above/below sea level, 38, 104, 105, 106, 119, 122, 156, 763
 fraction of Earth's water taken up by oceans, 278
 fraction of mountain peaks in India, 236
 fraction of mountains in Colorado, 384
 fraction of states adjacent to other states, 280
 highest dams in U.S., 574
 highest/lowest elevations, 119
 ice sheets on Earth, 483
 ice thickness needed on pond for skating, 667
 lake elevation differences, 122
 land area drained by river basins, 29
 land areas of continents, 560
 mean/median of longest rivers, 399
 mountain elevations, 64
 sediment thickness at bottom of creek, 667
 size of oceans, 562
 surface land area of each continent, 265–266
 surface temperatures of planets, 122
 volume of Mount Fuji, 652
 water flow volume over Niagara Falls, 764
 wildfires in U.S., 549–550

Geometry

area and perimeter of parallelogram, 747
 area of circle, 642–643, 654, 712
 area of geometric figures, 67, 206, 304, 407, 648–650, 655, 698, 702, 706, 708, 728, 744
 area of hemisphere, 652
 area of parallelogram, 750
 area of rectangle, 49, 50, 87, 92, 97, 170, 174, 184, 210, 252, 305, 315, 322, 325, 328, 386, 409, 639, 651, 698, 704, 728, 735
 area of square, 74, 93, 97, 322, 639, 698, 728, 735
 area of triangle, 383, 386, 412, 640, 728, 750
 circumference and area of circle, 782
 circumference of circle, 365, 406, 412, 633, 636–637, 638, 698, 702, 708
 circumference of geometric figures, 636–637
 circumference of telescope, 385
 complementary angles, 625, 628, 655, 701, 708
 diagonal lengths, 449, 451, 467, 710
 diameter of circle, 252, 278, 708
 length of sides of geometric shapes, 315, 324, 375, 460, 468
 lengths of composite figures, 355
 lengths of right triangles, 451, 471
 perimeter and area of rectangle, 654, 781
 perimeter of geometric figures, 28, 40, 49, 67, 75, 86, 90, 141, 173, 206, 210, 264, 277, 304, 323, 413, 526, 631–632, 634–636, 639, 655, 698, 701–702, 706, 708, 723
 perimeter of polygon, 22, 86
 perimeter of rectangle, 30, 40, 49, 97, 257, 315, 323, 328, 406, 634, 635, 698, 708, 750
 perimeter of square, 30, 74, 97, 257, 305, 353, 636, 698, 709

perimeter of triangle, 40, 170, 184, 323, 353, 406, 634, 635, 780
 radius of circle, 24, 162, 252, 655, 708
 ratio of perimeter to area, 711
 ratio of side length to perimeter, 423, 711
 ratio of sides of similar triangles, 458
 ratio of two diameters, 423
 ratio of width to length, 435, 465, 711
 ratio of width to perimeter, 422, 465
 ratios of corresponding sides of triangles, 616
 sail measurements for boat, 467
 supplementary angles, 625–626, 628, 655, 701, 708
 surface area of geometric figures, 654, 706
 total width/length of geometric object, 278
 unknown lengths, 723
 volume and surface area of ball, 645–646
 volume and surface area of box, 645, 651
 volume and surface area of solids, 649–650, 654, 703, 706
 volume and surface area of sphere, 644, 650
 volume of a can, 646, 703
 volume of a cone, 647, 650, 652
 volume of a cube, 650, 709
 volume of a cylinder, 646, 703, 709
 volume of a sphere, 653
 volume of a square-based pyramid, 647, 650, 651, 653, 703
 volume of geometric figures, 698
 width of geometric object, 460

Health and medicine

amount of medication in extra-strength version, 677
 aspirin usage, 265, 483
 average height of two-year-old, 693
 blood pressure drug testing, 593
 blood type ratios and distribution, 423, 484, 610
 body surface area calculations, 695
 components of bone, 484
 crutch lengths, 304
 disease cases estimates, 35
 distribution of blood types, 239, 324, 384
 dosage per weight of dog, 781
 dosage per weight of human, 438, 444, 711
 doses of medicine in bottle, 374
 female veterinarians in U.S., 509
 fluid ounces of medicine to be purchased, 694
 fraction of persons getting fewer than 8 hours sleep, 280
 hours of sleep per night, 601
 liquid dosage calculations, 444
 median/mean/mode pulse rates, 398
 medication prescriptions given in emergency rooms, 443
 medication received over time, 683
 medicine dosage conversion from milligrams to ounces, 692
 medicine dosage for dog, 471
 number of days medicine will last, 374
 number of registered nurses, 505
 number of teaspoons in medicine bottle, 374
 nursing home population, 443
 organ weight conversions to kilograms, 688
 oxygen supply provided by lawns, 442
 smoking products usage, 483
 syringe measurements, 687
 temperature conversions to Celsius, 691
 weight conversions from kilograms to pounds, 705
 weight of each lung of adult human, 693
 weight of man after being sick with the flu, 677
 weight of skin of adult human, 693

Home improvement

amount of paint needed for wall, 370–371, 373
 area of a wall, 653
 area of computer chip, 304

area of lawn, 409
 area of lawn in square feet, 412
 border material needed for garden, 354
 bricks needed for side of house, 652
 carpeting needed, 702
 circumference of circular spa, 633
 circumference of irrigation device, 633
 cost to hang wallpaper border on walls, 632, 636
 deck area, 304
 driveway sealant needed, 702
 fencing cost, 635
 fencing materials needed, 29, 323, 375, 630, 635
 grass seed needed for yard, 651
 gutters around house, 29, 75, 636
 insecticide amounts needed, 412
 insulation needed for attic, 653
 netting needed around trampoline, 637
 perimeter of a picture frame, 173
 perimeter of a room, 173
 perimeter of garden, 173
 railing amount needed for deck, 354
 scale drawings for triangular deck, 461
 shingles needed for roof, 652
 soil to fill rectangular hole, 709
 volume of birdbath, 651

Miscellaneous

amount of cloth remaining on bolt, 703
 amount of lotion needed for several people, 442
 area of plot of land, 651
 area of snow globe, 652
 average of several numbers, 92
 book categories at library, 561
 bricks laid in certain amount of time, 425
 card choosing from deck, 594
 charity collection amounts, 532
 choosing numbers from a bag, 614
 circumference of Pearl of Lao-tze, 637
 coin combinations, 356
 coin denomination random choices, 712
 coin toss probability, 588–590, 599, 605, 614
 conversion of inches to centimeters, 693
 depth of screw in wood after turning, 252
 die toss probability, 588–590, 594, 605, 616
 elevator shaft heights and depths, 155, 156
 elevator weight limit for each passenger, 674
 fabric needed for sashes/scarves, 703, 709
 fencing materials needed, 375
 formats of commercial radio stations, 386
 fraction of legal fireworks in states, 226
 fraction of marbles in certain colors, 226
 fraction of national monuments in New Mexico, 239
 fraction of national parks by state, 228
 fraction of wall as concrete, 239
 fundraiser amounts collected, 609
 geothermal energy use, 483
 height of trees after shortening, 709
 heights in inches from meters, 365
 inner diameter of tubing/pipe, 278, 281
 inner diameter of washer, 278
 length of logs for truck, 667
 length of metal strip around workbench, 635
 length of piece of rope, 659–660
 length of piece remaining after cutting off shorter pieces, 325, 328
 length of ropes after knot is tied, 666
 length of scarf in meters, 663
 lengths of two scarves, 324
 liters of shoe polish in bottles and boxes, 705
 mail in certain categories delivered by Postal Service, 279
 mail volume of Postal Service, 484
 marble choosing probability, 590, 593, 605, 608, 712
 Marine Corps training centers in California, 228

Miscellaneous (*continued*)

music preferences, 610
 netting needed around trampoline, 637
 newspaper circulation, 199
 nuclear-generated electricity in France, 484
 number of apartments in building, 50
 number of apartments on one floor, 50
 number of books sold per hour, 374
 number of boxes filled, 92, 94
 number of boxes of crayons, 373
 number of boxes on pallet, 50
 number of download cards given to friends, 60, 163
 number of dresses from bolt of material, 296
 number of items needed to fill box, 373
 number of libraries in Mississippi, 252
 number of meters in feet, 406
 number of meters in inches, 373
 number of pages in book, 29
 number of seats in lecture hall, 50
 number of windows in building, 52
 number of words on pages of book, 47
 numbers owning microwaves, 532
 oil remaining in container, 709
 percentage of shoppers paying with cash, 711
 perimeter of Coca Cola sign, 667
 perimeter of piece of land, 323
 postage costs, 377
 preferences for Coke and Pepsi, 441
 rates of various objects, 424
 ratio of men to total number of people present, 423
 ratio of women to men at a meeting, 423
 repeat contestants on television show, 252
 scale drawings for construction, 461, 471
 spinner spinning probability, 591–592, 593, 604–605, 614
 states with mandatory helmet laws, 426
 strips of metal from stock piece, 296
 tree girth comparisons, 295
 typing speed, 512
 unknown number, 213, 763
 usage rate of case of printer paper, 442
 volume of drawers in chest of drawers, 703
 volume of water storage tank, 350
 weight conversion from stone to pounds, 693
 weight of cinders to spread on roads, 704
 width of stacks of microwaves in warehouse, 667

Personal finance

charge account balances, 121
 checking account balances, 156, 161, 400, 553
 check writing, 333–334
 credit card balance transfer charges, 375
 debt repayment amounts, 157, 158, 406
 earnings shared by three persons, 60
 family monthly budget, 602
 households with no retirement plan, 535
 purchase price not covered by trade-in, 239
 savings account balance, 29, 90, 162
 total earnings during three years, 90
 total pay after taxes, 510

Politics and government

armed forces personnel, 558
 distribution of public lands, 562
 electoral votes for president, 198
 fraction of presidents born in Ohio, 225
 Marine Corps training centers in California, 228
 number of rulers in each country, 198
 registered voters for two senators, 424
 votes for candidates in election, 208

Real estate

home sales commissions, 201, 252
 lots in certain number of acres, 281
 new home construction, 599
 sales commissions, 516, 520, 521
 selling price of house, 510, 520

Sports

admission costs to baseball games, 51
 amount of lime powder needed for baseball foul lines, 635
 area of basketball court, 170, 173
 average speeds of Daytona winners, 354
 baseball average salaries, 37
 baseball caps sold at U.S. Open Golf Tournament, 239
 baseball hits, 441
 basketball free throws made, 531
 basketball points scored, 37, 201
 Beach Soccer World Cup goals made, 553–554
 betting losses, 157
 combined salary for 15 highest-paid Cubs players, 424
 distance between golf course holes, 63
 distance run around baseball bases, 635
 football average salaries, 37
 football passes attempted/completed, 466
 football yards lost, 130, 157
 fraction of sports team being boys, 225
 free throws made by basketball players, 211, 321
 golf scores, 114, 121, 127, 141, 156, 553
 golf wages earned by professionals, 374
 height of climbing wall, 511
 length and width of football field, 635
 length and width of soccer field, 638
 length of diagonals of football/soccer/polo fields, 449, 452, 460
 length of soccer field, 200
 medals awarded in Summer Olympics, 600
 number of female runners in race, 211, 781
 points scored during basketball season, 52
 races won by driver, 278
 ratio of Olympic medals won, 468
 ski run ratings, 509
 soccer field length per player age, 164
 stadium capacities, 200
 touchdowns made during season, 64
 volume of a Zorb, 652
 weight resistance for gym, 508
 width and height conversions to inches and feet, 692

Temperature and weather

average daily temperature, 544–546
 average monthly precipitation in California, 601
 average monthly temperature, 138
 average rainfall, 304, 353, 354
 average snowfall, 354
 average temperatures, 64
 boiling temperature, 107
 daily high temperatures, 575
 drops in temperature, 130, 157, 158, 705
 fraction of tropical storms as hurricanes, 225
 frequency of high temperatures, 601
 high and low temperatures, 105, 111–112, 115, 121, 133, 329, 555
 hottest temperature recorded in U.S., 695
 hurricanes making landfall in U.S., 550
 hurricane wind speeds per hour, 375
 monthly precipitation in Chicago, 609–610
 surface temperature of planets, 103, 122
 temperature at certain time of day, 122, 156, 158
 temperature conversions to Celsius, 82, 691, 695, 705
 temperature conversions to Fahrenheit, 82, 695, 700
 tornado wind speeds, 597
 wind speeds, 353, 375

Time and distance

antenna tracking distance to miles, 693
 conversion of centimeter to kilometer, 441
 conversion of miles to kilometers, 696

days for Neptune to orbit Sun, 37
 diagonal length of city block, 451
 distance between cities, 195–196, 199, 377, 406, 412, 441, 466, 694, 696
 distance between golf course holes, 63
 distance differences, 257
 distance estimates, 35, 38, 39, 90, 91
 distance from home after losing watch, 268
 distance light travels in 10,000 years, 764
 distance needed to balance board, 445
 distance object falls, 82
 distance remaining to inspect tracks, 265
 distance remaining to run, 265
 distance run around baseball bases, 635
 distance traveled in 3 days, 329
 feet in one rod, 64
 flight time remaining before landing, 273–274
 fraction of centimeters in one meter, 239
 fraction of days in month, 225
 fraction of feet in one mile, 239
 fraction of inches in one foot, 322
 height of antenna, 451
 height of building, 456, 459, 467, 721
 height of climbing wall, 511
 height of dropped/falling object, 718, 721, 722
 height of Empire State Building, 443
 height of fire hydrant from shadow length, 749
 height of fountain, 459
 height of human in meters, 663
 height of launch gantry, 460
 height of Space Needle deck, 459
 height of Statue of Liberty, 443
 height of tower from shadow length, 471
 height of tree, 451, 456, 459
 height of tree from shadow length, 749
 height of trees after shortening, 709
 heights of students, 611
 highway mileage by state, 31, 37
 inches as fraction of a foot, 225
 length differences of two scarves, 324
 length of diagonals of square land parcel, 451
 length of human index finger, 443
 length of pipe in feet, 304
 length of shadow, 460
 mileage shared by drivers on trip, 704
 miles hiked on trail, 252
 miles in certain number of yards, 92
 miles of Appalachian Trail in New York State, 503
 miles traveled on trip, 29
 miles walked on treadmill, 303
 number of steps run up in timed race, 468
 proportion of miles to inches, 538
 remaining length of cut board, 304
 rope lengths, 64
 speed limit conversion to kilometers per hour, 692
 speed of a fast sneeze, 693
 speed of truck after braking, 460
 time for elevator to reach top, 442
 times for completing mazes, 393
 time to travel 100 miles, 471
 200-meter time in Olympic swimming, 342
 unit rate of speed, 468
 yards in one mile, 64

Transportation

amount hauled by truck each trip, 63
 average speed needed to travel certain distance, 696
 bridge length, 67
 circumference of bridge caisson, 364
 distance remaining to inspect train tracks, 265
 distance traveled on gasoline amount, 324, 328, 425
 freight truck weight, 273
 gasoline mileage, 374, 425, 426, 441, 616, 781
 highway mileage by state, 31, 37
 highway speed limits, 265
 length of Dreamliner, 470
 length of logs for truck, 667
 length of sound barriers along highway, 667

length of truck in yards, 667
map scale, 437–438
mileage shared by drivers on trip, 704
miles driven in each category in one year, 253
miles driven per week, 374, 552
miles traveled on trip, 29, 38, 91
number of bridges, 63
number of lane dividers on highway, 63
number of light poles on highway, 64
number of roadway miles in selected states, 208
railroad standard gauges in U.S. and Spain/
Portugal, 303
span of bridge in meters, 709
speed limit conversion to kilometers per hour, 692
time to travel 100 miles, 471

World records

deepest bat colony, 100
driest place in world, 354
fastest tropical cockroach, 342
heaviest zucchini grown, 677
highest temperature produced in a laboratory,
696
highest wind speed, 353
largest American flag, 651
largest cities in world, 550
largest commercial building, 50
largest crater on near side of moon, 693
largest hotel lobby, 50
largest indoor illuminated sign, 651
largest optical telescope, 763

largest round barn, 637
largest suspension bridge, 364
largest yacht, 470
longest stairway, 426
longest truck, 667
number of steps run up in timed
race, 468
opening weekend movie revenue, 14
slowest mammal, 277
snowiest city in U.S., 354
tallest and shortest men, 666
tallest buildings, 397
tallest fountain, 459
tallest tree, 459
weight of premature baby, 677

This page intentionally left blank

The Whole Numbers

A Selection of Resources for Success in This Mathematics Course

Elayn Martin-Gay

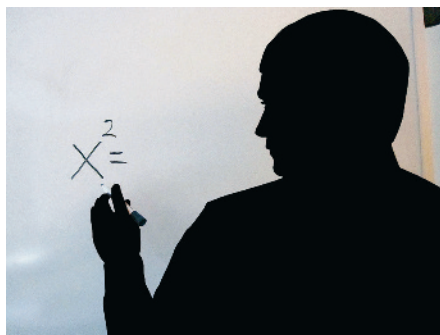
Prealgebra



Eighth Edition



Textbook



Instructor



MyLab Math and MathXL



Video Organizer



Interactive Lecture Series

For more information about the resources illustrated above, read Section 1.1.

1

Whole numbers are the basic building blocks of mathematics. The whole numbers answer the question “How many?”

This chapter covers basic operations on whole numbers. Knowledge of these operations provides a good foundation on which to build further mathematical skills.

Sections

- 1.1 Study Skill Tips for Success in Mathematics
- 1.2 Place Value, Names for Numbers, and Reading Tables
- 1.3 Adding and Subtracting Whole Numbers, and Perimeter
- 1.4 Rounding and Estimating
- 1.5 Multiplying Whole Numbers and Area
- 1.6 Dividing Whole Numbers
- Integrated Review**—Operations on Whole Numbers
- 1.7 Exponents and Order of Operations
- 1.8 Introduction to Variables, Algebraic Expressions, and Equations

Check Your Progress

- Vocabulary Check
- Chapter Highlights
- Chapter Review
- Getting Ready for the Test
- Chapter Test

1.1 Study Skill Tips for Success in Mathematics

Objectives

- A** Get Ready for This Course. 
- B** Understand Some General Tips for Success. 
- C** Know How to Use This Text. 
- D** Know How to Use Text Resources. 
- E** Get Help as Soon as You Need It. 
- F** Learn How to Prepare for and Take an Exam. 
- G** Develop Good Time Management. 

Before reading this section, ask yourself a few questions.

1. Were you satisfied—really satisfied—with your performance in your last math course? In other words, do you feel that your outcome represented your best effort?
2. When you took your last math course, were your notes and materials from that course organized and easy to find, or were they disorganized and hard to find—if you saved them at all?

If the answer is “no” to these questions, then it is time to make a change. To begin, continue reading this section.

Objective **A** Let's Get Ready for This Course

1. *Start With a Positive Attitude.* 

Now that you have decided to take this course, remember that a *positive attitude* will make all the difference in the world. Your belief that you can succeed is just as important as your commitment to this course. Make sure you are ready for this course by having the time and positive attitude that it takes to succeed.

2. *Understand How Your Course Material Is Presented—Lecture by Instructor, On-line With Computer, or Both?*

Make sure that you are familiar with the way that this course is being taught. Is it a traditional course, in which you have a printed textbook and meet with an instructor? Is it taught totally online, and your textbook is electronic and you e-mail your instructor? Or is your course structured somewhere in between these two methods? (Not all of the tips that follow will apply to all forms of instruction.)

3. *Schedule Your Class So That It Does Not Interfere With Other Commitments.*

Make sure that you have scheduled your math course for a time that will give you the best chance for success. For example, if you are also working, you may want to check with your employer to make sure that your work hours will not conflict with your course schedule.

Objective **B** Here are a Few General Tips for Success

Below are some general tips that will increase your chance for success in a mathematics class. Many of these tips will also help you in other courses you may be taking.

1. *Most Important! Organize Your Class Materials. Unless Told Otherwise, Use a 3-Ring Binder Solely for Your Mathematics Class.*

In the next couple pages, many ideas will be presented to help you organize your class materials—notes, any handouts, completed homework, previous tests, etc. In general, you **MUST** have these materials organized. All of them will be valuable references throughout your course and when studying for upcoming tests and the final exam. One way to make sure you can locate these materials when you need them is to use a three-ring binder. This binder should be used solely for your mathematics class and should be brought to each and every class or lab. This way, any material can be immediately inserted in a section of this binder and will be there when you need it.

2. *Choose to attend all class periods.*

If possible, sit near the front of the classroom. This way, you will see and hear the presentation better. It may also be easier for you to participate in classroom activities.

Helpful Hint

MyLab Math and MathXL

When assignments are turned in online, keep a hard copy of your complete written work. You will need to refer to your written work to be able to ask questions and to study for tests later.

3. Complete Your Homework. This Means: Attempt All of It, Check All of It, Correct Any Mistakes, and Ask for Help if Needed.

You've probably heard the phrase "practice makes perfect" in relation to music and sports. It also applies to mathematics. You will find that the more time you spend solving mathematics exercises, the easier the process becomes. Be sure to schedule enough time to complete your assignments before the due date assigned by your instructor.

Review the steps you took while working a problem. Learn to check your answers in the original exercises. You may also compare your answers with the "Answers to Selected Exercises" section in the back of the book. If you have made a mistake, try to figure out what went wrong. Then correct your mistake. If you can't find what went wrong, **don't** erase your work or throw it away. Show your work to your instructor, a tutor in a math lab, or a classmate. It is easier for someone to find where you had trouble if he or she looks at your original work.

It's all right to ask for help. In fact, it's a good idea to ask for help whenever there is something that you don't understand. Make sure you know when your instructor has office hours and how to find his or her office. Find out whether math tutoring services are available on your campus. Check on the hours, location, and requirements of the tutoring service.

4. Learn from your mistakes and be patient with yourself.

Everyone, even your instructor, makes mistakes. (That definitely includes me—Elayn Martin-Gay.) Use your errors to learn and to become a better math student. The key is finding and understanding your errors.

Was your mistake a careless one, or did you make it because you can't read your own math writing? If so, try to work more slowly or write more neatly and make a conscious effort to carefully check your work.

Did you make a mistake because you don't understand a concept? Take the time to review the concept or ask questions to better understand it.

Did you skip too many steps? Skipping steps or trying to do too many steps mentally may lead to preventable mistakes.

5. Turn in assignments on time.

This way, you can be sure that you will not lose points for being late. Show every step of a problem and be neat and organized. Also be sure that you understand which problems are assigned for homework. If allowed, you can always double-check the assignment with another student in your class.

Helpful Hint

MyLab Math and MathXL

If you are doing your homework online, you can work and re-work those exercises that you struggle with until you master them. Try working through all the assigned exercises twice before the due date.

Helpful Hint

MyLab Math and MathXL

If you are completing your homework online, it's important to work each exercise on paper before submitting the answer. That way, you can check your work and follow your steps to find and correct any mistakes.

Helpful Hint

MyLab Math and MathXL

Be aware of assignments and due dates set by your instructor. Don't wait until the last minute to submit work online.

Objective C Knowing and Using Your Text or e-Text

Flip through the pages of this text or view the e-text pages on a computer screen. Start noticing examples, exercise sets, end-of-chapter material, and so on. Learn the way this text is organized by finding an example in your text of each type of resource listed below. Finding and using these resources throughout your course will increase your chance of success.

- **Practice Exercises.** Each example in every section has a parallel Practice exercise. Work each Practice exercise after you've finished the corresponding example. Answers are at the bottom of the page. This "learn-by-doing" approach will help you grasp ideas before you move on to other concepts.
- **Objectives.** Every section of this text is divided into objectives, such as **A** or **B**. They are listed at the beginning of the section and noted in that section. The main section of exercises in each exercise set is also referenced by an objective, such as **A** or **B**, and also an example(s). There is also often a section of exercises entitled "Mixed Practice," which is referenced by two or more objectives or sections. These are mixed exercises written to prepare you for your next exam. Use all of this referencing if you have trouble completing an assignment from the exercise set.

- **Icons (Symbols).** Make sure that you understand the meaning of the icons that are beside many exercises.  tells you that the corresponding exercise may be viewed on the video Lecture Series that corresponds to that section.  tells you that this exercise is a writing exercise in which you should answer in complete sentences.  tells you that the exercise involves geometry.
- **Integrated Reviews.** Found in the middle of each chapter, these reviews offer you a chance to practice—in one place—the many concepts that you have learned separately over several sections.
- **End-of-Chapter Opportunities.** There are many opportunities at the end of each chapter to help you understand the concepts of the chapter.

Vocabulary Checks contain key vocabulary terms introduced in the chapter.

Chapter Highlights contain chapter summaries and examples.

Chapter Reviews contain review problems. The first part is organized section by section and the second part contains a set of mixed exercises.

Getting Ready for the Tests are multiple choice or matching exercises designed to check your knowledge of chapter concepts, before you attempt the chapter test. Video solutions are available for all these exercises.

Chapter Tests are sample tests to help you prepare for an exam. The Chapter Test Prep Videos found in MyLab Math and YouTube provide the video solution to each question on each Chapter Test.

Cumulative Reviews start at Chapter 2 and are reviews consisting of material from the beginning of the book to the end of that particular chapter.

- **Student Resources in Your Textbook.** You will find a **Student Resources** section at the back of this textbook. It contains the following to help you study and prepare for tests:

Study Skill Builders contain study skills advice. To increase your chance for success in the course, read these study tips, and answer the questions.

Bigger Picture—Study Guide Outline provides you with a study guide outline of the course, with examples.

Practice Final provides you with a Practice Final Exam to help you prepare for a final.

- **Resources to Check Your Work.** The **Answers to Selected Exercises** section provides answers to all odd-numbered section exercises and to all integrated review, chapter review, getting ready for the test, chapter test, and cumulative review exercises. Use the **Solutions to Selected Exercises** to see the worked-out solution to every other odd-numbered exercise in the section exercises and chapter tests.

Helpful Hint

MyLab Math

In MyLab Math, you have access to the following video resources:

- Lecture Videos for each section
- Getting Ready for the Test Videos
- Chapter Test Prep Videos
- Final Exam Videos

Use these videos provided by the author to prepare for class, review, and study for tests.

Objective D Knowing and Using Video and Notebook Organizer Resources

Video Resources

Below is a list of video resources that are all made by me—the author of your text, Elayn Martin-Gay. By making these videos, I can be sure that the methods presented are consistent with those in the text. All video resources may be found in MyLab Math and some also on YouTube.

- **Interactive Video Lecture Series.** Exercises marked with a  are fully worked out by the author. The lecture series provides approximately 20 minutes of instruction per section and is organized by Objective.
- **Getting Ready for the Test Videos.** These videos provide solutions to all of the Getting Ready for the Test exercises.

- **Chapter Test Prep Videos.** These videos provide solutions to all of the Chapter Test exercises worked out by the author. They can be found in MyLab Math and YouTube. This supplement is very helpful before a test or exam.
- **Tips for Success in Mathematics.** These video segments are about 3 minutes long and are daily reminders to help you continue practicing and maintaining good organizational and study habits.
- **Final Exam Videos.** These video segments provide solutions to each question.

Video Organizer

This organizer is in three-ring notebook ready form. It is to be inserted in a three-ring binder and completed. This organizer is numbered according to the sections in your text to which it refers.

It is closely tied to the Interactive (Video) Lecture Series. Each section should be completed while watching the lecture video on the same section. Once completed, you will have a set of notes to accompany the (Video) Lecture Series section by section.

Objective E Getting Help

If you have trouble completing assignments or understanding the mathematics, get help as soon as you need it! This tip is presented as an objective on its own because it is so important. In mathematics, usually the material presented in one section builds on your understanding of the previous section. This means that if you don't understand the concepts covered during a class period, there is a good chance that you will not understand the concepts covered during the next class period. If this happens to you, get help as soon as you can.

Where can you get help? Try your instructor, a tutoring center, or a math lab, or you may want to form a study group with fellow classmates. If you do decide to see your instructor or go to a tutoring center, make sure that you have a neat notebook and are ready with your questions.

Objective F Preparing for and Taking an Exam

Make sure that you allow yourself plenty of time to prepare for a test. If you think that you are a little “math anxious,” it may be that you are not preparing for a test in a way that will ensure success. The way that you prepare for a test in mathematics is important. To prepare for a test:

1. Review your previous homework assignments.
2. Review any notes from class and section-level quizzes you have taken. (If this is a final exam, also review chapter tests you have taken.)
3. Review concepts and definitions by reading the Chapter Highlights at the end of each chapter.
4. Practice working out exercises by completing the Chapter Review found at the end of each chapter. (If this is a final exam, go through a Cumulative Review. There is one found at the end of each chapter except Chapter 1. Choose the review found at the end of the latest chapter that you have covered in your course.) *Don't stop here!*
5. Take the Chapter Getting Ready for the Test. All answers to these exercises are available to you as well as video solutions.
6. Take a sample test with no notes, etc, available for help. It is important that you place yourself in conditions similar to test conditions to find out how you

Helpful Hint

MyLab Math and MathXL

- Use the **Help Me Solve This** button to get step-by-step help for the exercise you are working. You will need to work an additional exercise of the same type before you can get credit for having worked it correctly.
- Use the **Video** button to view a video clip of the author working a similar exercise.

Helpful Hint

MyLab Math and MathXL

Review your written work for previous assignments. Then, go back and re-work previous assignments. Open a previous assignment, and click **Similar Exercise** to generate new exercises. Re-work the exercises until you fully understand them and can work them without help features.

will perform. There is a Chapter Test available at the end of each chapter, or you can work selected problems from the Chapter Review. Your instructor may also provide you with a review sheet. Then check your sample test. If your sample test is the Chapter Test in the text, don't forget that the video solutions are in MyLab Math and YouTube.

7. On the day of the test, allow yourself plenty of time to arrive at where you will be taking your exam.

When taking your test:

1. Read the directions on the test carefully.
2. Read each problem carefully as you take the test. Make sure that you answer the question asked.
3. Watch your time and pace yourself so that you can attempt each problem on your test.
4. If you have time, check your work and answers.
5. Do not turn your test in early. If you have extra time, spend it double-checking your work.

Objective G Managing Your Time

As a college student, you know the demands that classes, homework, work, and family place on your time. Some days you probably wonder how you'll ever get everything done. One key to managing your time is developing a schedule. Here are some hints for making a schedule:

1. Make a list of all of your weekly commitments for the term. Include classes, work, regular meetings, extracurricular activities, etc. You may also find it helpful to list such things as laundry, regular workouts, grocery shopping, etc.
2. Next, estimate the time needed for each item on the list. Also make a note of how often you will need to do each item. Don't forget to include time estimates for the reading, studying, and homework you do outside of your classes. You may want to ask your instructor for help estimating the time needed.
3. In the exercise set that follows, you are asked to block out a typical week on the schedule grid given. Start with items with fixed time slots like classes and work.
4. Next, include the items on your list with flexible time slots. Think carefully about how best to schedule items such as study time.
5. Don't fill up every time slot on the schedule. Remember that you need to allow time for eating, sleeping, and relaxing! You should also allow a little extra time in case some items take longer than planned.
6. If you find that your weekly schedule is too full for you to handle, you may need to make some changes in your workload, classload, or other areas of your life. You may want to talk to your advisor, manager or supervisor at work, or someone in your college's academic counseling center for help with such decisions.



1.1 Exercise Set MyLab Math

1. What is your instructor's name?
2. What are your instructor's office location and office hours?
3. What is the best way to contact your instructor?
4. Do you have the name and contact information of at least one other student in class?
5. Will your instructor allow you to use a calculator in this class?
6. Why is it important that you write step-by-step solutions to homework exercises and keep a hard copy of all work submitted?
7. Is there a tutoring service available on campus? If so, what are its hours? What services are available?
8. Have you attempted this course before? If so, write down ways that you might improve your chances of success during this attempt.
9. List some steps that you can take if you begin having trouble understanding the material or completing an assignment. If you are completing your homework in MyLab Math and MathXL, list the resources you can use for help.
10. How many hours of studying does your instructor advise for each hour of instruction?
11. What does the  icon in this text mean?
12. What does the  icon in this text mean?
13. What does the  icon in this text mean?
14. Search the minor columns in your text. What are Practice exercises?
15. When might be the best time to work a Practice exercise?
16. Where are the answers to Practice exercises?
17. What answers are contained in this text and where are they?
18. What are Tips for Success in Mathematics and where are they located?
19. What and where are Integrated Reviews?
20. How many times is it suggested that you work through the homework exercises in MyLab Math or MathXL before the submission deadline?
21. How far in advance of the assigned due date is it suggested that homework be submitted online? Why?
22. Chapter Highlights are found at the end of each chapter. Find the Chapter 1 Highlights and explain how you might use it and how it might be helpful.
23. Chapter Reviews are found at the end of each chapter. Find the Chapter 1 Review and explain how you might use it and how it might be helpful.
24. Chapter Tests are found at the end of each chapter. Find the Chapter 1 Test and explain how you might use it and how it might be helpful when preparing for an exam on Chapter 1. Include how the Chapter Test Prep Videos may help. If you are working in MyLab Math and MathXL, how can you use previous homework assignments to study?
25. What is the Video Organizer? Explain the contents and how it might be used.
26. Read or reread objective  and fill out the schedule grid on the next page.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
4:00 a.m.							
5:00 a.m.							
6:00 a.m.							
7:00 a.m.							
8:00 a.m.							
9:00 a.m.							
10:00 a.m.							
11:00 a.m.							
12:00 p.m.							
1:00 p.m.							
2:00 p.m.							
3:00 p.m.							
4:00 p.m.							
5:00 p.m.							
6:00 p.m.							
7:00 p.m.							
8:00 p.m.							
9:00 p.m.							
10:00 p.m.							
11:00 p.m.							
Midnight							
1:00 a.m.							
2:00 a.m.							
3:00 a.m.							

1.2 Place Value, Names for Numbers, and Reading Tables

Objectives

- A** Find the Place Value of a Digit in a Whole Number. 
- B** Write a Whole Number in Words and in Standard Form. 
- C** Write a Whole Number in Expanded Form. 
- D** Read Tables. 

The **digits** 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 can be used to write numbers. For example, the **whole numbers** are

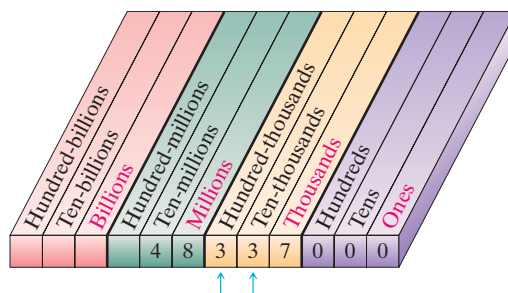
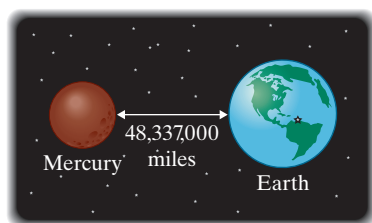
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, . . .

and the **natural numbers** are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, . . .

The three dots (. . .) after the 11 mean that both lists continue indefinitely. That is, there is no largest whole number and there is no largest natural number. Also, the smallest whole number is 0 and the smallest natural number is 1.

Objective **A** Finding the Place Value of a Digit in a Whole Number

The position of each digit in a number determines its **place value**. For example, the average distance (in miles) between the planet Mercury and the planet Earth can be represented by the whole number 48,337,000. A place-value chart for this whole number is on the next page.



The two 3s in 48,337,000, shown above, represent different amounts because of their different placements. The place value of the 3 on the left is hundred-thousands. The place value of the 3 on the right is ten-thousands.

Examples

Find the place value of the digit 3 in each whole number.

1. 396,418

↑
hundred-thousands

2. 93,192

↑
thousands

3. 534,275,866

↑
ten-millions

Work Practice 1–3

Practice 1–3

Find the place value of the digit 8 in each whole number.

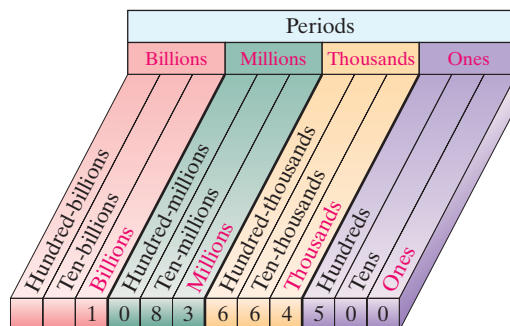
1. 38,760,005

2. 67,890

3. 481,922

Objective B Writing a Whole Number in Words and in Standard Form

A whole number such as 1,083,664,500 is written in **standard form**. Notice that commas separate the digits into groups of three, starting from the right. Each group of three digits is called a **period**. The names of the first four periods are shown below in red.



Writing a Whole Number in Words

To write a whole number in words, write the number in each period followed by the name of the period. (The ones period name is usually not written.) This same procedure can be used to read a whole number.

For example, we write 1,083,664,500 as

one billion,
eighty-three million,
six hundred sixty-four thousand,
five hundred

Helpful Hint

Notice the **commas** after the name of each period.

Answers

1. millions
2. hundreds
3. ten-thousands

Helpful Hint

The name of the ones period is not used when reading and writing whole numbers. For example,

9,265

is read as

“nine **thousand**, two hundred sixty-five.”

Practice 4–6

Write each whole number in words.

4. 54
5. 678
6. 93,205

Practice 7

Write 679,430,105 in words.

Practice 8–11

Write each whole number in standard form.

8. thirty-seven
9. two hundred twelve
10. eight thousand, two hundred seventy-four
11. five million, fifty-seven thousand, twenty-six

Answers

4. fifty-four
5. six hundred seventy-eight
6. ninety-three thousand, two hundred five
7. six hundred seventy-nine million, four hundred thirty thousand, one hundred five
8. 37
9. 212
10. 8,274 or 8274
11. 5,057,026

✓ **Concept Check Answer**
false

Examples

Write each whole number in words.

4. 72 seventy-two
5. 546 five hundred forty-six
6. 27,034 twenty-seven thousand, thirty-four

Work Practice 4–6**Helpful Hint**

The word “and” is *not* used when reading and writing whole numbers. It is used only when reading and writing mixed numbers and some decimal values, as shown later in this text.

Example 7

Write 308,063,557 in words.

Solution:

308,063,557 is written as

three hundred eight **million**, sixty-three **thousand**, five hundred fifty-seven

Work Practice 7

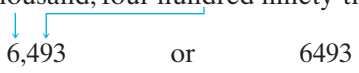
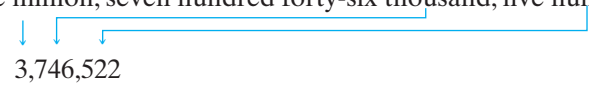
✓ **Concept Check** True or false? When writing a check for \$2600, the word name we write for the dollar amount of the check is “two thousand sixty.” Explain your answer.

Writing a Whole Number in Standard Form

To write a whole number in standard form, write the number in each period, followed by a comma.

Examples

Write each whole number in standard form.

8. forty-one 41
9. seven hundred eight 708
10. six thousand, four hundred ninety-three

11. three million, seven hundred forty-six thousand, five hundred twenty-two


Work Practice 8–11

Helpful Hint

A comma may or may not be inserted in a four-digit number. For example, both 6,493 and 6493 are acceptable ways of writing six thousand, four hundred ninety-three.

Objective C Writing a Whole Number in Expanded Form

The place value of a digit can be used to write a number in expanded form. The **expanded form** of a number shows each digit of the number with its place value. For example, 5672 is written in expanded form as

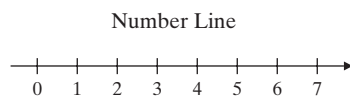
5 thousands ↑ digit place value	+	6 hundreds ↑ digit place value	+	7 tens ↑ digit place value	+	2 ones ↑ digit place value
↓		↓		↓		↓
5672 = 5000	+	600	+	70	+	2

Example 12 Write 5,207,034 in expanded form.

Solution: 5,000,000 + 200,000 + 7000 + 30 + 4

Work Practice 12

We can visualize whole numbers by points on a line. The line below is called a **number line**. This number line has equally spaced marks for each whole number. The arrow to the right simply means that the whole numbers continue indefinitely. In other words, there is no largest whole number.



We will study number lines further in Section 1.4.

Objective D Reading Tables

Now that we know about place value and names for whole numbers, we introduce one way that whole number data may be presented. **Tables** are often used to organize and display facts that contain numbers. The following table shows the countries that won the most medals during the Summer Olympic Games in Rio de Janeiro, Brazil in 2016. (Although the medals are truly won by athletes from the various countries, for simplicity we will state that countries have won the medals.)

Practice 12

Write 4,026,301 in expanded form.



Answer

12. 4,000,000 + 20,000 + 6000 + 300 + 1

2016 Rio de Janeiro Summer Olympics Medal Count

Country	Gold	Silver	Bronze	Total	Country	Gold	Silver	Bronze	Total
 United States	46	37	38	121	 Japan	12	8	21	41
 China	26	18	26	70	 Australia	8	11	10	29
 Great Britain	27	23	17	67	 Italy	8	12	8	28
 Russian Federation	19	18	19	56	 Canada	4	3	15	22
 Germany	17	10	15	42	 South Korea	9	3	9	21
 France	10	18	14	42	 Netherlands	8	7	4	19
(Source: International Olympic Committee)					 Brazil	7	6	6	19

For example, by reading from left to right along the row marked “France,” we find that France won 10 gold, 18 silver, and 14 bronze medals during the 2016 Summer Olympic Games.

Practice 13

Use the 2016 Summer Games table to answer each question.

- How many bronze medals did Australia win during the 2016 Summer Olympic Games?
- Which countries won more than 60 medals?

Answers

13. a. 10 b. United States, China, Great Britain

Example 13

Use the 2016 Summer Games table to answer each question.

- How many silver medals did Germany win during the 2016 Summer Olympic Games?
- Which countries shown won fewer gold medals than Japan?

Solution:

- Find “Germany” in the left-hand column. Then read from left to right until the “silver” column is reached. We find that Germany won 10 silver medals.
- Japan won 12 gold medals. Of the countries shown, France, Australia, Italy, Canada, South Korea, Netherlands, and Brazil each won fewer than 12 gold medals.

Work Practice 13

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

standard form
expanded form

period
place value

whole
words

- The numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, . . . are called _____ numbers.
- The number 1286 is written in _____.
- The number “twenty-one” is written in _____.
- The number $900 + 60 + 5$ is written in _____.
- In a whole number, each group of three digits is called a(n) _____.
- The _____ of the digit 4 in the whole number 264 is ones.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 1.2

- Objective A** 7. In Example 1, what is the place value of the digit 6?
- Objective B** 8. Complete this statement based on Example 3. To read (or write) a number, read from _____ to _____.
- Objective C** 9. In Example 5, what is the expanded form value of the digit 8?
- Objective D** 10. Use the table given in Example 6 to determine which breed shown has the fewest American Kennel Club registrations.

1.2 Exercise Set MyLab Math **Objective A** Determine the place value of the digit 5 in each whole number. See Examples 1 through 3.

1. 657 2. 905 3. 5423 4. 6527
5. 43,526,000 6. 79,050,000 7. 5,408,092 8. 51,682,700

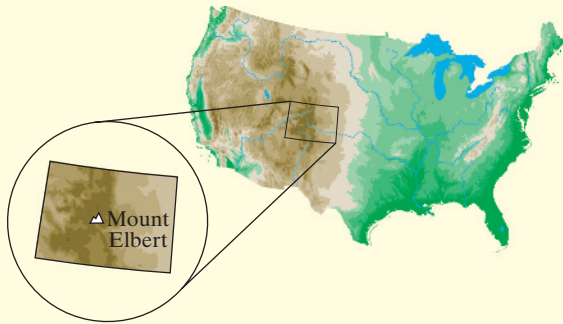
Objective B Write each whole number in words. See Examples 4 through 7.

9. 354 10. 316 11. 8279 12. 5445
13. 26,990 14. 42,009 15. 2,388,000 16. 3,204,000
17. 24,350,185 18. 47,033,107

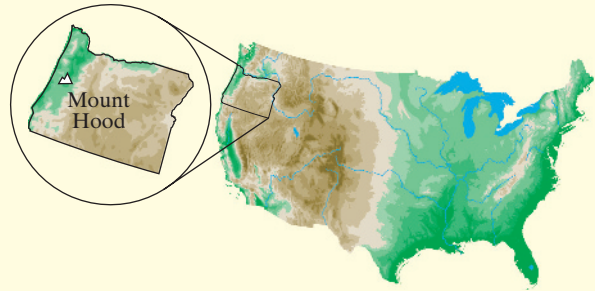
Write each number in the sentence in words. (Do not write the years in words.) See Examples 4 through 7.

19. In 2016, the population of Monaco was 37,917. (Source: CIA World Factbook)
20. The state of Texas is divided into 254 counties. (Source: Texas Association of counties)
21. The Burj Khalifa, in Dubai, United Arab Emirates, a hotel and office building, is currently the tallest in the world at a height of 2720 feet. (Source: Council on Tall Buildings and Urban Habitat)
22. In 2016, there were 119,926 patients in the United States waiting for an organ transplant. (Source: United Network for Organ Sharing)
23. Each day, UPS delivers an average of 18,300,000 packages and documents worldwide. (Source: UPS)
24. The annual revenue for FedEx during fiscal year 2016 was \$50,400,000,000. (Source: FedEx)

25. The highest point in Colorado is Mount Elbert, at an elevation of 14,433 feet. (Source: U.S. Geological Survey)



26. The highest point in Oregon is Mount Hood, at an elevation of 11,239 feet. (Source: U.S. Geological Survey)



27. The average low price for a 2017 Honda Civic sedan was \$17,410. (Source: Truecar.com)

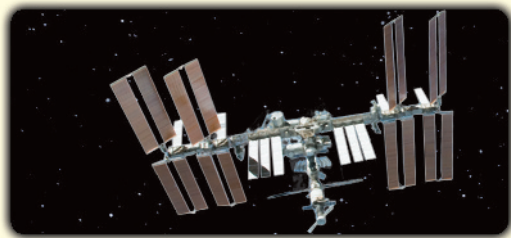
28. The Goodyear blimp *Eagle* holds 202,700 cubic feet of helium. (Source: The Goodyear Tire & Rubber Company)

Write each whole number in standard form. See Examples 8 through 11.

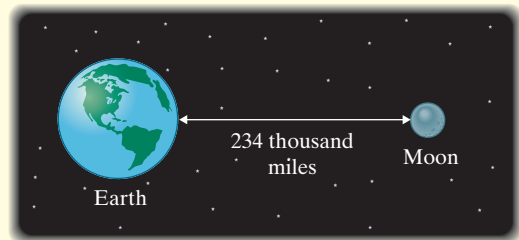
29. Six thousand, five hundred eighty-seven
31. Fifty-nine thousand, eight hundred
33. Thirteen million, six hundred one thousand, eleven
35. Seven million, seventeen
37. Two hundred sixty thousand, nine hundred ninety-seven
30. Four thousand, four hundred sixty-eight
32. Seventy-three thousand, two
34. Sixteen million, four hundred five thousand, sixteen
36. Two million, twelve
38. Six hundred forty thousand, eight hundred eighty-one

Write the whole number in each sentence in standard form. See Examples 8 through 11.

39. The International Space Station orbits above Earth at an average altitude of two hundred fifty kilometers. (Source: space.com)



40. The average distance between the surfaces of Earth and the moon is about two hundred thirty-four thousand miles.



41. By 2016, there were one thousand, twenty-four buildings worldwide that were over 200 meters tall. (Source: Council on Tall Buildings and Urban Habitat)

42. The CN Tower in Toronto, Canada, is the tallest telecommunications/observation tower in the world. Its height is one thousand, eight hundred fifteen feet. (Source: Council on Tall Buildings and Urban Habitat)

43. The film *Star Wars Episode VII: The Force Awakens* shattered the world record for opening weekend income when it took in two hundred forty-seven million, nine hundred sixty-six thousand, six hundred seventy-five dollars in 2015. (Source: Nash Information Services)

44. The film *Jurassic World* holds the record for the second-highest opening weekend income: it took in two hundred eight million, eight hundred six thousand, two hundred seventy dollars in 2015. (Source: Nash Information Service)

45. Morten Anderson, who played in the National Football League in 1982–2007, holds the record for most career field goals at five hundred sixty-five. (Source: NFL)

46. Morten Anderson also holds the record for the most field goals attempted in a career at seven hundred nine. (Source: NFL)

Objective C Write each whole number in expanded form. See Example 12.

47. 209

48. 789

49. 3470

50. 6040

▶ 51. 80,774

52. 20,215

53. 66,049

54. 99,032

55. 39,680,000

56. 47,703,029

Objective D The table shows the beginning year of recent eruptions of major volcanoes in the Cascade Mountains. Use this table to answer Exercises 57 through 62. See Example 13.



Recent Eruptions of Major Cascade Mountain Volcanoes (1750–present) *

Volcano	State Location	Year of Eruption (beginning year)					
		1750–1799	1800–1849	1850–1899	1900–1949	1950–1999	2000–present
Mt. Baker	Washington	1792	1843	1870, 1880			
Glacier Peak	Washington	1750(?)					
Mt. Rainier	Washington		1841, 1843	1854			
Mt. St. Helens	Washington		1800			1980	
Mt. Hood	Oregon			1854, 1859, 1865			
Three Sisters	Oregon			1853(?)			
Medicine Lake	California				1910		
Mt. Shasta	California	1786		1855			
Cinder Cone*	California			1850			
Lassen Peak	California				1914		
Chaos Crags*	California			1854			

Other major volcanoes in the Cascades have had no eruptions from 1750 to present; Source: Harris

*Cinder Cone and Chaos Crags are located by Lassen Peak.



57. Mount Shasta erupted in the 1700s. Locate, then write this eruption year in standard form.

58. Mount Baker erupted in the 1700s. Locate, then write this eruption year in standard form.

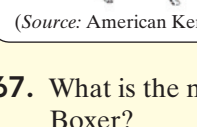
59. Which volcano in the table has had the most eruptions?

60. Which volcano(es) in the table has had two eruptions?

61. Which volcano in the table had the earliest eruption?

62. Which volcano in the table had the most recent eruption?

The table shows the top ten popular breeds of dogs in 2016 according to the American Kennel Club. The breeds are listed in the order of number of registrations with the most popular breed listed first. Use this table to answer Exercises 63 through 68. See Example 13.

Top Ten American Kennel Club Registrations in 2016			
Breed		Average Dog Maximum Height (in inches)	Average Dog Maximum Weight (in pounds)
	Labrador retriever	25	75
	German shepherd	26	95
	Golden retriever	24	80
	Bulldog	26	90
	Beagle	15	30
	French bulldog	12	24
	Yorkshire terrier	9	7
	Poodle (standard, miniature, and toy)	standard: 26	standard: 70
	Rottweiler	26	115
	Boxer	25	70

(Source: American Kennel Club)

- ▶ 63. Which breed has a greater average dog maximum weight, the Bulldog or the German shepherd?
64. Which breed has more dogs registered, Golden retriever or German shepherd?
- ▶ 65. Which breed is the most popular dog? Write the average dog maximum weight for this breed in words.
66. Which breed is second in popularity? Write the average dog maximum height for this breed in words.

- ▶ 67. What is the maximum weight of an average-size Boxer?

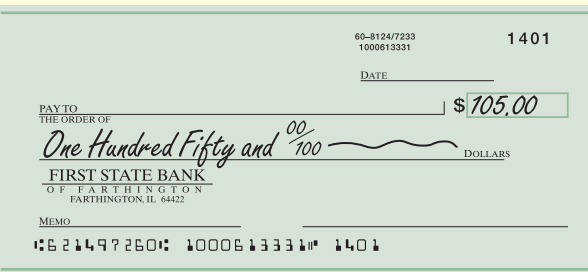
68. What is the maximum height of an average-size standard poodle?

Concept Extensions

69. Write the largest four-digit number that can be made from the digits 1, 9, 8, and 6 if each digit must be used once. _____

70. Write the largest five-digit number that can be made using the digits 5, 3, and 7 if each digit must be used at least once. _____

Check to see whether each number written in standard form matches the number written in words. If not, correct the number in words. See the Concept Check in this section.

71. 

60-8124/7233 1000613331 1401

DATE _____

PAY TO THE ORDER OF _____ \$105.00

One Hundred Fifty and ⁰⁰/₁₀₀ _____ DOLLARS

FIRST STATE BANK
OF FARTHINGTON
FARTHINGTON, IL 64422

MEMO _____

⑆ 621497260⑆ 1000613331⑆ 1401

72. 

60-8124/7233 1000613331 1402

DATE _____

PAY TO THE ORDER OF _____ \$7030.00

Seven Thousand Thirty and ⁰⁰/₁₀₀ _____ DOLLARS

FIRST STATE BANK
OF FARTHINGTON
FARTHINGTON, IL 64422

MEMO _____

⑆ 621497260⑆ 1000613331⑆ 1402

- ✎ 73. If a number is given in words, describe the process used to write this number in standard form.
75. How large is a trillion? To get an idea, one trillion seconds is over 31 thousand years. Look up “trillion” in a dictionary and use the definition to write this number in standard form.
76. How large is a quadrillion? To get an idea, if one quadrillion dollars were divided among the population of the United States, we would each receive over 9 million dollars. Look up “quadrillion” (in the American system) and write this number in standard form.
- ✎ 74. If a number is written in standard form, describe the process used to write this number in expanded form.

1.3 Adding and Subtracting Whole Numbers, and Perimeter

Objective A Adding Whole Numbers

An iPod is a hard drive–based portable audio player. In 2017, the iPod is still the most popular digital music player in the United States.

Suppose that an electronics store received a shipment of two boxes of iPods one day and an additional four boxes of iPods the next day. The **total** shipment in the two days can be found by adding 2 and 4.

$$2 \text{ boxes of iPods} + 4 \text{ boxes of iPods} = 6 \text{ boxes of iPods}$$

The **sum** (or total) is 6 boxes of iPods. Each of the numbers 2 and 4 is called an **addend**, and the process of finding the sum is called **addition**.

$$\begin{array}{ccc} 2 & + & 4 & = & 6 \\ \uparrow & & \uparrow & & \uparrow \\ \text{addend} & & \text{addend} & & \text{sum} \end{array}$$

To add whole numbers, we add the digits in the ones place, then the tens place, then the hundreds place, and so on. For example, let's add $2236 + 160$.

$$\begin{array}{r} 2236 \\ + 160 \\ \hline 2396 \end{array}$$

Line up numbers vertically so that the place values correspond. Then add digits in corresponding place values, starting with the ones place.

sum of ones
sum of tens
sum of hundreds
sum of thousands

Example 1 Add: $46 + 713$

Solution:

$$\begin{array}{r} 46 \\ + 713 \\ \hline 759 \end{array}$$

Work Practice 1

Adding by Carrying

When the sum of digits in corresponding place values is more than 9, **carrying** is necessary. For example, to add $365 + 89$, add the ones-place digits first.

Carrying

$$\begin{array}{r} 365 \\ + 89 \\ \hline \end{array}$$

5 ones + 9 ones = 14 ones or 1 ten + 4 ones
4 Write the 4 ones in the ones place and carry the 1 ten to the tens place.

Next, add the tens-place digits.

$$\begin{array}{r} 365 \\ + 89 \\ \hline \end{array}$$

1 ten + 6 tens + 8 tens = 15 tens or 1 hundred + 5 tens
5 4 Write the 5 tens in the tens place and carry the 1 hundred to the hundreds place.

Objectives

- A Add Whole Numbers.
- B Subtract Whole Numbers.
- C Find the Perimeter of a Polygon.
- D Solve Problems by Adding or Subtracting Whole Numbers.



Practice 1

Add: $4135 + 252$

Answer

1. 4387

Next, add the hundreds-place digits.

$$\begin{array}{r} 11 \\ 365 \\ + 89 \\ \hline 454 \end{array} \quad \begin{array}{l} 1 \text{ hundred} + 3 \text{ hundreds} = 4 \text{ hundreds} \\ \text{Write the 4 hundreds in the hundreds place.} \end{array}$$

Practice 2

Add: $47,364 + 135,898$

Example 2 Add: $46,278 + 124,931$

Solution:

$$\begin{array}{r} 111 \\ 46,278 \\ + 124,931 \\ \hline 171,209 \end{array}$$

Work Practice 2

✓ **Concept Check** What is wrong with the following computation?

$$\begin{array}{r} 394 \\ + 283 \\ \hline 577 \end{array}$$

Before we continue adding whole numbers, let's review some properties of addition that you may have already discovered. The first property that we will review is the **addition property of 0**. This property reminds us that the sum of 0 and any number is that same number.

Addition Property of 0

The sum of 0 and any number is that number. For example,

$$\begin{array}{l} 7 + 0 = 7 \\ 0 + 7 = 7 \end{array}$$

Next, notice that we can add any two whole numbers in any order and the sum is the same. For example,

$$4 + 5 = 9 \quad \text{and} \quad 5 + 4 = 9$$

We call this special property of addition the **commutative property of addition**.

Commutative Property of Addition

Changing the **order** of two addends does not change their sum. For example,

$$2 + 3 = 5 \quad \text{and} \quad 3 + 2 = 5$$

Another property that can help us when adding numbers is the **associative property of addition**. This property states that when adding numbers, the grouping of the numbers can be changed without changing the sum. We use parentheses to group numbers. They indicate which numbers to add first. For example, let's use two different groupings to find the sum of $2 + 1 + 5$.

$$(2 + 1) + 5 = 3 + 5 = 8$$

Also,

$$2 + (1 + 5) = 2 + 6 = 8$$

Both groupings give a sum of 8.

Answer

2. 183,262

✓ **Concept Check Answer**

forgot to carry 1 hundred to the hundreds place

Associative Property of Addition

Changing the **grouping** of addends does not change their sum. For example,

$$3 + (5 + 7) = 3 + 12 = 15 \quad \text{and} \quad (3 + 5) + 7 = 8 + 7 = 15$$

The commutative and associative properties tell us that we can add whole numbers using any order and grouping that we want.

When adding several numbers, it is often helpful to look for two or three numbers whose sum is 10, 20, and so on. Why? Adding multiples of 10 such as 10 and 20 is easier.

Example 3 Add: $13 + 2 + 7 + 8 + 9$

Solution:

$$13 + 2 + 7 + 8 + 9 = 39$$

Work Practice 3

Feel free to use the process of Example 3 any time when adding.

Example 4 Add: $1647 + 246 + 32 + 85$

Solution:

$$\begin{array}{r} 1647 \\ 246 \\ 32 \\ + 85 \\ \hline 2010 \end{array}$$

Work Practice 4

Practice 3

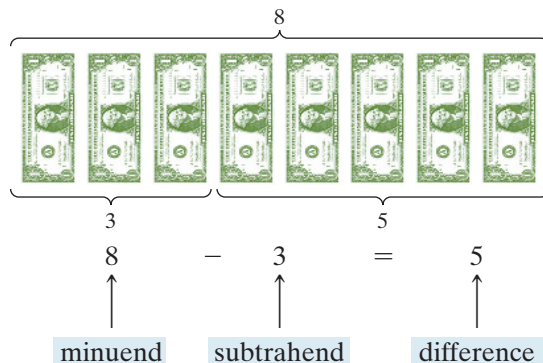
Add: $12 + 4 + 8 + 6 + 5$

Practice 4

Add: $6432 + 789 + 54 + 28$

Objective B Subtracting Whole Numbers

If you have \$5 and someone gives you \$3, you have a total of \$8, since $5 + 3 = 8$. Similarly, if you have \$8 and then someone borrows \$3, you have \$5 left. **Subtraction** is finding the **difference** of two numbers.



Answers

3. 35 4. 7303

In this example, 8 is the **minuend**, and 3 is the **subtrahend**. The **difference** between these two numbers, 8 and 3, is 5.

Notice that addition and subtraction are very closely related. In fact, subtraction is defined in terms of addition.

$$8 - 3 = 5 \text{ because } 5 + 3 = 8$$

This means that subtraction can be *checked* by addition, and we say that addition and subtraction are reverse operations.

Practice 5

Subtract. Check each answer by adding.

- a. $14 - 6$
- b. $20 - 8$
- c. $93 - 93$
- d. $42 - 0$

Example 5 Subtract. Check each answer by adding.

- a. $12 - 9$
- b. $22 - 7$
- c. $35 - 35$
- d. $70 - 0$

Solution:

- a. $12 - 9 = 3$ because $3 + 9 = 12$
- b. $22 - 7 = 15$ because $15 + 7 = 22$
- c. $35 - 35 = 0$ because $0 + 35 = 35$
- d. $70 - 0 = 70$ because $70 + 0 = 70$

Work Practice 5

Look again at Examples 5 (c) and 5(d).

$$\begin{array}{cc}
 \text{5(c)} & 35 - 35 = 0 \\
 \swarrow & \searrow \\
 \text{same} & \text{difference} \\
 \text{number} & \text{is 0}
 \end{array}
 \qquad
 \begin{array}{cc}
 \text{5(d)} & 70 - 0 = 70 \\
 \swarrow & \searrow \\
 \text{a number} & \text{difference is the} \\
 \text{minus 0} & \text{same number}
 \end{array}$$

These two examples illustrate the subtraction properties of 0.

Subtraction Properties of 0

The difference of any number and that same number is 0. For example,

$$11 - 11 = 0$$

The difference of any number and 0 is that same number. For example,

$$45 - 0 = 45$$

To subtract whole numbers we subtract the digits in the ones place, then the tens place, then the hundreds place, and so on. When subtraction involves numbers of two or more digits, it is more convenient to subtract vertically. For example, to subtract $893 - 52$,

$$\begin{array}{r}
 893 \\
 - 52 \\
 \hline
 841
 \end{array}
 \begin{array}{l}
 \leftarrow \text{minuend} \\
 \leftarrow \text{subtrahend} \\
 \leftarrow \text{difference}
 \end{array}$$

Line up the numbers vertically so that the minuend is on top and the place values correspond. Subtract in corresponding place values, starting with the ones place.

$$\begin{array}{r}
 841 \\
 \uparrow \quad \uparrow \quad \uparrow \\
 8 - 0 \\
 9 - 5 \\
 3 - 2
 \end{array}$$

To check, add.

$$\begin{array}{r}
 \text{difference} \\
 + \text{subtrahend} \\
 \hline
 \text{minuend}
 \end{array}
 \quad \text{or} \quad
 \begin{array}{r}
 841 \\
 + 52 \\
 \hline
 893
 \end{array}$$

← Since this is the original minuend, the problem checks.

Answers

5. a. 8 b. 12 c. 0 d. 42

Example 6 Subtract: $7826 - 505$. Check by adding.

Solution:

$$\begin{array}{r} 7826 \\ - 505 \\ \hline 7321 \end{array}$$

Check:

$$\begin{array}{r} 7321 \\ + 505 \\ \hline 7826 \end{array}$$

Work Practice 6

Subtracting by Borrowing

When subtracting vertically, if a digit in the second number (subtrahend) is larger than the corresponding digit in the first number (minuend), **borrowing** is necessary. For example, consider

$$\begin{array}{r} 81 \\ - 63 \\ \hline \end{array}$$

Since the 3 in the ones place of 63 is larger than the 1 in the ones place of 81, borrowing is necessary. We borrow 1 ten from the tens place and add it to the ones place.

Borrowing

$$\begin{array}{r} 8 \text{ tens} \\ - 1 \text{ ten} \\ \hline 7 \text{ tens} \end{array} \rightarrow \begin{array}{r} 7 \text{ tens} \\ 11 \text{ ones} \\ - 6 \text{ tens} \\ 3 \text{ ones} \\ \hline \end{array} \leftarrow 1 \text{ ten} + 1 \text{ one} = 11 \text{ ones}$$

Now we subtract the ones-place digits and then the tens-place digits.

$$\begin{array}{r} 7 \text{ tens} \\ 8 \text{ tens} \\ - 6 \text{ tens} \\ \hline 1 \text{ ten} \end{array} \begin{array}{r} 11 \text{ ones} \\ 11 \text{ ones} \\ - 3 \text{ ones} \\ \hline 8 \text{ ones} \end{array}$$

Check:

$$\begin{array}{r} 18 \\ + 63 \\ \hline 81 \end{array}$$

The original minuend.

Example 7 Subtract: $543 - 29$. Check by adding.

Solution:

$$\begin{array}{r} 543 \\ - 29 \\ \hline 514 \end{array}$$

Check:

$$\begin{array}{r} 514 \\ + 29 \\ \hline 543 \end{array}$$

Work Practice 7

Sometimes we may have to borrow from more than one place. For example, to subtract $7631 - 152$, we first borrow from the tens place.

$$\begin{array}{r} 7631 \\ - 152 \\ \hline 9 \end{array} \leftarrow 11 - 2 = 9$$

In the tens place, 5 is greater than 3, so we borrow again. This time we borrow from the hundreds place.

6 hundreds - 1 hundred = 5 hundreds

$$\begin{array}{r} 5 \text{ hundreds} \\ 7 \text{ tens} \\ 6 \text{ tens} \\ 3 \text{ tens} \\ 1 \text{ ones} \\ - 1 \text{ hundred} \\ 5 \text{ tens} \\ 2 \text{ ones} \\ \hline 7479 \end{array}$$

Check:

$$\begin{array}{r} 7479 \\ + 152 \\ \hline 7631 \end{array}$$

The original minuend.

Practice 6

Subtract. Check by adding.

a. $9143 - 122$

b. $978 - 851$

Practice 7

Subtract. Check by adding.

a. $\begin{array}{r} 697 \\ - 49 \\ \hline \end{array}$

b. $\begin{array}{r} 326 \\ - 245 \\ \hline \end{array}$

c. $\begin{array}{r} 1234 \\ - 822 \\ \hline \end{array}$

Answers

6. a. 9021 b. 127

7. a. 648 b. 81 c. 412

Practice 8

Subtract. Check by adding.

a.
$$\begin{array}{r} 400 \\ - 164 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 1000 \\ - 762 \\ \hline \end{array}$$

Example 8 Subtract: $900 - 174$. Check by adding.

Solution: In the ones place, 4 is larger than 0, so we borrow from the tens place. But the tens place of 900 is 0, so to borrow from the tens place, we must first borrow from the hundreds place.

$$\begin{array}{r} \\ \cancel{9} \cancel{0} 0 \\ - 174 \\ \hline \end{array}$$

Now borrow from the tens place.

$$\begin{array}{r} \\ \cancel{9} \cancel{0} \cancel{0} \\ - 174 \\ \hline 726 \end{array}$$

Check:
$$\begin{array}{r} 726 \\ + 174 \\ \hline 900 \end{array}$$

Work Practice 8**Objective C** Finding the Perimeter of a Polygon

In geometry, addition is used to find the perimeter of a polygon. A **polygon** can be described as a flat figure formed by line segments connected at their ends. (For more review, see Appendix A.1.) Geometric figures such as triangles, squares, and rectangles are called polygons.



Triangle



Square

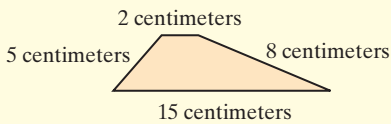
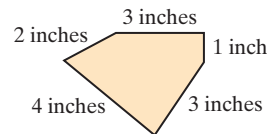


Rectangle

The **perimeter** of a polygon is the *distance around* the polygon, shown in red above. This means that the perimeter of a polygon is the sum of the lengths of its sides.

Practice 9

Find the perimeter of the polygon shown. (A centimeter is a unit of length in the metric system.)

**Example 9** Find the perimeter of the polygon shown.

Solution: To find the perimeter (distance around), we add the lengths of the sides.

$$2 \text{ in.} + 3 \text{ in.} + 1 \text{ in.} + 3 \text{ in.} + 4 \text{ in.} = 13 \text{ in.}$$

The perimeter is 13 inches.

Work Practice 9

To make the addition appear simpler, we will often not include units with the addends. If you do this, make sure units are included in the final answer.

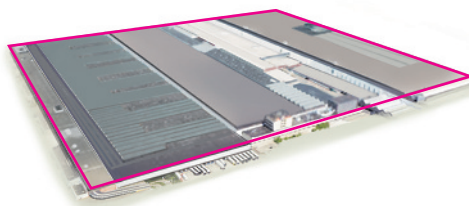
Answers

8. a. 236 b. 238

9. 30 cm

Example 10 Calculating the Perimeter of a Building

One way to compare large buildings is to calculate their footprint, or ground area. The second-largest building in the world by footprint is the Aalsmeer Flower Auction Building in Aalsmeer, Netherlands. The floor plan is a rectangle that measures 776 meters by 639 meters. Find the perimeter of this building. (A meter is a unit of length in the metric system.) (Source: *The Handy Science Answer Book*, Visible Ink Press and Internet research)



Solution: Recall that opposite sides of a rectangle have the same length. To find the perimeter of this building, we add the lengths of the sides. The sum of the lengths of its sides is

	$\begin{array}{r} 639 \\ 639 \\ 776 \\ + 776 \\ \hline 2830 \end{array}$
--	--

The perimeter of the building is 2830 meters.

(Note: upon completion, the largest footprint in the world will be the factory for Tesla vehicles, called the Gigafactory. It will have a footprint of about 100 football fields.)

Work Practice 10

Objective D Solving Problems by Adding or Subtracting

Often, real-life problems occur that can be solved by adding or subtracting. The first step in solving any word problem is to *understand* the problem by reading it carefully.

Descriptions in problems solved through addition or subtraction *may* include any of these key words or phrases:

Addition		
Key Words or Phrases	Examples	Symbols
added to	5 added to 7	$7 + 5$
plus	0 plus 78	$0 + 78$
increased by	12 increased by 6	$12 + 6$
more than	11 more than 25	$25 + 11$
total	the total of 8 and 1	$8 + 1$
sum	the sum of 4 and 133	$4 + 133$

Subtraction		
Key Words or Phrases	Examples	Symbols
subtract	subtract 5 from 8	$8 - 5$
difference	the difference of 10 and 2	$10 - 2$
less	17 less 3	$17 - 3$
less than	2 less than 20	$20 - 2$
take away	14 take away 9	$14 - 9$
decreased by	7 decreased by 5	$7 - 5$
subtracted from	9 subtracted from 12	$12 - 9$

✓ Concept Check In each of the following problems, identify which number is the minuend and which number is the subtrahend.

- What is the result when 6 is subtracted from 40?
- What is the difference of 15 and 8?
- Find a number that is 15 fewer than 23.

To solve a word problem that involves addition or subtraction, we first use the facts given to write an addition or subtraction statement. Next, we compute the addition or subtraction. Then we write the corresponding solution of the real-life problem. It is sometimes helpful to write the statement in words (brief phrases) and then translate to numbers.

Practice 10

A park is in the shape of a triangle. Each of the park's three sides is 647 feet. Find the perimeter of the park.

Helpful Hint

Be careful when solving applications that suggest subtraction. Although order *does not* matter when adding, order *does* matter when subtracting. For example, $20 - 15$ and $15 - 20$ do not simplify to the same number.

Answer

10. 1941 ft

✓ Concept Check Answers

- minuend: 40; subtrahend: 6
- minuend: 15; subtrahend: 8
- minuend: 23; subtrahend: 15

Practice 11

The radius of Uranus is 15,759 miles. The radius of Neptune is 458 miles less than the radius of Uranus. What is the radius of Neptune? (Source: National Space Science Data Center)

Helpful Hint

Since subtraction and addition are reverse operations, don't forget that a subtraction problem can be checked by adding.

Practice 12

Use the graph in Example 12 to answer the following:

- a. Which country shown has the fewest threatened amphibians?
- b. Find the total number of threatened amphibians for Madagascar, Peru, and Mexico.

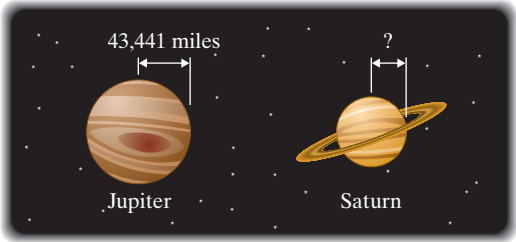


Answers

11. 15,301 miles
12. a. Madagascar b. 382

Example 11 Finding the Radius of a Planet

The radius of Jupiter is 43,441 miles. The radius of Saturn is 7257 miles less than the radius of Jupiter. Find the radius of Saturn. (Source: National Space Science Data Center)



Solution:	In Words	Translate to Numbers
	radius of Jupiter	43,441
	$- 7257$	$- 7257$
	radius of Saturn	36,184

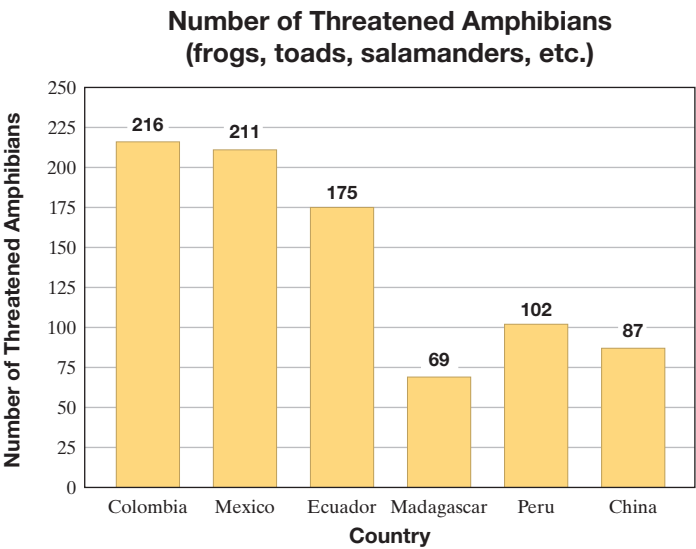
The radius of Saturn is 36,184 miles.

Work Practice 11

Graphs can be used to visualize data. The graph shown next is called a **bar graph**. For this bar graph, the height of each bar is labeled above the bar. To check this height, follow the top of each bar to the vertical line to the left. For example, the first bar is labeled 216. Follow the top of that bar to the left until the vertical line is reached, a bit more than halfway between 200 and 225, or 216.

Example 12 Reading a Bar Graph

The bar graph below contains the number of threatened amphibians for selected countries. Each bar represents a country and the height of each bar represents the number of endangered amphibians identified in that country.



Source: theguardian.com

- a. Which country shown has the greatest number of threatened amphibians?
- b. Find the total number of threatened amphibians for Ecuador, China, and Colombia.

Solution:

- a. The country with the greatest number of threatened amphibians corresponds to the tallest bar, which is Colombia.
- b. The key word here is “total.” To find the total number of threatened amphibians for Ecuador, China, and Colombia, we add.

In Words		Translate to Numbers
Ecuador	→	175
China	→	87
Colombia	→	+ 216
	Total	478

The total number of threatened amphibians for Ecuador, China, and Colombia is 478.

Work Practice 12**Calculator Explorations Adding and Subtracting Numbers****Adding Numbers**

To add numbers on a calculator, find the keys marked $+$ and $=$ or **ENTER**.

For example, to add 5 and 7 on a calculator, press the keys 5 $+$ 7 then $=$ or **ENTER**.

The display will read 12 .

Thus, $5 + 7 = 12$.

To add 687, 981, and 49 on a calculator, press the keys 687 $+$ 981 $+$ 49 then $=$ or **ENTER**.

The display will read 1717 .

Thus, $687 + 981 + 49 = 1717$. (Although entering 687, for example, requires pressing more than one key, here numbers are grouped together for easier reading.)

Use a calculator to add.

1. $89 + 45$

3. $285 + 55$

5. 985

1210

562

$+ 77$

2. $76 + 97$

4. $8773 + 652$

6. 465

9888

620

$+ 1550$

Subtracting Numbers

To subtract numbers on a calculator, find the keys marked $-$ and $=$ or **ENTER**.

For example, to find $83 - 49$ on a calculator, press the keys 83 $-$ 49 then $=$ or **ENTER**.

The display will read 34 . Thus, $83 - 49 = 34$.

Use a calculator to subtract.

7. $865 - 95$

8. $76 - 27$

9. $147 - 38$

10. $366 - 87$

11. $9625 - 647$

12. $10,711 - 8925$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once and some may not be used at all.

0	order	addend	associative
sum	number	grouping	commutative
perimeter	minuend	subtrahend	difference

- The sum of 0 and any number is the same _____.
- In $35 + 20 = 55$, the number 55 is called the _____ and 35 and 20 are each called a(n) _____.
- The difference of any number and that same number is _____.
- The difference of any number and 0 is the same _____.
- In $37 - 19 = 18$, the number 37 is the _____, the 19 is the _____, and the 18 is the _____.
- The distance around a polygon is called its _____.
- Since $7 + 10 = 10 + 7$, we say that changing the _____ in addition does not change the sum. This property is called the _____ property of addition.
- Since $(3 + 1) + 20 = 3 + (1 + 20)$, we say that changing the _____ in addition does not change the sum. This property is called the _____ property of addition.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 1.3

- Objective A** 9. Complete this statement based on the lecture before Example 1. To add whole numbers, we line up _____ values and add from _____ to _____.
- Objective B** 10. In Example 5, explain how we end up subtracting 7 from 12 in the ones place.
- Objective C** 11. In Example 7, the perimeter of what type of polygon is found? How many addends are in the resulting addition problem?
- Objective D** 12. Complete this statement based on Example 8. To find the sale price, subtract the _____ from the _____ price.

1.3 Exercise Set MyLab Math

Objective A Add. See Examples 1 through 4.

1. $14 + 22$

2. $27 + 31$

3.
$$\begin{array}{r} 62 \\ + 230 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 37 \\ + 542 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 12 \\ 13 \\ + 24 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 23 \\ 45 \\ + 30 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 5267 \\ + 132 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 236 \\ + 6243 \\ \hline \end{array}$$

9. $22,781 + 186,297$ 10. $17,427 + 821,059$

$$\begin{array}{r} 11. \quad 8 \\ 9 \\ 2 \\ 5 \\ + 1 \\ \hline \end{array}$$

$$\begin{array}{r} 12. \quad 3 \\ 5 \\ 8 \\ 5 \\ + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 13. \quad 81 \\ 17 \\ 23 \\ 79 \\ + 12 \\ \hline \end{array}$$

$$\begin{array}{r} 14. \quad 64 \\ 28 \\ 56 \\ 25 \\ + 32 \\ \hline \end{array}$$

$$15. \quad 24 + 9006 + 489 + 2407$$

$$16. \quad 16 + 1056 + 748 + 7770$$

$$\begin{array}{r} 17. \quad 6820 \\ 4271 \\ + 5626 \\ \hline \end{array}$$

$$\begin{array}{r} 18. \quad 6789 \\ 4321 \\ + 5555 \\ \hline \end{array}$$

$$\begin{array}{r} 19. \quad 49 \\ 628 \\ 5\,762 \\ + 29,462 \\ \hline \end{array}$$

$$\begin{array}{r} 20. \quad 26 \\ 582 \\ 4\,763 \\ + 62,511 \\ \hline \end{array}$$

$$\begin{array}{r} 21. \quad 121,742 \\ 57,279 \\ 26,586 \\ + 426,782 \\ \hline \end{array}$$

$$\begin{array}{r} 22. \quad 504,218 \\ 321,920 \\ 38,507 \\ + 594,687 \\ \hline \end{array}$$

Objective B Subtract. Check by adding. See Examples 5 through 8.

$$\begin{array}{r} 23. \quad 749 \\ - 149 \\ \hline \end{array}$$

$$\begin{array}{r} 24. \quad 957 \\ - 257 \\ \hline \end{array}$$

$$\begin{array}{r} 25. \quad 62 \\ - 37 \\ \hline \end{array}$$

$$\begin{array}{r} 26. \quad 55 \\ - 29 \\ \hline \end{array}$$

$$\begin{array}{r} 27. \quad 922 \\ - 634 \\ \hline \end{array}$$

$$\begin{array}{r} 28. \quad 674 \\ - 299 \\ \hline \end{array}$$

$$\begin{array}{r} 29. \quad 600 \\ - 432 \\ \hline \end{array}$$

$$\begin{array}{r} 30. \quad 300 \\ - 149 \\ \hline \end{array}$$

$$\begin{array}{r} 31. \quad 6283 \\ - 560 \\ \hline \end{array}$$

$$\begin{array}{r} 32. \quad 5349 \\ - 720 \\ \hline \end{array}$$

$$\begin{array}{r} 33. \quad 533 \\ - 29 \\ \hline \end{array}$$

$$\begin{array}{r} 34. \quad 724 \\ - 16 \\ \hline \end{array}$$

$$35. \quad 1983 - 1904$$

$$36. \quad 1983 - 1914$$

$$37. \quad 50,000 - 17,289$$

$$38. \quad 40,000 - 23,582$$

$$39. \quad 7020 - 1979$$

$$40. \quad 6050 - 1878$$

$$41. \quad 51,111 - 19,898$$

$$42. \quad 62,222 - 39,898$$

Objectives A B Mixed Practice Add or subtract as indicated. See Examples 1 through 8.

$$\begin{array}{r} 43. \quad 986 \\ + 48 \\ \hline \end{array}$$

$$\begin{array}{r} 44. \quad 986 \\ - 48 \\ \hline \end{array}$$

$$45. \quad 76 - 67$$

$$46. \quad 80 + 93 + 17 + 9 + 2$$

$$\begin{array}{r} 47. \quad 9000 \\ - 482 \\ \hline \end{array}$$

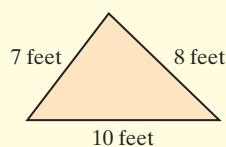
$$\begin{array}{r} 48. \quad 10,000 \\ - 1\,786 \\ \hline \end{array}$$

$$\begin{array}{r} 49. \quad 10,962 \\ 4\,851 \\ + 7\,063 \\ \hline \end{array}$$

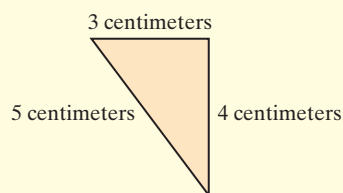
$$\begin{array}{r} 50. \quad 12,468 \\ 3\,211 \\ + 1\,988 \\ \hline \end{array}$$

Objective C Find the perimeter of each figure. See Examples 9 and 10.

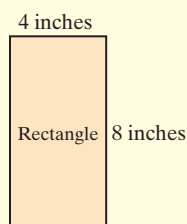
▶ 51.



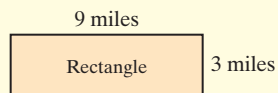
△ 52.



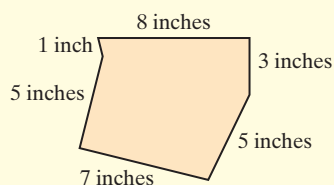
△ 53.



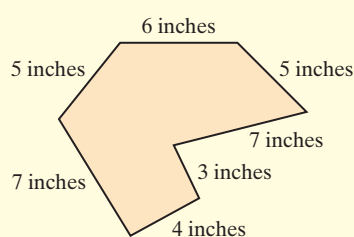
△ 54.



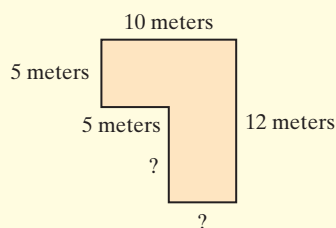
△ 55.



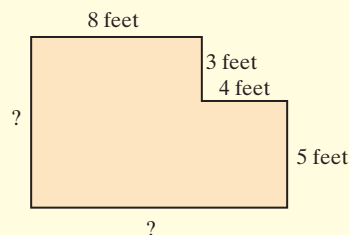
△ 56.



△ 57.



△ 58.



Objectives A B C D Mixed Practice—Translating Solve. See Examples 9 through 12.

59. Find the sum of 297 and 1796.

60. Find the sum of 802 and 6487.

61. Find the total of 76, 39, 8, 17, and 126.

62. Find the total of 89, 45, 2, 19, and 341.

63. Find the difference of 41 and 21.

64. Find the difference of 16 and 5.

65. What is 452 increased by 92?

66. What is 712 increased by 38?

67. Find 108 less 36.

68. Find 25 less 12.

69. Find 12 subtracted from 100.

70. Find 86 subtracted from 90.

Solve.

71. The population of Florida is projected to grow from 19,308 thousand in 2010 to 22,478 thousand in 2020. What is Florida's projected population increase over this time period?

72. The population of California is projected to grow from 39,136 thousand in 2010 to 44,126 thousand in 2020. What is California's projected population increase over this time period?

73. A new DVD player with remote control costs \$295. A college student has \$914 in her savings account. How much will she have left in her savings account after she buys the DVD player?

74. A stereo that regularly sells for \$547 is discounted by \$99 in a sale. What is the sale price?

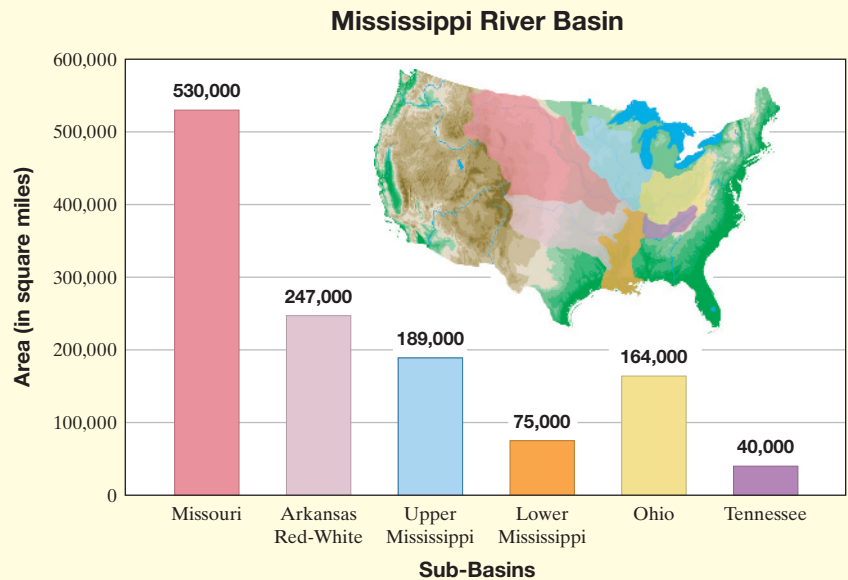
A river basin is the geographic area drained by a river and its tributaries. The Mississippi River Basin is the third largest in the world and is divided into six sub-basins, whose areas are shown in the following bar graph. Use this graph for Exercises 75 through 78.

75. Find the total U.S. land area drained by the Upper Mississippi and Lower Mississippi sub-basins.

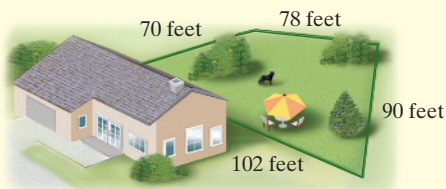
76. Find the total U.S. land area drained by the Ohio and Tennessee sub-basins.

77. How many more square miles of land are drained by the Missouri sub-basin than the Arkansas Red-White sub-basin?

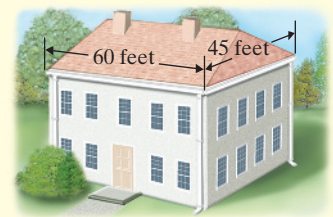
78. How many more square miles of land are drained by the Upper Mississippi sub-basin than the Lower Mississippi sub-basin?



79. A homeowner is installing a fence in his backyard. How many feet of fencing are needed to enclose the yard below?



80. A homeowner is considering installing gutters around her home. Find the perimeter of her rectangular home.



81. Professor Graham is reading a 503-page book. If she has just finished reading page 239, how many more pages must she read to finish the book?

82. When a couple began a trip, the odometer read 55,492. When the trip was over, the odometer read 59,320. How many miles did they drive on their trip?

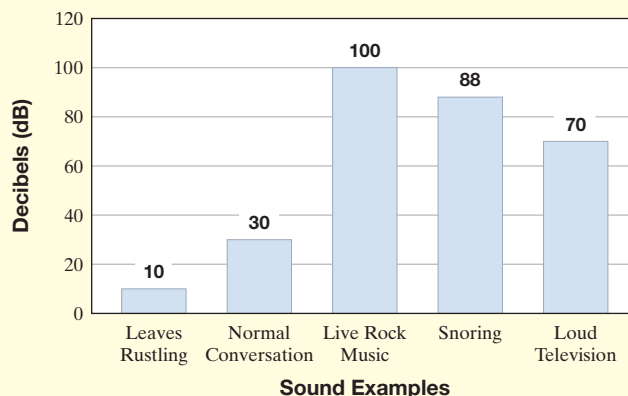
83. In 2016, the country of New Zealand had 22,867,835 more sheep than people. If the human population of New Zealand in 2016 was 4,573,567, what was the sheep population? (Source: www.stats.govt.nz)

84. During one month in 2016, the two top-selling vehicles in the United States were the Ford F-Series and the Chevrolet Silverado, both trucks. There were 65,542 F-Series trucks and 49,768 Silverados sold that month. What was the total number of these trucks sold in that month? (Source: www.goodcarbadcar.com)

The decibel (dB) is a unit of measurement for sound. Every increase of 10 dB is a tenfold increase in sound intensity. The bar graph shows the decibel levels for some common sounds. Use this graph for Exercises 85 through 88.

85. What is the dB level for live rock music?
86. Which is the quietest of all the sounds shown in the graph?
87. How much louder is the sound of snoring than normal conversation?
88. What is the difference in sound intensity between live rock music and loud television?

Decibel Levels for Common Sounds



89. In 2013, there were 2410 Gap Inc. (Gap, Banana Republic, Old Navy) stores located in the United States and 1034 located outside the United States. How many Gap Inc. stores were located worldwide? (Source: Gap, Inc.)
90. Automobile classes are defined by the amount of interior room. A subcompact car is defined as a car with a maximum interior space of 99 cubic feet. A midsize car is defined as a car with a maximum interior space of 119 cubic feet. What is the difference in volume between a midsize and a subcompact car?
91. The largest permanent Monopoly board is made of granite and is located in San Jose, California. It is in the shape of a square with side lengths of 31 ft. Find the perimeter of the square playing board.
92. The smallest commercially available jigsaw puzzle is a 1000-piece puzzle manufactured in Spain. It is in the shape of a rectangle with length of 18 inches and width of 12 inches. Find the perimeter of this rectangular-shaped puzzle.

The table shows the number of CVS pharmacies in ten states. Use this table to answer Exercises 93 through 98.

The Top Ten States for CVS Pharmacies in 2016	
State	Number of CVS Pharmacies
Massachusetts	356
California	867
Florida	756
Georgia	313
Indiana	301
New York	486
North Carolina	313
Ohio	309
Pennsylvania	408
Texas	659

(Source: CVS Pharmacy Inc.)

93. Which state has the most CVS pharmacies?
94. Which of the states listed in the table has the fewest CVS pharmacies?
95. What is the total number of CVS pharmacies located in the three states with the most CVS pharmacies?
96. How many CVS pharmacies are located in the 10 states listed in the table?
97. Which pair of neighboring states have more CVS pharmacies combined, Pennsylvania and New York or Florida and Georgia?
98. There are 3048 CVS pharmacies located in the states not listed in the table. How many CVS pharmacies are there in the 50 states?

- 99.** The state of Delaware has 2029 miles of urban highways and 3865 miles of rural highways. Find the total highway mileage in Delaware. (*Source:* U.S. Federal Highway Administration)

- 100.** The state of Rhode Island has 5193 miles of urban highways and 1222 miles of rural highways. Find the total highway mileage in Rhode Island. (*Source:* U.S. Federal Highway Administration)

Concept Extensions

For Exercises 101–104, identify which number is the minuend and which number is the subtrahend. See the second Concept Check in this section.

101.
$$\begin{array}{r} 48 \\ - 1 \\ \hline \end{array}$$

102.
$$\begin{array}{r} 2863 \\ - 1904 \\ \hline \end{array}$$

- 103.** Subtract 7 from 70.

- 104.** Find 86 decreased by 25.

-  **105.** In your own words, explain the commutative property of addition.

-  **106.** In your own words, explain the associative property of addition.

Check each addition below. If it is incorrect, find the correct answer. See the first Concept Check in this section.

107.
$$\begin{array}{r} 566 \\ 932 \\ + 871 \\ \hline 2369 \end{array}$$

108.
$$\begin{array}{r} 773 \\ 659 \\ + 481 \\ \hline 1913 \end{array}$$

109.
$$\begin{array}{r} 14 \\ 173 \\ 86 \\ + 257 \\ \hline 520 \end{array}$$

110.
$$\begin{array}{r} 19 \\ 214 \\ 49 \\ + 651 \\ \hline 923 \end{array}$$

Identify each answer as correct or incorrect. Use addition to check. If the answer is incorrect, write the correct answer.

111.
$$\begin{array}{r} 741 \\ - 56 \\ \hline 675 \end{array}$$

112.
$$\begin{array}{r} 478 \\ - 89 \\ \hline 389 \end{array}$$

113.
$$\begin{array}{r} 1029 \\ - 888 \\ \hline 141 \end{array}$$

114.
$$\begin{array}{r} 7615 \\ - 547 \\ \hline 7168 \end{array}$$

Fill in the missing digits in each problem.

115.
$$\begin{array}{r} 526\_ \\ - 285 \\ \hline 28\_4 \end{array}$$

116.
$$\begin{array}{r} 10,\_4\_ \\ - 85\_4 \\ \hline \_710 \end{array}$$

-  **117.** Is there a commutative property of subtraction? In other words, does order matter when subtracting? Why or why not?

-  **118.** Explain why the phrase “Subtract 7 from 10” translates to “ $10 - 7$.”

-  **119.** The local college library is having a Million Pages of Reading promotion. The freshmen have read a total of 289,462 pages; the sophomores have read a total of 369,477 pages; the juniors have read a total of 218,287 pages; and the seniors have read a total of 121,685 pages. Have they reached the goal of one million pages? If not, how many more pages need to be read?

1.4 Rounding and Estimating

Objectives

- A** Round Whole Numbers.
- B** Use Rounding to Estimate Sums and Differences.
- C** Solve Problems by Estimating.



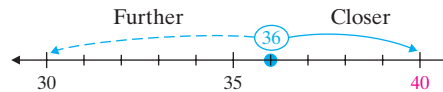
Objective A Rounding Whole Numbers

Rounding a whole number means approximating it. A rounded whole number is often easier to use, understand, and remember than the precise whole number. For example, instead of trying to remember the Colorado state population as 5,188,000, it is much easier to remember it rounded to the nearest million: 5,000,000, or 5 million people. (*Source*: U.S. census)

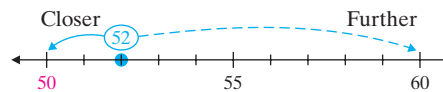
Recall from Section 1.2 that the line below is called a number line. To **graph** a whole number on this number line, we darken the point representing the location of the whole number. For example, the number 4 is graphed below.



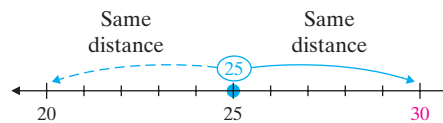
On the number line, the whole number 36 is closer to 40 than 30, so 36 rounded to the nearest ten is **40**.



The whole number 52 is closer to 50 than 60, so 52 rounded to the nearest ten is **50**.



In trying to round 25 to the nearest ten, we see that 25 is halfway between 20 and 30. It is not closer to either number. In such a case, we round to the larger ten, that is, to **30**.



Here, we round "up."

To round a whole number without using a number line, follow these steps:

Rounding a Whole Number to a Given Place Value

- Step 1:** Locate the digit to the right of the given place value.
- Step 2:** If this digit is 5 or greater, add 1 to the digit in the given place value and replace each digit to its right by 0.
- Step 3:** If this digit is less than 5, replace it and each digit to its right by 0.

Example 1 Round 568 to the nearest ten.

Solution: 5 6 **(8)**
 ↑
 tens place
 The digit to the right of the tens place is the ones place, which is circled.

5 6 **(8)**
 ↑ ↙
 Add 1. Replace with 0.
 Since the circled digit is 5 or greater, add 1 to the 6 in the tens place and replace the digit to the right by 0.

We find that 568 rounded to the nearest ten is 570.

Work Practice 1**Example 2** Round 278,362 to the nearest thousand.

Solution: Thousands place
 ↓ ↙
 278, **(3)** 62
 Do not add 1. Replace with zeros.
 3 is less than 5.

The number 278,362 rounded to the nearest thousand is 278,000.

Work Practice 2**Example 3** Round 248,982 to the nearest hundred.

Solution: Hundreds place
 ↓ ↙
 248,9 **(8)** 2
 Add 1. $9 + 1 = 10$, so replace the digit 9 by 0 and carry 1 to the place value to the left.
 2 4 ⁸⁺¹ ⁰ 8, ~~9~~ ⁰ 8 2
 Add 1. Replace with zeros.
 8 is greater than or equal to 5.

The number 248,982 rounded to the nearest hundred is 249,000.

Work Practice 3

✓ **Concept Check** Round each of the following numbers to the nearest *hundred*. Explain your reasoning.

- a. 59 b. 29

Objective B Estimating Sums and Differences

By rounding addends, minuends, and subtrahends, we can **estimate** sums and differences. An estimated sum or difference is appropriate when the exact number is not necessary. Also, an estimated sum or difference can help us determine if we made

Practice 1

Round to the nearest ten.

- a. 57
 b. 641
 c. 325

Practice 2

Round to the nearest thousand.

- a. 72,304
 b. 9222
 c. 671,800

Practice 3

Round to the nearest hundred.

- a. 3474
 b. 76,243
 c. 978,965

Answers

1. a. 60 b. 640 c. 330
 2. a. 72,000 b. 9000 c. 672,000
 3. a. 3500 b. 76,200 c. 979,000

✓ **Concept Check Answers**

- a. 100 b. 0

a mistake in calculating an exact amount. To estimate the sum below, round each number to the nearest hundred and then add.

$$\begin{array}{r}
 768 \text{ rounds to } 800 \\
 1952 \text{ rounds to } 2000 \\
 225 \text{ rounds to } 200 \\
 + 149 \text{ rounds to } + 100 \\
 \hline
 3100
 \end{array}$$

The estimated sum is 3100, which is close to the **exact** sum of 3094.

Practice 4

Round each number to the nearest ten to find an estimated sum.

$$\begin{array}{r}
 49 \\
 25 \\
 32 \\
 51 \\
 + 98 \\
 \hline
 \end{array}$$

Practice 5

Round each number to the nearest thousand to find an estimated difference.

$$\begin{array}{r}
 3785 \\
 - 2479 \\
 \hline
 \end{array}$$

Example 4

Round each number to the nearest hundred to find an estimated sum.

$$\begin{array}{r}
 294 \\
 625 \\
 1071 \\
 + 349 \\
 \hline
 \end{array}$$

Solution:

Exact:	Estimate:
294 rounds to	300
625 rounds to	600
1071 rounds to	1100
+ 349 rounds to	+ 300
	<u>2300</u>

The estimated sum is 2300. (The exact sum is 2339.)

Work Practice 4

Example 5

Round each number to the nearest hundred to find an estimated difference.

$$\begin{array}{r}
 4725 \\
 - 2879 \\
 \hline
 \end{array}$$

Solution:

Exact:	Estimate:
4725 rounds to	4700
- 2879 rounds to	- 2900
	<u>1800</u>

The estimated difference is 1800. (The exact difference is 1846.)

Work Practice 5

Objective C Solving Problems by Estimating

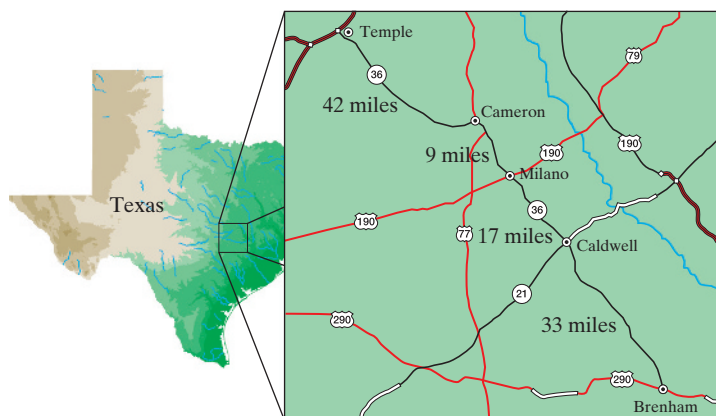
Making estimates is often the quickest way to solve real-life problems when solutions do not need to be exact.

Answers

4. 260
5. 2000

Example 6 Estimating Distances

A driver is trying to quickly estimate the distance from Temple, Texas, to Brenham, Texas. Round each distance given on the map to the nearest ten to estimate the total distance.



Solution:

Exact Distance:		Estimate:
42	rounds to	40
9	rounds to	10
17	rounds to	20
<u>+33</u>	rounds to	<u>+30</u>
		100

It is approximately 100 miles from Temple to Brenham. (The exact distance is 101 miles.)

Work Practice 6

Example 7 Estimating Data

In three recent years the numbers of tons of air cargo and mail that went through Hartsfield-Jackson Atlanta International Airport were 629,700, 685,550, and 737,655. Round each number to the nearest thousand to estimate the tons of mail that passed through this airport.

Solution:

Exact Tons of Cargo Mail:		Estimate:
629,700	rounds to	630,000
685,550	rounds to	686,000
<u>+737,655</u>	rounds to	<u>+738,000</u>
		2,054,000



The approximate tonnage of mail that moved through Atlanta's airport over this 3-year period was 2,054,000 tons. (The exact tonnage was 2,052,905 tons.)

Work Practice 7

Practice 6

Tasha Kilbey is trying to estimate how far it is from Gove, Kansas, to Hays, Kansas. Round each given distance on the map to the nearest ten to estimate the total distance.

**Practice 7**

In 2015, there were 2930 reported cases of mumps, 18,166 reported cases of pertussis (whooping cough), and 189 reported cases of measles. Round each number to the nearest thousand to estimate the total number of cases reported for these preventable diseases. (*Source:* Centers for Disease Control)

Answers

6. 80 mi
7. 21,000 total cases

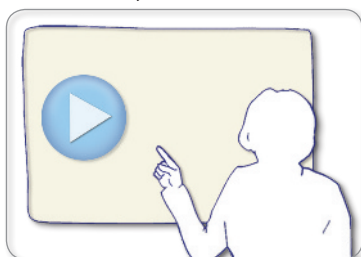
Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

60	rounding	exact
70	estimate	graph

- To _____ a number on a number line, darken the point representing the location of the number.
- Another word for approximating a whole number is _____.
- The number 65 rounded to the nearest ten is _____, but the number 61 rounded to the nearest ten is _____.
- A(n) _____ number of products is 1265, but a(n) _____ is 1000.

Martin-Gay Interactive Videos Watch the section lecture video and answer the following questions.



See Video 1.4 

Objective A 5. In  Example 1, when rounding the number to the nearest ten, why do we replace the digit 3 with a 4? 

Objective B 6. As discussed in  Example 3, explain how a number line can help us understand how to round 22 to the nearest ten. 

Objective C 7. What is the significance of the circled digit in each height value in  Example 5? 

1.4 Exercise Set MyLab Math

Objective A Round each whole number to the given place value. See Examples 1 through 3.

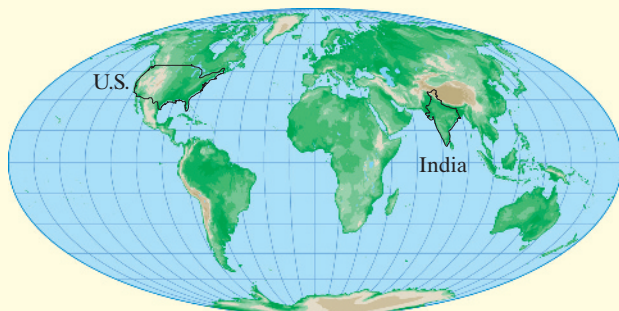
- | | | |
|---|---------------------------------------|---|
| 1. 423 to the nearest ten | 2. 273 to the nearest ten |  3. 635 to the nearest ten |
| 4. 846 to the nearest ten | 5. 2791 to the nearest hundred | 6. 8494 to the nearest hundred |
| 7. 495 to the nearest ten | 8. 898 to the nearest ten | 9. 21,094 to the nearest thousand |
| 10. 82,198 to the nearest thousand | 11. 33,762 to the nearest thousand | 12. 42,682 to the nearest ten-thousand |
| 13. 328,495 to the nearest hundred | 14. 179,406 to the nearest hundred |  15. 36,499 to the nearest thousand |
| 16. 96,501 to the nearest thousand | 17. 39,994 to the nearest ten | 18. 99,995 to the nearest ten |
| 19. 29,834,235 to the nearest ten-million | 20. 39,523,698 to the nearest million | |

Complete the table by estimating the given number to the given place value.

		Tens	Hundreds	Thousands
21.	5281			
22.	7619			
23.	9444			
24.	7777			
25.	14,876			
26.	85,049			

Round each number to the indicated place value.

27. The state of Texas contains 310,850 miles of urban and rural highways. Round this number to the nearest thousand. (Source: U.S. Federal Highway Administration)
28. The state of California contains 171,874 miles of urban and rural highways. Round this number to the nearest thousand. (Source: U.S. Federal Highway Administration)
29. It takes 60,149 days for Neptune to make a complete orbit around the Sun. Round this number to the nearest hundred. (Source: National Space Science Data Center)
30. Kareem Abdul-Jabbar holds the NBA record for points scored, a total of 38,387 over his NBA career. Round this number to the nearest thousand. (Source: National Basketball Association)
31. In 2016, the most valuable brand in the world was Apple, Inc. The estimated brand value of Apple at this time was \$154,118,000,000. Round this to the nearest ten-billion. (Source: Forbes)
32. According to the U.S. Population Clock, the population of the United States was 322,962,019 in November 2016. Round this population figure to the nearest million. (Source: U.S. Census Population Clock)
33. The average salary for a professional baseball player in 2016 was \$4,155,907. Round this average salary to the nearest hundred-thousand. (Source: ESPN)
34. The average salary for a professional football player in 2016 was \$2,110,000. Round this average salary to the nearest million. (Source: ESPN)
35. The United States currently has 219,600,000 smart-phone users, while India has 292,300,000 users. Round each user number to the nearest million. (Source: Pew Internet Research)
36. U.S. farms produced 15,226,000,000 bushels of corn in 2016. Round the corn production figure to the nearest ten-million. (Source: U.S. Department of Agriculture)



Objective B Estimate the sum or difference by rounding each number to the nearest ten. See Examples 4 and 5.

$$\begin{array}{r} 37. \quad 39 \\ 45 \\ 22 \\ + 17 \\ \hline \end{array}$$

$$\begin{array}{r} 38. \quad 52 \\ 33 \\ 15 \\ + 29 \\ \hline \end{array}$$

$$\begin{array}{r} 39. \quad 449 \\ - 373 \\ \hline \end{array}$$

$$\begin{array}{r} 40. \quad 555 \\ - 235 \\ \hline \end{array}$$

Estimate the sum or difference by rounding each number to the nearest hundred. See Examples 4 and 5.

$$\begin{array}{r} 41. \quad 1913 \\ 1886 \\ + 1925 \\ \hline \end{array}$$

$$\begin{array}{r} 42. \quad 4050 \\ 3133 \\ + 1220 \\ \hline \end{array}$$

$$\begin{array}{r} 43. \quad 1774 \\ - 1492 \\ \hline \end{array}$$

$$\begin{array}{r} 44. \quad 1989 \\ - 1870 \\ \hline \end{array}$$

$$\begin{array}{r} 45. \quad 3995 \\ 2549 \\ + 4944 \\ \hline \end{array}$$

$$\begin{array}{r} 46. \quad 799 \\ 1655 \\ + 271 \\ \hline \end{array}$$

Three of the given calculator answers below are incorrect. Find them by estimating each sum.

$$47. \quad 463 + 219 \quad 600$$

$$48. \quad 522 + 785 \quad 1307$$

$$49. \quad 229 + 443 + 606 \quad 1278$$

$$50. \quad 542 + 789 + 198 \quad 2139$$

$$51. \quad 7806 + 5150 \quad 12,956$$

$$52. \quad 5233 + 4988 \quad 9011$$

Helpful Hint

Estimation is useful to check for incorrect answers when using a calculator. For example, pressing a key too hard may result in a double digit, while pressing a key too softly may result in the digit not appearing in the display.

Objective C Solve each problem by estimating. See Examples 6 and 7.

53. An appliance store advertises three refrigerators on sale at \$899, \$1499, and \$999. Round each cost to the nearest hundred to estimate the total cost.

54. Suppose you scored 89, 97, 100, 79, 75, and 82 on your biology tests. Round each score to the nearest ten to estimate your total score.

55. The distance from Kansas City to Boston is 1429 miles and from Kansas City to Chicago is 530 miles. Round each distance to the nearest hundred to estimate how much farther Boston is from Kansas City than Chicago is.

56. The Gonzales family took a trip and traveled 588, 689, 277, 143, 59, and 802 miles on six consecutive days. Round each distance to the nearest hundred to estimate the distance they traveled.

57. The peak of Denali, in Alaska, is 20,320 feet above sea level. The top of Mt. Rainier, in Washington, is 14,410 feet above sea level. Round each height to the nearest thousand to estimate the difference in elevation of these two peaks. (Source: U.S. Geological Survey) (Note: Denali was formerly known as Mt. McKinley.)

58. A student is pricing new car stereo systems. One system sells for \$1895 and another system sells for \$1524. Round each price to the nearest hundred dollars to estimate the difference in price of these systems.

59. In 2016, the population of Springfield, Illinois, was 117,006, and the population of Champaign, Illinois, was 83,424. Round each population to the nearest ten-thousand to estimate how much larger Springfield was than Champaign. (*Source: U.S. Census Bureau*)



60. Round each distance given on the map to the nearest ten to estimate the total distance from North Platte, Nebraska, to Lincoln, Nebraska.



61. Head Start is a national program that provides developmental and social services for America's low-income preschool children ages three to five. Enrollment figures in Head Start programs showed an decrease from 1,128,030 in 2012 to 946,357 in 2016. Round each number of children to the nearest thousand to estimate this decrease. (*Source: Office of Head Start*)
62. Enrollment figures at a local community college showed an increase from 49,713 credit hours in 2005 to 51,746 credit hours in 2006. Round each number to the nearest thousand to estimate the increase.

Mixed Practice (Sections 1.2 and 1.4) The following table shows a few of the airports in the United States with the largest volumes of passengers. Complete this table. The first line is completed for you. (*Source: Airports Council International*)

	City Location of Airport	Total passengers in 2015 (in hundred-thousands of passengers)	Amount Written in Standard Form	Standard Form Rounded to the Nearest Million	Standard Form Rounded to the Nearest Ten-Million
	Atlanta, GA	1015	101,500,000	102,000,000	100,000,000
63.	Los Angeles, CA	749			
64.	Chicago, IL	769			
65.	Dallas/Fort Worth, TX	641			
66.	New York JFK	568			

Concept Extensions

67. Find one number that when rounded to the nearest hundred is 5700.
68. Find one number that when rounded to the nearest ten is 5700.

69. A number rounded to the nearest hundred is 8600.

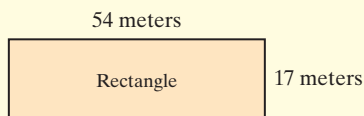
- Determine the smallest possible number.
- Determine the largest possible number.

70. On August 23, 1989, it was estimated that 1,500,000 people joined hands in a human chain stretching 370 miles to protest the fiftieth anniversary of the pact that allowed what was then the Soviet Union to annex the Baltic nations in 1939. If the estimate of the number of people is to the nearest hundred-thousand, determine the largest possible number of people in the chain.

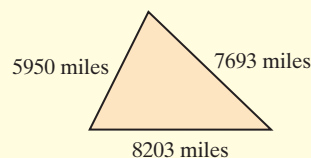
71. In your own words, explain how to round a number to the nearest thousand.

72. In your own words, explain how to round 9660 to the nearest thousand.

73. Estimate the perimeter of the rectangle by first rounding the length of each side to the nearest ten.



74. Estimate the perimeter of the triangle by first rounding the length of each side to the nearest hundred.

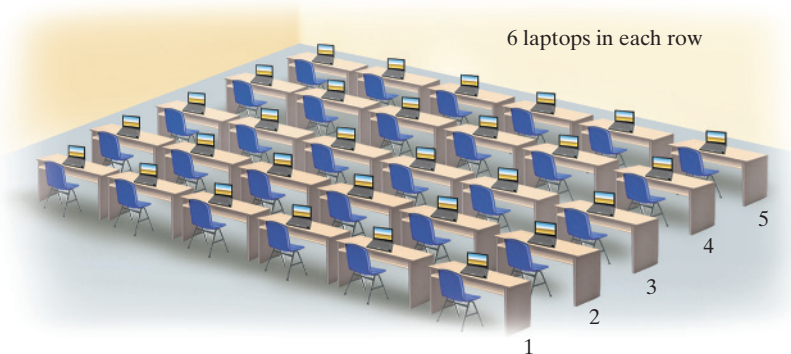


Objectives

- Use the Properties of Multiplication. ▶
- Multiply Whole Numbers. ▶
- Find the Area of a Rectangle. ▶
- Solve Problems by Multiplying Whole Numbers. ▶

1.5 Multiplying Whole Numbers and Area ▶

Multiplication Shown as Repeated Addition Suppose that we wish to count the number of laptops provided in a computer class. The laptops are arranged in 5 rows, and each row has 6 laptops.



Adding 5 sixes gives the total number of laptops. We can write this as $6 + 6 + 6 + 6 + 6 = 30$ laptops. When each addend is the same, we refer to this as **repeated addition**.

Multiplication is repeated addition but with different notation.

$$6 + 6 + 6 + 6 + 6 = 5 \times 6 = 30$$

5 addends; each addend is 6 (number of addends) factor (each addend) factor product

The $\times$ is called a **multiplication sign**. The numbers 5 and 6 are called **factors**. The number 30 is called the **product**. The notation 5×6 is read as “five times six.” The symbols $\cdot$ and $()$ can also be used to indicate multiplication.

$$5 \times 6 = 30, \quad 5 \cdot 6 = 30, \quad (5)(6) = 30, \quad \text{and} \quad 5(6) = 30$$

✓ **Concept Check**

- a.** Rewrite $5 + 5 + 5 + 5 + 5 + 5 + 5$ using multiplication.
- b.** Rewrite 3×16 as repeated addition. Is there more than one way to do this? If so, show all ways.

Objective A Using the Properties of Multiplication

As with addition, we memorize products of one-digit whole numbers and then use certain properties of multiplication to multiply larger numbers. (If necessary, review the multiplication of one-digit numbers.)

Notice that when any number is multiplied by 0, the result is always 0. This is called the **multiplication property of 0**.

Multiplication Property of 0

The product of 0 and any number is 0. For example,

$$5 \cdot 0 = 0 \quad \text{and} \quad 0 \cdot 8 = 0$$

Also notice that when any number is multiplied by 1, the result is always the original number. We call this result the **multiplication property of 1**.

Multiplication Property of 1

The product of 1 and any number is that same number. For example,

$$1 \cdot 9 = 9 \quad \text{and} \quad 6 \cdot 1 = 6$$

Example 1 Multiply.

- a.** 4×1 **b.** $0(3)$ **c.** $1 \cdot 64$ **d.** $(48)(0)$

Solution:

- a.** $4 \times 1 = 4$ **b.** $0(3) = 0$
c. $1 \cdot 64 = 64$ **d.** $(48)(0) = 0$

Work Practice 1

Practice 1

Multiply.

- a. 6×0
- b. $(1)8$
- c. $(50)(0)$
- d. $75 \cdot 1$

✓ Concept Check Answers

- a. 7×5
- b. $16 + 16 + 16$; yes,
 $3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 +$
 $3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$

Answers

- 1. a. 0 b. 8 c. 0 d. 75**

Like addition, multiplication is commutative and associative. Notice that when multiplying two numbers, the order of these numbers can be changed without changing the product. For example,

$$3 \cdot 5 = 15 \quad \text{and} \quad 5 \cdot 3 = 15$$

This property is called the **commutative property of multiplication**.

Commutative Property of Multiplication

Changing the **order** of two factors does not change their product. For example,

$$9 \cdot 2 = 18 \quad \text{and} \quad 2 \cdot 9 = 18$$

Another property that can help us when multiplying is the **associative property of multiplication**. This property states that when multiplying numbers, the grouping of the numbers can be changed without changing the product. For example,

$$(2 \cdot 3) \cdot 4 = 6 \cdot 4 = 24$$

Also,

$$2 \cdot (3 \cdot 4) = 2 \cdot 12 = 24$$

Both groupings give a product of 24.

Associative Property of Multiplication

Changing the **grouping** of factors does not change their product. From the previous work, we know that, for example,

$$(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$$

With these properties, along with the **distributive property**, we can find the product of any whole numbers. The distributive property says that multiplication **distributes** over addition. For example, notice that $3(2 + 5)$ simplifies to the same number as $3 \cdot 2 + 3 \cdot 5$.

$$3(2 + 5) = 3(7) = 21$$

$$3 \cdot 2 + 3 \cdot 5 = 6 + 15 = 21$$

Since $3(2 + 5)$ and $3 \cdot 2 + 3 \cdot 5$ both simplify to 21, then

$$3(2 + 5) = 3 \cdot 2 + 3 \cdot 5$$

Notice in $3(2 + 5) = 3 \cdot 2 + 3 \cdot 5$ that each number inside the parentheses is multiplied by 3.

Distributive Property

Multiplication distributes over addition. For example,

$$2(3 + 4) = 2 \cdot 3 + 2 \cdot 4$$

Example 2 Rewrite each using the distributive property.

- a. $5(6 + 5)$ b. $20(4 + 7)$ c. $2(7 + 9)$

Solution: Using the distributive property, we have

- a. $5(6 + 5) = 5 \cdot 6 + 5 \cdot 5$
 b. $20(4 + 7) = 20 \cdot 4 + 20 \cdot 7$
 c. $2(7 + 9) = 2 \cdot 7 + 2 \cdot 9$

Work Practice 2

Objective B Multiplying Whole Numbers

Let's use the distributive property to multiply $7(48)$. To do so, we begin by writing the expanded form of 48 (see Section 1.2) and then applying the distributive property.

$$\begin{aligned} 7(48) &= 7(40 + 8) && \text{Write 48 in expanded form.} \\ &= 7 \cdot 40 + 7 \cdot 8 && \text{Apply the distributive property.} \\ &= 280 + 56 && \text{Multiply.} \\ &= 336 && \text{Add.} \end{aligned}$$

This is how we multiply whole numbers. When multiplying whole numbers, we will use the following notation.

First:

$$\begin{array}{r} \overset{5}{4}8 \\ \times 7 \\ \hline 336 \end{array} \quad \leftarrow 7 \cdot 8 = 56$$

Write 6 in the ones place and carry 5 to the tens place.

Next:

$$\begin{array}{r} \overset{5}{4}8 \\ \times 7 \\ \hline 336 \end{array} \quad 7 \cdot 4 + 5 = 28 + 5 = 33$$

The product of 48 and 7 is 336.

Example 3 Multiply:

- a. $\begin{array}{r} 25 \\ \times 8 \\ \hline \end{array}$ b. $\begin{array}{r} 246 \\ \times 5 \\ \hline \end{array}$

Solution:

- a. $\begin{array}{r} \overset{4}{2}5 \\ \times 8 \\ \hline 200 \end{array}$ b. $\begin{array}{r} \overset{23}{2}46 \\ \times 5 \\ \hline 1230 \end{array}$

Work Practice 3

Practice 2

Rewrite each using the distributive property.

- a. $6(4 + 5)$
 b. $30(2 + 3)$
 c. $7(2 + 8)$

Practice 3

Multiply.

- a. $\begin{array}{r} 29 \\ \times 6 \\ \hline \end{array}$ b. $\begin{array}{r} 648 \\ \times 5 \\ \hline \end{array}$

Answers

2. a. $6(4 + 5) = 6 \cdot 4 + 6 \cdot 5$
 c. $30(2 + 3) = 30 \cdot 2 + 30 \cdot 3$
 d. $7(2 + 8) = 7 \cdot 2 + 7 \cdot 8$
 3. a. 174 b. 3240

To multiply larger whole numbers, use the following similar notation. Multiply 89×52 .

Step 1

$$\begin{array}{r} 1 \\ 89 \\ \times 52 \\ \hline 178 \end{array} \leftarrow \text{Multiply } 89 \times 2.$$

Step 2

$$\begin{array}{r} 4 \\ 89 \\ \times 52 \\ \hline 178 \\ \times 4450 \end{array} \leftarrow \text{Multiply } 89 \times 50.$$

Step 3

$$\begin{array}{r} 89 \\ \times 52 \\ \hline 178 \\ 4450 \\ \hline 4628 \end{array} \text{ Add.}$$

The numbers 178 and 4450 are called **partial products**. The sum of the partial products, 4628, is the product of 89 and 52.

Practice 4

Multiply.

$$\begin{array}{r} 306 \\ \times 81 \\ \hline \end{array}$$

Practice 5

Multiply.

$$\begin{array}{r} 726 \\ \times 142 \\ \hline \end{array}$$

Example 4 Multiply: 236×86

Solution:

$$\begin{array}{r} 236 \\ \times 86 \\ \hline 1416 \leftarrow 6(236) \\ 18880 \leftarrow 80(236) \\ \hline 20,296 \text{ Add.} \end{array}$$

Work Practice 4

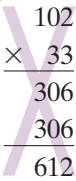
Example 5 Multiply: 631×125

Solution:

$$\begin{array}{r} 631 \\ \times 125 \\ \hline 3155 \leftarrow 5(631) \\ 12620 \leftarrow 20(631) \\ 63100 \leftarrow 100(631) \\ \hline 78,875 \text{ Add.} \end{array}$$

Work Practice 5

 **Concept Check** Find and explain the error in the following multiplication problem.


$$\begin{array}{r} 102 \\ \times 33 \\ \hline 306 \\ 306 \\ \hline 612 \end{array}$$

Answers

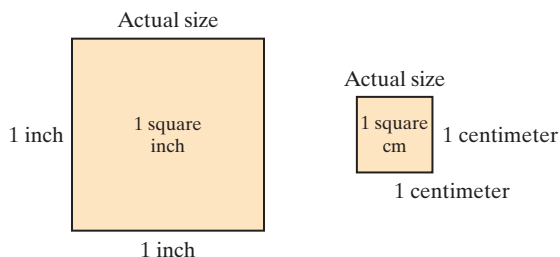
4. 24,786 5. 103,092

 Concept Check Answer

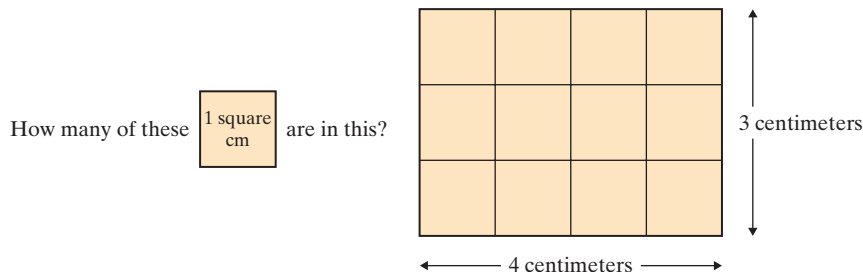
$$\begin{array}{r} 102 \\ \times 33 \\ \hline 306 \\ 3060 \\ \hline 3366 \end{array}$$

Objective Finding the Area of a Rectangle

A special application of multiplication is finding the **area** of a region. Area measures the amount of surface of a region. For example, we measure a plot of land or the living space of a home by its area. The figures on the next page show two examples of units of area measure. (A centimeter is a unit of length in the metric system.)



For example, to measure the area of a geometric figure such as the rectangle below, count the number of square units that cover the region.



This rectangular region contains 12 square units, each 1 square centimeter. Thus, the area is 12 square centimeters. This total number of squares can be found by counting or by multiplying $4 \cdot 3$ (length $\cdot$ width).

$$\begin{aligned}\text{Area of a rectangle} &= \text{length} \cdot \text{width} \\ &= (4 \text{ centimeters})(3 \text{ centimeters}) \\ &= 12 \text{ square centimeters}\end{aligned}$$

In this section, we find the areas of rectangles only. In later sections, we will find the areas of other geometric regions.

Helpful Hint

Notice that area is measured in **square** units while perimeter is measured in units.

Example 6 Finding the Area of a State

The state of Colorado is in the shape of a rectangle whose length is 380 miles and whose width is 280 miles. Find its area.

Solution: The area of a rectangle is the product of its length and its width.

$$\begin{aligned}\text{Area} &= \text{length} \cdot \text{width} \\ &= (380 \text{ miles})(280 \text{ miles}) \\ &= 106,400 \text{ square miles}\end{aligned}$$



The area of Colorado is 106,400 square miles.

Work Practice 6

Practice 6

The state of Wyoming is in the shape of a rectangle whose length is 360 miles and whose width is 280 miles. Find its area.

Answer

6. 100,800 sq mi

Objective D Solving Problems by Multiplying

There are several words or phrases that indicate the operation of multiplication. Some of these are as follows:

Multiplication		
Key Words or Phrases	Examples	Symbols
multiply	multiply 5 by 7	$5 \cdot 7$
product	the product of 3 and 2	$3 \cdot 2$
times	10 times 13	$10 \cdot 13$

Many key words or phrases describing real-life problems that suggest addition might be better solved by multiplication instead. For example, to find the **total** cost of 8 shirts, each selling for \$27, we can either add

$$27 + 27 + 27 + 27 + 27 + 27 + 27 + 27$$

or we can multiply $8(27)$.

Practice 7

A particular computer printer can print 16 pages per minute in color. How many pages can it print in 45 minutes?

Practice 8

A professor of history purchased DVDs and CDs through a club. Each DVD was priced at \$11 and each CD cost \$9. He bought eight DVDs and five CDs. Find the total cost of the order.

Example 7 Finding DVD Space

A digital video disc (DVD) can hold about 4800 megabytes (MB) of information. How many megabytes can 12 DVDs hold?

Solution: Twelve DVDs will hold 12×4800 megabytes.

In Words	Translate to Numbers
megabytes per DVD $\rightarrow$	4800
$\times$ number of DVDs $\rightarrow$	$\times 12$
	9600
	48000
total megabytes	57,600

Twelve DVDs will hold 57,600 megabytes.

Work Practice 7

Example 8 Budgeting Money

A woman and her friend plan to take their children to the Georgia Aquarium in Atlanta, the world's largest aquarium. The ticket price for each child is \$30 and for each adult, \$36. If five children and two adults plan to go, how much money is needed for admission? (Source: GeorgiaAquarium.org)

Solution: If the price of one child's ticket is \$30, the price for 5 children is $5 \times \$30 = \$150$. The price of one adult ticket is \$36, so the price for two adults is $2 \times \$36 = \$72$. The total cost is:

In Words	Translate to Numbers
cost for 5 children $\rightarrow$	150
$+$ cost for 2 adults $\rightarrow$	$+ 72$
total cost	222

The total cost is \$222.

Work Practice 8



Answers

7. 720 pages 8. \$133

Example 9 Estimating Word Count

The average page of a book contains 259 words. Estimate, by rounding each number to the nearest hundred, the total number of words contained on 212 pages.

Solution: The exact number of words is 259×212 . Estimate this product by rounding each factor to the nearest hundred.

$$\begin{array}{rcl} 259 & \text{rounds to} & 300 \\ \times 212 & \text{rounds to} & \times 200 \\ \hline & & 300 \times 200 = 60,000 \\ & & 3 \cdot 2 = 6 \end{array}$$

There are approximately 60,000 words contained on 212 pages.

Work Practice 9**Practice 9**

If an average page in a book contains 163 words, estimate, by rounding each number to the nearest hundred, the total number of words contained on 391 pages.

Answer

9. 80,000 words

**Calculator Explorations** Multiplying Numbers

To multiply numbers on a calculator, find the keys marked $\times$ and $=$ or **ENTER**. For example, to find $31 \cdot 66$ on a calculator, press the keys $\boxed{31}$ $\boxed{\times}$ $\boxed{66}$ then $\boxed{=}$ or **ENTER**. The display will read $\boxed{2046}$. Thus, $31 \cdot 66 = 2046$.

Use a calculator to multiply.

- | | |
|-------------------|--------------------|
| 1. 72×48 | 2. 81×92 |
| 3. $163 \cdot 94$ | 4. $285 \cdot 144$ |
| 5. $983(277)$ | 6. $1562(843)$ |

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

area	grouping	commutative	1	product	length
factor	order	associative	0	distributive	number

- The product of 0 and any number is _____.
- The product of 1 and any number is the _____.
- In $8 \cdot 12 = 96$, the 96 is called the _____ and 8 and 12 are each called a(n) _____.
- Since $9 \cdot 10 = 10 \cdot 9$, we say that changing the _____ in multiplication does not change the product. This property is called the _____ property of multiplication.
- Since $(3 \cdot 4) \cdot 6 = 3 \cdot (4 \cdot 6)$, we say that changing the _____ in multiplication does not change the product. This property is called the _____ property of multiplication.
- _____ measures the amount of surface of a region.
- Area of a rectangle = _____ $\cdot$ width.
- We know $9(10 + 8) = 9 \cdot 10 + 9 \cdot 8$ by the _____ property.

Martin-Gay Interactive Videos Watch the section lecture video and answer the following questions.



See Video 1.5 

Objective A 9. The expression in  Example 3 is rewritten using what property? 

Objective B 10. During the multiplication process for  Example 5, why is a single zero placed at the end of the second partial product? 

Objective C 11. Why are the units to the answer to  Example 6 not just meters? What are the correct units? 

Objective D 12. In  Example 7, why can “total” imply multiplication as well as addition? 

1.5 Exercise Set MyLab Math

Objective A Multiply. See Example 1.

 1. $1 \cdot 24$

2. $55 \cdot 1$

 3. $0 \cdot 19$

4. $27 \cdot 0$

5. $8 \cdot 0 \cdot 9$

6. $7 \cdot 6 \cdot 0$

7. $87 \cdot 1$

8. $1 \cdot 41$

Use the distributive property to rewrite each expression. See Example 2.

9. $6(3 + 8)$

10. $5(8 + 2)$

11. $4(3 + 9)$

12. $6(1 + 4)$

 13. $20(14 + 6)$

14. $12(12 + 3)$

Objective B Multiply. See Example 3.

15.
$$\begin{array}{r} 64 \\ \times 8 \\ \hline \end{array}$$

16.
$$\begin{array}{r} 79 \\ \times 3 \\ \hline \end{array}$$

17.
$$\begin{array}{r} 613 \\ \times 6 \\ \hline \end{array}$$

18.
$$\begin{array}{r} 638 \\ \times 5 \\ \hline \end{array}$$

 19. 277×6

20. 882×2

21. 1074×6

22. 9021×3

Objective A B Mixed Practice Multiply. See Examples 1 through 5.

23.
$$\begin{array}{r} 89 \\ \times 13 \\ \hline \end{array}$$

24.
$$\begin{array}{r} 91 \\ \times 72 \\ \hline \end{array}$$

25.
$$\begin{array}{r} 421 \\ \times 58 \\ \hline \end{array}$$

26.
$$\begin{array}{r} 526 \\ \times 23 \\ \hline \end{array}$$

27.
$$\begin{array}{r} 306 \\ \times 81 \\ \hline \end{array}$$

28.
$$\begin{array}{r} 708 \\ \times 21 \\ \hline \end{array}$$

29. $(780)(20)$

30. $(720)(80)$

31. $(495)(13)(0)$

32. $(593)(47)(0)$

33. $(640)(1)(10)$

34. $(240)(1)(20)$

35. 1234×39

36. 1357×79

37. 609×234

38. 807×127

▶ 39.
$$\begin{array}{r} 8649 \\ \times 274 \\ \hline \end{array}$$

40.
$$\begin{array}{r} 1234 \\ \times 567 \\ \hline \end{array}$$

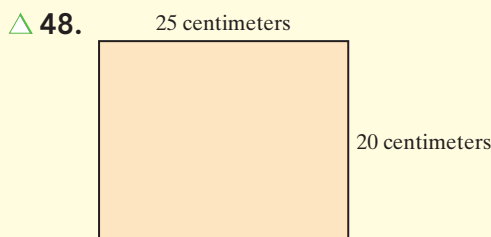
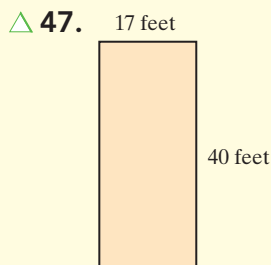
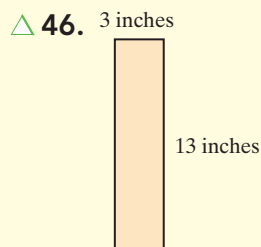
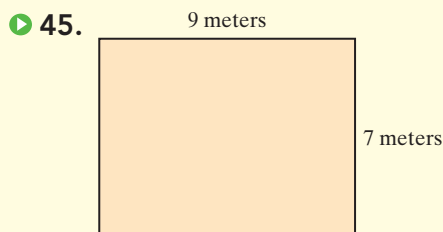
41.
$$\begin{array}{r} 589 \\ \times 110 \\ \hline \end{array}$$

42.
$$\begin{array}{r} 426 \\ \times 110 \\ \hline \end{array}$$

43.
$$\begin{array}{r} 1941 \\ \times 2035 \\ \hline \end{array}$$

44.
$$\begin{array}{r} 1876 \\ \times 1407 \\ \hline \end{array}$$

Objective C **Mixed Practice (Sections 1.3 and 1.5)** Find the area and the perimeter of each rectangle. See Example 6.



Objective D Estimate the products by rounding each factor to the nearest hundred. See Example 9.

49. 576×354

50. 982×650

51. 604×451

52. 111×999

Without actually calculating, mentally round, multiply, and choose the best estimate.

53. $38 \times 42 =$

- a. 16
- b. 160
- c. 1600
- d. 16,000

54. $2872 \times 12 =$

- a. 2872
- b. 28,720
- c. 287,200
- d. 2,872,000

55. $612 \times 29 =$

- a. 180
- b. 1800
- c. 18,000
- d. 180,000

56. $706 \times 409 =$

- a. 280
- b. 2800
- c. 28,000
- d. 280,000

Objectives C D **Mixed Practice—Translating** Solve. See Examples 6 through 9.

57. Multiply 80 by 11.

58. Multiply 70 by 12.

59. Find the product of 6 and 700.

60. Find the product of 9 and 900.

61. Find 2 times 2240.

62. Find 3 times 3310.

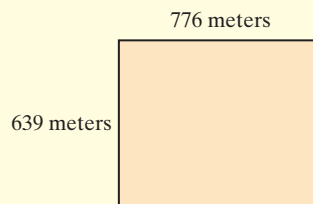
- ▶ 63. One tablespoon of olive oil contains 125 calories. How many calories are in 3 tablespoons of olive oil? (Source: *Home and Garden Bulletin No. 72*, U.S. Department of Agriculture).

64. One ounce of hulled sunflower seeds contains 14 grams of fat. How many grams of fat are in 8 ounces of hulled sunflower seeds? (Source: *Home and Garden Bulletin No. 72*, U.S. Department of Agriculture).

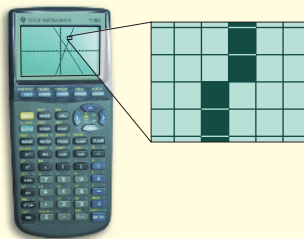
65. The textbook for a course in biology costs \$94. There are 35 students in the class. Find the total cost of the biology books for the class.
66. The seats in a large lecture hall are arranged in 14 rows with 34 seats in each row. Find how many seats are in this room.
67. Cabot Creamery is packing a pallet of 20-lb boxes of cheddar cheese to send to a local restaurant. There are five layers of boxes on the pallet, and each layer is four boxes wide by five boxes deep.
- How many boxes are in one layer?
 - How many boxes are on the pallet?
 - What is the weight of the cheese on the pallet?
68. An apartment building has *three floors*. Each floor has five rows of apartments with four apartments in each row.
- How many apartments are on 1 floor?
 - How many apartments are in the building?

- △ 69. A plot of land measures 80 feet by 110 feet. Find its area.
- △ 70. A house measures 45 feet by 60 feet. Find the floor area of the house.

- △ 71. The largest hotel lobby can be found at the Hyatt Regency in San Francisco, CA. It is in the shape of a rectangle that measures 350 feet by 160 feet. Find its area.
- △ 72. Recall from an earlier section that the second-largest commercial building in the world under one roof is the flower auction building of the cooperative VBA in Aalsmeer, Netherlands. The floor plan is a rectangle that measures 776 meters by 639 meters. Find the area of this building. (A meter is a unit of length in the metric system.) (Source: *The Handy Science Answer Book*, Visible Ink Press)



73. A pixel is a rectangular dot on a graphing calculator screen. If a graphing calculator screen contains 62 pixels in a row and 94 pixels in a column, find the total number of pixels on a screen.
74. A certain compact disc (CD) can hold 700 megabytes (MB) of information. How many MBs can 17 discs hold?



75. A line of print on a computer contains 60 characters (letters, spaces, punctuation marks). Find how many characters there are in 35 lines.
76. An average cow eats 3 pounds of grain per day. Find how much grain a cow eats in a year. (Assume 365 days in 1 year.)

- 77.** One ounce of Planters Dry Roasted Peanuts has 160 calories. How many calories are in 8 ounces? (*Source:* RJR Nabisco, Inc.)

- 79.** The Thespian club at a local community college is ordering T-shirts. T-shirts size S, M, or L cost \$10 each and T-shirts size XL or XXL cost \$12 each. Complete the table below and use it to find the total cost. (The first row is filled in for you.)

T-Shirt Size	Number of Shirts Ordered	Cost per Shirt	Cost per Size Ordered
S	4	\$10	\$40
M	6		
L	20		
XL	3		
XXL	3		

- 78.** One ounce of Planters Dry Roasted Peanuts has 13 grams of fat. How many grams of fat are in 16 ounces? (*Source:* RJR Nabisco, Inc.)

- 80.** The student activities group at North Shore Community College is planning a trip to see the local minor league baseball team. Tickets cost \$5 for students, \$7 for nonstudents, and \$2 for children under 12. Complete the following table and use it to find the total cost.

Person	Number of Persons	Cost per Person	Cost per Category
Student	24	\$5	\$120
Nonstudent	4		
Children under 12	5		

- 81.** Celestial Seasonings of Boulder, Colorado, is a tea company that specializes in herbal teas. Their plant in Boulder has bagging machines capable of bagging over 1000 bags of tea per minute. If the plant runs 24 hours day, how many tea bags are produced in one day? (*Source:* Celestial Seasonings)

- 82.** There were about 3 million “older” Americans (ages 65 and older) in 1900. By 2020, this number is projected to increase eighteen times. Find this projected number of “older” Americans in 2020. (*Source:* Administration on Aging, U.S. Census Bureau)

Mixed Practice (Sections 1.3 and 1.5) Perform each indicated operation.

83.
$$\begin{array}{r} 128 \\ + 7 \\ \hline \end{array}$$

84.
$$\begin{array}{r} 126 \\ - 8 \\ \hline \end{array}$$

85.
$$\begin{array}{r} 134 \\ \times 16 \\ \hline \end{array}$$

86. $47 + 26 + 10 + 231 + 50$

87. Find the sum of 19 and 4.

88. Find the product of 19 and 4.

89. Find the difference of 19 and 4.

90. Find the total of 14 and 9.

Concept Extensions

Solve. See the first Concept Check in this section.

91. Rewrite $6 + 6 + 6 + 6 + 6$ using multiplication.

92. Rewrite $11 + 11 + 11 + 11 + 11 + 11$ using multiplication.

-  **93. a.** Rewrite $3 \cdot 5$ as repeated addition.
b. Explain why there is more than one way to do this.

-  **94. a.** Rewrite $4 \cdot 5$ as repeated addition.
b. Explain why there is more than one way to do this.

Find and explain the error in each multiplication problem. See the second Concept Check in this section.

$$\begin{array}{r} 95. \quad 203 \\ \times 14 \\ \hline 812 \\ 203 \\ \hline 1015 \end{array}$$

$$\begin{array}{r} 96. \quad 31 \\ \times 50 \\ \hline 155 \end{array}$$

Fill in the missing digits in each problem.

$$\begin{array}{r} 97. \quad 4 \_ \\ \times \_ 3 \\ \hline 126 \\ 3780 \\ \hline 3906 \end{array}$$

$$\begin{array}{r} 98. \quad \_ 7 \\ \times 6 \_ \\ \hline 171 \\ 3420 \\ \hline 3591 \end{array}$$

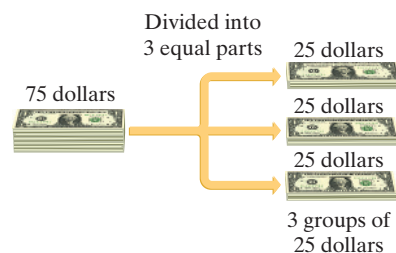
99. Explain how to multiply two 2-digit numbers using partial products.
100. In your own words, explain the meaning of the area of a rectangle and how this area is measured.
101. A window washer in New York City is bidding for a contract to wash the windows of a 23-story building. To write a bid, the number of windows in the building is needed. If there are 7 windows in each row of windows on 2 sides of the building and 4 windows per row on the other 2 sides of the building, find the total number of windows.
102. During the NBA's 2015–2016 regular season, LeBron James of the Cleveland Cavaliers scored 87 three-point field goals, 650 two-point field goals, and 359 free throws (worth one point each). How many points did LeBron James score during the 2015–2016 regular season? (Source: NBA)

1.6 Dividing Whole Numbers

Objectives

- A** Divide Whole Numbers.
- B** Perform Long Division.
- C** Solve Problems That Require Dividing by Whole Numbers.
- D** Find the Average of a List of Numbers.

Suppose three people pooled their money and bought a raffle ticket at a local fundraiser. Their ticket was the winner and they won a \$75 cash prize. They then divided the prize into three equal parts so that each person received \$25.



Objective A Dividing Whole Numbers

The process of separating a quantity into equal parts is called **division**. The division above can be symbolized by several notations.

$$\begin{array}{r} \text{quotient} \\ 25 \\ 3 \overline{)75} \\ \text{divisor} \end{array} \leftarrow \text{dividend}$$

$$\begin{array}{r} \text{dividend} \\ 75 \\ \frac{75}{3} = 25 \leftarrow \text{quotient} \\ \text{divisor} \end{array}$$

$$\begin{array}{r} \text{quotient} \\ 75 \div 3 = 25 \\ \text{dividend} \quad \text{divisor} \end{array}$$

$$\begin{array}{r} \text{dividend} \quad \text{quotient} \\ 75/3 = 25 \\ \text{divisor} \end{array}$$

(In the notation $\frac{75}{3}$, the bar separating 75 and 3 is called a **fraction bar**.) Just as subtraction is the reverse of addition, division is the reverse of multiplication. This means that division can be checked by multiplication.

$$\begin{array}{r} 25 \\ 3 \overline{)75} \end{array} \text{ because } 25 \cdot 3 = 75$$

$$\boxed{\text{Quotient}} \cdot \boxed{\text{Divisor}} = \boxed{\text{Dividend}}$$

Since multiplication and division are related in this way, you can use your knowledge of multiplication facts to review quotients of one-digit divisors if necessary.

Example 1 Find each quotient. Check by multiplying.

a. $42 \div 7$

b. $\frac{64}{8}$

c. $3 \overline{)21}$

Solution:

a. $42 \div 7 = 6$ because $6 \cdot 7 = 42$

b. $\frac{64}{8} = 8$ because $8 \cdot 8 = 64$

c. $3 \overline{)21}$ because $7 \cdot 3 = 21$

Work Practice 1

Example 2 Find each quotient. Check by multiplying.

a. $1 \overline{)7}$

► b. $25 \div 1$

c. $\frac{6}{6}$

► d. $12 \div 12$

e. $\frac{20}{1}$

f. $18 \overline{)18}$

Solution:

a. $1 \overline{)7}$ because $7 \cdot 1 = 7$

b. $25 \div 1 = 25$ because $25 \cdot 1 = 25$

c. $\frac{6}{6} = 1$ because $1 \cdot 6 = 6$

d. $12 \div 12 = 1$ because $1 \cdot 12 = 12$

e. $\frac{20}{1} = 20$ because $20 \cdot 1 = 20$

f. $18 \overline{)18}$ because $1 \cdot 18 = 18$

Work Practice 2

Practice 1

Find each quotient. Check by multiplying.

a. $9 \overline{)72}$

b. $40 \div 5$

c. $\frac{24}{6}$

Practice 2

Find each quotient. Check by multiplying.

a. $\frac{7}{7}$

b. $5 \div 1$

c. $1 \overline{)11}$

d. $4 \div 1$

e. $\frac{10}{1}$

f. $21 \div 21$

Answers

1. a. 8 b. 8 c. 4 2. a. 1 b. 5
c. 11 d. 4 e. 10 f. 1

Example 2 illustrates the important properties of division described next:

Division Properties of 1

The quotient of any number (except 0) and that same number is 1. For example,

$$8 \div 8 = 1 \quad \frac{5}{5} = 1 \quad 4 \overline{)4}$$

The quotient of any number and 1 is that same number. For example,

$$9 \div 1 = 9 \quad \frac{6}{1} = 6 \quad 1 \overline{)3} \quad \frac{0}{1} = 0$$

Practice 3

Find each quotient. Check by multiplying.

- a. $\frac{0}{7}$ b. $8 \overline{)0}$
c. $7 \div 0$ d. $0 \div 14$

Example 3

Find each quotient. Check by multiplying.

- a. $9 \overline{)0}$ b. $0 \div 12$ c. $\frac{0}{5}$ d. $\frac{3}{0}$

Solution:

- a. $9 \overline{)0}$ because $0 \cdot 9 = 0$
b. $0 \div 12 = 0$ because $0 \cdot 12 = 0$
c. $\frac{0}{5} = 0$ because $0 \cdot 5 = 0$
d. If $\frac{3}{0} = \text{a number}$, then the *number* times $0 = 3$. Recall from Section 1.5 that any number multiplied by 0 is 0 and not 3. We say, then, that $\frac{3}{0}$ is **undefined**.

Work Practice 3

Example 3 illustrates important division properties of 0.

Division Properties of 0

The quotient of 0 and any number (except 0) is 0. For example,

$$0 \div 9 = 0 \quad \frac{0}{5} = 0 \quad 14 \overline{)0}$$

The quotient of any number and 0 is not a number. We say that

$$\frac{3}{0}, \quad 0 \overline{)3}, \quad \text{and} \quad 3 \div 0$$

are **undefined**.

Objective B Performing Long Division

When dividends are larger, the quotient can be found by a process called **long division**. For example, let's divide 2541 by 3.

$$\begin{array}{r} \text{divisor} \rightarrow 3 \overline{)2541} \\ \uparrow \\ \text{dividend} \end{array}$$

We can't divide 3 into 2, so we try dividing 3 into the first two digits.

$$\begin{array}{r} 8 \\ 3 \overline{)2541} \end{array} \quad \begin{array}{l} 25 \div 3 = 8 \text{ with } 1 \text{ left, so our best estimate is } 8. \\ \text{We place 8 over the 5 in 25.} \end{array}$$

Answers

3. a. 0 b. 0 c. undefined d. 0

Next, multiply 8 and 3 and subtract this product from 25. Make sure that this difference is less than the divisor.

$$\begin{array}{r} 8 \\ 3 \overline{)2541} \\ \underline{-24} \\ 1 \end{array} \quad \begin{array}{l} 8(3) = 24 \\ 25 - 24 = 1, \text{ and } 1 \text{ is less than the divisor } 3. \end{array}$$

Bring down the next digit and go through the process again.

$$\begin{array}{r} 84 \\ 3 \overline{)2541} \\ \underline{-24} \downarrow \\ 14 \\ \underline{-12} \\ 2 \end{array} \quad \begin{array}{l} 14 \div 3 = 4 \text{ with } 2 \text{ left} \\ 4(3) = 12 \\ 14 - 12 = 2 \end{array}$$

Once more, bring down the next digit and go through the process.

$$\begin{array}{r} 847 \\ 3 \overline{)2541} \\ \underline{-24} \downarrow \\ 14 \downarrow \\ \underline{-12} \\ 21 \\ \underline{-21} \\ 0 \end{array} \quad \begin{array}{l} 21 \div 3 = 7 \\ 7(3) = 21 \\ 21 - 21 = 0 \end{array}$$

The quotient is 847. To check, see that $847 \times 3 = 2541$.

Example 4 Divide: $3705 \div 5$. Check by multiplying.

Solution:

$$\begin{array}{r} 7 \\ 5 \overline{)3705} \\ \underline{-35} \downarrow \\ 20 \uparrow \\ \text{Bring down the 0.} \end{array} \quad \begin{array}{l} 37 \div 5 = 7 \text{ with } 2 \text{ left. Place this estimate, } 7, \text{ over the } 7 \text{ in } 37. \\ 7(5) = 35 \\ 37 - 35 = 2, \text{ and } 2 \text{ is less than the divisor } 5. \end{array}$$

$$\begin{array}{r} 74 \\ 5 \overline{)3705} \\ \underline{-35} \downarrow \\ 20 \downarrow \\ \underline{-20} \\ 05 \uparrow \\ \text{Bring down the 5.} \end{array} \quad \begin{array}{l} 20 \div 5 = 4 \\ 4(5) = 20 \\ 20 - 20 = 0, \text{ and } 0 \text{ is less than the divisor } 5. \end{array}$$

$$\begin{array}{r} 741 \\ 5 \overline{)3705} \\ \underline{-35} \downarrow \\ 20 \downarrow \\ \underline{-20} \\ 5 \\ \underline{-5} \\ 0 \end{array} \quad \begin{array}{l} 5 \div 5 = 1 \\ 1(5) = 5 \\ 5 - 5 = 0 \end{array}$$

Practice 4

Divide. Check by multiplying.

- a. $4908 \div 6$
- b. $2212 \div 4$
- c. $753 \div 3$

Answers

4. a. 818 b. 553 c. 251

(Continued on next page)

Helpful Hint

Since division and multiplication are reverse operations, don't forget that a division problem can be checked by multiplying.

Practice 5

Divide and check by multiplying.

a. $7 \overline{)2128}$

b. $9 \overline{)45,900}$

Check:

$$\begin{array}{r} 741 \\ \times 5 \\ \hline 3705 \end{array}$$

Work Practice 4

Example 5

Divide and check: $1872 \div 9$

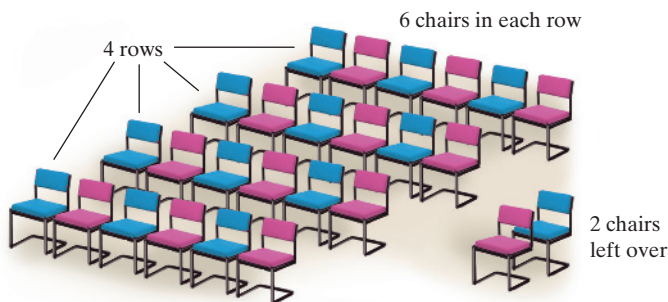
Solution:

$$\begin{array}{r} 208 \\ 9 \overline{)1872} \\ \underline{-18} \\ 07 \\ \underline{-0} \\ 72 \\ \underline{-72} \\ 0 \end{array} \quad \begin{array}{l} 2(9) = 18 \\ 18 - 18 = 0; \text{bring down the 7.} \\ 0(9) = 0 \\ 7 - 0 = 7; \text{bring down the 2.} \\ 8(9) = 72 \\ 72 - 72 = 0 \end{array}$$

Check: $208 \cdot 9 = 1872$

Work Practice 5

Naturally, quotients don't always "come out even." Making 4 rows out of 26 chairs, for example, isn't possible if each row is supposed to have exactly the same number of chairs. Each of 4 rows can have 6 chairs, but 2 chairs are left over.



We signify "leftovers" or **remainders** in this way:

$$\begin{array}{r} 6 \text{ R } 2 \\ 4 \overline{)26} \end{array}$$

The **whole number part of the quotient** is 6; the **remainder part of the quotient** is 2. Checking by multiplying,

whole number part	·	divisor	+	remainder part	=	dividend
↓		↓		↓		↓
6	·	4	+	2	=	
		24	+	2	=	26

Answers

5. a. 304 b. 5100

Example 6 Divide and check: $2557 \div 7$ **Solution:**

$$\begin{array}{r}
 365 \text{ R } 2 \\
 7 \overline{)2557} \\
 \underline{-21} \\
 45 \\
 \underline{-42} \\
 37 \\
 \underline{-35} \\
 2
 \end{array}$$

$3(7) = 21$
 $25 - 21 = 4$; bring down the 5.
 $6(7) = 42$
 $45 - 42 = 3$; bring down the 7.
 $5(7) = 35$
 $37 - 35 = 2$; the remainder is 2.

Check:

365	•	7	+	2	=	2557
↑		↑		↑		↑
whole number part	•	divisor	+	remainder part	=	dividend

Work Practice 6**Example 7** Divide and check: $56,717 \div 8$ **Solution:**

$$\begin{array}{r}
 7089 \text{ R } 5 \\
 8 \overline{)56717} \\
 \underline{-56} \\
 07 \\
 \underline{-0} \\
 71 \\
 \underline{-64} \\
 77 \\
 \underline{-72} \\
 5
 \end{array}$$

$7(8) = 56$
 Subtract and bring down the 7.
 $0(8) = 0$
 Subtract and bring down the 1.
 $8(8) = 64$
 Subtract and bring down the 7.
 $9(8) = 72$
 Subtract. The remainder is 5.

Check:

7089	•	8	+	5	=	56,717
↓		↓		↓		↓
whole number part	•	divisor	+	remainder part	=	dividend

Work Practice 7

When the divisor has more than one digit, the same pattern applies. For example, let's find $1358 \div 23$.

$$\begin{array}{r}
 5 \\
 23 \overline{)1358} \\
 \underline{-115} \\
 208
 \end{array}$$

$135 \div 23 = 5$ with 20 left over. Our estimate is 5.
 $5(23) = 115$
 $135 - 115 = 20$. Bring down the 8.

Practice 6

Divide and check.

a. $4 \overline{)939}$

b. $5 \overline{)3287}$

Practice 7

Divide and check.

a. $9 \overline{)81,605}$

b. $4 \overline{)23,310}$

Answers

6. a. 234 R 3 b. 657 R 2

7. a. 9067 R 2 b. 5827 R 2

(Continued on next page)

Now we continue estimating.

$$\begin{array}{r}
 59 \text{ R } 1 \quad 208 \div 23 = 9 \text{ with } 1 \text{ left over.} \\
 23 \overline{)1358} \\
 \underline{-115} \\
 208 \\
 \underline{-207} \\
 1
 \end{array}$$

$9(23) = 207$
 $208 - 207 = 1$. The remainder is 1.

To check, see that $59 \cdot 23 + 1 = 1358$.

Practice 8

Divide: $8920 \div 17$

Example 8

Divide: $6819 \div 17$

Solution:

$$\begin{array}{r}
 401 \text{ R } 2 \\
 17 \overline{)6819} \\
 \underline{-68} \quad 4(17) = 68 \\
 01 \quad \text{Subtract and bring down the 1.} \\
 \underline{-0} \quad 0(17) = 0 \\
 19 \quad \text{Subtract and bring down the 9.} \\
 \underline{-17} \quad 1(17) = 17 \\
 2 \quad \text{Subtract. The remainder is 2.}
 \end{array}$$

To check, see that $401 \cdot 17 + 2 = 6819$.

Work Practice 8

Practice 9

Divide: $33,282 \div 678$

Example 9

Divide: $51,600 \div 403$

Solution:

$$\begin{array}{r}
 128 \text{ R } 16 \\
 403 \overline{)51600} \\
 \underline{-403} \quad 1(403) = 403 \\
 1130 \quad \text{Subtract and bring down the 0.} \\
 \underline{-806} \quad 2(403) = 806 \\
 3240 \quad \text{Subtract and bring down the 0.} \\
 \underline{-3224} \quad 8(403) = 3224 \\
 16 \quad \text{Subtract. The remainder is 16.}
 \end{array}$$

To check, see that $128 \cdot 403 + 16 = 51,600$.

Work Practice 9

Division Shown as Repeated Subtraction To further understand division, recall from Section 1.5 that addition and multiplication are related in the following manner:

$$\underbrace{3 + 3 + 3 + 3}_{4 \text{ addends; each addend is } 3} = 4 \times 3 = 12$$

In other words, multiplication is repeated addition. Likewise, division is repeated subtraction.

For example, let's find

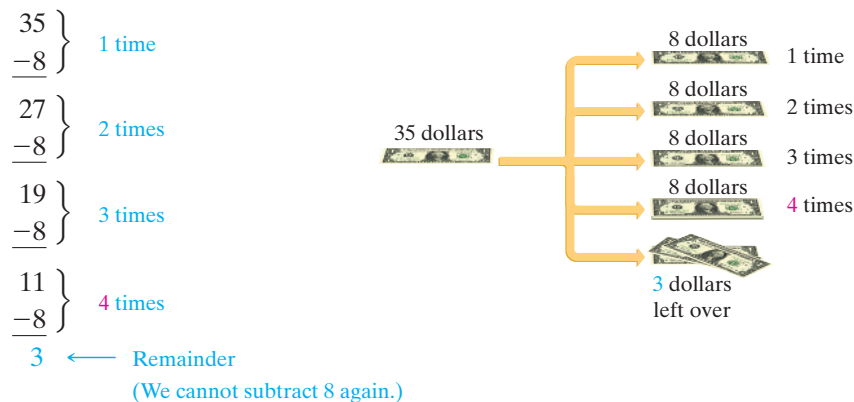
$$35 \div 8$$

Answers

8. 524 R 12 9. 49 R 60

by repeated subtraction. Keep track of the number of times 8 is subtracted from 35. We are through when we can subtract no more because the difference is less than 8.

$35 \div 8$: Repeated subtraction



Thus, $35 \div 8 = 4 \text{ R } 3$.

To check, perform the same multiplication as usual, but finish by adding in the remainder.

whole number part of quotient	•	divisor	+	remainder	=	dividend
↓		↓		↓		↓
4	•	8	+	3	=	35

Objective C Solving Problems by Dividing

Below are some key words and phrases that may indicate the operation of division:

Division		
Key Words or Phrases	Examples	Symbols
divide	divide 10 by 5	$10 \div 5$ or $\frac{10}{5}$
quotient	the quotient of 64 and 4	$64 \div 4$ or $\frac{64}{4}$
divided by	9 divided by 3	$9 \div 3$ or $\frac{9}{3}$
divided or shared equally among	\$100 divided equally among five people	$100 \div 5$ or $\frac{100}{5}$
per	100 miles per 2 hours	$\frac{100 \text{ miles}}{2 \text{ hours}}$

✓ **Concept Check** Which of the following is the correct way to represent “the quotient of 60 and 12”? Or are both correct? Explain your answer.

- a. $12 \div 60$
- b. $60 \div 12$

✓ **Concept Check Answers**

- a. incorrect
- b. correct

Practice 10

Three students bought 171 blank CDs to share equally. How many CDs did each person get?

Practice 11

Printers can be packed 12 to a box. If 532 printers are to be packed but only full boxes are shipped, how many full boxes will be shipped? How many printers are left over and not shipped?

Answers

10. 57 CDs

11. 44 full boxes; 4 printers left over

Example 10 Finding Shared Earnings

Three college freshmen share a paper route to earn money for expenses. The total in their fund after expenses is \$2895. How much is each person's equal share?

Solution:

$$\begin{array}{l} \text{In words:} \quad \boxed{\text{Each person's share}} = \boxed{\text{total money}} \div \boxed{\text{number of persons}} \\ \quad \quad \quad \downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow \\ \text{Translate:} \quad \text{Each person's share} = 2895 \div 3 \end{array}$$

$$\begin{array}{r} \text{Then} \quad 965 \\ 3 \overline{)2895} \\ \underline{-27} \\ 19 \\ \underline{-18} \\ 15 \\ \underline{-15} \\ 0 \end{array}$$

Each person's share is \$965.

Work Practice 10**Example 11** Dividing Number of Downloads

As part of a promotion, an executive receives 238 cards, each good for one free song download. If she wants to share them evenly with 19 friends, how many download cards will each friend receive? How many will be left over?

Solution:

$$\begin{array}{l} \text{In words:} \quad \boxed{\text{Number of cards for each person}} = \boxed{\text{number of cards}} \div \boxed{\text{number of friends}} \\ \quad \quad \quad \downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow \\ \text{Translate:} \quad \text{Number of cards for each person} = 238 \div 19 \end{array}$$

$$\begin{array}{r} 12 \text{ R } 10 \\ 19 \overline{)238} \\ \underline{-19} \\ 48 \\ \underline{-38} \\ 10 \end{array}$$

Each friend will receive 12 download cards. The cards cannot be divided equally among her friends since there is a nonzero remainder. There will be 10 download cards left over.

Work Practice 11

Objective D Finding Averages

A special application of division (and addition) is finding the average of a list of numbers. The **average** of a list of numbers is the sum of the numbers divided by the *number* of numbers.

$$\text{average} = \frac{\text{sum of numbers}}{\text{number of numbers}}$$

Example 12 Averaging Scores

A mathematics instructor is checking a simple program she wrote for averaging the scores of her students. To do so, she averages a student's scores of 75, 96, 81, and 88 by hand. Find this average score.

Solution: To find the average score, we find the sum of the student's scores and divide by 4, the number of scores.

$$\begin{array}{r} 75 \\ 96 \\ 81 \\ + 88 \\ \hline 340 \quad \text{sum} \end{array} \quad \text{average} = \frac{340}{4} = 85$$

$$\begin{array}{r} 85 \\ 4 \overline{)340} \\ \underline{-32} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

The average score is 85.

Work Practice 12

Practice 12

To compute a safe time to wait for reactions to occur after allergy shots are administered, a lab technician is given a list of elapsed times between administered shots and reactions. Find the average of the times 4 minutes, 7 minutes, 35 minutes, 16 minutes, 9 minutes, 3 minutes, and 52 minutes.

Answer

12. 18 min



Calculator Explorations Dividing Numbers

To divide numbers on a calculator, find the keys marked $\div$ and $=$ or **ENTER**. For example, to find $435 \div 5$ on a calculator, press the keys $\boxed{435} \boxed{\div} \boxed{5}$ then $\boxed{=}$ or **ENTER**. The display will read $\boxed{87}$. Thus, $435 \div 5 = 87$.

Use a calculator to divide.

- | | |
|--------------------------|--------------------------|
| 1. $848 \div 16$ | 2. $564 \div 12$ |
| 3. $95 \overline{)5890}$ | 4. $27 \overline{)1053}$ |
| 5. $\frac{32,886}{126}$ | 6. $\frac{143,088}{264}$ |
| 7. $0 \div 315$ | 8. $315 \div 0$ |

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

1	number	divisor	dividend
0	undefined	average	quotient

- In $90 \div 2 = 45$, the answer 45 is called the _____, 90 is called the _____, and 2 is called the _____.
- The quotient of any number and 1 is the same _____.
- The quotient of any number (except 0) and the same number is _____.
- The quotient of 0 and any number (except 0) is _____.
- The quotient of any number and 0 is _____.
- The _____ of a list of numbers is the sum of the numbers divided by the _____ of numbers.

Martin-Gay Interactive Videos Watch the section lecture video and answer the following questions.



See Video 1.6

Objective A 7. Look at Examples 6–8. What number can never be the divisor in division?

Objective B 8. In Example 10, how many 102s are in 21? How does this result affect the quotient?

9. What calculation would you use to check the answer in Example 10?

Objective C 10. In Example 11, what is the importance of knowing that the distance to each hole is the same?

Objective D 11. As shown in Example 12, what two operations are used when finding an average?

1.6 Exercise Set MyLab Math

Objective A Find each quotient. See Examples 1 through 3.

1. $54 \div 9$

2. $72 \div 9$

3. $36 \div 3$

4. $24 \div 3$

5. $0 \div 8$

6. $0 \div 4$

7. $31 \div 1$

8. $38 \div 1$

9. $\frac{18}{18}$

10. $\frac{49}{49}$

11. $\frac{24}{3}$

12. $\frac{45}{9}$

13. $26 \div 0$

14. $\frac{12}{0}$

15. $26 \div 26$

16. $6 \div 6$

17. $0 \div 14$

18. $7 \div 0$

19. $18 \div 2$

20. $18 \div 3$

Objectives A B Mixed Practice Divide and then check by multiplying. See Examples 1 through 5.

21. $3 \overline{)87}$

22. $5 \overline{)85}$

23. $3 \overline{)222}$

24. $8 \overline{)640}$

25. $3 \overline{)1014}$

26. $4 \overline{)2104}$

27. $\frac{30}{0}$

28. $\frac{0}{30}$

29. $63 \div 7$

30. $56 \div 8$

31. $150 \div 6$

32. $121 \div 11$

Divide and then check by multiplying. See Examples 6 and 7.

33. $7 \overline{)479}$

34. $7 \overline{)426}$

35. $6 \overline{)1421}$

36. $3 \overline{)1240}$

37. $305 \div 8$

38. $167 \div 3$

39. $2286 \div 7$

40. $3333 \div 4$

Divide and then check by multiplying. See Examples 8 and 9.

41. $55 \overline{)715}$ 42. $23 \overline{)736}$ 43. $23 \overline{)1127}$ 44. $42 \overline{)2016}$ 45. $97 \overline{)9417}$
 46. $44 \overline{)1938}$ 47. $3146 \div 15$ 48. $7354 \div 12$ 49. $6578 \div 13$ 50. $5670 \div 14$
 51. $9299 \div 46$ 52. $2505 \div 64$ 53. $\frac{12,744}{236}$ 54. $\frac{5781}{123}$ 55. $\frac{10,297}{103}$
 56. $\frac{23,092}{240}$ 57. $20,619 \div 102$ 58. $40,853 \div 203$ 59. $244,989 \div 423$ 60. $164,592 \div 543$

Divide. See Examples 1 through 9.

61. $7 \overline{)119}$ 62. $8 \overline{)104}$ 63. $7 \overline{)3580}$ 64. $5 \overline{)3017}$
 65. $40 \overline{)85,312}$ 66. $50 \overline{)85,747}$ 67. $142 \overline{)863,360}$ 68. $214 \overline{)650,560}$

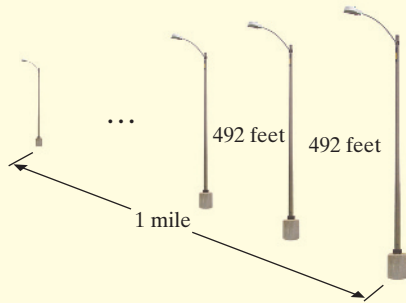
Objective C Translating Solve. See Examples 10 and 11.

69. Find the quotient of 117 and 5. 70. Find the quotient of 94 and 7.
 71. Find 200 divided by 35. 72. Find 116 divided by 32.
 73. Find the quotient of 62 and 3. 74. Find the quotient of 78 and 5.

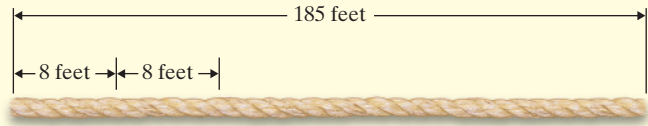
Solve.

75. Amad Ford teaches American Sign Language classes for \$65 per student for a 7-week session. He collects \$2145 from the group of students. Find how many students are in the group.
 76. Kathy Gomez teaches Spanish lessons for \$85 per student for a 5-week session. From one group of students, she collects \$4930. Find how many students are in the group.
 77. The gravity of Jupiter is 318 times as strong as the gravity of Earth, so objects on Jupiter weigh 318 times as much as they weigh on Earth. If a person would weigh 52,470 pounds on Jupiter, find how much the person weighs on Earth.
 78. Twenty-one people pooled their money and bought lottery tickets. One ticket won a prize of \$5,292,000. Find how many dollars each person received.
 79. An 18-hole golf course is 5580 yards long. If the distance to each hole is the same, find the distance between holes.
 80. A truck hauls wheat to a storage granary. It carries a total of 5768 bushels of wheat in 14 trips. How much does the truck haul each trip if each trip it hauls the same amount?
 81. There is a bridge over highway I-35 every three miles. The first bridge is at the beginning of a 265-mile stretch of highway. Find how many bridges there are over 265 miles of I-35.
 82. The white stripes dividing the lanes on a highway are 25 feet long, and the spaces between them are 25 feet long. Let's call a "lane divider" a stripe followed by a space. Find how many whole "lane dividers" there are in 1 mile of highway. (A mile is 5280 feet.)

83. Light poles along a highway are placed 492 feet apart. The first light pole is at the beginning of a 1-mile strip. Find how many poles should be ordered for the 1-mile strip of highway. (A mile is 5280 feet.)



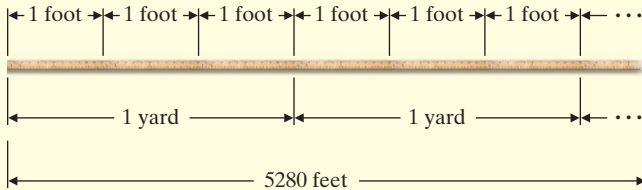
84. A piece of rope 185 feet long is to be cut into pieces for an experiment in a physics class. Each piece of rope is to be 8 feet long. Determine whether there is enough rope for 22 students. Determine the amount extra or the amount short.



85. Broad Peak in Pakistan is the twelfth-tallest mountain in the world. Its elevation is 26,400 feet. A mile is 5280 feet. How many miles tall is Broad Peak? (Source: National Geographic Society)

86. Doug Baldwin of the Seattle Seahawks, Brandon Marshall of the New York Jets, and Allen Robinson of the Jacksonville Jaguars tied to lead the NFL in scoring touchdowns during the 2015 regular football season, scoring a total of 84 points each. If a touchdown is worth 6 points, how many touchdowns did each of these players make during the 2015 season? (Source: NFL)

87. Find how many yards are in 1 mile. (A mile is 5280 feet; a yard is 3 feet.)



88. Find how many whole feet are in 1 rod. (A mile is 5280 feet; 1 mile is 320 rods.)

Objective D Find the average of each list of numbers. See Example 12.

89. 10, 24, 35, 22, 17, 12

90. 37, 26, 15, 29, 51, 22

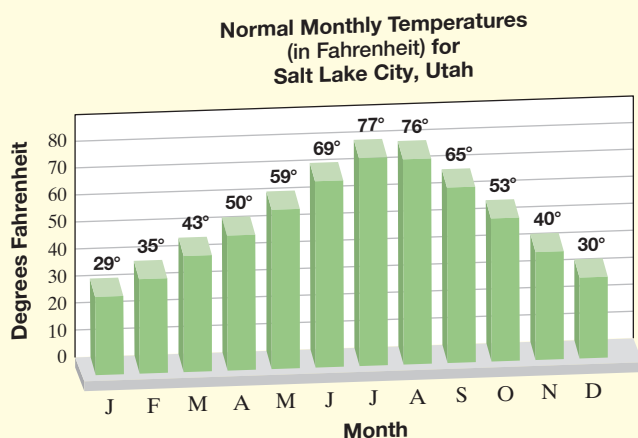
91. 205, 972, 210, 161

92. 121, 200, 185, 176, 163

93. 86, 79, 81, 69, 80

94. 92, 96, 90, 85, 92, 79

The normal monthly temperatures in degrees Fahrenheit for Salt Lake City, Utah, are given in the graph. Use this graph to answer Exercises 95 and 96. (Source: National Climatic Data Center)



95. Find the average temperature for June, July, and August.

96. Find the average temperature for October, November, and December.

Mixed Practice (Sections 1.3, 1.5, and 1.6) Perform each indicated operation. Watch the operation symbol.

97. $82 + 463 + 29 + 8704$

98. $23 + 407 + 92 + 7011$

99.
$$\begin{array}{r} 546 \\ \times 28 \\ \hline \end{array}$$

100.
$$\begin{array}{r} 712 \\ \times 54 \\ \hline \end{array}$$

101.
$$\begin{array}{r} 722 \\ - 43 \\ \hline \end{array}$$

102.
$$\begin{array}{r} 712 \\ - 54 \\ \hline \end{array}$$

103.
$$\frac{45}{0}$$

104.
$$\frac{0}{23}$$

105. $228 \div 24$

106. $304 \div 31$

Concept Extensions

Match each word phrase to the correct translation. (Not all letter choices will be used.) See the Concept Check in this section.

107. The quotient of 40 and 8

108. The quotient of 200 and 20

a. $20 \div 200$

b. $200 \div 20$

109. 200 divided by 20

110. 40 divided by 8

c. $40 \div 8$

d. $8 \div 40$

The following table shows the top five automakers and the amount of money they spent on U.S. television advertising during a recent year. Use this table to answer Exercises 111 and 112. (Source: Automotive News)

Automotive Company	Amount Spent on Television Advertising During a Recent Year
General Motors	\$3,500,000,000
Ford	\$2,680,000,000
Fiat Chrysler Automobiles	\$2,250,000,000
Toyota	\$1,800,000,000
Nissan	\$1,140,000,000

111. Find the average amount of money spent on television ads by the top two advertisers.

112. Find the average amount of money spent on television ads by the top four advertisers.

In Example 12 in this section, we found that the average of 75, 96, 81, and 88 is 85. Use this information to answer Exercises 113 and 114.

 113. If the number 75 is removed from the list of numbers, does the average increase or decrease? Explain why.

 114. If the number 96 is removed from the list of numbers, does the average increase or decrease? Explain why.

 115. Without computing it, tell whether the average of 126, 135, 198, and 113 is 86. Explain why it is possible or why it is not.

 116. Without computing it, tell whether the average of 38, 27, 58, and 43 is 17. Explain why it is possible or why it is not.

 117. If the area of a rectangle is 60 square feet and its width is 5 feet, what is its length?

 118. If the area of a rectangle is 84 square inches and its length is 21 inches, what is its width?

119. Write down any two numbers whose quotient is 25.

120. Write down any two numbers whose quotient is 1.

121. Find $26 \div 5$ using the process of repeated subtraction.

122. Find $86 \div 10$ using the process of repeated subtraction.

Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. _____
24. _____
25. _____
26. _____
27. _____
28. _____
29. _____
30. _____

Operations on Whole Numbers

Perform each indicated operation.

$$\begin{array}{r} 1. \quad 42 \\ \quad 63 \\ + 89 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 7006 \\ - 451 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 87 \\ \times 52 \\ \hline \end{array}$$

$$4. \quad 8 \overline{)4496}$$

$$5. \quad 1 \cdot 67$$

$$6. \quad \frac{36}{0}$$

$$7. \quad 16 \div 16$$

$$8. \quad 5 \div 1$$

$$9. \quad 0 \cdot 21$$

$$10. \quad 7 \cdot 0 \cdot 8$$

$$11. \quad 0 \div 7$$

$$12. \quad 12 \div 4$$

$$13. \quad 9 \cdot 7$$

$$14. \quad 45 \div 5$$

$$\begin{array}{r} 15. \quad 207 \\ - 69 \\ \hline \end{array}$$

$$\begin{array}{r} 16. \quad 207 \\ + 69 \\ \hline \end{array}$$

$$17. \quad 3718 - 2549$$

$$18. \quad 1861 + 7965$$

$$19. \quad 7 \overline{)1278}$$

$$\begin{array}{r} 20. \quad 1259 \\ \times 63 \\ \hline \end{array}$$

$$21. \quad 7 \overline{)7695}$$

$$22. \quad 9 \overline{)1000}$$

$$23. \quad 32 \overline{)21,240}$$

$$24. \quad 65 \overline{)70,000}$$

$$25. \quad 4000 - 2963$$

$$26. \quad 10,000 - 101$$

$$\begin{array}{r} 27. \quad 303 \\ \times 101 \\ \hline \end{array}$$

$$28. \quad (475)(100)$$

$$29. \quad \text{Find the total of 62 and 9.}$$

$$30. \quad \text{Find the product of 62 and 9.}$$

31. Find the quotient of 62 and 9.

32. Find the difference of 62 and 9.

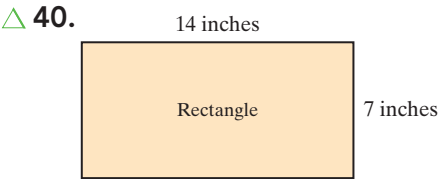
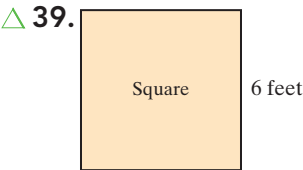
33. Subtract 17 from 200.

34. Find the difference of 432 and 201.

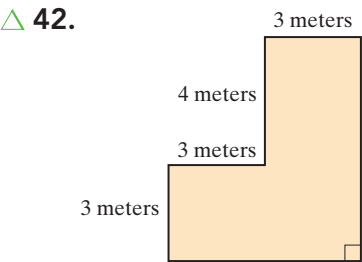
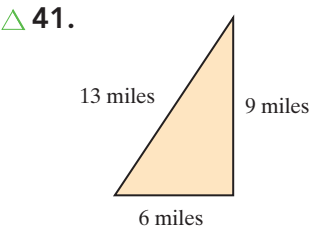
Complete the table by rounding the given number to the given place value.

		Tens	Hundrds	Thousands
35.	9735			
36.	1429			
37.	20,801			
38.	432,198			

Find the perimeter and area of each figure.



Find the perimeter of each figure.

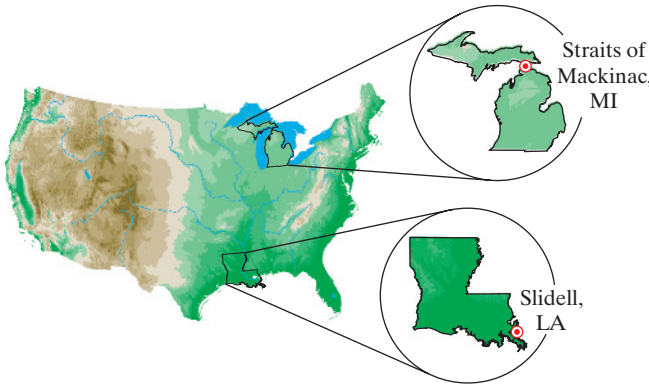


Find the average of each list of numbers.

43. 19, 15, 25, 37, 24

44. 108, 131, 98, 159

45. The Mackinac Bridge is a suspension bridge that connects the lower and upper peninsulas of Michigan across the Straits of Mackinac. Its total length is 26,372 feet. The Lake Pontchartrain Bridge is a twin concrete trestle bridge in Slidell, Louisiana. Its total length is 28,547 feet. Which bridge is longer and by how much? (Sources: Mackinac Bridge Authority and Federal Highway Administration, Bridge Division)



46. In the United States, the average toy expenditure per child is \$309 per year. On average, how much is spent on toys for a child by the time he or she reaches age 18? (Source: statista)

31. _____
32. _____
33. _____
34. _____
35. _____
36. _____
37. _____
38. _____
39. _____
40. _____
41. _____
42. _____
43. _____
44. _____
45. _____
46. _____

1.7 Exponents and Order of Operations

Objectives

- A** Write Repeated Factors Using Exponential Notation.
- B** Evaluate Expressions Containing Exponents.
- C** Use the Order of Operations.
- D** Find the Area of a Square.

Objective A Using Exponential Notation

In the product $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$, notice that 3 is a factor several times. When this happens, we can use a shorthand notation, called an **exponent**, to write the repeated multiplication.

$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ can be written as

3 is a factor 5 times

3^5 Read as "three to the fifth power."

base

This is called **exponential notation**. The **exponent**, 5, indicates how many times the **base**, 3, is a factor.

The table below shows examples of reading exponential notation in words.

Expression	In Words
5^2	"five to the second power" or "five squared"
5^3	"five to the third power" or "five cubed"
5^4	"five to the fourth power"

Usually, an exponent of 1 is not written, so when no exponent appears, we assume that the exponent is 1. For example, $2 = 2^1$ and $7 = 7^1$.

Practice 1–4

Write using exponential notation.

- $8 \cdot 8 \cdot 8 \cdot 8$
- $3 \cdot 3 \cdot 3$
- $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$
- $5 \cdot 5 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

Examples

Write using exponential notation.

- $7 \cdot 7 \cdot 7 = 7^3$
- $3 \cdot 3 = 3^2$
- $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 6^5$
- $3 \cdot 3 \cdot 3 \cdot 3 \cdot 9 \cdot 9 \cdot 9 = 3^4 \cdot 9^3$

Work Practice 1–4

Objective B Evaluating Exponential Expressions

To **evaluate** an exponential expression, we write the expression as a product and then find the value of the product.

Examples

Evaluate.

- $9^2 = 9 \cdot 9 = 81$
- $6^1 = 6$
- $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$
- $5 \cdot 6^2 = 5 \cdot 6 \cdot 6 = 180$

Work Practice 5–8

Practice 5–8

Evaluate.

- 4^2
- 11^1
- 7^3
- $2 \cdot 3^2$

Answers

- 8^4
- 3^3
- 10^5
- $5^2 \cdot 4^6$
- 16
- 343
- 11
- 18

Example 8 illustrates an important property: An exponent applies only to its base. The exponent 2, in $5 \cdot 6^2$, applies only to its base, 6.

Helpful Hint

An exponent applies only to its base. For example, $4 \cdot 2^3$ means $4 \cdot 2 \cdot 2 \cdot 2$.

Helpful Hint

Don't forget that 2^4 , for example, is *not* $2 \cdot 4$. The expression 2^4 means repeated multiplication of the same factor.

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16, \text{ whereas } 2 \cdot 4 = 8$$

✓ **Concept Check** Which of the following statements is correct?

- a. 3^5 is the same as $5 \cdot 5 \cdot 5$.
- b. “Ten cubed” is the same as 10^2 .
- c. “Six to the fourth power” is the same as 6^4 .
- d. 12^2 is the same as $12 \cdot 2$.

Objective C Using the Order of Operations

Suppose that you are in charge of taking inventory at a local cell phone store. An employee has given you the number of a certain cell phone in stock as the expression

$$6 + 2 \cdot 30$$

To calculate the value of this expression, do you add first or multiply first? If you add first, the answer is 240. If you multiply first, the answer is 66.



Mathematical symbols wouldn't be very useful if two values were possible for one expression. Thus, mathematicians have agreed that, given a choice, we multiply first.

$$\begin{aligned} 6 + 2 \cdot 30 &= 6 + 60 && \text{Multiply.} \\ &= 66 && \text{Add.} \end{aligned}$$

This agreement is one of several **order of operations** agreements.

Order of Operations

1. Perform all operations within parentheses (), brackets [], or other grouping symbols such as fraction bars, starting with the innermost set.
2. Evaluate any expressions with exponents.
3. Multiply or divide in order from left to right.
4. Add or subtract in order from left to right.

Below we practice using order of operations to simplify expressions.

Practice 9

Simplify: $9 \cdot 3 - 8 \div 4$

Practice 10

Simplify: $48 \div 3 \cdot 2^2$

Practice 11

Simplify: $(10 - 7)^4 + 2 \cdot 3^2$

Practice 12

Simplify:

$36 \div [20 - (4 \cdot 2)] + 4^3 - 6$

Example 9 Simplify: $2 \cdot 4 - 3 \div 3$

Solution: There are no parentheses and no exponents, so we start by multiplying and dividing, from left to right.

$$2 \cdot 4 - 3 \div 3 = 8 - 3 \div 3 \quad \text{Multiply.}$$

$$= 8 - 1 \quad \text{Divide.}$$

$$= 7 \quad \text{Subtract.}$$

Work Practice 9

Example 10 Simplify: $4^2 \div 2 \cdot 4$

Solution: We start by evaluating 4^2 .

$$4^2 \div 2 \cdot 4 = 16 \div 2 \cdot 4 \quad \text{Write } 4^2 \text{ as } 16.$$

Next we multiply or divide *in order* from left to right. Since division appears before multiplication from left to right, we divide first, then multiply.

$$16 \div 2 \cdot 4 = 8 \cdot 4 \quad \text{Divide.}$$

$$= 32 \quad \text{Multiply.}$$

Work Practice 10

Example 11 Simplify: $(8 - 6)^2 + 2^3 \cdot 3$

Solution: $(8 - 6)^2 + 2^3 \cdot 3 = 2^2 + 2^3 \cdot 3$ Simplify inside parentheses.

$$= 4 + 8 \cdot 3 \quad \text{Write } 2^2 \text{ as } 4 \text{ and } 2^3 \text{ as } 8.$$

$$= 4 + 24 \quad \text{Multiply.}$$

$$= 28 \quad \text{Add.}$$

Work Practice 11

Example 12 Simplify: $4^3 + [3^2 - (10 \div 2)] - 7 \cdot 3$

Solution: Here we begin with the innermost set of parentheses.

$$4^3 + [3^2 - (10 \div 2)] - 7 \cdot 3 = 4^3 + [3^2 - 5] - 7 \cdot 3 \quad \text{Simplify inside parentheses.}$$

$$= 4^3 + [9 - 5] - 7 \cdot 3 \quad \text{Write } 3^2 \text{ as } 9.$$

$$= 4^3 + 4 - 7 \cdot 3 \quad \text{Simplify inside brackets.}$$

$$= 64 + 4 - 7 \cdot 3 \quad \text{Write } 4^3 \text{ as } 64.$$

$$= 64 + 4 - 21 \quad \text{Multiply.}$$

$$= 47 \quad \text{Add and subtract from left to right.}$$

Work Practice 12

Answers

9. 25 10. 64 11. 99 12. 61

Example 13 Simplify: $\frac{7 - 2 \cdot 3 + 3^2}{5(2 - 1)}$

Solution: Here, the fraction bar is a grouping symbol. We simplify above and below the fraction bar separately.

$$\begin{aligned}\frac{7 - 2 \cdot 3 + 3^2}{5(2 - 1)} &= \frac{7 - 2 \cdot 3 + 9}{5(1)} && \text{Evaluate } 3^2 \text{ and } (2 - 1). \\ &= \frac{7 - 6 + 9}{5} && \text{Multiply } 2 \cdot 3 \text{ in the numerator and multiply 5 and 1 in the denominator.} \\ &= \frac{10}{5} && \text{Add and subtract from left to right.} \\ &= 2 && \text{Divide.}\end{aligned}$$

Work Practice 13

Example 14 Simplify: $64 \div 8 \cdot 2 + 4$

$$\begin{aligned}\text{Solution: } 64 \div 8 \cdot 2 + 4 &= \underbrace{8 \cdot 2} + 4 && \text{Divide.} \\ &= 16 + 4 && \text{Multiply.} \\ &= 20 && \text{Add.}\end{aligned}$$

Work Practice 14

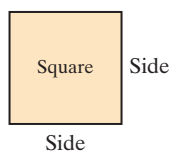
Objective D Finding the Area of a Square

Since a square is a special rectangle, we can find its area by finding the product of its length and its width.

$$\text{Area of a rectangle} = \text{length} \cdot \text{width}$$

By recalling that each side of a square has the same measurement, we can use the following procedure to find its area:

$$\begin{aligned}\text{Area of a square} &= \text{length} \cdot \text{width} \\ &= \text{side} \cdot \text{side} \\ &= (\text{side})^2\end{aligned}$$

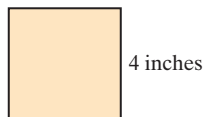


Helpful Hint

Recall from Section 1.5 that area is measured in **square** units while perimeter is measured in units.

Example 15 Find the area of a square whose side measures 4 inches.

$$\begin{aligned}\text{Solution: } \text{Area of a square} &= (\text{side})^2 \\ &= (4 \text{ inches})^2 \\ &= 16 \text{ square inches}\end{aligned}$$



The area of the square is 16 square inches.

Work Practice 15

Practice 13

Simplify: $\frac{25 + 8 \cdot 2 - 3^3}{2(3 - 2)}$

Practice 14

Simplify: $36 \div 6 \cdot 3 + 5$

Practice 15

Find the area of a square whose side measures 12 centimeters.

Answers

13. 7 14. 23 15. 144 sq cm



Calculator Explorations Exponents

To evaluate an exponential expression such as 4^7 on a calculator, find the keys marked y^x or $\wedge$ and $=$ or ENTER . To evaluate 4^7 , press the keys 4 y^x (or $\wedge$) 7 then $=$ or ENTER . The display will read 16384. Thus, $4^7 = 16,384$.

Use a calculator to evaluate.

1. 4^6
2. 5^6
3. 5^5
4. 7^6
5. 2^{11}
6. 6^8

Order of Operations

To see whether your calculator has the order of operations built in, evaluate $5 + 2 \cdot 3$ by pressing the keys 5 $+$ 2 $\times$ 3 then $=$ or ENTER . If the display reads 11, your calculator does have the order of operations built in. This means that most of the time, you can key in a problem exactly as it is written,

and the calculator will perform operations in the proper order. When evaluating an expression containing parentheses, key in the parentheses. (If an expression contains brackets, key in parentheses.) For example, to evaluate $2[25 - (8 + 4)] - 11$, press the keys 2 $\times$ $($ 25 $-$ $($ 8 $+$ 4 $)$ $)$ $-$ 11 then $=$ or ENTER .

The display will read 15.

Use a calculator to evaluate.

7. $7^4 + 5^3$
8. $12^4 - 8^4$
9. $63 \cdot 75 - 43 \cdot 10$
10. $8 \cdot 22 + 7 \cdot 16$
11. $4(15 \div 3 + 2) - 10 \cdot 2$
12. $155 - 2(17 + 3) + 185$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

addition	multiplication	exponent
subtraction	division	base

1. In $2^5 = 32$, the 2 is called the _____ and the 5 is called the _____.
2. To simplify $8 + 2 \cdot 6$, which operation should be performed first? _____
3. To simplify $(8 + 2) \cdot 6$, which operation should be performed first? _____
4. To simplify $9(3 - 2) \div 3 + 6$, which operation should be performed first? _____
5. To simplify $8 \div 2 \cdot 6$, which operation should be performed first? _____

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 1.7

- Objective A** 6. In the Example 1 expression, what is the 3 called and what is the 12 called?
- Objective B** 7. As mentioned in Example 4, what “understood exponent” does any number we’ve worked with before have?
- Objective C** 8. List the three operations needed to evaluate Example 7 in the order they should be performed.
- Objective D** 9. As explained in the lecture before Example 10, why does the area of a square involve an exponent whereas the area of a rectangle usually does not?

1.7 Exercise Set MyLab Math

Objective A Write using exponential notation. See Examples 1 through 4.

1. $4 \cdot 4 \cdot 4$

2. $5 \cdot 5 \cdot 5 \cdot 5$

3. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

4. $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

▶ 5. $12 \cdot 12 \cdot 12$

6. $10 \cdot 10 \cdot 10$

▶ 7. $6 \cdot 6 \cdot 5 \cdot 5 \cdot 5$

8. $4 \cdot 4 \cdot 3 \cdot 3 \cdot 3$

9. $9 \cdot 8 \cdot 8$

10. $7 \cdot 4 \cdot 4 \cdot 4$

11. $3 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

12. $4 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

13. $3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

14. $6 \cdot 6 \cdot 2 \cdot 9 \cdot 9 \cdot 9 \cdot 9$

Objective B Evaluate. See Examples 5 through 8.

15. 8^2

16. 6^2

▶ 17. 5^3

18. 6^3

19. 2^5

20. 3^5

21. 1^{10}

22. 1^{12}

▶ 23. 7^1

24. 8^1

25. 2^7

26. 5^4

27. 2^8

28. 3^3

29. 4^4

30. 4^3

31. 9^3

32. 8^3

33. 12^2

34. 11^2

▶ 35. 10^2

36. 10^3

37. 20^1

38. 14^1

39. 3^6

40. 4^5

41. $3 \cdot 2^6$

42. $5 \cdot 3^2$

43. $2 \cdot 3^4$

44. $2 \cdot 7^2$

Objective C Simplify. See Examples 9 through 14.

▶ 45. $15 + 3 \cdot 2$

46. $24 + 6 \cdot 3$

▶ 47. $14 \div 7 \cdot 2 + 3$

48. $100 \div 10 \cdot 5 + 4$

49. $32 \div 4 - 3$

50. $42 \div 7 - 6$

51. $13 + \frac{24}{8}$

52. $32 + \frac{8}{2}$

53. $6 \cdot 5 + 8 \cdot 2$

54. $3 \cdot 4 + 9 \cdot 1$

55. $\frac{5 + 12 \div 4}{1^7}$

56. $\frac{6 + 9 \div 3}{3^2}$

57. $(7 + 5^2) \div 4 \cdot 2^3$

58. $6^2 \cdot (10 - 8)$

59. $5^2 \cdot (10 - 8) + 2^3 + 5^2$

60. $5^3 \div (10 + 15) + 9^2 + 3^3$

61. $\frac{18 + 6}{2^4 - 2^2}$

62. $\frac{40 + 8}{5^2 - 3^2}$

63. $(3 + 5) \cdot (9 - 3)$

64. $(9 - 7) \cdot (12 + 18)$

▶ 65. $\frac{7(9 - 6) + 3}{3^2 - 3}$

66. $\frac{5(12 - 7) - 4}{5^2 - 18}$

67. $8 \div 0 + 37$

68. $18 - 7 \div 0$

69. $2^4 \cdot 4 - (25 \div 5)$

70. $2^3 \cdot 3 - (100 \div 10)$

71. $3^4 - [35 - (12 - 6)]$

72. $[40 - (8 - 2)] - 2^5$

73. $(7 \cdot 5) + [9 \div (3 \div 3)]$

74. $(18 \div 6) + [(3 + 5) \cdot 2]$

75. $8 \cdot [2^2 + (6 - 1) \cdot 2] - 50 \cdot 2$

76. $35 \div [3^2 + (9 - 7) - 2^2] + 10 \cdot 3$

77. $\frac{9^2 + 2^2 - 1^2}{8 \div 2 \cdot 3 \cdot 1 \div 3}$

78. $\frac{5^2 - 2^3 + 1^4}{10 \div 5 \cdot 4 \cdot 1 \div 4}$

79. $\frac{2 + 4^2}{5(20 - 16) - 3^2 - 5}$

80. $\frac{3 + 9^2}{3(10 - 6) - 2^2 - 1}$

81. $9 \div 3 + 5^2 \cdot 2 - 10$

82. $10 \div 2 + 3^3 \cdot 2 - 20$

83. $[13 \div (20 - 7) + 2^5] - (2 + 3)^2$

84. $[15 \div (11 - 6) + 2^2] + (5 - 1)^2$

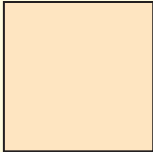
85. $7^2 - \{18 - [40 \div (5 \cdot 1) + 2] + 5^2\}$

86. $29 - \{5 + 3[8 \cdot (10 - 8)] - 50\}$

△ **Objective D Mixed Practice (Sections 1.3 and 1.7)** Find the area and perimeter of each square.
See Example 15.

▶ 87.  7 meters

△ 88.  9 centimeters

△ 89.  23 miles

△ 90.  41 feet

Concept Extensions

Answer the following true or false. See the Concept Check in this section.

91. “Six to the fifth power” is the same as 6^5 .

92. “Seven squared” is the same as 7^2 .

93. 2^5 is the same as $5 \cdot 5$.

94. 4^9 is the same as $4 \cdot 9$.

Insert grouping symbols (parentheses) so that each given expression evaluates to the given number.

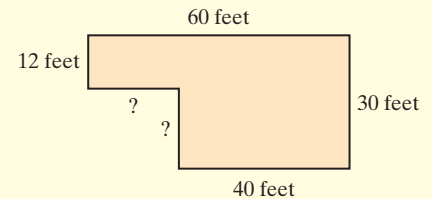
95. $2 + 3 \cdot 6 - 2$; evaluates to 28

96. $2 + 3 \cdot 6 - 2$; evaluates to 20

97. $24 \div 3 \cdot 2 + 2 \cdot 5$; evaluates to 14

98. $24 \div 3 \cdot 2 + 2 \cdot 5$; evaluates to 15

- △ 99. A building contractor is bidding on a contract to install gutters on seven homes in a retirement community, all in the shape shown. To estimate the cost of materials, she needs to know the total perimeter of all seven homes. Find the total perimeter.



100. The building contractor from Exercise 99 plans to charge \$4 per foot for installing vinyl gutters. Find the total charge for the seven homes given the total perimeter answer to Exercise 99.

Simplify.

101. $(7 + 2^4)^5 - (3^5 - 2^4)^2$

102. $25^3 \cdot (45 - 7 \cdot 5) \cdot 5$

- ✎ 103. Write an expression that simplifies to 5. Use multiplication, division, addition, subtraction, and at least one set of parentheses. Explain the process you would use to simplify the expression.
- ✎ 104. Explain why $2 \cdot 3^2$ is not the same as $(2 \cdot 3)^2$.

1.8 Introduction to Variables, Algebraic Expressions, and Equations

Objective A Evaluating Algebraic Expressions

Perhaps the most important quality of mathematics is that it is a science of patterns. Communicating about patterns is often made easier by using a letter to represent all the numbers fitting a pattern. We call such a letter a **variable**. For example, in Section 1.3 we presented the addition property of 0, which states that the sum of 0 and any number is that number. We might write

$$\begin{aligned} 0 + 1 &= 1 \\ 0 + 2 &= 2 \\ 0 + 3 &= 3 \\ 0 + 4 &= 4 \\ 0 + 5 &= 5 \\ 0 + 6 &= 6 \\ &\vdots \end{aligned}$$

Objectives

- A** Evaluate Algebraic Expressions Given Replacement Values.
- B** Identify Solutions of Equations.
- C** Translate Phrases into Variable Expressions.

(Continued on next page)

continuing indefinitely. This is a pattern, and all whole numbers fit the pattern. We can communicate this pattern for all whole numbers by letting a letter, such as a , represent all whole numbers. We can then write

$$0 + a = a$$

Using variable notation is a primary goal of learning **algebra**. We now take some important first steps in beginning to use variable notation.

A combination of operations on letters (variables) and numbers is called an **algebraic expression** or simply an **expression**.

Algebraic Expressions

$$3 + x \quad 5 \cdot y \quad 2 \cdot z - 1 + x$$

If two variables or a number and a variable are next to each other, with no operation sign between them, the operation is multiplication. For example,

$$2x \text{ means } 2 \cdot x$$

and

$$xy \text{ or } x(y) \text{ means } x \cdot y$$

Also, the meaning of an exponent remains the same when the base is a variable. For example,

$$x^2 = \underbrace{x \cdot x}_{\text{2 factors of } x} \quad \text{and} \quad y^5 = \underbrace{y \cdot y \cdot y \cdot y \cdot y}_{\text{5 factors of } y}$$

Algebraic expressions such as $3x$ have different values depending on replacement values for x . For example, if x is 2, then $3x$ becomes

$$\begin{aligned} 3x &= 3 \cdot 2 \\ &= 6 \end{aligned}$$

If x is 7, then $3x$ becomes

$$\begin{aligned} 3x &= 3 \cdot 7 \\ &= 21 \end{aligned}$$

Replacing a variable in an expression by a number and then finding the value of the expression is called **evaluating the expression**. When finding the value of an expression, remember to follow the order of operations given in Section 1.7.

Practice 1

Evaluate $x - 2$ if x is 7.

Example 1 Evaluate $x + 6$ if x is 8.

Solution: Replace x with 8 in the expression $x + 6$.

$$\begin{aligned} x + 6 &= 8 + 6 && \text{Replace } x \text{ with 8.} \\ &= 14 && \text{Add.} \end{aligned}$$

Work Practice 1

When we write a statement such as “ x is 5,” we can use an equal sign ($=$) to represent “is” so that

$$x \text{ is 5 can be written as } x = 5.$$

Answer

1. 5

Example 2 Evaluate $2(x - y)$ for $x = 6$ and $y = 3$.

Solution: $2(x - y) = 2(6 - 3)$ Replace x with 6 and y with 3.
 $= 2(3)$ Subtract.
 $= 6$ Multiply.

Work Practice 2

Example 3 Evaluate $\frac{x - 5y}{y}$ for $x = 35$ and $y = 5$.

Solution: $\frac{x - 5y}{y} = \frac{35 - 5(5)}{5}$ Replace x with 35 and y with 5.
 $= \frac{35 - 25}{5}$ Multiply.
 $= \frac{10}{5}$ Subtract.
 $= 2$ Divide.

Work Practice 3

Example 4 Evaluate $x^2 + z - 3$ for $x = 5$ and $z = 4$.

Solution: $x^2 + z - 3 = 5^2 + 4 - 3$ Replace x with 5 and z with 4.
 $= 25 + 4 - 3$ Evaluate 5^2 .
 $= 26$ Add and subtract from left to right.

Work Practice 4

Helpful Hint

If you are having difficulty replacing variables with numbers, first replace each variable with a set of parentheses, then insert the replacement number within the parentheses.

Example:

$$\begin{aligned} x^2 + z - 3 &= ()^2 + () - 3 \\ &= (5)^2 + (4) - 3 \\ &= 25 + 4 - 3 \\ &= 26 \end{aligned}$$

 **Concept Check** What's wrong with the solution to the following problem?
 Evaluate $3x + 2y$ for $x = 2$ and $y = 3$.

Solution: $3x + 2y = 3(3) + 2(2)$
 $= 9 + 4$
 $= 13$

Practice 2

Evaluate $y(x - 3)$ for $x = 8$ and $y = 4$.

Practice 3

Evaluate $\frac{y + 6}{x}$ for $x = 6$ and $y = 18$.

Practice 4

Evaluate $25 - z^3 + x$ for $z = 2$ and $x = 1$.

Answers

2. 20 3. 4 4. 18

✓ Concept Check Answer

$$\begin{aligned} 3x + 2y &= 3(2) + 2(3) \\ &= 6 + 6 \\ &= 12 \end{aligned}$$

Practice 5

Evaluate $\frac{5(F - 32)}{9}$ for $F = 41$.

Example 5

The expression $\frac{5(F - 32)}{9}$ can be used to write degrees Fahrenheit F as degrees Celsius C . Find the value of this expression for $F = 86$.

Solution:

$$\begin{aligned}\frac{5(F - 32)}{9} &= \frac{5(86 - 32)}{9} \\ &= \frac{5(54)}{9} \\ &= \frac{270}{9} \\ &= 30\end{aligned}$$

Thus 86°F is the same temperature as 30°C.

Work Practice 5**Objective B** Identifying Solutions of Equations

In Objective **A**, we learned that a combination of operations on variables and numbers is called an algebraic expression or simply an expression. Frequently in this book, we have written statements like $7 + 4 = 11$ or $\text{area} = \text{length} \cdot \text{width}$. Each of these statements is called an **equation**. An equation is of the form

$$\text{expression} = \text{expression}$$

An equation can be labeled as

$$\begin{array}{c} \text{equal sign} \\ \downarrow \\ x + 7 = 10 \\ \uparrow \quad \uparrow \\ \text{left side} \quad \text{right side} \end{array}$$

Helpful Hint

An equation contains “=,” while an expression does not.

When an equation contains a variable, deciding which values of the variable make the equation a true statement is called **solving** the equation for the variable. A **solution** of an equation is a value for the variable that makes the equation a true statement. For example, 2 is a solution of the equation $x + 5 = 7$, since replacing x with 2 results in the *true* statement $2 + 5 = 7$. Similarly, 3 is not a solution of $x + 5 = 7$, since replacing x with 3 results in the *false* statement $3 + 5 = 7$.

Practice 6

Determine whether 8 is a solution of the equation $3(y - 6) = 6$.

Example 6

Determine whether 6 is a solution of the equation $4(x - 3) = 12$.

Solution: We replace x with 6 in the equation.

$$\begin{aligned}4(x - 3) &= 12 \\ \downarrow \\ 4(6 - 3) &\stackrel{?}{=} 12 && \text{Replace } x \text{ with 6.} \\ 4(3) &\stackrel{?}{=} 12 \\ 12 &= 12 && \text{True}\end{aligned}$$

Since $12 = 12$ is a true statement, 6 is a solution of the equation.

Work Practice 6**Answers**

5. 5 6. yes

A collection of numbers enclosed by braces is called a set. For example,

$$\{0, 1, 2, 3, \dots\}$$

is the set of whole numbers that we are studying about in this chapter. The three dots after the number 3 in the set mean that this list of numbers continues in the same manner indefinitely.

The next example contains set notation.

Example 7 Determine which numbers in the set $\{26, 40, 20\}$ are solutions of the equation $2n - 30 = 10$.

Solution: Replace n with each number from the set to see if a true statement results.

Let n be 26.	Let n be 40.	Let n be 20.
$2n - 30 = 10$	$2n - 30 = 10$	$2n - 30 = 10$
$2 \cdot 26 - 30 \stackrel{?}{=} 10$	$2 \cdot 40 - 30 \stackrel{?}{=} 10$	$2 \cdot 20 - 30 \stackrel{?}{=} 10$
$52 - 30 \stackrel{?}{=} 10$	$80 - 30 \stackrel{?}{=} 10$	$40 - 30 \stackrel{?}{=} 10$
$22 = 10$ False	$50 = 10$ False	$10 = 10$ True ✓

Thus, 20 is a solution while 26 and 40 are not solutions.

Work Practice 7

Objective C Translating Phrases into Variable Expressions

To aid us in solving problems later, we practice translating verbal phrases into algebraic expressions. Certain key words and phrases suggesting addition, subtraction, multiplication, or division are reviewed next.

Addition (+)	Subtraction (-)	Multiplication (·)	Division (÷)
sum	difference	product	quotient
plus	minus	times	divide
added to	subtract	multiply	shared equally among
more than	less than	multiply by	per
increased by	decreased by	of	divided by
total	less	double/triple	divided into

Example 8 Write as an algebraic expression. Use x to represent “a number.”

- 7 increased by a number
- 15 decreased by a number
- The product of 2 and a number
- The quotient of a number and 5
- 2 subtracted from a number

Solution:

- In words: 7 increased by a number
Translate: 7 + x

(Continued on next page)

Practice 7

Determine which numbers in the set $\{10, 6, 8\}$ are solutions of the equation $5n + 4 = 34$.

Practice 8

Write as an algebraic expression. Use x to represent “a number.”

- Twice a number
- 8 increased by a number
- 10 minus a number
- 10 subtracted from a number
- The quotient of 6 and a number

Answers

7. 6 is a solution.

8. a. $2x$ b. $8 + x$ c. $10 - x$

d. $x - 10$ e. $6 \div x$ or $\frac{6}{x}$

b. In words: 15 decreased by a number
Translate: $15 - x$

c. In words: The product of 2 and a number
Translate: $2 \cdot x$ or $2x$

d. In words: The quotient of a number and 5
Translate: $x \div 5$ or $\frac{x}{5}$

e. In words: 2 subtracted from a number
Translate: $x - 2$

Work Practice 8

Helpful Hint

Remember that order is important when subtracting. Study the order of numbers and variables below.

Phrase	Translation
a number <i>decreased by</i> 5	$x - 5$
a number <i>subtracted from</i> 5	$5 - x$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. You may use each choice more than once.

evaluating the expression variable(s) expression equation solution

- 1. A combination of operations on letters (variables) and numbers is a(n) _____.
- 2. A letter that represents a number is a(n) _____.
- 3. $3x - 2y$ is called a(n) _____ and the letters x and y are _____.
- 4. Replacing a variable in an expression by a number and then finding the value of the expression is called _____.
- 5. A statement of the form “expression = expression” is called a(n) _____.
- 6. A value for the variable that makes an equation a true statement is called a(n) _____.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 1.8

- Objective A** 7. Complete this statement based on the lecture before
 Example 1: When a letter and a variable are next to each other, the operation is an understood _____.
- Objective B** 8. In Example 5, why is a question mark written over the equal sign?
- Objective C** 9. In Example 6, what phrase translates to subtraction?

1.8 Exercise Set MyLab Math

Objective A Complete the table. The first row has been done for you. See Examples 1 through 5.

	a	b	$a + b$	$a - b$	$a \cdot b$	$a \div b$
	45	9	54	36	405	5
1.	21	7				
2.	24	6				
3.	152	0				
4.	298	0				
5.	56	1				
6.	82	1				

Evaluate each following expression for $x = 2$, $y = 5$, and $z = 3$. See Examples 1 through 5.

7. $3 + 2z$ 8. $7 + 3z$ 9. $3xz - 5x$ 10. $4yz + 2x$
 11. $z - x + y$ 12. $x + 5y - z$ 13. $4x - z$ 14. $2y + 5z$
 15. $y^3 - 4x$ 16. $y^3 - z$ 17. $2xy^2 - 6$ 18. $3yz^2 + 1$
 19. $8 - (y - x)$ 20. $3 + (2y - 4)$ 21. $x^5 + (y - z)$ 22. $x^4 - (y - z)$
 23. $\frac{6xy}{z}$ 24. $\frac{8yz}{15}$ 25. $\frac{2y - 2}{x}$ 26. $\frac{6 + 3x}{z}$
 27. $\frac{x + 2y}{z}$ 28. $\frac{2z + 6}{3}$ 29. $\frac{5x}{y} - \frac{10}{y}$ 30. $\frac{70}{2y} - \frac{15}{z}$
 31. $2y^2 - 4y + 3$ 32. $3x^2 + 2x - 5$ 33. $(4y - 5z)^3$ 34. $(4y + 3z)^2$
 35. $(xy + 1)^2$ 36. $(xz - 5)^4$ 37. $2y(4z - x)$ 38. $3x(y + z)$

39. $xy(5 + z - x)$

40. $xz(2y + x - z)$

41. $\frac{7x + 2y}{3x}$

42. $\frac{6z + 2y}{4}$
43. The expression $16t^2$ gives the distance in feet that an object falls after t seconds. Complete the table by evaluating $16t^2$ for each given value of t .

44. The expression $\frac{5(F - 32)}{9}$ gives the equivalent degrees Celsius for F degrees Fahrenheit. Complete the table by evaluating this expression for each given value of F .
- | | | | | |
|---------|---|---|---|---|
| t | 1 | 2 | 3 | 4 |
| $16t^2$ | | | | |

F	50	59	68	77
$\frac{5(F - 32)}{9}$				

Objective B Decide whether the given number is a solution of the given equation. See Example 6.

45. Is 10 a solution of $n - 8 = 2$?

46. Is 9 a solution of $n - 2 = 7$?
47. Is 3 a solution of $24 = 80n$?

48. Is 50 a solution of $250 = 5n$?
49. Is 7 a solution of $3n - 5 = 10$?

50. Is 8 a solution of $11n + 3 = 91$?
51. Is 20 a solution of $2(n - 17) = 6$?

52. Is 0 a solution of $5(n + 9) = 40$?
53. Is 0 a solution of $5x + 3 = 4x + 13$?

54. Is 2 a solution of $3x - 6 = 5x - 10$?
55. Is 8 a solution of $7f = 64 - f$?

56. Is 5 a solution of $8x - 30 = 2x$?

Determine which numbers in each set are solutions to the corresponding equations. See Example 7.

57. $n - 2 = 10$; {10, 12, 14}

58. $n + 3 = 16$; {9, 11, 13}
59. $5n = 30$; {6, 25, 30}

60. $3n = 45$; {15, 30, 45}
61. $6n + 2 = 26$; {0, 2, 4}

62. $4n - 14 = 6$; {0, 5, 10}
63. $3(n - 4) = 10$; {5, 7, 10}

64. $6(n + 2) = 23$; {1, 3, 5}
65. $7x - 9 = 5x + 13$; {3, 7, 11}

66. $9x - 15 = 5x + 1$; {2, 4, 11}

Objective C Translating Write each phrase as a variable expression. Use x to represent “a number.” See Example 8.

67. Eight more than a number

68. The sum of three and a number
69. The total of a number and eight

70. The difference of a number and five hundred

71. Twenty decreased by a number
72. A number less thirty
73. The product of 512 and a number
74. A number times twenty
75. The quotient of eight and a number
76. A number divided by 11
77. The sum of seventeen and a number added to the product of five and the number
78. The quotient of twenty and a number, decreased by three
79. The product of five and a number
80. The difference of twice a number, and four
81. A number subtracted from 11
82. Twelve subtracted from a number
83. A number less 5
84. The sum of a number and 7
85. 6 divided by a number
86. The product of a number and 7
87. Fifty decreased by eight times a number
88. Twenty decreased by twice a number

Concept Extensions

For Exercises 89 through 92, find and correct the error when evaluating each expression for $x = 3$ and $y = 5$. See the Concept Check in this section.

89. $2x + 3y = 2(5) + 3(3)$
 $= 10 + 9$
 $= 19$

90. $6y + 3x = 6(3) + 3(5)$
 $= 18 + 15$
 $= 33$

91. $7x + y^2 = 7(3) + 5^2$
 $= 21 + 10$
 $= 31$

92. $x^3 + 4y = 3^3 + 4(5)$
 $= 9 + 20$
 $= 29$

For Exercises 93 through 100, use a calculator to evaluate each expression for $x = 23$ and $y = 72$.

 93. $x^4 - y^2$

 94. $2(x + y)^2$

 95. $x^2 + 5y - 112$

 96. $16y - 20x + x^3$

-  97. If x is a whole number, which expression is the largest: $2x$, $5x$, or $\frac{x}{3}$?
 Explain your answer.

-  98. If x is a whole number, which expression is the smallest: $2x$, $5x$, or $\frac{x}{3}$?
 Explain your answer.

-  99. In Exercise 43, what do you notice about the value of $16t^2$ as t gets larger?

-  100. In Exercise 44, what do you notice about the value of $\frac{5(F - 32)}{9}$ as F gets larger?

Chapter 1 Group Activity

Investigating Endangered and Threatened Species

An **endangered** species is one that is thought to be in danger of becoming extinct throughout all or a major part of its habitat. A **threatened** species is one that may become endangered. The Division of Endangered Species at the U.S. Fish and Wildlife Service keeps close tabs on the state of threatened and endangered wildlife in the United States and around the world. The table below was compiled from early 2017 data in the Division of Endangered Species' box score. The "Total Species" column gives the total number of endangered and threatened species for each group.

1. Round each number of *endangered animal species* to the nearest ten to estimate the Animal Total.
2. Round each number of *endangered plant species* to the nearest ten to estimate the Plant Total.
3. Add the exact numbers of endangered animal species to find the exact Animal Total and record it in the table in the Endangered Species column. Add the exact numbers of endangered plant species to find the Plant Total and record it in the table in the Endangered Species column. Then find the total number of endangered species (animals and plants combined) and record this number in the table as the Grand Total in the Endangered Species column.
4. Find the Animal Total, Plant Total, and Grand Total for the Total Species column. Record these values in the table.
5. Use the data in the table to complete the Threatened Species column.
6. Write a paragraph discussing the conclusions that can be drawn from the table.

Endangered and Threatened Species Worldwide				
Animals	Group	Endangered Species	Threatened Species	Total Species
	Mammals	329		377
	Birds	299		337
	Reptiles	87		139
	Amphibians	28		44
	Fishes	114		191
	Snails	41		53
	Clams	77		91
	Crustaceans	23		27
	Insects	69		80
	Arachnids	12		12
	Corals	1		23
	Animal Total			
Plants	Flowering Plants	704		867
	Conifers	1		6
	Ferns and others	29		31
	Lichens	2		2
	Plant Total			
Grand Total				

Chapter 1 Vocabulary Check

Fill in each blank with one of the words or phrases listed below.

difference	factor	perimeter	dividend	minuend	
place value	whole numbers	equation	divisor	variable	
sum	set	addend	exponent	expression	
solution	quotient	subtrahend	product	digits	area

- The _____ are 0, 1, 2, 3, . . .
- The _____ of a polygon is its distance around or the sum of the lengths of its sides.
- The position of each digit in a number determines its _____.
- A(n) _____ is a shorthand notation for repeated multiplication of the same factor.
- To find the _____ of a rectangle, multiply length times width.
- The _____ used to write numbers are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.
- A letter used to represent a number is called a(n) _____.
- A(n) _____ can be written in the form “expression = expression.”
- A combination of operations on variables and numbers is called a(n) _____.
- A(n) _____ of an equation is a value of the variable that makes the equation a true statement.
- A collection of numbers (or objects) enclosed by braces is called a(n) _____.

Use the facts below for Exercises 12 through 21.

$$2 \cdot 3 = 6 \quad 4 + 17 = 21 \quad 20 - 9 = 11 \quad \begin{array}{r} 7 \\ 5 \overline{)35} \end{array}$$

- The 21 above is called the _____.
- The 5 above is called the _____.
- The 35 above is called the _____.
- The 7 above is called the _____.
- The 3 above is called a(n) _____.
- The 6 above is called the _____.
- The 20 above is called the _____.
- The 9 above is called the _____.
- The 11 above is called the _____.
- The 4 above is called a(n) _____.

Helpful Hint

▶ Are you preparing for your test?

To help, don't forget to take these:

- Chapter 1 Getting Ready for the Test on page 95
- Chapter 1 Test on page 96

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

1

Chapter Highlights

Definitions and Concepts

Section 1.2 Place Value, Names for Numbers, and Reading Tables

The **whole numbers** are 0, 1, 2, 3, 4, 5, ...

The position of each digit in a number determines its **place value**. A place-value chart is shown next with the names of the periods given.

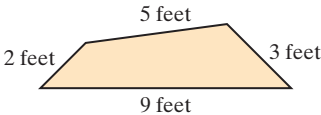
Periods												
Billions				Millions			Thousands			Ones		
Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones	
1	0	8	3	6	6	4	5	0	0			

Examples

Examples of whole numbers:

0, 14, 968, 5, 268, 619

(continued)

Definitions and Concepts	Examples
Section 1.2 Place Value, Names for Numbers, and Reading Tables (continued)	
To write a whole number in words, write the number in each period followed by the name of the period. (The name of the ones period is not included.) To write a whole number in standard form, write the number in each period, followed by a comma.	9,078,651,002 is written as nine billion, seventy-eight million, six hundred fifty-one thousand, two. Four million, seven hundred six thousand, twenty-eight is written as 4,706,028.
Section 1.3 Adding and Subtracting Whole Numbers, and Perimeter	
To add whole numbers, add the digits in the ones place, then the tens place, then the hundreds place, and so on, carrying when necessary. To subtract whole numbers, subtract the digits in the ones place, then the tens place, then the hundreds place, and so on, borrowing when necessary. The perimeter of a polygon is its distance around or the sum of the lengths of its sides.	Find the sum: $\begin{array}{r} 211 \\ 2689 \\ 1735 \\ + 662 \\ \hline 5086 \end{array}$ ← addend ← addend ← addend ← sum Subtract: $\begin{array}{r} 815 \\ 7954 \\ - 5673 \\ \hline 2281 \end{array}$ ← minuend ← subtrahend ← difference △ Find the perimeter of the polygon shown.  The perimeter is 5 feet + 3 feet + 9 feet + 2 feet = 19 feet.
Section 1.4 Rounding and Estimating	
Rounding Whole Numbers to a Given Place Value Step 1: Locate the digit to the right of the given place value. Step 2: If this digit is 5 or greater, add 1 to the digit in the given place value and replace each digit to its right with 0. Step 3: If this digit is less than 5, replace it and each digit to its right with 0.	Round 15,721 to the nearest thousand. 15, <u>7</u> 21 Add 1 ↑ Replace with zeros. ↑ Since the circled digit is 5 or greater, add 1 to the given place value and replace digits to its right with zeros. 15,721 rounded to the nearest thousand is 16,000.

Definitions and Concepts

Examples

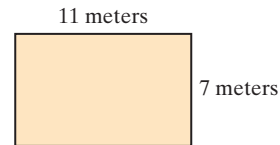
Section 1.5 Multiplying Whole Numbers and Area

To multiply 73 and 58, for example, multiply 73 and 8, then 73 and 50. The sum of these partial products is the product of 73 and 58. Use the notation to the right.

To find the **area** of a rectangle, multiply length times width.

$$\begin{array}{r} 73 \leftarrow \text{factor} \\ \times 58 \leftarrow \text{factor} \\ \hline 584 \leftarrow 73 \times 8 \\ 3650 \leftarrow 73 \times 50 \\ \hline 4234 \leftarrow \text{product} \end{array}$$

△ Find the area of the rectangle shown.



$$\begin{aligned} \text{area of rectangle} &= \text{length} \cdot \text{width} \\ &= (11 \text{ meters})(7 \text{ meters}) \\ &= 77 \text{ square meters} \end{aligned}$$

Section 1.6 Dividing Whole Numbers

Division Properties of 0

The quotient of 0 and any number (except 0) is 0.

The quotient of any number and 0 is not a number. We say that this quotient is undefined.

To divide larger whole numbers, use the process called **long division** as shown to the right.

The **average** of a list of numbers is

$$\text{average} = \frac{\text{sum of numbers}}{\text{number of numbers}}$$

$$\frac{0}{5} = 0$$

$$\frac{7}{0} \text{ is undefined}$$

$$\begin{array}{r} 507 \text{ R } 2 \leftarrow \text{quotient} \\ \text{divisor} \rightarrow 14 \overline{) 7100} \leftarrow \text{dividend} \\ \underline{-70} \\ 10 \\ \underline{-0} \\ 100 \\ \underline{-98} \\ 2 \end{array}$$

$5(14) = 70$
 Subtract and bring down the 0.
 $0(14) = 0$
 Subtract and bring down the 0.
 $7(14) = 98$
 Subtract. The remainder is 2.

To check, see that $507 \cdot 14 + 2 = 7100$.

Find the average of 23, 35, and 38.

$$\text{average} = \frac{23 + 35 + 38}{3} = \frac{96}{3} = 32$$

Definitions and Concepts	Examples
Section 1.7 Exponents and Order of Operations	
An exponent is a shorthand notation for repeated multiplication of the same factor. Order of Operations 1. Perform all operations within parentheses (), brackets [], or other grouping symbols such as fraction bars, starting with the innermost set. 2. Evaluate any expressions with exponents. 3. Multiply or divide in order from left to right. 4. Add or subtract in order from left to right. The area of a square is (side)².	↙ exponent $3^4 = \underbrace{3 \cdot 3 \cdot 3 \cdot 3}_{\substack{\uparrow \text{base} \quad 4 \text{ factors of } 3}} = 81$ Simplify: $\frac{5 + 3^2}{2(7 - 6)}$ Simplify above and below the fraction bar separately. $\frac{5 + 3^2}{2(7 - 6)} = \frac{5 + 9}{2(1)}$ Evaluate 3² above the fraction bar. Subtract 7 - 6 below the fraction bar. $= \frac{14}{2}$ Add. Multiply. $= 7$ Divide. Find the area of a square with side length 9 inches. Area of the square = (side)² = (9 inches)² = 81 square inches
Section 1.8 Introduction to Variables, Algebraic Expressions, and Equations	
A letter used to represent a number is called a variable. A combination of operations on variables and numbers is called an algebraic expression. Replacing a variable in an expression by a number, and then finding the value of the expression, is called evaluating the expression. A statement written in the form “expression = expression” is an equation. A solution of an equation is a value for the variable that makes the equation a true statement.	Variables: x, y, z, a, b Algebraic expressions: $3 + x, 7y, x^3 + y - 10$ Evaluate $2x + y$ for $x = 22$ and $y = 4$. $2x + y = 2 \cdot 22 + 4$ Replace x with 22 and y with 4. $= 44 + 4$ Multiply. $= 48$ Add. Equations: $n - 8 = 12$ $2(20 - 7n) = 32$ Area = length · width Determine whether 2 is a solution of the equation $4(x - 1) = 7$. $4(2 - 1) \stackrel{?}{=} 7$ Replace x with 2. $4(1) \stackrel{?}{=} 7$ Subtract. $4 = 7$ False No, 2 is not a solution.

(1.2) Determine the place value of the digit 4 in each whole number.

1. 7640

2. 46,200,120

Write each whole number in words.

3. 7640

4. 46,200,120

Write each whole number in expanded form.

5. 3158

6. 403,225,000

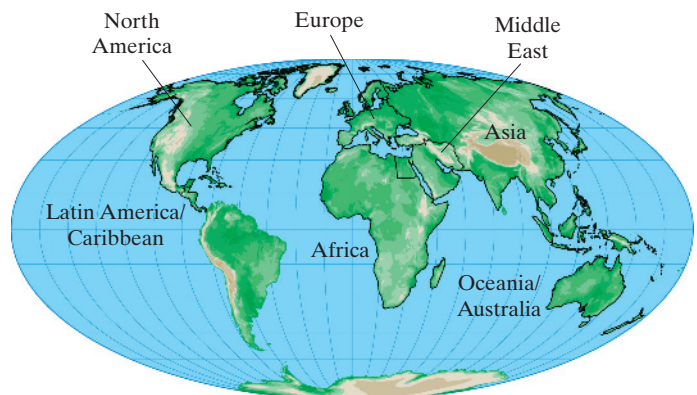
Write each whole number in standard form.

7. Eighty-one thousand, nine hundred

8. Six billion, three hundred four million

The following table shows the Internet use by world regions. Use this table to answer Exercises 9 through 12 and other exercises throughout this review. (Source: International Telecommunications Union and Internet World Stats)

Internet Use by World Regions (in millions)			
World Region	2016	2013	2008
Africa	341	140	51
Asia	1846	1268	579
Europe	615	467	385
Middle East	141	141	42
North America	320	302	248
Latin America/Caribbean	385	280	139
Oceania/Australia	28	24	20



9. Find the number of Internet users in 2016 in Europe. Write your answer in standard form.

10. Find the number of Internet users in Oceania/Australia in 2016. Write your answer in standard form.

11. Which world region had the smallest number of Internet users in 2013?

12. Which world region had the greatest number of Internet users in 2008?

(1.3) Add or subtract as indicated.

13. $18 + 49$

14. $28 + 39$

15. $462 - 397$

16. $583 - 279$

17. $428 + 21$

18. $819 + 21$

19. $4000 - 86$

20. $8000 - 92$

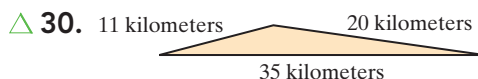
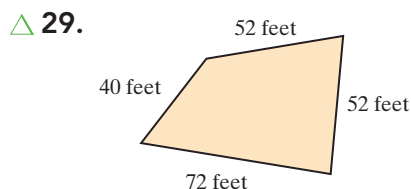
21. $91 + 3623 + 497$

22. $82 + 1647 + 238$

Translating *Solve.*

- 23.** Find the sum of 74, 342, and 918.
- 24.** Find the sum of 49, 529, and 308.
- 25.** Subtract 7965 from 25,862.
- 26.** Subtract 4349 from 39,007.
- 27.** The distance from Washington, DC, to New York City is 205 miles. The distance from New York City to New Delhi, India, is 7318 miles. Find the total distance from Washington, DC, to New Delhi if traveling by air through New York City.
- 28.** Susan Summerline earned salaries of \$62,589, \$65,340, and \$69,770 during the years 2004, 2005, and 2006, respectively. Find her total earnings during those three years.

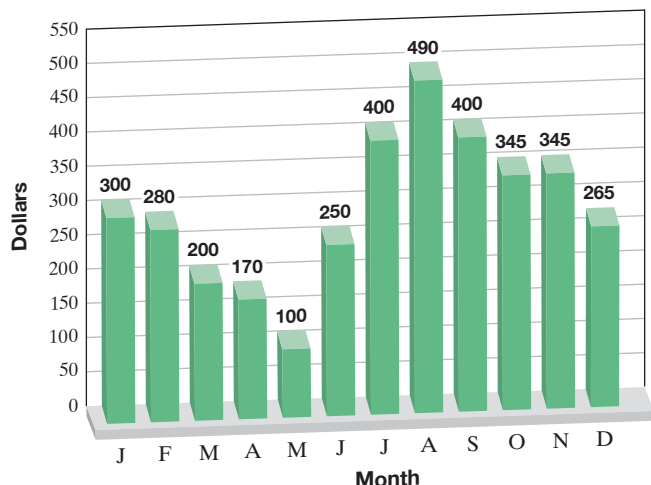
Find the perimeter of each figure.



Use the Internet Use by World Regions table on page 89 for Exercises 31 and 32.

- 31.** Find the increase in Internet users in Europe from 2008 to 2013.
- 32.** Find the difference in the number of Internet users in 2016 between the Middle East and Oceania/Australia.

The following bar graph shows the monthly savings account balances for a freshman attending a local community college. Use this graph to answer Exercises 33 through 36.



- 33.** During what month was the balance the least?
- 34.** During what month was the balance the greatest?
- 35.** By how much did the balance decrease from February to April?
- 36.** By how much did the balance increase from June to August?

(1.4) Round to the given place value.

37. 43 to the nearest ten

38. 45 to the nearest ten

39. 876 to the nearest ten

40. 493 to the nearest hundred

41. 3829 to the nearest hundred

42. 57,534 to the nearest thousand

43. 39,583,819 to the nearest million

44. 768,542 to the nearest hundred-thousand

Estimate the sum or difference by rounding each number to the nearest hundred.

45. $3785 + 648 + 2866$

46. $5925 - 1787$

47. A group of students took a week-long driving trip and traveled 630, 192, 271, 56, 703, 454, and 329 miles on seven consecutive days. Round each distance to the nearest hundred to estimate the distance they traveled.

48. In 2016, the population of Europe was 832,073,224 while the population of Latin America/Caribbean was 626,119,788. Round each number to the nearest million, and estimate the difference in population in 2016. (*Source:* Internet World Stats)

(1.5) Multiply.

$$\begin{array}{r} 49. \quad 276 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 50. \quad 349 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 51. \quad 57 \\ \times 40 \\ \hline \end{array}$$

$$\begin{array}{r} 52. \quad 69 \\ \times 42 \\ \hline \end{array}$$

$$53. \quad 20(7)(4)$$

$$54. \quad 25(9)(4)$$

$$55. \quad 26 \cdot 34 \cdot 0$$

$$56. \quad 62 \cdot 88 \cdot 0$$

$$\begin{array}{r} 57. \quad 586 \\ \times 29 \\ \hline \end{array}$$

$$\begin{array}{r} 58. \quad 242 \\ \times 37 \\ \hline \end{array}$$

$$\begin{array}{r} 59. \quad 642 \\ \times 177 \\ \hline \end{array}$$

$$\begin{array}{r} 60. \quad 347 \\ \times 129 \\ \hline \end{array}$$

$$\begin{array}{r} 61. \quad 1026 \\ \times 401 \\ \hline \end{array}$$

$$\begin{array}{r} 62. \quad 2107 \\ \times 302 \\ \hline \end{array}$$

Translating Solve.

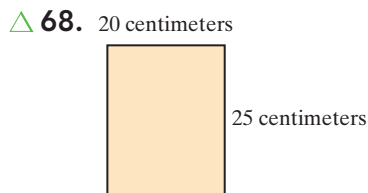
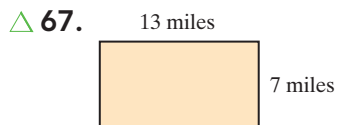
63. Find the product of 6 and 250.

64. Find the product of 6 and 820.

65. A golf pro orders shirts for the company sponsoring a local charity golfing event. Shirts size large cost \$32 while shirts size extra large cost \$38. If 15 large shirts and 11 extra-large shirts are ordered, find the cost.

66. The cost for a South Dakota resident to attend Black Hills State University full-time is \$20,199 per year. Determine the cost for 20 students to attend full-time. (*Source:* Black Hills State University)

Find the area of each rectangle.



(1.6) Divide and then check.

69. $\frac{49}{7}$

70. $\frac{36}{9}$

71. $27 \div 5$

72. $18 \div 4$

73. $918 \div 0$

74. $0 \div 668$

75. $5 \overline{)167}$

76. $8 \overline{)159}$

77. $26 \overline{)626}$

78. $19 \overline{)680}$

79. $47 \overline{)23,792}$

80. $53 \overline{)48,111}$

81. $207 \overline{)578,291}$

82. $306 \overline{)615,732}$

Translating Solve.

83. Find the quotient of 92 and 5.

84. Find the quotient of 86 and 4.

85. A box can hold 24 cans of corn. How many boxes can be filled with 648 cans of corn?

86. One mile is 1760 yards. Find how many miles there are in 22,880 yards.

87. Find the average of the numbers 76, 49, 32, and 47.

88. Find the average of the numbers 23, 85, 62, and 66.

(1.7) Simplify.

89. 8^2

90. 5^3

91. $5 \cdot 9^2$

92. $4 \cdot 10^2$

93. $18 \div 2 + 7$

94. $12 - 8 \div 4$

95. $\frac{5(6^2 - 3)}{3^2 + 2}$

96. $\frac{7(16 - 8)}{2^3}$

97. $48 \div 8 \cdot 2$

98. $27 \div 9 \cdot 3$

99. $2 + 3[1^5 + (20 - 17) \cdot 3] + 5 \cdot 2$

100. $21 - [2^4 - (7 - 5) - 10] + 8 \cdot 2$

101. $19 - 2(3^2 - 2^2)$

102. $16 - 2(4^2 - 3^2)$

103. $4 \cdot 5 - 2 \cdot 7$

104. $8 \cdot 7 - 3 \cdot 9$

105. $(6 - 4)^3 \cdot [10^2 \div (3 + 17)]$

106. $(7 - 5)^3 \cdot [9^2 \div (2 + 7)]$

107. $\frac{5 \cdot 7 - 3 \cdot 5}{2(11 - 3^2)}$

108. $\frac{4 \cdot 8 - 1 \cdot 11}{3(9 - 2^3)}$

Find the area of each square.

△ 109. A square with side length of 7 meters.



(1.8) Evaluate each expression for $x = 5$, $y = 0$, and $z = 2$.

111. $\frac{2x}{z}$

112. $4x - 3$

113. $\frac{x + 7}{y}$

114. $\frac{y}{5x}$

115. $x^3 - 2z$

116. $\frac{7 + x}{3z}$

117. $(y + z)^2$

118. $\frac{100}{x} + \frac{y}{3}$

Translating Translate each phrase into a variable expression. Use x to represent a number.

119. Five subtracted from a number

120. Seven more than a number

121. Ten divided by a number

122. The product of 5 and a number

Decide whether the given number is a solution of the given equation.

123. Is 5 a solution of $n + 12 = 20 - 3$?

124. Is 23 a solution of $n - 8 = 10 + 6$?

125. Is 14 a solution of $30 = 3(n - 3)$?

126. Is 20 a solution of $5(n - 7) = 65$?

Determine which numbers in each set are solutions to the corresponding equations.

127. $7n = 77$; {6, 11, 20}

128. $n - 25 = 150$; {125, 145, 175}

129. $5(n + 4) = 90$; {14, 16, 26}

130. $3n - 8 = 28$; {3, 7, 15}

Mixed Review

Perform the indicated operations.

131. $485 - 68$

132. $729 - 47$

133. 732×3

134. 629×4

135. $374 + 29 + 698$

136. $593 + 52 + 766$

137. $13 \overline{)5962}$

138. $18 \overline{)4267}$

139. 1968×36

140. 5324×18

141. $2000 - 356$

142. $9000 - 519$

Round to the given place value.

143. 842 to the nearest ten

144. 258,371 to the nearest hundred-thousand

Simplify.

145. $24 \div 4 \cdot 2$

146.
$$\frac{(15 + 3) \cdot (8 - 5)}{2^3 + 1}$$

Solve.

147. Is 9 a solution of $5n - 6 = 40$?

148. Is 3 a solution of $2n - 6 = 5n - 15$?

149. A manufacturer of drinking glasses ships his delicate stock in special boxes that can hold 32 glasses. If 1714 glasses are manufactured, how many full boxes are filled? Are there any glasses left over?

150. A teacher orders 2 small whiteboards for \$27 each and 8 boxes of dry erase pens for \$4 each. What is her total bill before taxes?

MULTIPLE CHOICE All the exercises below are **Multiple Choice**. Choose the correct letter. Some letters may be used more than once or not at all.

For Exercises 1 through 4, name the place value for the given digit. The number is 28,690,357,004 and choices are:

A. tens B. millions C. ten-millions D. ten-thousands E. billions F. hundred-millions

- ▶ 1. the digit 5
- ▶ 2. the digit 8
- ▶ 3. the digit 6
- ▶ 4. the digit 0 to the far left

For Exercises 5 through 8, identify the first operation to be performed to simplify the expression.

A. add B. subtract C. multiply D. divide

- ▶ 5. $6 - 3 \cdot 2$
- ▶ 6. $(6 - 3) \cdot 2$
- ▶ 7. $6 \div 3 \cdot 2$
- ▶ 8. $6 + 3 - 2$
- ▶ 9. The expression $5 \cdot 2^3$ simplifies to:

A. 1000 B. 30 C. 40 D. 13

For Exercises 10 through 13, let $a = 35$ and $b = 5$. Choose the expression that gives each answer.

A. $a - b$ B. $a \div b$ C. $a + b$ D. ab

- ▶ 10. answer: 7
- ▶ 11. answer: 175
- ▶ 12. answer: 30
- ▶ 13. answer: 40

Answers

Simplify.

▶ 1. Write 82,426 in words.

▶ 2. Write “four hundred two thousand, five hundred fifty” in standard form.

▶ 3. $59 + 82$

▶ 4. $600 - 487$

▶ 5.
$$\begin{array}{r} 496 \\ \times 30 \\ \hline \end{array}$$

▶ 6. $52,896 \div 69$

▶ 7. $2^3 \cdot 5^2$

▶ 8. $98 \div 1$

▶ 9. $0 \div 49$

▶ 10. $62 \div 0$

▶ 11. $(2^4 - 5) \cdot 3$

▶ 12. $16 + 9 \div 3 \cdot 4 - 7$

▶ 13. $6^1 \cdot 2^3$

▶ 14. $2[(6 - 4)^2 + (22 - 19)^2] + 10$

▶ 15. $5698 \cdot 1000$

▶ 16. Find the average of 62, 79, 84, 90, and 95.

▶ 17. Round 52,369 to the nearest thousand.

Estimate each sum or difference by rounding each number to the nearest hundred.

▶ 18. $6289 + 5403 + 1957$

▶ 19. $4267 - 2738$

Solve.

▶ 20. Subtract 15 from 107.

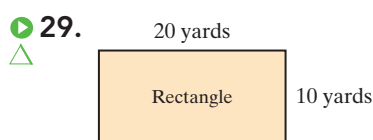
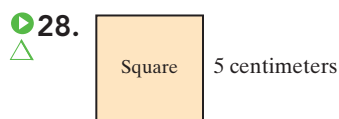
▶ 21. Find the sum of 15 and 107.

▶ 22. Find the product of 15 and 107.

▶ 23. Find the quotient of 107 and 15.

- ▶24. Twenty-nine cans of Sherwin-Williams paint cost \$493. How much was each can?
- ▶25. Jo McElory is looking at two new refrigerators for her apartment. One costs \$599 and the other costs \$725. How much more expensive is the higher-priced one?
- ▶26. One tablespoon of white granulated sugar contains 45 calories. How many calories are in 8 tablespoons of white granulated sugar? (*Source: Home and Garden Bulletin No. 72, U.S. Department of Agriculture*)
- ▶27. A small business owner recently ordered 16 digital cameras that cost \$430 each and 5 printers that cost \$205 each. Find the total cost for these items.

Find the perimeter and the area of each figure.



- ▶30. Evaluate $5(x^3 - 2)$ for $x = 2$.
- ▶31. Evaluate $\frac{3x - 5}{2y}$ for $x = 7$ and $y = 8$.
- ▶32. Translate the following phrases into mathematical expressions. Use x to represent "a number."
- The quotient of a number and 17
 - Twice a number, decreased by 20
- ▶33. Is 6 a solution of the equation $5n - 11 = 19$?
- ▶34. Determine which number in the set is a solution to the given equation.
 $n + 20 = 4n - 10$; $\{0, 10, 20\}$

24. _____

25. _____

26. _____

27. _____

28. _____

29. _____

30. _____

31. _____

32. a. _____

b. _____

33. _____

34. _____

2

Integers and Introduction to Solving Equations

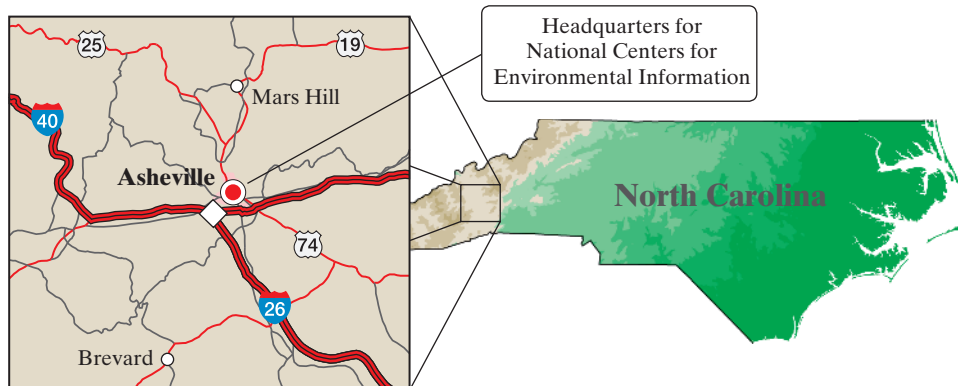
Thus far, we have studied whole numbers, but these numbers are not sufficient for representing many situations in real life. For example, to express 5 degrees below zero or \$100 in debt, numbers less than 0 are needed. This chapter is devoted to integers, which include numbers less than 0, and operations on these numbers.

Sections

- 2.1 Introduction to Integers
- 2.2 Adding Integers
- 2.3 Subtracting Integers
- 2.4 Multiplying and Dividing Integers
- Integrated Review**—Integers
- 2.5 Order of Operations
- 2.6 Solving Equations: The Addition and Multiplication Properties

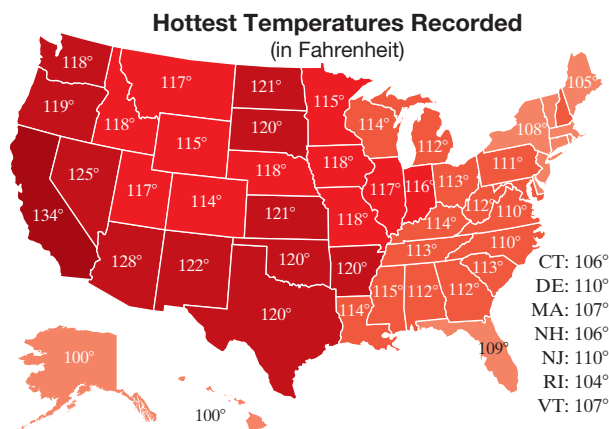
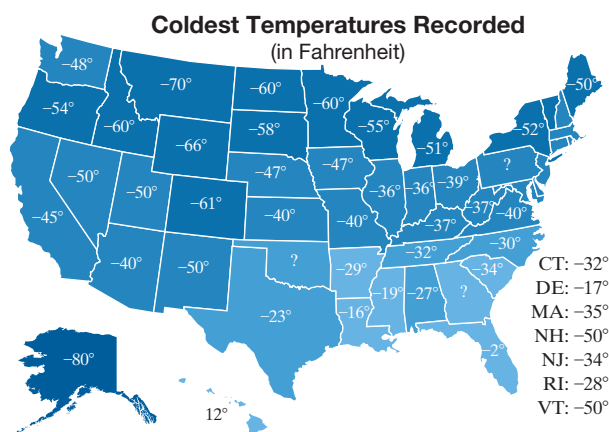
Check Your Progress

Vocabulary Check
Chapter Highlights
Chapter Review
Getting Ready for the Test
Chapter Test
Cumulative Review



Where Is U.S. Weather Data Stored?

To express whole degrees below zero, we need the set of integers. Below, we show the states of the United States with record minimum temperatures per state on one map and maximum temperatures on the other. How long have these temperatures been recorded and stored, and who stores this data? Much of temperature data recording in the U.S. started in 1895. In 1934, a unit was established in New Orleans, Louisiana, to store and process past weather records. This organization was moved to Asheville, North Carolina, in 1951 and was eventually called the National Climatic Data Center (NCDC). In 2015, the NCDC merged with two other centers to form the National Centers for Environmental Information (NCEI), headquartered in Asheville, North Carolina. See temperature exercises throughout this chapter.



2.1 Introduction to Integers

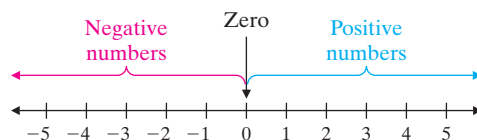
Objective A Representing Real-Life Situations

Thus far in this text, all numbers have been 0 or greater than 0. Numbers greater than 0 are called **positive numbers**. However, sometimes situations exist that cannot be represented by a number greater than 0. For example,



To represent these situations, we need numbers less than 0.

Extending the number line to the left of 0 allows us to picture **negative numbers**, which are numbers that are less than 0.



When a single + sign or no sign is in front of a number, the number is a positive number. When a single - sign is in front of a number, the number is a negative number. Together, we call positive numbers, negative numbers, and zero the **signed numbers**.

-5 indicates "negative five."

5 and +5 both indicate "positive five."

The number 0 is neither positive nor negative.

Some signed numbers are integers. The **integers** consist of the numbers labeled on the number line above. The integers are

$\dots, -3, -2, -1, 0, 1, 2, 3, \dots$

Now we have numbers to represent the situations previously mentioned.

5 degrees below 0 -5°

20 feet below sea level -20 feet

Objectives

- A Represent Real-Life Situations with Integers.
- B Graph Integers on a Number Line.
- C Compare Integers.
- D Find the Absolute Value of a Number.
- E Find the Opposite of a Number.
- F Read Bar Graphs Containing Integers.

Helpful Hint

Notice that 0 is neither positive nor negative.

Helpful Hint

A - sign, such as the one in -1, tells us that the number is to the left of 0 on the number line. -1 is read "negative one."

A + sign or no sign tells us that a number lies to the right of 0 on the number line. For example, 3 and +3 both mean "positive three."

Practice 1

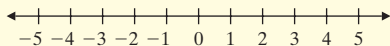
- a. The world's deepest bat colony spends each winter in a New York zinc mine at a depth of 3805 feet. Represent this position with an integer. (Source: Guinness Book of World Records)



- b. The tamarack tree, a type of conifer, commonly grows at the edge of the arctic tundra and survives winter temperatures of 85 degrees below zero Fahrenheit. Represent this temperature with an integer in degrees Fahrenheit.

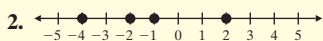
Practice 2

Graph -4 , -1 , 2 , and -2 on the number line.



Answers

1. a. -3805 b. -85°F

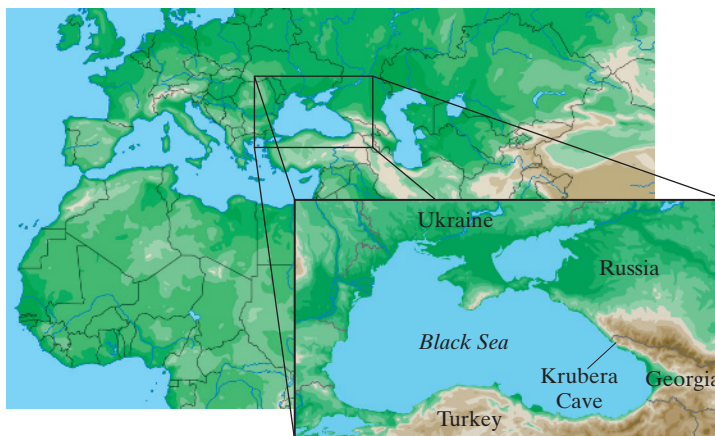


✓ **Concept Check Answer**
no

Example 1 Representing Depth with an Integer

The world's deepest cave is Krubera (or Voronja), in the country of Georgia, located by the Black Sea in Asia. It has been explored to a depth of 7188 feet below the surface of Earth. Represent this position using an integer. (Source: MessagetoEagle.com and Wikipedia)

Solution: If 0 represents the surface of Earth, then 7188 feet below the surface can be represented by -7188 .



Work Practice 1

Objective B Graphing Integers

Example 2 Graph 0, -3 , 5, and -5 on the number line.

Solution:



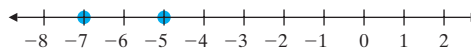
Work Practice 2

Objective C Comparing Integers

We can compare integers by using a number line. For any two numbers graphed on a number line, the number to the **right** is the **greater number** and the number to the **left** is the **smaller number**. Also, the symbols $<$ and $>$ are called **inequality symbols**.

The inequality symbol $>$ means “**is greater than**” and the inequality symbol $<$ means “**is less than**.”

For example, both -5 and -7 are graphed on the number line below.



On the graph, -7 is **to the left of** -5 , so -7 is **less than** -5 , written as

$$-7 < -5$$

We can also write

$$-5 > -7$$

since -5 is **to the right of** -7 , so -5 is **greater than** -7 .

✓ **Concept Check** Is there a largest positive number? Is there a smallest negative number? Explain.

Example 3

Insert $<$ or $>$ between each pair of numbers to make a true statement.

a. -7 7

b. 0 -4

c. -9 -11

Solution:

a. -7 is to the left of 7 on a number line, so $-7 < 7$.

b. 0 is to the right of -4 on a number line, so $0 > -4$.

c. -9 is to the right of -11 on a number line, so $-9 > -11$.

Work Practice 3**Helpful Hint**

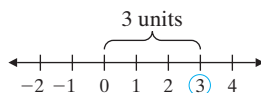
If you think of $<$ and $>$ as arrowheads, notice that in a true statement the arrow always points to the smaller number.

$$\begin{array}{ccc} 5 > -4 & & -3 < -1 \\ \uparrow & & \uparrow \\ \text{smaller} & & \text{smaller} \\ \text{number} & & \text{number} \end{array}$$

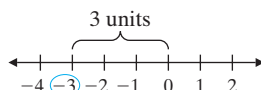
Objective D Finding the Absolute Value of a Number

The **absolute value** of a number is the number's distance from 0 on the number line. The symbol for absolute value is $|$. For example, $|3|$ is read as “the absolute value of 3.”

$$|3| = 3 \text{ because } 3 \text{ is } 3 \text{ units from } 0.$$



$$|-3| = 3 \text{ because } -3 \text{ is } 3 \text{ units from } 0.$$

**Example 4**

Simplify.

a. $|-9|$

b. $|8|$

c. $|0|$

Solution:

a. $|-9| = 9$ because -9 is 9 units from 0.

b. $|8| = 8$ because 8 is 8 units from 0.

c. $|0| = 0$ because 0 is 0 units from 0.

Work Practice 4**Helpful Hint**

Since the absolute value of a number is that number's *distance* from 0, the absolute value of a number is always 0 or positive. It is never negative.

$$\begin{array}{ccc} |0| = 0 & & |-6| = 6 \\ \uparrow & & \uparrow \\ \text{zero} & & \text{a positive number} \end{array}$$

Practice 3

Insert $<$ or $>$ between each pair of numbers to make a true statement.

a. 0 -5

b. -3 3

c. -7 -12

Practice 4

Simplify.

a. $|-6|$

b. $|4|$

c. $|-12|$

Answers

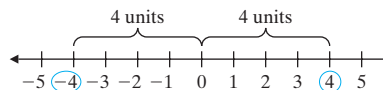
3. a. $>$ b. $<$ c. $>$

4. a. 6 b. 4 c. 12

Objective E Finding Opposites

Two numbers that are the same distance from 0 on the number line but are on opposite sides of 0 are called **opposites**.

4 and -4 are opposites.



When two numbers are opposites, we say that each is the opposite of the other. Thus **4 is the opposite of -4 and -4 is the opposite of 4.**

The phrase “the opposite” of is written in symbols as “ $-$ ”. For example,

The opposite of	5	is	-5
↓	↓	↓	↓
$-$	(5)	=	-5 , or $-(5) = -5$
The opposite of	-3	is	3
↓	↓	↓	↓
$-$	(-3)	=	3 or $-(-3) = 3$

In general, we have the following:

Opposites

If a is a number, then $-(-a) = a$.

Notice that because “the opposite of” is written as “ $-$ ”, to find the opposite of a number we place a “ $-$ ” sign in front of the number.

Practice 5

Find the opposite of each number.

- a. 14 b. -9

Example 5 Find the opposite of each number.

- a. 13 b. -2 c. 0

Solution:

- a. The opposite of 13 is -13 .
 b. The opposite of -2 is $-(-2)$ or 2.
 c. The opposite of 0 is 0.

Helpful Hint

Remember that 0 is neither positive nor negative.

Work Practice 5

 **Concept Check** True or false? The number 0 is the only number that is its own opposite.

Practice 6

Simplify.

- a. $-|-7|$
 b. $-|4|$
 c. $-(-12)$

Answers

5. a. -14 b. 9
 6. a. -7 b. -4 c. 12

 **Concept Check Answer**
 true

Example 6 Simplify.

- a. $-(-4)$ b. $-|-5|$ c. $-|6|$

Solution:

- a. $-(-4) = 4$ The opposite of negative 4 is 4.
 b. $-|-5| = -5$ The opposite of the absolute value of -5 is the opposite of 5, or -5 .
 c. $-|6| = -6$ The opposite of the absolute value of 6 is the opposite of 6, or -6 .

Work Practice 6

Example 7 Evaluate $-|-x|$ if $x = -2$.

Solution: Carefully replace x with -2 ; then simplify.

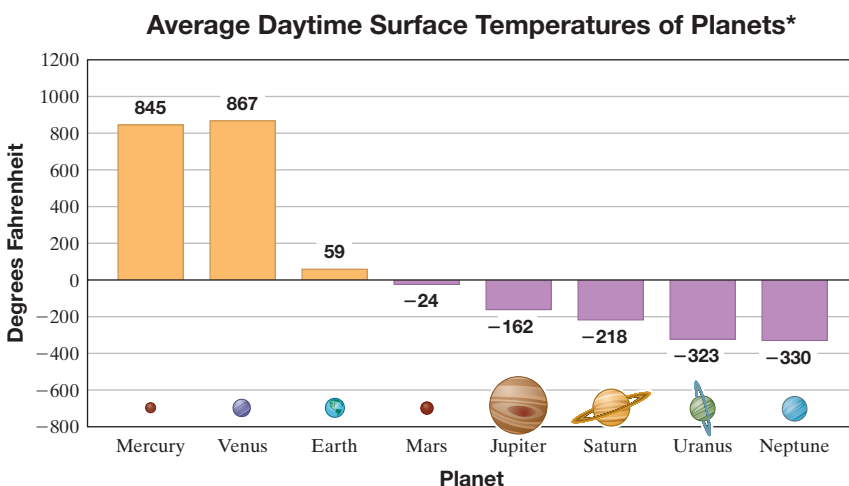
$$-|-x| = -|-(-2)| \quad \text{Replace } x \text{ with } -2.$$

$$\text{Then } -|-(-2)| = -|2| = -2.$$

Work Practice 7

Objective F Reading Bar Graphs Containing Integers

The bar graph below shows the average daytime surface temperatures (in degrees Fahrenheit) of the eight planets, excluding the newly classified “dwarf planet,” Pluto. Notice that a negative temperature is illustrated by a bar below the horizontal line representing 0°F , and a positive temperature is illustrated by a bar above the horizontal line representing 0°F .



Source: The World Almanac

*For some planets, the temperature given is the temperature where the atmospheric pressure equals 1 Earth atmosphere.

Example 8 Which planet has the lowest average daytime surface temperature?

Solution: The planet with the lowest average daytime surface temperature is the one that corresponds to the bar that extends the farthest in the negative direction (downward). Neptune has the lowest average daytime surface temperature, -330°F .

Work Practice 8

Practice 7

Evaluate $-|x|$ if $x = -6$.

Practice 8

Which planet has the highest average daytime surface temperature?

Answers

7. -6 8. Venus

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

opposites	absolute value	right	is less than
inequality symbols	negative	positive	left
signed	integers	is greater than	

- The numbers $\dots -3, -2, -1, 0, 1, 2, 3, \dots$ are called _____.
- Positive numbers, negative numbers, and zero together are called _____ numbers.
- The symbols “ $<$ ” and “ $>$ ” are called _____.

4. Numbers greater than 0 are called _____ numbers while numbers less than 0 are called _____ numbers.
5. The sign “<” means _____ and “>” means _____.
6. On a number line, the greater number is to the _____ of the lesser number.
7. A number’s distance from 0 on a number line is the number’s _____.
8. The numbers -5 and 5 are called _____.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 2.1

- Objective A** 9. In Example 1, what application is used to represent a negative number?
- Objective B** 10. In Example 2, the tick marks are labelled with what numbers on the number line?
- Objective C** 11. From Example 3 and your knowledge of a number line, complete this statement: 0 will always be greater than any of the _____ integers.
- Objective D** 12. What is the answer to Example 5? The absolute value of what other integer has this same answer?
- Objective E** 13. Complete this statement based on Example 10: A negative sign can be translated to the phrase “_____.”
- Objective F** 14. In Examples 13 and 14, what other lake has a negative integer elevation?

2.1 Exercise Set MyLab Math

Objective A Represent each quantity by an integer. See Example 1.

1. A worker in a silver mine in Nevada works 1235 feet underground.
2. A scuba diver is swimming 25 feet below the surface of the water in the Gulf of Mexico.



3. The Challenger Deep in the Mariana Trench (the deepest-known part of the Pacific Ocean) is the lowest known point in the Earth’s seabed hydrosphere. The bottom is estimated to be 35,814 feet below sea level.
4. The lowest elevation in the United States is found at Death Valley, California, at an elevation of 282 feet below sea level. (Source: U.S. Geological Survey)

5. The record high temperature in Arkansas is 120 degrees above zero Fahrenheit. (Source: National Centers for Environmental Information)
6. The record high temperature in California is 134 degrees above zero Fahrenheit. (Source: National Centers for Environmental Information.)
7. The average depth of the Atlantic Ocean is 11,810 feet below its surface. (Source: *The World Almanac*, 2013)
8. The average depth of the Pacific Ocean is 14,040 feet below its surface. (Source: *The World Almanac*, 2013)



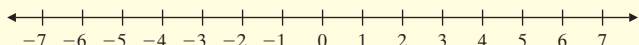
9. Uber lost \$1270 million in the first half of 2016. (Source: Bloomberg)
10. Occidental Petroleum reported a loss of \$241 million for the third quarter of 2016. (Source: Occidental Petroleum Corp.)
11. Two divers are exploring the wreck of the *Andrea Doria*, south of Nantucket Island, Massachusetts. Guillermo is 160 feet below the surface of the ocean and Luigi is 147 feet below the surface. Represent each quantity by an integer and determine who is deeper.
12. The temperature on one January day in Minneapolis was 10° below 0° Celsius. Represent this quantity by an integer and tell whether this temperature is cooler or warmer than 5° below 0° Celsius.

Use the Chapter 2 Opener for Exercises 13 and 14. First, locate the state. Then use the blue map to locate the coldest temperature on record for that state.

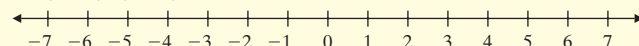
13. Find the coldest temperature on record for the state of Idaho.
14. Find the coldest temperature on record for the state of Arkansas.

Objective B Graph each integer in the list on the same number line. See Example 2.

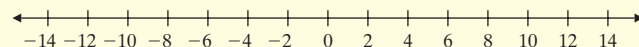
15. 0, 3, 4, 6



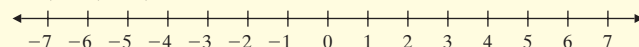
- ▶ 17. 1, -1, 2, -2, -4



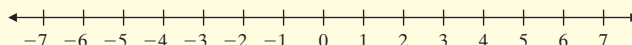
19. 0, 1, 9, 14



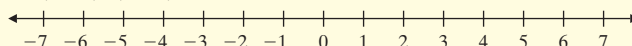
21. 0, -2, -7, -5



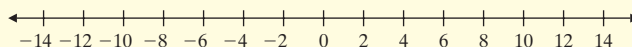
16. 7, 5, 2, 0



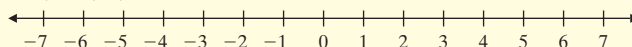
18. 3, -3, 5, -5, 6



20. 0, 3, 10, 11



22. 0, -7, 3, -6



Objective C Insert $<$ or $>$ between each pair of integers to make a true statement. See Example 3.

- | | | | |
|-----------------|-----------------|-----------------|-----------------|
| ▶ 23. 0 -7 | 24. -8 0 | ▶ 25. -7 -5 | 26. -12 -10 |
| 27. -30 -35 | 28. -27 -29 | 29. -26 26 | 30. 13 -13 |

Objective D Simplify. See Example 4.

- | | | | |
|-------------|-------------|--------------|-------------|
| ▶ 31. $ 5 $ | 32. $ 7 $ | ▶ 33. $ -8 $ | 34. $ -19 $ |
| ▶ 35. $ 0 $ | 36. $ 100 $ | 37. $ -55 $ | 38. $ -10 $ |

Objective E Find the opposite of each integer. See Example 5.

39. 5
40. 8
41. -4
42. -6
43. 23
44. 123
45. -85
46. -13

Objectives D E Mixed Practice Simplify. See Example 6.

47. $|-7|$
48. $|-11|$
49. $-|20|$
50. $-|43|$
51. $-|-3|$
52. $-|-18|$
53. $-(-43)$
54. $-(-27)$
55. $|-15|$
56. $-(-14)$
57. $-(-33)$
58. $-|-29|$

Evaluate. See Example 7.

59. $|-x|$ if $x = -6$
60. $-|x|$ if $x = -8$
61. $-|-x|$ if $x = 2$
62. $-|-x|$ if $x = 10$
63. $|x|$ if $x = -32$
64. $|x|$ if $x = 32$
65. $-|x|$ if $x = 7$
66. $-|x|$ if $x = 1$

Insert $<$, $>$, or $=$ between each pair of numbers to make a true statement. See Examples 3 through 6.

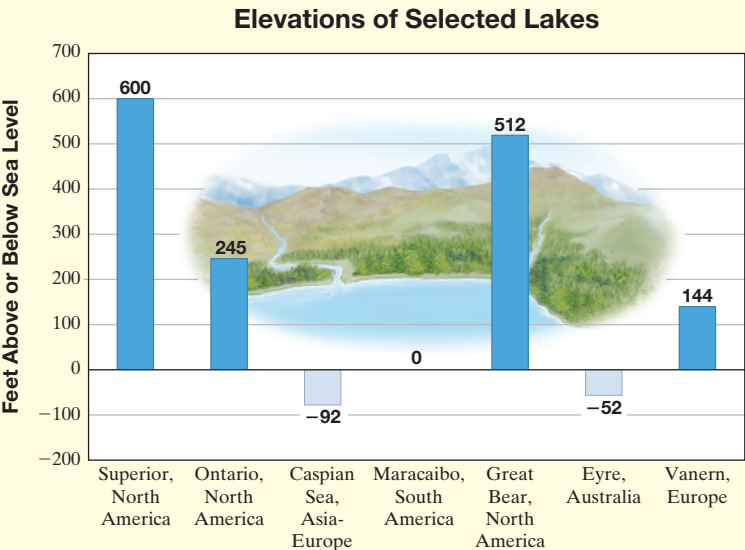
67. -12 -6
68. -4 -17
69. $|-8|$ $|-11|$
70. $|-8|$ $|-4|$
71. $|-47|$ $-(-47)$
72. $-|17|$ $-(-17)$
73. $-|-12|$ $-(-12)$
74. $|-24|$ $-(-24)$
75. 0 -9
76. -45 0
77. $|0|$ $|-9|$
78. $|-45|$ $|0|$
79. $-|-2|$ $-|-10|$
80. $-|-8|$ $-|-4|$
81. $-(-12)$ $-(-18)$
82. -22 $-(-38)$

Objectives D E Mixed Practice Fill in the chart. See Examples 4 through 6.

	Number	Absolute Value of Number	Opposite of Number
83.	31		
85.			-28

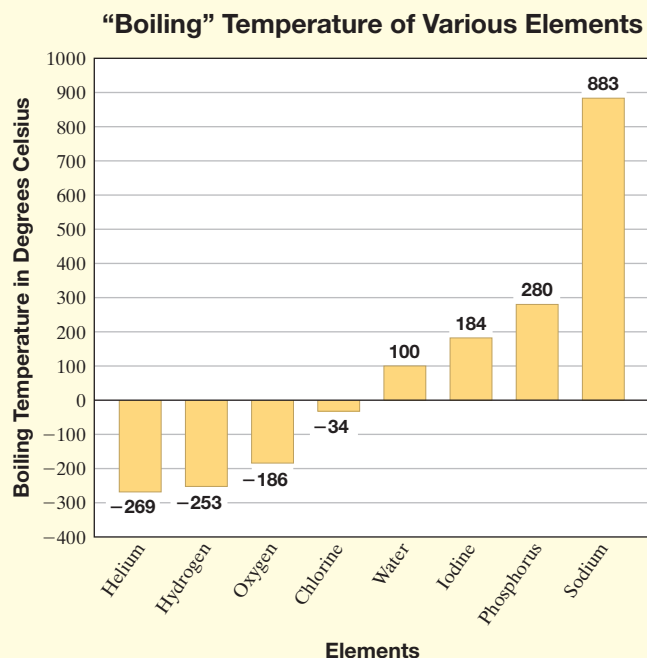
	Number	Absolute Value of Number	Opposite of Number
84.	-13		
86.			90

Objective F The bar graph shows the elevations of selected lakes. Use this graph For Exercises 87 through 90.
(Source: U.S. Geological Survey) See Example 8.



87. Which lake shown has the lowest elevation?
88. Which lake has an elevation at sea level?
89. Which lake shown has the highest elevation?
90. Which lake shown has the second-lowest elevation?

The following bar graph represents the boiling temperature, the temperature at which a substance changes from liquid to gas at standard atmospheric pressure. Use this graph to answer Exercises 91 through 94.



91. Which element has a positive boiling temperature closest to that of water?
92. Which element has the lowest boiling temperature?
93. Which element has a boiling temperature closest to -200°C ?
94. Which element has an average boiling temperature closest to $+300^{\circ}\text{C}$?

Review

Add. See Section 1.3.

95. $0 + 13$

96. $9 + 0$

97. $15 + 20$

98. $20 + 15$

99. $47 + 236 + 77$

100. $362 + 37 + 90$

Concept Extensions

Write the given numbers in order from least to greatest.

101. $2^2, -|3|, -(-5), -|-8|$

102. $|10|, 2^3, -|-5|, -(-4)$

103. $|-1|, -|-6|, -(-6), -|1|$

104. $1^4, -(-3), -|7|, |-20|$

105. $-(-2), 5^2, -10, -|-9|, |-12|$

106. $3^3, -|-11|, -(-10), -4, -|2|$

Choose all numbers for x from each given list that make each statement true.

107. $|x| > 8$

a. -9

b. -5

c. 8

d. -12

108. $|x| > 4$

a. 0

b. -4

c. 5

d. -100

109. Evaluate: $-(-|-8|)$

110. Evaluate: $(-|-(-7)|)$

Answer true or false for Exercises 111 through 115.

111. If $a > b$, then a must be a positive number.

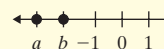
112. The absolute value of a number is *always* a positive number.

113. A positive number is always greater than a negative number.

114. Zero is always less than a positive number.

115. The number $-a$ is always a negative number.
(Hint: Read “ $-$ ” as “the opposite of.”)

116. Given the number line is it true that $b < a$?



117. Write in your own words how to find the absolute value of a signed number.

118. Explain how to determine which of two signed numbers is larger.

For Exercises 119 and 120, see the first Concept Check in this section.

119. Is there a largest negative number? If so, what is it?

120. Is there a smallest positive number? If so, what is it?

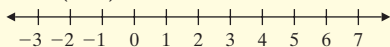
Objectives

- A** Add Integers.
- B** Evaluate an Algebraic Expression by Adding.
- C** Solve Problems by Adding Integers.

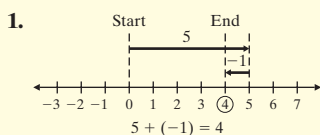
Practice 1

Add using a number line:

$$5 + (-1)$$



Answer

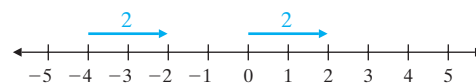


2.2 Adding Integers

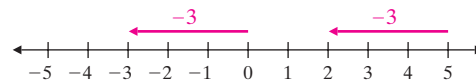
Objective A Adding Integers

Adding integers can be visualized using a number line. A positive number can be represented on the number line by an arrow of appropriate length pointing to the right, and a negative number by an arrow of appropriate length pointing to the left.

Both arrows represent 2 or $+2$.
They both point to the right and they are both 2 units long.

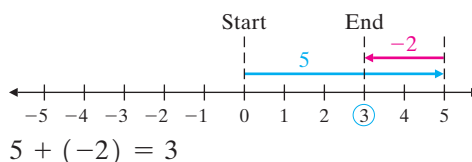


Both arrows represent -3 .
They both point to the left and they are both 3 units long.



Example 1 Add using a number line: $5 + (-2)$

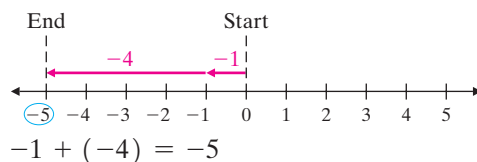
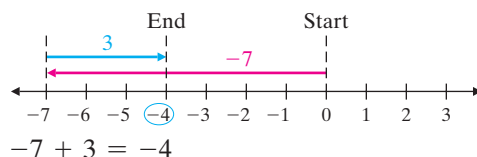
Solution: To add integers on a number line, such as $5 + (-2)$, we start at 0 on the number line and draw an arrow representing 5. From the tip of this arrow, we draw another arrow representing -2 . The tip of the second arrow ends at their sum, 3.



Work Practice 1

Example 2 Add using a number line: $-1 + (-4)$

Start at 0 and draw an arrow representing -1 . From the tip of this arrow, we draw another arrow representing -4 . The tip of the second arrow ends at their sum, -5 .

Solution:**Work Practice 2****Example 3** Add using a number line: $-7 + 3$ **Solution:****Work Practice 3**

Using a number line each time we add two numbers can be time consuming. Instead, we can notice patterns in the previous examples and write rules for adding signed numbers.

Rules for adding signed numbers depend on whether we are adding numbers with the same sign or different signs. When adding two numbers with the same sign, as in Example 2, notice that the sign of the sum is the same as the sign of the addends.

Adding Two Numbers with the Same Sign**Step 1:** Add their absolute values.**Step 2:** Use their common sign as the sign of the sum.**Example 4** Add: $-2 + (-21)$ **Solution:****Step 1:** $|-2| = 2$, $|-21| = 21$, and $2 + 21 = 23$.**Step 2:** Their common sign is negative, so the sum is negative:

$$-2 + (-21) = -23$$

Work Practice 4**Examples** Add.

5. $-15 + (-10) = -25$

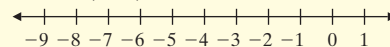
6. $2 + 6 = 8$

Work Practice 5–6

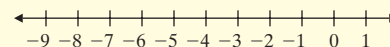
When adding two numbers with different signs, as in Examples 1 and 3, the sign of the result may be positive or negative, or the result may be 0.

Practice 2

Add using a number line:
 $-6 + (-2)$

**Practice 3**

Add using a number line:
 $-8 + 3$

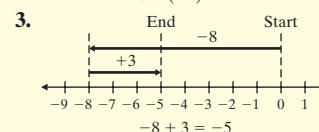
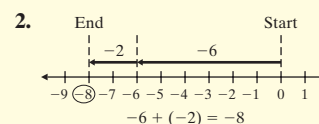
**Practice 4**

Add: $(-3) + (-19)$

Practice 5–6

Add.

5. $-12 + (-30)$ 6. $9 + 4$

Answers

4. -22 5. -42 6. 13

Practice 7Add: $-1 + 26$ **Practice 8**Add: $2 + (-18)$ **Practice 9–11**

Add.

9. $-54 + 20$

10. $7 + (-2)$

11. $-3 + 0$

Practice 12–13

Add.

12. $18 + (-18)$

13. $-64 + 64$

Answers

7. 25 8. -16 9. -34 10. 5

11. -3 12. 0 13. 0

✓ **Concept Check Answer**
 $5 + (-22) = -17$

Adding Two Numbers with Different Signs**Step 1:** Find the larger absolute value minus the smaller absolute value.**Step 2:** Use the sign of the number with the larger absolute value as the sign of the sum.**Example 7** Add: $-2 + 25$ **Solution:****Step 1:** $|-2| = 2$, $|25| = 25$, and $25 - 2 = 23$.**Step 2:** 25 has the larger absolute value and its sign is an understood +:
 $-2 + 25 = +23$ or 23**Work Practice 7****Example 8** Add: $3 + (-17)$ **Solution:****Step 1:** $|3| = 3$, $|-17| = 17$, and $17 - 3 = 14$.**Step 2:** -17 has the larger absolute value and its sign is -:
 $3 + (-17) = -14$ **Work Practice 8****Examples** Add.

9. $-18 + 10 = -8$

10. $12 + (-8) = 4$

11. $0 + (-5) = -5$ The sum of 0 and any number is the number.

Work Practice 9–11

Recall that numbers such as 7 and -7 are called opposites. In general, the sum of a number and its opposite is always 0.

$$\begin{array}{ccc}
 7 + (-7) = 0 & -26 + 26 = 0 & 1008 + (-1008) = 0 \\
 \swarrow \quad \searrow & \swarrow \quad \searrow & \swarrow \quad \searrow \\
 \text{opposites} & \text{opposites} & \text{opposites}
 \end{array}$$

If a is a number, then $-a$ is its opposite. Also,

$$\left. \begin{array}{l} a + (-a) = 0 \\ -a + a = 0 \end{array} \right\} \text{The sum of a number and its opposite is 0.}$$

Examples Add.

12. $-21 + 21 = 0$

13. $36 + (-36) = 0$

Work Practice 12–13✓ **Concept Check** What is wrong with the following calculation?

$$5 + (-22) = 17$$

In the following examples, we add three or more integers. Remember that by the associative and commutative properties for addition, we may add numbers in any order that we wish. In Examples 14 and 15, let's add the numbers from left to right.

Example 14 Add: $(-3) + 4 + (-11)$

Solution: $(-3) + 4 + (-11) = 1 + (-11)$
 $= -10$

Work Practice 14

Example 15 Add: $1 + (-10) + (-8) + 9$

Solution: $1 + (-10) + (-8) + 9 = -9 + (-8) + 9$
 $= -17 + 9$
 $= -8$

Work Practice 15

A sum is the same if we add the numbers in any order. To see this, let's add the numbers in Example 15 by first adding the positive numbers together and the negative numbers together.

$$1 + (-10) + (-8) + 9 = 10 + (-18) \quad \begin{array}{l} \text{Add the positive numbers: } 1 + 9 = 10. \\ \text{Add the negative numbers: } (-10) + (-8) = -18. \\ \text{Add these results.} \end{array}$$

$$= -8$$

The sum is -8 .

Objective B Evaluating Algebraic Expressions

We can continue our work with algebraic expressions by evaluating expressions given integer replacement values.

Example 16 Evaluate $2x + y$ for $x = 3$ and $y = -5$.

Solution: Replace x with 3 and y with -5 in $2x + y$.
 $2x + y = 2 \cdot 3 + (-5)$
 $= 6 + (-5)$
 $= 1$

Work Practice 16

Example 17 Evaluate $x + y$ for $x = -2$ and $y = -10$.

Solution: $x + y = (-2) + (-10)$ Replace x with -2 and y with -10 .
 $= -12$

Work Practice 17

Objective C Solving Problems by Adding Integers

Next, we practice solving problems that require adding integers.

Example 18 Calculating Temperature

In Pennsylvania, the record extreme high temperature is 111°F . Add -153° to this temperature and the result is the record extreme low temperature. Find this extreme low temperature. (Source: National Centers for Environmental Information)

(Continued on next page)

Practice 14

Add: $6 + (-2) + (-15)$

Practice 15

Add: $5 + (-3) + 12 + (-14)$

Helpful Hint

Don't forget that addition is commutative and associative. In other words, numbers may be added in any order.

Practice 16

Evaluate $x + 3y$ for $x = -6$ and $y = 2$.

Practice 17

Evaluate $x + y$ for $x = -13$ and $y = -9$.

Practice 18

If the temperature was -7°F at 6 a.m., and it rose 4 degrees by 7 a.m. and then rose another 7 degrees in the hour from 7 a.m. to 8 a.m., what was the temperature at 8 a.m.?

Answers

14. -11 15. 0 16. 0 17. -22
 18. 4°F

**Solution:**

In words: $\begin{array}{c} \text{extreme low} \\ \text{temperature} \end{array} = \begin{array}{c} \text{extreme high} \\ \text{temperature} \end{array} + \begin{array}{c} \text{decrease of} \\ \text{of } -153^\circ \end{array}$

Translate: $\begin{array}{c} \text{extreme low} \\ \text{temperature} \end{array} = 111 + (-153)$

$= -42$

The record extreme low temperature in Pennsylvania, is -42°F .

Work Practice 18



Calculator Explorations Entering Negative Numbers

To enter a negative number on a calculator, find the key marked $+/-$. (Some calculators have a key marked CHS and some calculators have a special key $(-)$ for entering a negative sign.) To enter the number -2 , for example, press the keys 2 $+/-$. The display will read -2 .

To find $-32 + (-131)$, press the keys

32 $+/-$ $+$ 131 $+/-$ $=$ or
 $(-)$ 32 $+$ $(-)$ 131 ENTER

The display will read -163 .

Thus $-32 + (-131) = -163$.

Use a calculator to perform each indicated operation.

- $-256 + 97$
- $811 + (-1058)$
- $6(15) + (-46)$
- $-129 + 10(48)$
- $-108,650 + (-786,205)$
- $-196,662 + (-129,856)$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

$-a$ a 0 commutative associative

- If n is a number, then $-n + n =$ _____.
- Since $x + n = n + x$, we say that addition is _____.
- If a is a number, then $-(-a) =$ _____.
- Since $n + (x + a) = (n + x) + a$, we say that addition is _____.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



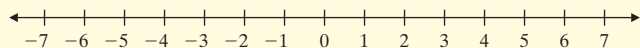
See Video 2.2

- Objective A** 5. What is the sign of the sum in Example 6 and why?
- Objective B** 6. What is the sign of the sum in Example 8 and why?
- Objective C** 7. What does the answer to Example 10, -231 , mean in the context of the application?

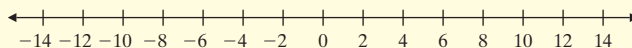
2.2 Exercise Set MyLab Math

Objective A Add using a number line. See Examples 1 through 3.

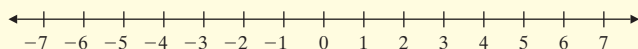
▶ 1. $-1 + (-6)$



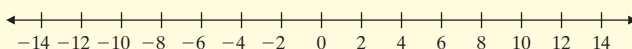
2. $-6 + (-5)$



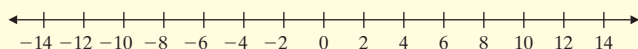
▶ 3. $-4 + 7$



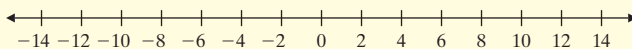
4. $10 + (-3)$



5. $-13 + 7$



6. $9 + (-4)$



Add. See Examples 4 through 13.

7. $46 + 21$

8. $15 + 42$

9. $-8 + (-2)$

10. $-5 + (-4)$

11. $-43 + 43$

12. $-62 + 62$

▶ 13. $6 + (-2)$

14. $8 + (-3)$

15. $-6 + 0$

16. $-8 + 0$

17. $3 + (-5)$

18. $5 + (-9)$

▶ 19. $-2 + (-7)$

20. $-6 + (-1)$

21. $-12 + (-12)$

22. $-23 + (-23)$

23. $-640 + (-200)$

24. $-400 + (-256)$

25. $12 + (-5)$

26. $24 + (-10)$

▶ 27. $-6 + 3$

28. $-8 + 4$

29. $-56 + 26$

30. $-89 + 37$

31. $-45 + 85$

32. $-32 + 62$

33. $124 + (-144)$

34. $325 + (-375)$

▶ 35. $-82 + (-43)$

36. $-56 + (-33)$

Add. See Examples 14 and 15.

▶ 37. $-4 + 2 + (-5)$

38. $-1 + 5 + (-8)$

39. $-52 + (-77) + (-117)$

40. $-103 + (-32) + (-27)$

41. $12 + (-4) + (-4) + 12$

42. $18 + (-9) + 5 + (-2)$

43. $(-10) + 14 + 25 + (-16)$

44. $34 + (-12) + (-11) + 213$

Objective A Mixed Practice Add. See Examples 1 through 15.

45. $-6 + (-15) + (-7)$

46. $-12 + (-3) + (-5)$

47. $-26 + 15$

48. $-35 + (-12)$

49. $5 + (-2) + 17$

50. $3 + (-23) + 6$

51. $-13 + (-21)$

52. $-100 + 70$

53. $3 + 14 + (-18)$

54. $(-45) + 22 + 20$

55. $-92 + 92$

56. $-87 + 0$

57. $-13 + 8 + (-10) + (-27)$

58. $-16 + 6 + (-14) + (-20)$

Objective B Evaluate $x + y$ for the given replacement values. See Examples 16 and 17.

▶ 59. $x = -20$ and $y = -50$

60. $x = -1$ and $y = -29$

Evaluate $3x + y$ for the given replacement values. See Examples 16 and 17.

▶ 61. $x = 2$ and $y = -3$

62. $x = 7$ and $y = -11$

63. $x = 3$ and $y = -30$

64. $x = 13$ and $y = -17$

Objective C Translating Translate each phrase; then simplify. See Example 18.

65. Find the sum of -6 and 25 .66. Find the sum of -30 and 15 .67. Find the sum of -31 , -9 , and 30 .68. Find the sum of -49 , -2 , and 40 .

Solve. See Example 18.

- ▶ 69. Suppose a deep-sea diver dives from the surface to 215 feet below the surface. He then dives down 16 more feet. Use signed numbers to represent this situation. Then find the diver's present depth.

70. Suppose a diver dives from the surface to 248 meters below the surface and then swims up 8 meters, down 16 meters, down another 28 meters, and then up 32 meters. Use signed numbers to represent this situation. Then find the diver's depth after these movements.

In golf, it is possible to have positive and negative scores. The following table shows the results of the eighteen-hole Round 4 for Jimmy Walker and Hideki Matsuyama at the 2016 PGA Championship in Baltusrol Golf Club in Springfield, New Jersey. Use the table to answer Exercises 71 and 72.

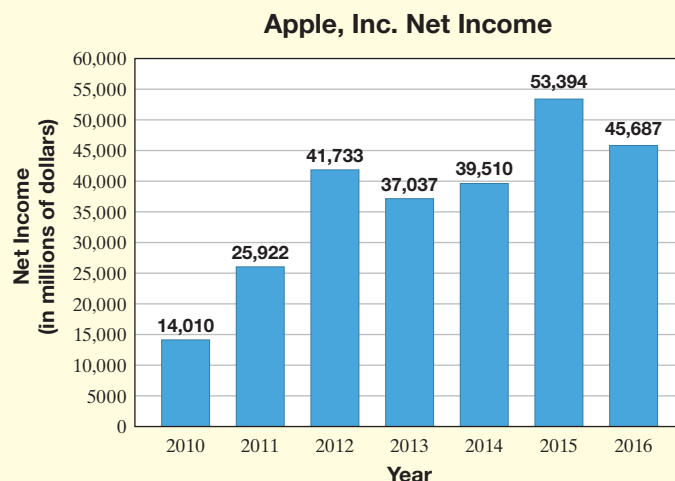
Player/Hole	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Walker	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	-1	0
Matsuyama	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	-1	0

71. Find the total score for each of the athletes in the round.

72. In golf, the lower score is the winner. Use the result of Exercise 71 to determine who won Round 4.

The following bar graph shows the yearly net income for Apple, Inc. Net income is one indication of a company's health. It measures revenue (money taken in) minus cost (money spent). Use this graph to answer Exercises 73 through 76. (Source: Apple, Inc.)

73. What was the net income (in dollars) for Apple, Inc. in 2016?
74. What was the net income (in dollars) for Apple, Inc. in 2015?
75. What was the difference in net income (in dollars) for Apple, Inc. from 2015 to 2016?
76. Find the total net income for the years 2014, 2015, and 2016.
77. The temperature at 4 p.m. on February 2 was -10° Celsius. By 11 p.m. the temperature had risen 12 degrees. Find the temperature at 11 p.m.



78. In some card games, it is possible to have both positive and negative scores. After four rounds of play, Michelle had scores of 14, -5 , -8 , and 7. What was her total score for the game?

A small business reports the following net incomes. Use this table to answer Exercises 79 and 80.

Year	Net Income (in dollars)
2013	$-\$10,412$
2014	$-\$1786$
2015	$\$15,395$
2016	$\$31,418$

79. Find the sum of the net incomes for 2014 and 2015.
80. Find the sum of the net incomes for all four years shown.

81. The all-time record low temperature for Colorado is -61°F . Oklahoma's all-time record low temperature is 30°F higher than Colorado's record low. What is Oklahoma's record low temperature? (Source: National Centers for Environmental Information)
82. The all-time record low temperature for Minnesota is -60°F . In Georgia, the lowest temperature ever recorded is 43°F higher than Minnesota's all-time low temperature. What is the all-time record low temperature for Georgia? (Source: National Centers for Environmental Information)
83. The deepest spot in the Atlantic Ocean is the Puerto Rico Trench, which has an elevation of 8605 meters below sea level. The bottom of the Atlantic's Cayman Trench has an elevation 1070 meters above the level of the Puerto Rico Trench. Use a negative number to represent the depth of the Cayman Trench. (Source: Defense Mapping Agency)



84. The deepest spot in the Pacific Ocean is the Mariana Trench, which has an elevation of 10,924 meters below sea level. The bottom of the Pacific's Aleutian Trench has an elevation 3245 meters higher than that of the Mariana Trench. Use a negative number to represent the depth of the Aleutian Trench. (Source: Defense Mapping Agency)

Review

Subtract. See Section 1.3.

85. $44 - 0$

86. $91 - 0$

87. $200 - 59$

88. $400 - 18$

Concept Extensions

89. Name 2 numbers whose sum is -17 .90. Name 2 numbers whose sum is -30 .

Each calculation below is incorrect. Find the error and correct it. See the Concept Check in this section.

91. $7 + (-10) \stackrel{?}{=} 17$

92. $-4 + 14 \stackrel{?}{=} -18$

93. $-10 + (-12) \stackrel{?}{=} -120$

94. $-15 + (-17) \stackrel{?}{=} 32$

For Exercises 95 through 98, determine whether each statement is true or false.

95. The sum of two negative numbers is always a negative number.

96. The sum of two positive numbers is always a positive number.

97. The sum of a positive number and a negative number is always a negative number.

98. The sum of zero and a negative number is always a negative number.

 99. In your own words, explain how to add two negative numbers.

 100. In your own words, explain how to add a positive number and a negative number.

Objectives

A Subtract Integers. 

B Add and Subtract Integers. 

C Evaluate an Algebraic Expression by Subtracting. 

D Solve Problems by Subtracting Integers. 

2.3 Subtracting Integers

In Section 2.1, we discussed the opposite of an integer.

The opposite of 3 is -3 .

The opposite of -6 is 6.

In this section, we use opposites to subtract integers.

Objective A Subtracting Integers

To subtract integers, we will write the subtraction problem as an addition problem. To see how to do this, study the examples below.

$$10 - 4 = 6$$

$$10 + (-4) = 6$$

Since both expressions simplify to 6, this means that

$$10 - 4 = 10 + (-4) = 6$$

Also,

$$3 - 2 = 3 + (-2) = 1$$

$$15 - 1 = 15 + (-1) = 14$$

Thus, to subtract two numbers, we add the first number to the opposite of the second number. (The opposite of a number is also known as its **additive inverse**.)

Subtracting Two Numbers

If a and b are numbers, then $a - b = a + (-b)$.

Examples Subtract.

	subtraction	=	first number	+	opposite of the second number	=	
	↓		↓	↓	↓		
1.	$8 - 5$	=	8	+	(-5)	=	3
2.	$-4 - 10$	=	-4	+	(-10)	=	-14
3.	$6 - (-5)$	=	6	+	5	=	11
4.	$-11 - (-7)$	=	-11	+	7	=	-4

Work Practice 1–4

Examples Subtract.

5. $-10 - 5 = -10 + (-5) = -15$
6. $8 - 15 = 8 + (-15) = -7$
7. $-4 - (-5) = -4 + 5 = 1$

Work Practice 5–7

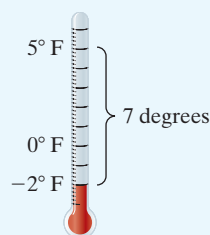
Helpful Hint

To visualize subtraction, try the following:

The difference between 5°F and -2°F can be found by subtracting. That is,

$$5 - (-2) = 5 + 2 = 7$$

Can you visually see from the thermometer on the right that there are actually 7 degrees between 5°F and -2°F ?



✓ Concept Check What is wrong with the following calculation?

$$-9 - (-5) = -14$$

Example 8 Subtract 7 from -3.

Solution: To subtract 7 from -3, we find

$$-3 - 7 = -3 + (-7) = -10$$

Work Practice 8

Practice 1–4

Subtract.

- $13 - 4$
- $-8 - 2$
- $11 - (-15)$
- $-9 - (-1)$

Practice 5–7

Subtract.

- $6 - 9$
- $-14 - 5$
- $-3 - (-4)$

Practice 8

Subtract 6 from -15.

Answers

- 9
- 10
- 26
- 8
- 3
- 19
- 1
- 21

✓ Concept Check Answer

$$-9 - (-5) = -9 + 5 = -4$$

Objective B Adding and Subtracting Integers

If a problem involves adding or subtracting more than two integers, we rewrite differences as sums and add. Recall that by associative and commutative properties, we may add numbers in any order. In Examples 9 and 10, we will add from left to right.

Example 9 Simplify: $7 - 8 - (-5) - 1$

$$\begin{aligned}\text{Solution: } 7 - 8 - (-5) - 1 &= 7 + (-8) + 5 + (-1) \\ &= \underbrace{-1}_{-1} + 5 + (-1) \\ &= 4 + (-1) \\ &= \underbrace{3}_{3}\end{aligned}$$

 **Work Practice 9**

Example 10 Simplify: $7 + (-12) - 3 - (-8)$

$$\begin{aligned}\text{Solution: } 7 + (-12) - 3 - (-8) &= 7 + (-12) + (-3) + 8 \\ &= \underbrace{-5}_{-5} + (-3) + 8 \\ &= -8 + 8 \\ &= 0\end{aligned}$$

 **Work Practice 10**

Objective C Evaluating Expressions

Now let's practice evaluating expressions when the replacement values are integers.

Example 11 Evaluate $x - y$ for $x = -3$ and $y = 9$.

Solution: Replace x with -3 and y with 9 in $x - y$.

$$\begin{aligned}&\begin{array}{ccc} x & - & y \\ \downarrow & & \downarrow \\ = & (-3) - & 9 \\ & = & (-3) + (-9) \\ & = & -12 \end{array}\end{aligned}$$

 **Work Practice 11**

Example 12 Evaluate $2a - b$ for $a = 8$ and $b = -6$.

Solution: Watch your signs carefully!

$$\begin{aligned}&\begin{array}{ccc} 2a & - & b \\ \downarrow & & \downarrow \\ = & 2 \cdot 8 - & (-6) \\ & = & 16 + 6 \\ & = & 22 \end{array}\end{aligned}$$

Replace a with 8 and b with -6 . Multiply. Write subtraction as equivalent addition. Add.

Helpful Hint

Watch carefully when replacing variables in the expression $2a - b$. Make sure that all symbols are inserted and accounted for.

 **Work Practice 12**

Practice 9

Simplify: $-6 - 5 - 2 - (-3)$

Practice 10

Simplify: $8 + (-2) - 9 - (-7)$

Practice 11

Evaluate $x - y$ for $x = -5$ and $y = 13$.

Practice 12

Evaluate $3y - z$ for $y = 9$ and $z = -4$.

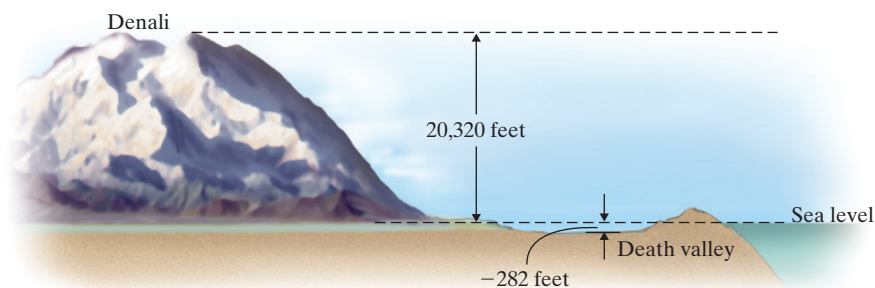
Answers

9. -10 10. 4 11. -18 12. 31

Solving problems often requires subtraction of integers. For Example 13 below, notice we begin to organize our solving process by numbering and labelling steps.

The highest point in the United States is the top of Denali, at a height of 20,320 feet above sea level. The lowest point is Death Valley, California, which is 282 feet below sea level. How much higher is Denali than Death Valley? (*Source*: U.S. Geological Survey)

1. UNDERSTAND. Read and reread the problem. To find “how much higher,” we subtract. Don’t forget that since Death Valley is 282 feet *below* sea level, we represent its height by -282 . Draw a diagram to help visualize the problem.



In words:	how much higher is Denali	=	height of Denali	minus	height of Death Valley
	↓	↓	↓	↓	↓
Translate:	how much higher is Denali	=	20,320	−	(−282)

$$20,320 - (-282) = 20,320 + 282 = 20,602$$

The highest point in Asia is the top of Mount Everest, at a height of 29,028 feet above sea level. The lowest point is the Dead Sea, which is 1312 feet below sea level. How much higher is Mount Everest than the Dead Sea? (*Source: National Geographic Society*)

13. 30,340 ft

Vocabulary, Readiness & Video Check

Multiple choice: Select the correct lettered response following each exercise.

- It is true that $a - b =$ _____.
a. $b - a$ **b.** $a + (-b)$ **c.** $a + b$
- The opposite of n is _____.
a. $-n$ **b.** $-(-n)$ **c.** n
- To evaluate $x - y$ for $x = -10$ and $y = -14$, we replace x with -10 and y with -14 and evaluate _____.
a. $10 - 14$ **b.** $-10 - 14$ **c.** $-14 - 10$ **d.** $-10 - (-14)$
- The expression $-5 - 10$ equals _____.
a. $5 - 10$ **b.** $5 + 10$ **c.** $-5 + (-10)$ **d.** $10 - 5$

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 2.3

Objective A 5. In the lecture before Example 1, what can the “opposite” of a number also be called? ▶

Objective B 6. In Example 7, how is the example rewritten in the first step of simplifying and why? ▶

Objective C 7. In Example 8, why do we multiply first? ▶

Objective D 8. What does the answer to Example 9, 263, mean in the context of the application? ▶

2.3 Exercise Set MyLab Math ▶

Objective A Subtract. See Examples 1 through 7.

1. $-8 - (-8)$

2. $-6 - (-6)$

3. $19 - 16$

4. $15 - 12$

▶ 5. $3 - 8$

6. $2 - 5$

7. $11 - (-11)$

8. $12 - (-12)$

9. $-4 - (-7)$

10. $-25 - (-25)$

11. $-16 - 4$

12. $-2 - 42$

13. $3 - 15$

14. $8 - 9$

15. $42 - 55$

16. $17 - 63$

17. $478 - (-30)$

18. $844 - (-20)$

▶ 19. $-4 - 10$

20. $-5 - 8$

▶ 21. $-7 - (-3)$

22. $-12 - (-5)$

23. $17 - 29$

24. $16 - 45$

Translating Translate each phrase; then simplify. See Example 8.

▶ 25. Subtract 17 from -25 .

26. Subtract 10 from -22 .

27. Find the difference of -22 and -3 .

28. Find the difference of -8 and -13 .

29. Subtract -12 from 2.

30. Subtract -50 from -50 .

Mixed Practice (Sections 2.2 and 2.3) Add or subtract as indicated.

31. $-37 + (-19)$

32. $-35 + (-11)$

33. $8 - 13$

34. $4 - 21$

35. $-56 - 89$

36. $-105 - 68$

37. $30 - 67$

38. $86 - 98$

Objective B Simplify. See Examples 9 and 10.

39. $8 - 3 - 2$

40. $8 - 4 - 1$

41. $13 - 5 - 7$

42. $30 - 18 - 12$

43. $-5 - 8 - (-12)$

44. $-10 - 6 - (-9)$

45. $-11 + (-6) - 14$

46. $-15 + (-8) - 4$

47. $18 - (-32) + (-6)$

48. $23 - (-17) + (-9)$

49. $-(-5) - 21 + (-16)$

50. $-(-9) - 14 + (-23)$

51. $-10 - (-12) + (-7) - 4$

52. $-6 - (-8) + (-12) - 7$

53. $-3 + 4 - (-23) - 10$

54. $5 + (-18) - (-21) - 2$

Objective C Evaluate $x - y$ for the given replacement values. See Examples 11 and 12.

55. $x = -4$ and $y = 7$

56. $x = -7$ and $y = 1$

57. $x = 8$ and $y = -23$

58. $x = 9$ and $y = -2$

Evaluate $2x - y$ for the given replacement values. See Examples 11 and 12.

59. $x = 4$ and $y = -4$

60. $x = 8$ and $y = -10$

61. $x = 1$ and $y = -18$

62. $x = 14$ and $y = -12$

Objective D Solve. See Example 13.

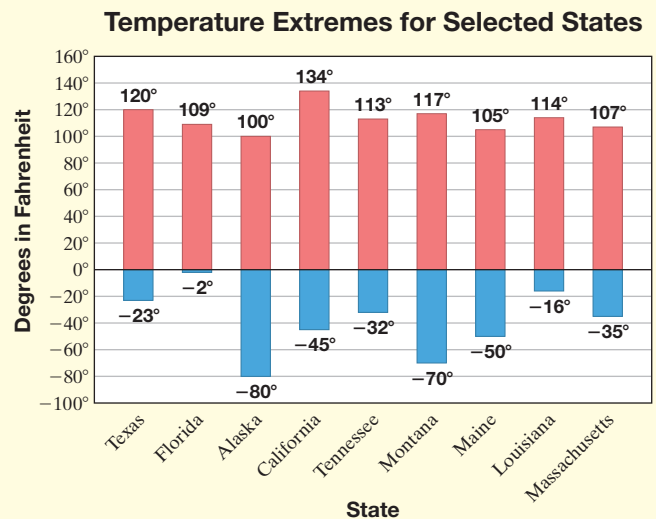
The bar graph shows the minimum and maximum temperatures for selected states. A temperature extreme is found by calculating maximum minus minimum. Find the temperature extreme for the states in Exercises 63 through 66.

63. Find the temperature extreme (difference in max and min) for the state of Florida.

64. Find the temperature extreme for the state of Texas.

65. Find the temperature extreme for the state with the coldest minimum.

66. Find the temperature extreme for the state with the hottest maximum.



Source: National Centers for Environmental Information

Solve.

67. The coldest temperature ever recorded on Earth was -129°F in Antarctica. The warmest temperature ever recorded was 134°F in Death Valley, California. How many degrees warmer is 134°F than -129°F ? (Source: *The World Almanac*)

69. Charley Hull from England finished first in the 2016 LPGA CME Group Tour Championship at the Tiburon Golf Club in Naples, Florida, with a score of -19 , or nineteen strokes under par. In 63rd place with a $+7$ was Christina Kim of the United States. What was the difference in scores between Hull and Kim? (Source: LPGA)

68. The coldest temperature ever recorded in the United States was -80°F in Alaska. The warmest temperature ever recorded was 134°F in California. How many degrees warmer is 134°F than -80°F ? (Source: *The World Almanac*)

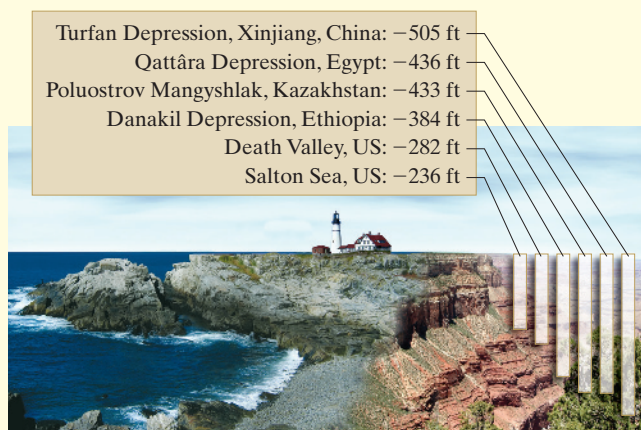
70. A woman received a statement of her charge account at Old Navy. She spent \$93 on purchases last month. She returned an \$18 blouse because she didn't like the color. She also returned a \$26 pajama set because it was damaged. What does she actually owe on her account?

71. The temperature on a February morning was -4° Celsius at 6 a.m. If the temperature drops 3 degrees by 7 a.m., rises 4 degrees between 7 a.m. and 8 a.m., and then drops 7 degrees between 8 a.m. and 9 a.m., find the temperature at 9 a.m.

72. Mauna Kea in Hawaii has an elevation of 13,796 feet above sea level. The Mid-America Trench in the Pacific Ocean has an elevation of 21,857 feet below sea level. Find the difference in elevation between those two points. (Source: National Geographic Society and Defense Mapping Agency)

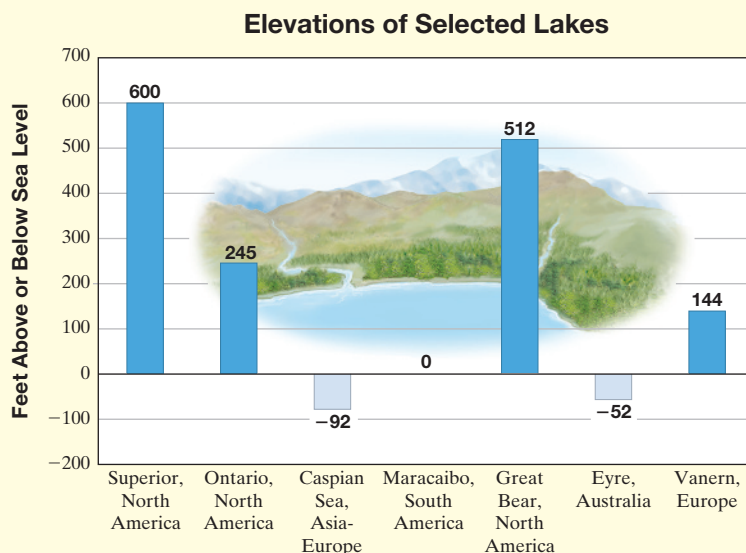
Some places on Earth lie below sea level, which is the average level of the surface of the oceans. Use this diagram to answer Exercises 73 through 76. (Source: *Fantastic Book of Comparisons*, Russell Ash)

73. Find the difference in elevation between Death Valley and Qattâra Depression.
74. Find the difference in elevation between the Danakil and Turfan Depressions.
75. Find the difference in elevation between the two lowest elevations shown.
76. Find the difference in elevation between the highest elevation shown and the lowest elevation shown.



The bar graph from Section 2.1 shows elevations of selected lakes. For Exercises 77 through 80, find the difference in elevation for the lakes listed. (Source: U.S. Geological Survey)

77. Lake Superior and Lake Eyre
78. Great Bear Lake and Caspian Sea
79. Lake Maracaibo and Lake Vanern
80. Lake Eyre and Caspian Sea



Solve.

81. The average daytime surface temperature of the hottest planet, Venus, is 867°F , while the average daytime surface temperature of the coldest planet, Neptune, is -330°F . Find the difference in temperatures.
82. The average daytime surface temperature of Mercury is 845°F , while the average daytime surface temperature of Jupiter is -162°F . Find the difference in temperatures.

A country's exports minus its imports is called the country's trade balance. Use this for Exercises 83 and 84.

- 83.** In November 2016, the United States had \$186 billion in exports and \$231 billion in imports. What was the U.S. trade balance in November 2016? (Source: U.S. Department of Commerce)
- 84.** In 2016, the United States exported an average of 5101 thousand barrels of petroleum products per day and imported an average of 10,044 thousand barrels of petroleum products per day. What was the average U.S. trade balance for petroleum products per day in 2016? (Source: U.S. Energy Information Administration)

Mixed Practice–Translating (Sections 2.2 and 2.3) Translate each phrase to an algebraic expression. Use “ x ” to represent “a number.”

- 85.** The sum of -5 and a number.
- 86.** The difference of -3 and a number.
- 87.** Subtract a number from -20 .
- 88.** Add a number and -36 .

Review

Multiply or divide as indicated. See Sections 1.5 and 1.6.

89. $\frac{100}{20}$

90. $\frac{96}{3}$

91. $\begin{array}{r} 23 \\ \times 46 \\ \hline \end{array}$

92. $\begin{array}{r} 51 \\ \times 89 \\ \hline \end{array}$

Concept Extensions

- 93.** Name two numbers whose difference is -3 .
- 94.** Name two numbers whose difference is -10 .

Each calculation below is **incorrect**. Find the error and correct it. See the Concept Check in this section.

95. $9 - (-7) \stackrel{?}{=} 2$

96. $-4 - 8 \stackrel{?}{=} 4$

97. $10 - 30 \stackrel{?}{=} 20$

98. $-3 - (-10) \stackrel{?}{=} -13$

Simplify. (Hint: Find the absolute values first.)

99. $|-3| - |-7|$

100. $|-12| - |-5|$

101. $|-5| - |5|$

102. $|-8| - |8|$

103. $|-15| - |-29|$

104. $|-23| - |-42|$

For Exercises 105 and 106, determine whether each statement is true or false.

105. $|-8 - 3| = 8 - 3$

106. $|-2 - (-6)| = |-2| - |-6|$

-  **107.** In your own words, explain how to subtract one signed number from another.

-  **108.** A student explains to you that the first step to simplify $8 + 12 \cdot 5 - 100$ is to add 8 and 12. Is the student correct? Explain why or why not.

2.4 Multiplying and Dividing Integers

Objectives

- A** Multiply Integers. 
- B** Divide Integers. 
- C** Evaluate an Algebraic Expression by Multiplying or Dividing. 
- D** Solve Problems by Multiplying or Dividing Integers. 

Multiplying and dividing integers is similar to multiplying and dividing whole numbers. One difference is that we need to determine whether the result is a positive number or a negative number.

Objective A Multiplying Integers

Consider the following pattern of products.

First factor decreases by 1 each time.	$3 \cdot 2 = 6$	Product decreases by 2 each time.
	$2 \cdot 2 = 4$	
	$1 \cdot 2 = 2$	
	$0 \cdot 2 = 0$	

This pattern can be continued, as follows.

$$\begin{aligned} -1 \cdot 2 &= -2 \\ -2 \cdot 2 &= -4 \\ -3 \cdot 2 &= -6 \end{aligned}$$

This suggests that the product of a negative number and a positive number is a negative number.

What is the sign of the product of two negative numbers? To find out, we form another pattern of products. Again, we decrease the first factor by 1 each time, but this time the second factor is negative.

First factor decreases by 1 each time.	$2 \cdot (-3) = -6$	Product increases by 3 each time.
	$1 \cdot (-3) = -3$	
	$0 \cdot (-3) = 0$	

This pattern continues as:

$$\begin{aligned} -1 \cdot (-3) &= 3 \\ -2 \cdot (-3) &= 6 \\ -3 \cdot (-3) &= 9 \end{aligned}$$

This suggests that the product of two negative numbers is a positive number. Thus we can determine the sign of a product when we know the signs of the factors.

Multiplying Numbers

The product of two numbers having the same sign is a positive number.

Product of Like Signs

$$\begin{aligned} (+)(+) &= + \\ (-)(-) &= + \end{aligned}$$

The product of two numbers having different signs is a negative number.

Product of Different Signs

$$\begin{aligned} (-)(+) &= - \\ (+)(-) &= - \end{aligned}$$

Practice 1–4

Multiply.

1. $-3 \cdot 8$
2. $-5(-2)$
3. $0 \cdot (-20)$
4. $10(-5)$

Answers

1. -24
2. 10
3. 0
4. -50

Examples

Multiply.

1. $-7 \cdot 3 = -21$
2. $-3(-5) = 15$
3. $0 \cdot (-4) = 0$
4. $10(-8) = -80$

Work Practice 1–4

Recall that by the associative and commutative properties for multiplication, we may multiply numbers in any order that we wish. In Examples 5 and 6, we multiply from left to right.

Examples Multiply.

$$5. \overbrace{7(-6)}^{(-42)}(-2) = -42(-2) \\ = 84$$

$$6. (-2)\overbrace{(-3)(-4)}^{12} = 6(-4) \\ = -24$$

$$7. (-1)(-2)(-3)(-4) = -1(-24) \quad \text{We have } -24 \text{ from Example 6.} \\ = 24$$

Work Practice 5–7

Helpful Hint

Have you noticed a pattern when multiplying signed numbers?

If we let $(-)$ represent a negative number and $(+)$ represent a positive number, then

The product of an even number of negative numbers is a positive result.	$(-)(-) = (+)$	The product of an odd number of negative numbers is a negative result.
	$(-)(-)(-) = (-)$	
	$(-)(-)(-)(-) = (+)$	
	$(-)(-)(-)(-)(-) = (-)$	

✓ **Concept Check** What is the sign of the product of five negative numbers? Explain.

Recall from our study of exponents that $2^3 = 2 \cdot 2 \cdot 2 = 8$. We can now work with bases that are negative numbers. For example,

$$(-2)^3 = (-2)(-2)(-2) = -8$$

Example 8 Evaluate: $(-5)^2$

Solution: Remember that $(-5)^2$ means 2 factors of -5 .

$$(-5)^2 = (-5)(-5) = 25$$

Work Practice 8

Notice in Example 8 the parentheses around -5 in $(-5)^2$. With these parentheses, -5 is the base that is squared. Without parentheses, such as -5^2 , only the 5 is squared. In other words, $-5^2 = -(5 \cdot 5) = -25$.

Example 9 Evaluate: -7^2

Solution: Remember that without parentheses, only the 7 is squared.

$$-7^2 = -(7 \cdot 7) = -49$$

Work Practice 9

Practice 5–7

Multiply.

5. $8(-6)(-2)$

6. $(-9)(-2)(-1)$

7. $(-3)(-4)(-5)(-1)$

Practice 8

Evaluate $(-2)^4$.

Practice 9

Evaluate: -8^2

Answers

5. 96 6. -18 7. 60 8. 16 9. -64

✓ **Concept Check Answer**
negative; answers may vary

Helpful Hint

Make sure you understand the difference between Examples 8 and 9.

→ parentheses, so -5 is squared

$$(-5)^2 = (-5)(-5) = 25$$

→ no parentheses, so only the 7 is squared

$$-7^2 = -(7 \cdot 7) = -49$$

Objective B Dividing Integers

Division of integers is related to multiplication of integers. The sign rules for division can be discovered by writing a related multiplication problem. For example,

$$\left. \begin{array}{ll} \frac{6}{2} = 3 & \text{because } 3 \cdot 2 = 6 \\ \frac{-6}{2} = -3 & \text{because } -3 \cdot 2 = -6 \\ \frac{6}{-2} = -3 & \text{because } -3 \cdot (-2) = 6 \\ \frac{-6}{-2} = 3 & \text{because } 3 \cdot (-2) = -6 \end{array} \right\}$$

Helpful Hint

Just as for whole numbers, division can be checked by multiplication.

Dividing Numbers

The quotient of two numbers having the same sign is a positive number.

The quotient of two numbers having different signs is a negative number.

Quotient of Like Signs

$$\frac{(+)}{(+)} = + \quad \frac{(-)}{(-)} = +$$

Quotient of Different Signs

$$\frac{(+)}{(-)} = - \quad \frac{(-)}{(+)} = -$$

Practice 10–12

Divide.

10. $\frac{42}{-7}$

11. $-16 \div (-2)$

12. $\frac{-80}{10}$

Answers

10. -6 11. 8 12. -8

✓ Concept Check Answer

$$\frac{-36}{-9} = 4$$

Examples

Divide.

10. $\frac{-12}{6} = -2$

11. $-20 \div (-4) = 5$

12. $\frac{48}{-3} = -16$

Work Practice 10–12

✓ Concept Check What is wrong with the following calculation?

$$\frac{-36}{-9} = -4$$

Examples Divide, if possible.

13. $\frac{0}{-5} = 0$ because $0 \cdot -5 = 0$

14. $\frac{-7}{0}$ is undefined because there is no number that gives a product of -7 when multiplied by 0.

Work Practice 13–14**Objective C** Evaluating Expressions

Next, we practice evaluating expressions given integer replacement values.

Example 15 Evaluate xy for $x = -2$ and $y = 7$.

Solution: Recall that xy means $x \cdot y$.

Replace x with -2 and y with 7 .

$$\begin{aligned} xy &= -2 \cdot 7 \\ &= -14 \end{aligned}$$

Work Practice 15**Example 16** Evaluate $\frac{x}{y}$ for $x = -24$ and $y = 6$.

Solution: $\frac{x}{y} = \frac{-24}{6}$ Replace x with -24 and y with 6 .

$$= -4$$

Work Practice 16**Objective D** Solving Problems by Multiplying and Dividing Integers

Many real-life problems involve multiplication and division of signed numbers.

Example 17 Calculating a Total Golf Score

A professional golfer finished seven strokes under par (-7) for each of three days of a tournament. What was his total score for the tournament?

Solution:

1. UNDERSTAND. Read and reread the problem. Although the key word is “total,” since this is repeated addition of the same number, we multiply.
2. TRANSLATE.

In words:	golfer's total score	=	number of days	·	score each day
	↓	↓	↓	↓	↓
Translate:	golfer's total	=	3	·	(-7)

3. SOLVE: $3 \cdot (-7) = -21$
4. INTERPRET. Check and state your conclusion: The golfer's total score was -21 , or 21 strokes under par.

Work Practice 17**Practice 13–14**

Divide, if possible.

13. $\frac{-6}{0}$

14. $\frac{0}{-7}$

Practice 15

Evaluate xy for $x = 5$ and $y = -8$.

Practice 16

Evaluate $\frac{x}{y}$ for $x = -12$ and $y = -3$.

Practice 17

A card player had a score of -13 for each of four games. Find the total score.

**Answers**

13. undefined 14. 0 15. -40
16. 4 17. -52

Vocabulary, Readiness & Video Check

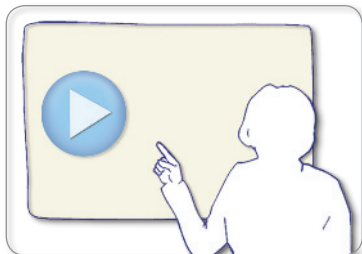
Use the choices below to fill in each blank. Each choice may be used more than once.

negative	0
positive	undefined

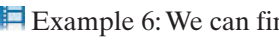
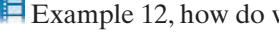
- The product of a negative number and a positive number is a(n) _____ number.
- The product of two negative numbers is a(n) _____ number.
- The quotient of two negative numbers is a(n) _____ number.
- The quotient of a negative number and a positive number is a(n) _____ number.
- The product of a negative number and zero is _____.
- The quotient of 0 and a negative number is _____.
- The quotient of a negative number and 0 is _____.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 2.4 

- Objective A** 8. Explain the role of parentheses when comparing  Examples 3 and 4. 
- Objective B** 9. Complete this statement based on the lecture before  Example 6: We can find out about sign rules for division because we know sign rules for _____. 
- Objective C** 10. In  Example 10, what are you asked to remember about the algebraic expression ab ? 
- Objective D** 11. In  Example 12, how do we know the example will involve a negative number? 

2.4 Exercise Set MyLab Math

Objective A Multiply. See Examples 1 through 4.

- | | | | |
|-------------|------------|--|-------------|
| 1. $-6(-2)$ | 2. $5(-3)$ |  3. $-4(9)$ | 4. $-7(-2)$ |
| 5. $9(-9)$ | 6. $-9(7)$ | 7. $0(-11)$ | 8. $-6(0)$ |

Multiply. See Examples 5 through 7.

- | | | | |
|---------------------|---------------------|---------------------|--------------------|
| 9. $6(-2)(-4)$ | 10. $-2(3)(-7)$ | 11. $-1(-3)(-4)$ | 12. $-8(-3)(-3)$ |
| 13. $-4(4)(-5)$ | 14. $2(-5)(-4)$ | 15. $10(-5)(0)(-7)$ | 16. $3(0)(-4)(-8)$ |
| 17. $-5(3)(-1)(-1)$ | 18. $-2(-1)(3)(-2)$ | | |

Evaluate. See Examples 8 and 9.

▶ 19. -3^2

20. -2^4

21. $(-3)^3$

22. $(-1)^4$

23. -6^2

24. -4^3

25. $(-4)^3$

▶ 26. $(-3)^2$

Objective B Find each quotient. See Examples 10 through 14.

27. $-24 \div 3$

28. $90 \div (-9)$

▶ 29. $\frac{-30}{6}$

30. $\frac{56}{-8}$

31. $\frac{-77}{-11}$

32. $\frac{-32}{4}$

33. $\frac{0}{-21}$

34. $\frac{-13}{0}$

▶ 35. $\frac{-10}{0}$

▶ 36. $\frac{0}{-15}$

37. $\frac{56}{-4}$

38. $\frac{-24}{-12}$

Objectives A B **Mixed Practice** Multiply or divide as indicated. See Examples 1 through 14.

39. $-14(0)$

40. $0(-100)$

41. $-5(3)$

42. $-6 \cdot 2$

43. $-9 \cdot 7$

44. $-12(13)$

▶ 45. $-7(-6)$

46. $-9(-5)$

▶ 47. $-3(-4)(-2)$

48. $-7(-5)(-3)$

49. $(-7)^2$

50. $(-5)^2$

51. $-\frac{25}{5}$

52. $-\frac{30}{5}$

53. $-\frac{72}{8}$

54. $-\frac{49}{7}$

55. $-18 \div 3$

56. $-15 \div 3$

57. $4(-10)(-3)$

58. $6(-5)(-2)$

59. $-30(6)(-2)(-3)$

60. $-20 \cdot 5 \cdot (-5) \cdot (-3)$

61. $\frac{-25}{0}$

62. $\frac{0}{-14}$

63. $\frac{120}{-20}$

64. $\frac{63}{-9}$

65. $280 \div (-40)$

66. $480 \div (-8)$

▶ 67. $\frac{-12}{-4}$

68. $\frac{-36}{-3}$

69. -1^4

70. -2^3

71. $(-2)^5$

72. $(-11)^2$

73. $-2(3)(5)(-6)$

74. $-1(2)(7)(-3)$

75. $(-1)^{32}$

76. $(-1)^{33}$

77. $-2(-3)(-5)$

78. $-2(-2)(-3)(-2)$

79. $-48 \cdot 23$

80. $-56 \cdot 43$

81. $35 \cdot (-82)$

82. $70 \cdot (-23)$

Objective C Evaluate ab for the given replacement values. See Example 15.

83. $a = -8$ and $b = 7$

84. $a = 5$ and $b = -1$

▶ 85. $a = 9$ and $b = -2$

86. $a = -8$ and $b = 8$

87. $a = -7$ and $b = -5$

88. $a = -9$ and $b = -6$

Evaluate $\frac{x}{y}$ for the given replacement values. See Example 16.

▶ 89. $x = 5$ and $y = -5$

90. $x = 9$ and $y = -3$

91. $x = -15$ and $y = 0$

92. $x = 0$ and $y = -5$

93. $x = -36$ and $y = -6$

94. $x = -10$ and $y = -10$

Evaluate xy and also $\frac{x}{y}$ for the given replacement values. See Examples 15 and 16.

95. $x = -8$ and $y = -2$

96. $x = 20$ and $y = -5$

97. $x = 0$ and $y = -8$

98. $x = -3$ and $y = 0$

Objective D Translating Translate each phrase; then simplify. See Example 17.

99. Find the quotient of -54 and 9 .

100. Find the quotient of -63 and -3 .

101. Find the product of -42 and -6 .

102. Find the product of -49 and 5 .

Translating Translate each phrase to an expression. Use x to represent “a number.” See Example 17.

103. The product of -71 and a number

104. The quotient of -8 and a number

105. Subtract a number from -16 .

106. The sum of a number and -12

107. -29 increased by a number

108. The difference of a number and -10

109. Divide a number by -33 .

110. Multiply a number by -17 .

Solve. See Example 17.

- ▶ 111. A football team lost four yards on each of three consecutive plays. Represent the total loss as a product of signed numbers and find the total loss.

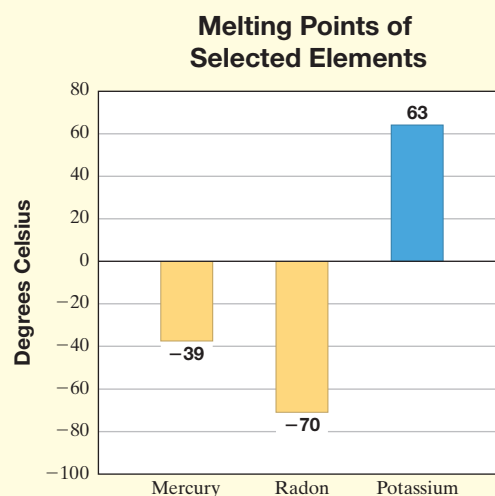
112. An investor lost \$400 on each of seven consecutive days in the stock market. Represent his total loss as a product of signed numbers and find his total loss.

113. A deep-sea diver must move up or down in the water in short steps in order to keep from getting a physical condition called the “bends.” Suppose a diver moves down from the surface in five steps of 20 feet each. Represent his total movement as a product of signed numbers and find the product.

114. A weather forecaster predicts that the temperature will drop five degrees each hour for the next six hours. Represent this drop as a product of signed numbers and find the total drop in temperature.

The graph shows melting points in degrees Celsius of selected elements. Use this graph to answer Exercises 115 through 118.

- 115.** The melting point of nitrogen is 3 times the melting point of radon. Find the melting point of nitrogen.
- 116.** The melting point of rubidium is -1 times the melting point of mercury. Find the melting point of rubidium.
- 117.** The melting point of argon is -3 times the melting point of potassium. Find the melting point of argon.
- 118.** The melting point of strontium is -11 times the melting point of radon. Find the melting point of strontium.



Solve. See Example 17.

- 119.** For the second quarter of 2016, Sears Holding Corporation posted a loss of \$396 million due to a decline in sales. If this trend were consistent for each month of the quarter, how much would you expect this loss to have been for each month? (Source: Sears Holdings Corporation)
- 120.** From 2003 through 2015, the estimated population of mourning doves in the United States dropped by 73,023,000 birds. If the loss in birds were evenly spaced over these years, by how many birds would you expect the population to drop each year? (Source: U.S. Fish and Wildlife Service)
- 121.** In 2010 there were 24,814 analog (nondigital) U.S. movie screens. In 2015, this number of screens dropped to 1109. (Source: Motion Picture Association of America)
- Find the change in the number of U.S. analog screens from 2010 to 2015.
 - Find the average change per year in the number of analog movie screens over this period.
- 122.** In 1987, the California condor was all but extinct in the wild, with about 30 condors in the world. The U.S. Fish and Wildlife Service captured the condors in the wild in an aggressive move to rebuild the population by breeding them in captivity and releasing the chicks into the wild. The condor population increased to approximately 436 by 2016. (Source: U.S. Fish and Wildlife Service)
- Find the change in the number of California condors from 1987 to 2016.
 - Find the average change per year in the California condor population over the period in part **a**.



Review

Perform each indicated operation. See Section 1.7.

123. $90 + 12^2 - 5^3$

124. $3 \cdot (7 - 4) + 2 \cdot 5^2$

125. $12 \div 4 - 2 + 7$

126. $12 \div (4 - 2) + 7$

Concept Extensions

Mixed Practice (Sections 2.2, 2.3, and 2.4) Perform the indicated operations.

127. $-57 \div 3$

128. $-9(-11)$

129. $-8 - 20$

130. $-4 + (-3) + 21$

131. $-4 - 15 - (-11)$

132. $-16 - (-2)$

Solve. For Exercises 133 and 134, see the first Concept Check in this section.

133. What is the sign of the product of seven negative numbers?

134. What is the sign of the product of ten negative numbers?

Without actually finding the product, write the list of numbers in Exercises 135 and 136 in order from least to greatest. For help, see a helpful hint box in this section.

135. $(-2)^{12}, (-2)^{17}, (-5)^{12}, (-5)^{17}$

136. $(-1)^{50}, (-1)^{55}, 0^{15}, (-7)^{20}, (-7)^{23}$

 **137.** In your own words, explain how to divide two integers.

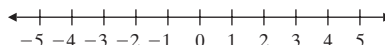
 **138.** In your own words, explain how to multiply two integers.

Integers

1. The record low temperature in New Mexico is 50 degrees Fahrenheit below zero. The highest temperature in that state is 122 degrees above zero. Represent each quantity by an integer.



2. Graph the signed numbers on the given number line. $-4, 0, -1, 3$



Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____

Insert $<$ or $>$ between each pair of numbers to make a true statement.

3. 0 -10 4. -4 4 5. -15 -5 6. -2 -7

Simplify.

7. $|-3|$ 8. $|-9|$ 9. $-|-4|$ 10. $-(-5)$

Find the opposite of each number.

11. 11 12. -3 13. 64 14. 0

Perform the indicated operation.

15. $-3 + 15$ 16. $-9 + (-11)$ 17. $-8(-6)(-1)$ 18. $-18 \div 2$
19. $65 + (-55)$ 20. $1000 - 1002$ 21. $53 - (-53)$ 22. $-2 - 1$

23. _____

24. _____

25. _____

26. _____

27. _____

28. _____

29. _____

30. _____

31. _____

32. _____

33. _____

34. _____

35. _____

36. _____

37. _____

38. _____

39. _____

40. _____

41. _____

42. _____

43. _____

44. _____

45. _____

46. _____

23. $\frac{0}{-47}$

24. $\frac{-36}{-9}$

25. $-17 - (-59)$

26. $-8 + (-6) + 20$

27. $\frac{-95}{-5}$

28. $-9(100)$

29. $-12 - 6 - (-6)$

30. $-4 + (-8) - 16 - (-9)$

31. $\frac{-105}{0}$

32. $7(-16)(0)(-3)$

Translating *Translate each phrase; then simplify.*33. Subtract -8 from -12 .34. Find the sum of -17 and -27 .35. Find the product of -5 and -25 .36. Find the quotient of -100 and -5 .**Translating** *Translate each phrase to an expression. Use x to represent "a number."*37. Divide a number by -17 38. The sum of -3 and a number39. A number decreased by -18 40. The product of -7 and a number*Evaluate the expressions below for $x = -3$ and $y = 12$.*

41. $x + y$

42. $x - y$

43. $2y - x$

44. $3y + x$

45. $5x$

46. $\frac{y}{x}$

2.5 Order of Operations

Objective A Simplifying Expressions

We first discussed the order of operations in Chapter 1. In this section, you are given an opportunity to practice using the order of operations when expressions contain signed numbers. The rules for the order of operations from Section 1.7 are repeated here.

Order of Operations

1. Perform all operations within parentheses (), brackets [], or other grouping symbols such as fraction bars, starting with the innermost set.
2. Evaluate any expressions with exponents.
3. Multiply or divide in order from left to right.
4. Add or subtract in order from left to right.

Before simplifying other expressions, make sure you are confident simplifying Examples 1 through 3.

Examples Find the value of each expression.

1. $(-3)^2 = (-3)(-3) = 9$ The base of the exponent is -3 .
2. $-3^2 = -(3)(3) = -9$ The base of the exponent is 3 .
3. $2 \cdot 5^2 = 2 \cdot (5 \cdot 5) = 2 \cdot 25 = 50$ The base of the exponent is 5 .

Work Practice 1–3

Helpful Hint

When simplifying expressions with exponents, remember that parentheses make an important difference.

$(-3)^2$ and -3^2 **do not** mean the same thing.

$(-3)^2$ means $(-3)(-3) = 9$.

-3^2 means the opposite of $3 \cdot 3$, or -9 .

Only with parentheses around it is the -3 squared.

Example 4 Simplify: $\frac{-6(2)}{-3}$

Solution: First we multiply -6 and 2 . Then we divide.

$$\begin{aligned}\frac{-6(2)}{-3} &= \frac{-12}{-3} \\ &= 4\end{aligned}$$

Work Practice 4

Objectives

- A** Simplify Expressions by Using the Order of Operations. 
- B** Evaluate an Algebraic Expression. 
- C** Find the Average of a List of Numbers. 

Practice 1–3

Find the value of each expression.

1. $(-2)^4$
2. -2^4
3. $3 \cdot 6^2$

Practice 4

Simplify: $\frac{-25}{5(-1)}$

Answers

1. 16 2. -16 3. 108 4. 5

Practice 5

Simplify: $\frac{-18 + 6}{-3 - 1}$

Practice 6

Simplify: $30 + 50 + (-4)^3$

Practice 7

Simplify: $-2^3 + (-4)^2 + 1^5$

Practice 8

Simplify:
 $2(2 - 9) + (-12) - 3$

Practice 9

Simplify:
 $(-5) \cdot |-8| + (-3) + 2^3$

Answers

5. 3 6. 16 7. 9 8. -29 9. -35

Example 5

Simplify: $\frac{12 - 16}{-1 + 3}$

Solution: We simplify above and below the fraction bar separately. Then we divide.

$$\frac{12 - 16}{-1 + 3} = \frac{-4}{2}$$

$$= -2$$

Work Practice 5**Example 6**

Simplify: $60 + 30 + (-2)^3$

Solution: $60 + 30 + (-2)^3 = 60 + 30 + (-8)$ Write $(-2)^3$ as -8 .
 $= 90 + (-8)$ Add from left to right.
 $= 82$

Work Practice 6**Example 7**

Simplify: $-4^2 + (-3)^2 - 1^3$

Solution:

$$-4^2 + (-3)^2 - 1^3 = -16 + 9 - 1$$
 Simplify expressions with exponents.
 $= -7 - 1$ Add or subtract from left to right.
 $= -8$

Work Practice 7**Example 8**

Simplify: $3(4 - 7) + (-2) - 5$

Solution:

$$3(4 - 7) + (-2) - 5 = 3(-3) + (-2) - 5$$
 Simplify inside parentheses.
 $= -9 + (-2) - 5$ Multiply.
 $= -11 - 5$ Add or subtract from left to right.
 $= -16$

Work Practice 8**Example 9**

Simplify: $(-3) \cdot |-5| - (-2) + 4^2$

Solution:

$$(-3) \cdot |-5| - (-2) + 4^2 = (-3) \cdot 5 - (-2) + 4^2$$
 Write $|-5|$ as 5.
 $= (-3) \cdot 5 - (-2) + 16$ Write 4^2 as 16.
 $= -15 - (-2) + 16$ Multiply.
 $= -13 + 16$ Add or subtract from left to right.
 $= 3$

Work Practice 9

Example 10 Simplify: $-2[-3 + 2(-1 + 6)] - 5$

Solution: Here we begin with the innermost set of parentheses.

$$\begin{aligned}
 -2[-3 + 2(-1 + 6)] - 5 &= -2[-3 + 2(5)] - 5 && \text{Write } -1 + 6 \text{ as } 5. \\
 &= -2[-3 + 10] - 5 && \text{Multiply.} \\
 &= -2(7) - 5 && \text{Add.} \\
 &= -14 - 5 && \text{Multiply.} \\
 &= -19 && \text{Subtract.}
 \end{aligned}$$

Work Practice 10

 **Concept Check** True or false? Explain your answer. The result of

$$-4(3 - 7) - 8(9 - 6)$$

is positive because there are four negative signs.

Objective B Evaluating Expressions

Now we practice evaluating expressions.

Example 11 Evaluate x^2 and $-x^2$ for $x = -11$.

Solution: $x^2 = (-11)^2 = (-11)(-11) = 121$
 $-x^2 = -(-11)^2 = -(-11)(-11) = -121$

Work Practice 11

Example 12 Evaluate $6z^2$ for $z = 2$ and $z = -2$.

Solution: $6z^2 = 6(2)^2 = 6(4) = 24$
 $6z^2 = 6(-2)^2 = 6(4) = 24$

Work Practice 12

Example 13 Evaluate $x + 2y - z$ for $x = 3$, $y = -5$, and $z = -4$.

Solution: Replace x with 3, y with -5 , z with -4 , and simplify.

Helpful Hint

Remember to rewrite the subtraction sign.

$$\begin{aligned}
 x + 2y - z &= 3 + 2(-5) - (-4) && \text{Let } x = 3, y = -5, \text{ and } z = -4. \\
 &= 3 + (-10) + 4 && \text{Replace } 2(-5) \text{ with its product, } -10. \\
 &= -3 && \text{Add.}
 \end{aligned}$$

Work Practice 13

Practice 10

Simplify:

$$-4[-6 + 5(-3 + 5)] - 7$$

Practice 11

Evaluate x^2 and $-x^2$ for $x = -15$.

Practice 12

Evaluate $5y^2$ for $y = 4$ and $y = -4$.

Practice 13

Evaluate $x^2 + y$ for $x = -6$ and $y = -3$.

Answers

10. -23 11. $225; -225$ 12. $80; 80$
 13. 33

 **Concept Check Answer**

false; $-4(3 - 7) - 8(9 - 6) = -8$

Practice 14Evaluate $4 - x^2$ for $x = -8$.**Practice 15**

Find the average of the temperatures for the months October through April.

Answers14. -60 15. -5°F **Example 14** Evaluate $7 - x^2$ for $x = -4$.**Solution:** Replace x with -4 and simplify carefully!

$$\begin{aligned}
 7 - x^2 &= 7 - (-4)^2 \\
 &\quad \downarrow \quad \downarrow \\
 &= 7 - 16 \qquad (-4)^2 = (-4)(-4) = 16 \\
 &= -9 \qquad \text{Subtract.}
 \end{aligned}$$

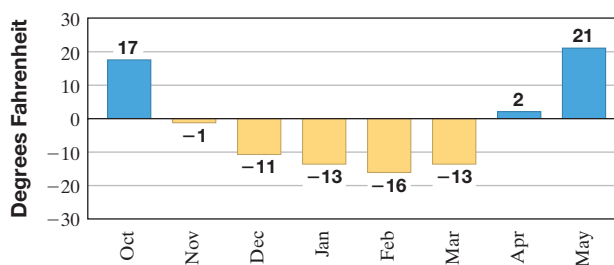
Work Practice 14**Objective C** Finding Averages

Recall from Chapter 1 that the average of a list of numbers is

$$\text{average} = \frac{\text{sum of numbers}}{\text{number of numbers}}$$

Example 15

The graph shows some monthly normal temperatures for Barrow, Alaska. Use this graph to find the average of the temperatures for the months January through April.

Monthly Normal Temperatures for Barrow, Alaska**Solution:** By reading the graph, we have

$$\begin{aligned}
 \text{average} &= \frac{-13 + (-16) + (-13) + 2}{4} \quad \text{There are 4 months from January through April.} \\
 &= \frac{-40}{4} \\
 &= -10
 \end{aligned}$$

The average of the temperatures is -10°F .**Work Practice 15****Calculator Explorations** Simplifying an Expression Containing a Fraction Bar

Recall that even though most calculators follow the order of operations, parentheses must sometimes be inserted. For example, to simplify $\frac{-8 + 6}{-2}$ on a calculator, enter parentheses around the expression above the fraction bar so that it is simplified separately.

To simplify $\frac{-8 + 6}{-2}$, press the keys

([8] [+/−] [+] [6] [)] [÷] [2] [+/−] [=] or
 ([(−)] [8] [+] [6] [)] [÷] [(−)] [2] [ENTER]

The display will read 1.

$$\text{Thus, } \frac{-8 + 6}{-2} = 1.$$

Use a calculator to simplify.

$$1. \quad \frac{-120 - 360}{-10}$$

$$3. \quad \frac{-316 + (-458)}{28 + (-25)}$$

$$2. \quad \frac{4750}{-2 + (-17)}$$

$$4. \quad \frac{-234 + 86}{-18 + 16}$$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

average subtraction division $-7 - 3(1)$
 addition multiplication $-7 - 3(-1)$

- To simplify $-2 \div 2 \cdot (3)$, which operation should be performed first? _____
- To simplify $-9 - 3 \cdot 4$, which operation should be performed first? _____
- The _____ of a list of numbers is $\frac{\text{sum of numbers}}{\text{number of numbers}}$.
- To simplify $5[-9 + (-3)] \div 4$, which operation should be performed first? _____
- To simplify $-2 + 3(10 - 12) \cdot (-8)$, which operation should be performed first? _____
- To evaluate $x - 3y$ for $x = -7$ and $y = -1$, replace x with -7 and y with -1 and evaluate _____.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 2.5 

Objective A 7. In  Example 1, what two things about the fraction bar are we reminded of? 

Objective B 8. In  Example 5, why is it important to place the replacement value for x within parentheses? 

Objective C 9. From the lecture before  Example 6, explain why finding the average is a good example of an application for this section. 

2.5 Exercise Set MyLab Math

Objective A Simplify. See Examples 1 through 10.

- | | | | |
|--------------------------|--------------------------|--|--------------------------|
| 1. $(-5)^3$ | 2. -2^4 | 3. -4^3 | 4. $(-2)^4$ |
| 5. $8 \cdot 2^2$ | 6. $5 \cdot 2^3$ | 7. $8 - 12 - 4$ | 8. $10 - 23 - 12$ |
| 9. $7 + 3(-6)$ | 10. $-8 + 4(3)$ | 11. $5(-9) + 2$ | 12. $7(-6) + 3$ |
| 13. $-10 + 4 \div 2$ | 14. $-12 + 6 \div 3$ | 15. $6 + 7 \cdot 3 - 10$ | 16. $5 + 9 \cdot 4 - 20$ |
| 17. $\frac{16 - 13}{-3}$ | 18. $\frac{20 - 15}{-1}$ |  19. $\frac{24}{10 + (-4)}$ | 20. $\frac{88}{-8 - 3}$ |
| 21. $5(-3) - (-12)$ | 22. $7(-4) - (-6)$ | 23. $[8 + (-4)]^2$ | 24. $[9 + (-2)]^3$ |

25. $8 \cdot 6 - 3 \cdot 5 + (-20)$

26. $7 \cdot 6 - 6 \cdot 5 + (-10)$

27. $4 - (-3)^4$

28. $7 - (-5)^2$

29. $|7 + 3| \cdot 2^3$

30. $|-3 + 7| \cdot 7^2$

31. $7 \cdot 6^2 + 4$

32. $10 \cdot 5^3 + 7$

33. $7^2 - (4 - 2^3)$

34. $8^2 - (5 - 2)^4$

35. $|3 - 15| \div 3$

36. $|12 - 19| \div 7$

37. $-(-2)^6$

38. $-(-2)^3$

39. $(5 - 9)^2 \div (4 - 2)^2$

40. $(2 - 7)^2 \div (4 - 3)^4$

41. $|8 - 24| \cdot (-2) \div (-2)$

42. $|3 - 15| \cdot (-4) \div (-16)$

43. $(-12 - 20) \div 16 - 25$

44. $(-20 - 5) \div 5 - 15$

45. $5(5 - 2) + (-5)^2 - 6$

46. $3 \cdot (8 - 3) + (-4) - 10$

47. $(2 - 7) \cdot (6 - 19)$

48. $(4 - 12) \cdot (8 - 17)$

49. $(-36 \div 6) - (4 \div 4)$

50. $(-4 \div 4) - (8 \div 8)$

51. $(10 - 4^2)^2$

52. $(11 - 3^2)^3$

53. $2(8 - 10)^2 - 5(1 - 6)^2$

54. $-3(4 - 8)^2 + 5(14 - 16)^3$

55. $3(-10) \div [5(-3) - 7(-2)]$

56. $12 - [7 - (3 - 6)] + (2 - 3)^3$

57. $\frac{(-7)(-3) - (4)(3)}{3[7 \div (3 - 10)]}$

58. $\frac{10(-1) - (-2)(-3)}{2[-8 \div (-2 - 2)]}$

59. $-3[5 + 2(-4 + 9)] + 15$

60. $-2[6 + 4(2 - 8)] - 25$

Objective B Evaluate each expression for $x = -2$, $y = 4$, and $z = -1$. See Examples 11 through 14.

61. $x + y + z$

62. $x - y - z$

63. $2x - 3y - 4z$

64. $5x - y + 4z$

65. $x^2 - y$

66. $x^2 + z$

67. $\frac{5y}{z}$

68. $\frac{4x}{y}$

Evaluate each expression for $x = -3$ and $z = -4$. See Examples 11 through 14.

69. x^2

70. z^2

71. $-z^2$

72. $-x^2$

73. $2z^3$

74. $3x^2$

75. $10 - x^2$

76. $3 - z^2$

77. $2x^3 - z$

78. $3z^2 - x$

Objective C Find the average of each list of numbers. See Example 15.

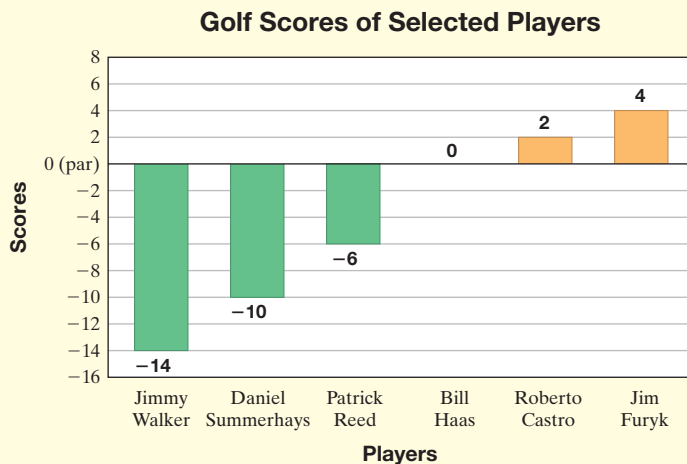
79. $-10, 8, -4, 2, 7, -5, -12$

80. $-18, -8, -1, -1, 0, 4$

81. $-17, -26, -20, -13$

82. $-40, -20, -10, -15, -5$

Scores in golf can be 0 (also called par), a positive integer (also called above par), or a negative integer (also called below par). The bar graph shows final scores of selected golfers from a 2016 tournament. Use this graph for Exercises 83 through 88. (Source: PGA)



83. Find the difference between the lowest score shown and the highest score shown.

84. Find the difference between the two lowest scores.

85. Find the average of the scores for Reed, Haas, Castro, and Furyk. (*Hint:* Here, the average is the sum of the scores divided by the number of players).

86. Find the average of the scores for Walker, Summerhays, and Reed.

87. Can the average for the scores in Exercise 85 be greater than the highest score, 4? Explain why or why not.

88. Can the average of the scores in Exercise 86 be less than the lowest score, -14 ? Explain why or why not.

Review

Perform each indicated operation. See Sections 1.3, 1.5, and 1.6.

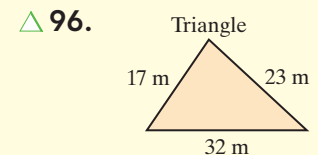
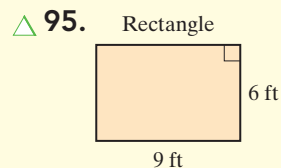
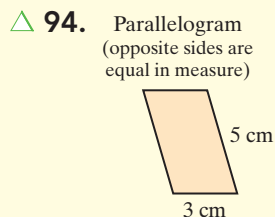
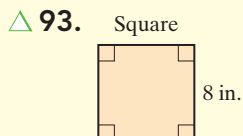
89. $45 \cdot 90$

90. $90 \div 45$

91. $90 - 45$

92. $45 + 90$

Find the perimeter of each figure. See Section 1.3.



Concept Extensions

Insert parentheses where needed so that each expression evaluates to the given number.

97. $2 \cdot 7 - 5 \cdot 3$; evaluates to 12

98. $7 \cdot 3 - 4 \cdot 2$; evaluates to 34

99. $-6 \cdot 10 - 4$; evaluates to -36

100. $2 \cdot 8 \div 4 - 20$; evaluates to -36

 101. Are parentheses necessary in the expression $3 + (4 \cdot 5)$? Explain your answer.

 102. Are parentheses necessary in the expression $(3 + 4) \cdot 5$? Explain your answer.

 103. Discuss the effect parentheses have in an exponential expression. For example, what is the difference between $(-6)^2$ and -6^2 ?

 104. Discuss the effect parentheses have in an exponential expression. For example, what is the difference between $(2 \cdot 4)^2$ and $2 \cdot 4^2$?

Evaluate.

 105. $(-12)^4$

 106. $(-17)^6$

 107. $x^3 - y^2$ for $x = 21$ and $y = -19$

 108. $3x^2 + 2x - y$ for $x = -18$ and $y = 2868$

109. $(xy + z)^x$ for $x = 2$, $y = -5$, and $z = 7$

110. $5(ab + 3)^b$ for $a = -2$, $b = 3$

2.6 Solving Equations: The Addition and Multiplication Properties

Objectives

- A** Identify Solutions of Equations. 
- B** Use the Addition Property of Equality to Solve Equations. 
- C** Use the Multiplication Property of Equality to Solve Equations. 

In this section, we introduce properties of equations and we use these properties to begin solving equations. Now that we know how to perform operations on integers, this is an excellent way to practice these operations.

First, let's recall the difference between an equation and an expression. From Section 1.8, a combination of operations on variables and numbers is an expression, and an equation is of the form “expression = expression.”

Equations	Expressions
$3x - 1 = -17$	$3x - 1$
area = length $\cdot$ width	$5(20 - 3) + 10$
$8 + 16 = 16 + 8$	y^3
$-9a + 11b = 14b + 3$	$-x^2 + y - 2$

Helpful Hint

Simply stated, an equation contains “=” while an expression does not. Also, we *simplify* expressions and *solve* equations.

Objective A Identifying Solutions of Equations

Let's practice identifying solutions of equations. Recall from Section 1.8 that a solution of an equation is a number that when substituted for the variable makes the equation a true statement.

For example,

-8 is a solution of

$$\frac{x}{2} = -4, \text{ because}$$

$$\frac{-8}{2} = -4, \text{ or}$$

$$-4 = -4 \text{ is true.}$$

Also,

-8 is not a solution of

$$x + 6 = 2, \text{ because}$$

$$-8 + 6 = 2 \text{ is false.}$$

Let's practice determining whether a number is a solution of an equation. In this section, we will be performing operations on integers.

Example 1 Determine whether -1 is a solution of the equation $3y + 1 = 3$.

Solution:

$$3y + 1 = 3$$

$$3(-1) + 1 \stackrel{?}{=} 3$$

$$-3 + 1 \stackrel{?}{=} 3$$

$$-2 = 3 \quad \text{False}$$

Since $-2 = 3$ is false, -1 is *not* a solution of the equation.

Work Practice 1

Now we know how to check whether a number is a solution. But, given an equation, how do we find its **solution**? In other words, how do we find a number that makes the equation true? How do we solve an equation?

Objective B Using the Addition Property to Solve Equations

To solve an equation, we use properties of equality to write simpler equations, all equivalent to the original equation, until the final equation has the form

$$x = \text{number} \quad \text{or} \quad \text{number} = x$$

Equivalent equations have the same solution, so the word “number” above represents the solution of the original equation. The first property of equality to help us write simpler, equivalent equations is the **addition property of equality**.

Practice 1

Determine whether -2 is a solution of the equation $-4x - 3 = 5$.

Answer

1. yes

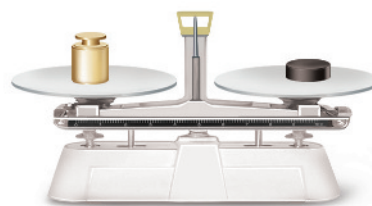
Addition Property of Equality

Let a , b , and c represent numbers. Then

$a = b$ and $a + c = b + c$ are equivalent equations.	Also, $a = b$ and $a - c = b - c$ are equivalent equations.
---	---

In other words, the same number may be added to or subtracted from both sides of an equation without changing the solution of the equation. (Recall from Section 2.3 that we defined subtraction as addition of the first number and the opposite of the second number. Because of this, the addition property of equality also allows us to subtract the same number from both sides.)

A good way to visualize a true equation is to picture a balanced scale. Since it is balanced, the objects on each side of the scale weigh the same amount. Similarly, in a true equation the expressions on each side have the same value. Picturing our balanced scale, if we add the same weight to each side, the scale remains balanced.

**Practice 2**

Solve the equation for y : $y - 6 = -2$

Example 2 Solve: $x - 2 = -1$ for x .

Solution: To solve the equation for x , we need to rewrite the equation in the form $x = \text{number}$. In other words, our goal is to get x alone on one side of the equation. To do so, we add 2 to both sides of the equation.

$$\begin{aligned}
 x - 2 &= -1 \\
 x - 2 + 2 &= -1 + 2 && \text{Add 2 to both sides of the equation.} \\
 x + 0 &= 1 && \text{Replace } -2 + 2 \text{ with 0.} \\
 x &= 1 && \text{Simplify by replacing } x + 0 \text{ with } x.
 \end{aligned}$$

Check: To check, we replace x with 1 in the *original* equation.

$$\begin{aligned}
 x - 2 &= -1 && \text{Original equation} \\
 1 - 2 &\stackrel{?}{=} -1 && \text{Replace } x \text{ with 1.} \\
 -1 &= -1 && \text{True}
 \end{aligned}$$

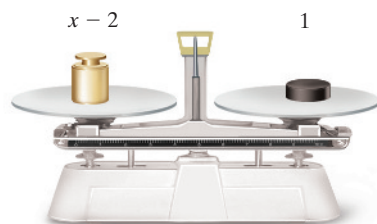
Since $-1 = -1$ is a true statement, 1 is the solution of the equation.

Work Practice 2

Helpful Hint

Note that it is always a good idea to check the solution in the *original* equation to see that it makes the equation a true statement.

Let's visualize how we used the addition property of equality to solve an equation. Picture the equation $x - 2 = 1$ as a balanced scale. The left side of the equation has the same value as the right side.



If the same weight is added to each side of a scale, the scale remains balanced. Likewise, if the same number is added to each side of an equation, the left side continues to have the same value as the right side.



Example 3 Solve: $-8 = n + 1$

Solution: To get n alone on one side of the equation, we subtract 1 from both sides of the equation.

$$-8 = n + 1$$

$$-8 - 1 = n + 1 - 1 \quad \text{Subtract 1 from both sides.}$$

$$-9 = n + 0 \quad \text{Replace } 1 - 1 \text{ with } 0.$$

$$-9 = n \quad \text{Simplify.}$$

Check:

$$-8 = n + 1$$

$$-8 \stackrel{?}{=} -9 + 1 \quad \text{Replace } n \text{ with } -9.$$

$$-8 = -8 \quad \text{True}$$

The solution is -9 .

Work Practice 3

Helpful Hint

Remember that we can get the variable alone on either side of the equation. For example, the equations $-9 = n$ and $n = -9$ both have the solution of -9 .

✓ **Concept Check** What number should be added to or subtracted from both sides of the equation in order to solve the equation $-3 = y + 2$?

Example 4 Solve: $x = -60 + 4 + 10$

Solution: Study this equation for a moment. Notice that our variable x is alone on the left side. Thus, we only need to add on the right side to find the value of x .

$$x = -60 + 4 + 10$$

$$x = -56 + 10 \quad \text{Add } -60 \text{ and } 4.$$

$$x = -46 \quad \text{Add } -56 \text{ and } 10.$$

Check to see that -46 is the solution.

Work Practice 4

Practice 3

Solve: $-2 = z + 8$

Practice 4

Solve: $x = -2 + 90 + (-100)$

Answers

3. -10 4. -12

✓ **Concept Check Answer**
Subtract 2 from both sides.

Objective C Using the Multiplication Property to Solve Equations

Although the addition property of equality is a powerful tool for helping us solve equations, it cannot help us solve all types of equations. For example, it cannot help us solve an equation such as $2x = 6$. To solve this equation, we use a second property of equality called the **multiplication property of equality**.

Multiplication Property of Equality

Let a , b , and c represent numbers and let $c \neq 0$. Then

$a = b$	Also, $a = b$
and $a \cdot c = b \cdot c$	and $\frac{a}{c} = \frac{b}{c}$
are equivalent equations.	are equivalent equations.

In other words, both sides of an equation may be multiplied or divided by the same nonzero number without changing the solution of the equation. (We will see in Chapter 4 how the multiplication property allows us to divide both sides of an equation by the same nonzero number.)

To solve an equation like $2x = 6$ for x , notice that 2 is *multiplied* by x . To get x alone, we use the multiplication property of equality to *divide* both sides of the equation by 2, and simplify as follows:

$$2x = 6$$

$$\frac{2 \cdot x}{2} = \frac{6}{2} \quad \text{Divide both sides by 2.}$$

Then it can be shown that an expression such as $\frac{2 \cdot x}{2}$ is equivalent to $\frac{2}{2} \cdot x$, so

$$\frac{2 \cdot x}{2} = \frac{6}{2} \quad \text{can be written as} \quad \frac{2}{2} \cdot x = \frac{6}{2}$$

$$1 \cdot x = 3 \quad \text{or} \quad x = 3$$

Picturing again our balanced scale, if we multiply or divide the weight on each side by the same nonzero number, the scale (or equation) remains balanced.



Practice 5

Solve: $3y = -18$

Answer
5. -6

Example 5 Solve: $-5x = 15$

Solution: To get x alone, divide both sides by -5 .

$$-5x = 15 \quad \text{Original equation}$$

$$\frac{-5x}{-5} = \frac{15}{-5} \quad \text{Divide both sides by } -5.$$

$$\frac{-5}{-5} \cdot x = \frac{15}{-5}$$

$$1x = -3 \quad \text{or} \quad x = -3 \quad \text{Simplify.}$$

Check: To check, replace x with -3 in the original equation.

$$\begin{aligned} -5x &= 15 && \text{Original equation} \\ -5(-3) &\stackrel{?}{=} 15 && \text{Let } x = -3. \\ 15 &= 15 && \text{True} \end{aligned}$$

The solution is -3 .

Work Practice 5

Example 6 Solve: $-27 = 3y$

Solution: To get y alone, divide both sides of the equation by 3.

$$\begin{aligned} -27 &= 3y \\ \frac{-27}{3} &= \frac{3y}{3} && \text{Divide both sides by 3.} \\ \frac{-27}{3} &= \frac{3}{3} \cdot y \\ -9 &= 1y \quad \text{or} \quad y = -9 \end{aligned}$$

Check to see that -9 is the solution.

Work Practice 6

Example 7 Solve: $-12x = -36$

Solution: To get x alone, divide both sides of the equation by -12 .

$$\begin{aligned} -12x &= -36 \\ \frac{-12x}{-12} &= \frac{-36}{-12} \\ \frac{-12}{-12} \cdot x &= \frac{-36}{-12} \\ x &= 3 \end{aligned}$$

Check: To check, replace x with 3 in the original equation.

$$\begin{aligned} -12x &= -36 \\ -12(3) &\stackrel{?}{=} -36 && \text{Let } x = 3. \\ -36 &= -36 && \text{True} \end{aligned}$$

Since $-36 = -36$ is a true statement, the solution is 3.

Work Practice 7

 **Concept Check** Which operation is appropriate for solving each of the following equations, addition or division?

- a. $12 = x - 3$
- b. $12 = 3x$

The multiplication property also allows us to solve equations like

$$\frac{x}{5} = 2$$

Practice 6

Solve: $-32 = 8x$

Practice 7

Solve: $-3y = -27$

Answers

6. -4 7. 9

Concept Check Answers

- a. addition
- b. division

Here, x is *divided* by 5. To get x alone, we use the multiplication property to *multiply* both sides by 5.

$$5 \cdot \frac{x}{5} = 5 \cdot 2 \quad \text{Multiply both sides by 5.}$$

Then it can be shown that

$$5 \cdot \frac{x}{5} = 5 \cdot 2 \text{ can be written as } \frac{5}{5} \cdot x = 5 \cdot 2$$

$$1 \cdot x = 10 \quad \text{or} \quad x = 10$$

Practice 8

Solve: $\frac{x}{-4} = 7$

Answer

8. -28

Example 8 Solve: $\frac{x}{3} = -2$

Solution: To get x alone, multiply both sides by 3.

$$\frac{x}{3} = -2$$

$$3 \cdot \frac{x}{3} = 3 \cdot (-2) \quad \text{Multiply both sides by 3.}$$

$$\frac{3}{3} \cdot x = 3 \cdot (-2)$$

$$1x = -6 \quad \text{or} \quad x = -6 \quad \text{Simplify.}$$

Check: Replace x with -6 in the original equation.

$$\frac{x}{3} = -2 \quad \text{Original equation}$$

$$\frac{-6}{3} \stackrel{?}{=} -2 \quad \text{Let } x = -6.$$

$$-2 = -2 \quad \text{True}$$

The solution is -6 .

 **Work Practice 8**

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

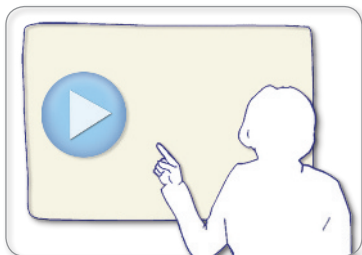
equation	multiplication	addition
expression	solution	equivalent

1. A combination of operations on variables and numbers is called a(n) _____.
2. A statement of the form “expression = expression” is called a(n) _____.
3. A(n) _____ contains an equal sign (=) while a(n) _____ does not.
4. A(n) _____ may be simplified and evaluated while a(n) _____ may be solved.
5. A(n) _____ of an equation is a number that when substituted for the variable makes the equation a true statement.
6. _____ equations have the same solution.

7. By the _____ property of equality, the same number may be added to or subtracted from both sides of an equation without changing the solution of the equation.
8. By the _____ property of equality, both sides of an equation may be multiplied or divided by the same nonzero number without changing the solution of the equation.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 2.6

- Objective A** 9. From the lecture before Example 1, what does an equation have that an expression does not?
- Objective B** 10. In the lecture before Example 2, what does the addition property of equality mean in words?
- Objective C** 11. Complete this statement based on Example 7: To check a solution, we go back to the _____ equation, replace the variable with the proposed solution, and see if we get a _____ statement.

2.6 Exercise Set MyLab Math

Objective A Determine whether the given number is a solution of the given equation. See Example 1.

1. Is 6 a solution of $x - 8 = -2$?
2. Is 9 a solution of $y - 16 = -7$?
3. Is -5 a solution of $x + 12 = 17$?
4. Is -7 a solution of $a + 23 = -16$?
5. Is -8 a solution of $-9f = 64 - f$?
6. Is -6 a solution of $-3k = 12 - k$?
7. Is 3 a solution of $5(c - 5) = -10$?
8. Is 1 a solution of $2(b - 3) = 10$?

Objective B Solve. Check each solution. See Examples 2 through 4.

9. $a + 5 = 23$
10. $f + 4 = -6$
11. $d - 9 = -21$
12. $s - 7 = -15$
13. $7 = y - 2$
14. $1 = y + 7$
15. $-7 + 10 - 20 = x$
16. $-50 + 40 - 5 = z$

Objective C Solve. Check each solution. See Examples 5 through 8.

17. $5x = 20$
18. $6y = 48$
19. $-3z = 12$
20. $-2x = 26$

21. $\frac{n}{7} = -2$

22. $\frac{n}{11} = -5$

23. $2z = -34$

24. $7y = -21$

25. $-4y = 0$

26. $-9x = 0$

27. $-10x = -10$

28. $-31x = -31$

Objectives B C Mixed Practice Solve. See Examples 2 through 8.

29. $5x = -35$

30. $3y = -27$

31. $n - 5 = -55$

32. $n - 4 = -48$

33. $-15 = y + 10$

34. $-36 = y + 12$

35. $\frac{x}{-6} = -6$

36. $\frac{x}{-9} = -9$

37. $n = -10 + 31$

38. $z = -28 + 36$

39. $-12y = -144$

40. $-11x = -121$

 41. $\frac{n}{4} = -20$

42. $\frac{n}{5} = -20$

43. $-64 = 32y$

44. $-81 = 27x$

Review

Translate each phrase to an algebraic expression. Use x to represent “a number.” See Section 1.8.

45. A number decreased by -2 46. A number increased by -5 47. The product of -6 and a number48. The quotient of a number and -20 49. The sum of -15 and a number50. -32 multiplied by a number51. -8 divided by a number52. Subtract a number from -18 .

Concept Extensions

Solve.

53. $n - 42,860 = -1286$

54. $n + 961 = 120$

55. $-38x = 15,542$

56. $\frac{y}{-18} = 1098$

 57. Explain the differences between an equation and an expression. 58. Explain the differences between the addition property of equality and the multiplication property of equality. 59. Write an equation that can be solved using the addition property of equality. 60. Write an equation that can be solved using the multiplication property of equality.

Chapter 2 Group Activity

Magic Squares

Sections 2.1–2.3

A magic square is a set of numbers arranged in a square table so that the sum of the numbers in each column, row, and diagonal is the same. For instance, in the magic square below, the sum of each column, row, and diagonal is 15. Notice that no number is used more than once in the magic square.

2	9	4
7	5	3
6	1	8

The properties of magic squares have been known for a very long time and once were thought to be good luck charms. The ancient Egyptians and Greeks understood their patterns. A magic square even made it into a famous work of art. The engraving titled *Melencolia I*, created by German artist Albrecht Dürer in 1514, features the following four-by-four magic square on the building behind the central figure.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1



Exercises

1. Verify that what is shown in the Dürer engraving is, in fact, a magic square. What is the common sum of the columns, rows, and diagonals?
2. Negative numbers can also be used in magic squares. Complete the following magic square:

	-1	
0		-4

3. Use the numbers -16 , -12 , -8 , -4 , 0 , 4 , 8 , 12 , and 16 to form a magic square:

Chapter 2 Vocabulary Check

Fill in each blank with one of the words or phrases listed below.

- inequality symbols
- addition
- solution
- is less than
- integers
- expression
- average
- negative
- absolute value
- equation
- positive
- opposites
- is greater than
- multiplication

1. Two numbers that are the same distance from 0 on the number line but are on opposite sides of 0 are called _____.
2. The _____ of a number is that number’s distance from 0 on a number line.
3. The _____ are $\dots, -3, -2, -1, 0, 1, 2, 3, \dots$.
4. The _____ numbers are numbers less than zero.
5. The _____ numbers are numbers greater than zero.
6. The symbols “<” and “>” are called _____.
7. A(n) _____ of an equation is a number that when substituted for the variable makes the equation a true statement.
8. The _____ of a list of numbers is $\frac{\text{sum of numbers}}{\text{number of numbers}}$.
9. A combination of operations on variables and numbers is called a(n) _____.
10. A statement of the form “expression = expression” is called a(n) _____.
11. The sign “<” means _____ and “>” means _____.
12. By the _____ property of equality, the same number may be added to or subtracted from both sides of an equation without changing the solution of the equation.
13. By the _____ property of equality, both sides of an equation may be multiplied or divided by the same nonzero number without changing the solution of the equation.

Helpful Hint

Are you preparing for your test? To help, don’t forget to take these:

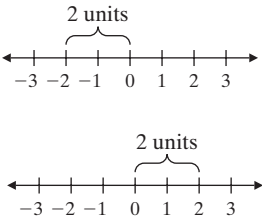
- Chapter 2 Getting Ready for the Test on page 159
- Chapter 2 Test on page 160

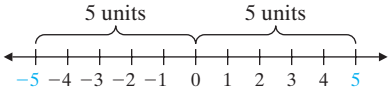
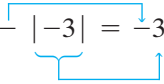
Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

2

Chapter Highlights

Definitions and Concepts	Examples
Section 2.1 Introduction to Integers	
Together, positive numbers, negative numbers, and 0 are called signed numbers .	$-432, -10, 0, 15$
The integers are $\dots, -3, -2, -1, 0, 1, 2, 3, \dots$.	$ -2 = 2$
The absolute value of a number is that number’s distance from 0 on a number line. The symbol for absolute value is $ \quad $.	$ 2 = 2$



Definitions and Concepts	Examples
Section 2.1 Introduction to Integers (continued)	
Two numbers that are the same distance from 0 on the number line but are on opposite sides of 0 are called opposites .	5 and -5 are opposites.
If a is a number, then $-(-a) = a$.	 $-(-11) = 11.$ Do not confuse with $- -3 = -3$ 
Section 2.2 Adding Integers	
Adding Two Numbers with the Same Sign Step 1: Add their absolute values. Step 2: Use their common sign as the sign of the sum.	Add: $-3 + (-2) = -5$ $-7 + (-15) = -22$ $-6 + 4 = -2$ $17 + (-12) = 5$ $-32 + (-2) + 14 = -34 + 14 = -20$
Adding Two Numbers with Different Signs Step 1: Find the larger absolute value minus the smaller absolute value. Step 2: Use the sign of the number with the larger absolute value as the sign of the sum.	
Section 2.3 Subtracting Integers	
Subtracting Two Numbers If a and b are numbers, then $a - b = a + (-b)$.	Subtract: $-35 - 4 = -35 + (-4) = -39$ $3 - 8 = 3 + (-8) = -5$ $-10 - (-12) = -10 + 12 = 2$ $7 - 20 - 18 - (-3) = 7 + (-20) + (-18) + (+3) = -13 + (-18) + 3 = -31 + 3 = -28$
Section 2.4 Multiplying and Dividing Integers	
Multiplying Numbers The product of two numbers having the same sign is a positive number. The product of two numbers having different signs is a negative number.	Multiply: $(-7)(-6) = 42$ $9(-4) = -36$
Dividing Numbers The quotient of two numbers having the same sign is a positive number. The quotient of two numbers having different signs is a negative number.	Evaluate: $(-3)^2 = (-3)(-3) = 9$ Divide: $-100 \div (-10) = 10$ $\frac{14}{-2} = -7, \quad \frac{0}{-3} = 0, \quad \frac{22}{0} \text{ is undefined.}$

Definitions and Concepts	Examples
Section 2.5 Order of Operations	
Order of Operations 1. Perform all operations within parentheses (), brackets [], or other grouping symbols such as fraction bars, starting with the innermost set. 2. Evaluate any expressions with exponents. 3. Multiply or divide in order from left to right. 4. Add or subtract in order from left to right.	Simplify: $3 + 2 \cdot (-5) = 3 + (-10)$ $= -7$ $\frac{-2(5 - 7)}{-7 + -3 } = \frac{-2(-2)}{-7 + 3}$ $= \frac{4}{-4}$ $= -1$
Section 2.6 Solving Equations: The Addition and Multiplication Properties	
Addition Property of Equality Let a, b, and c represent numbers. If $a = b$, then $a + c = b + c \quad \text{and} \quad a - c = b - c$ In other words, the same number may be added to or subtracted from both sides of an equation without changing the solution of the equation. Multiplication Property of Equality Let a, b, and c represent numbers and let $c \neq 0$. If $a = b$, then $a \cdot c = b \cdot c \quad \text{and} \quad \frac{a}{c} = \frac{b}{c}$ In other words, both sides of an equation may be multiplied or divided by the same nonzero number without changing the solution of the equation.	Solve: $x + 8 = 1$ $x + 8 - 8 = 1 - 8 \quad \text{Subtract 8 from both sides.}$ $x = -7 \quad \text{Simplify.}$ The solution is -7. Solve: $-6y = 30$ $\frac{-6y}{-6} = \frac{30}{-6} \quad \text{Divide both sides by } -6.$ $\frac{-6}{-6} \cdot y = \frac{30}{-6}$ $y = -5 \quad \text{Simplify.}$ The solution is -5.

Chapter 2

Review

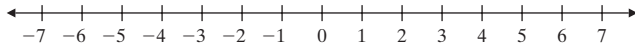
(2.1) Represent each quantity by an integer.

1. A gold miner is working 1572 feet down in a mine.
2. Mount Hood, in Oregon, has an elevation of 11,239 feet.

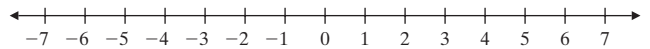


Graph each integer in the list on the same number line.

3. $-3, -5, 0, 7$



4. $-6, -1, 0, 5$



Simplify.

5. $|-11|$

6. $|0|$

7. $-|8|$

8. $-(-9)$

9. $-|-16|$

10. $-(-2)$

Insert $<$ or $>$ between each pair of integers to make a true statement.

11. -18 -20

12. -5 5

13. $|-123|$ $-|-198|$

14. $|-12|$ $-|-16|$

Find the opposite of each integer.

15. -18

16. 42

Answer true or false for each statement.

17. If $a < b$, then a must be a negative number.

18. The absolute value of an integer is always 0 or a positive number.

19. A negative number is always less than a positive number.

20. If a is a negative number, then $-a$ is a positive number.

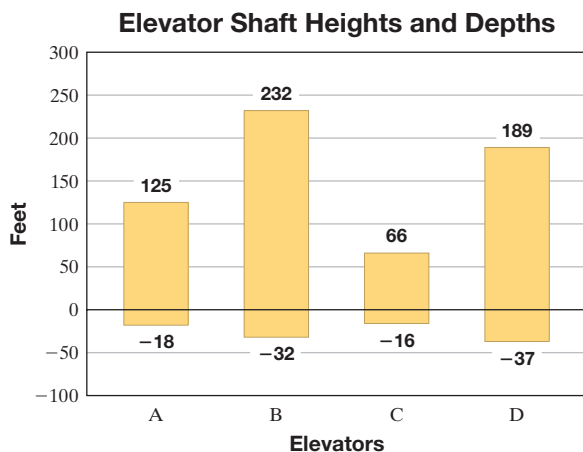
Evaluate.

21. $|y|$ if $y = -2$

22. $|-x|$ if $x = -3$

23. $-|-z|$ if $z = -5$

24. $-|-n|$ if $n = -10$



Elevator shafts in some buildings extend not only above ground, but in many cases below ground to accommodate basements, underground parking, etc. The bar graph shows four such elevators and their shaft distance above and below ground. Use the bar graph to answer Exercises 25 and 26.

25. Which elevator shaft extends the farthest below ground?

26. Which elevator shaft extends the highest above ground?

(2.2) *Add.*

27. $5 + (-3)$

28. $18 + (-4)$

29. $-12 + 16$

30. $-23 + 40$

31. $-8 + (-15)$

32. $-5 + (-17)$

33. $-24 + 3$

34. $-89 + 19$

35. $15 + (-15)$

36. $-24 + 24$

37. $-43 + (-108)$

38. $-100 + (-506)$

Solve.

- 39.** The temperature at 5 a.m. on a day in January was -15° Celsius. By 6 a.m. the temperature had fallen 5 degrees. Use a signed number to represent the temperature at 6 a.m.

- 41.** During the LPGA 2016 CME Group Tour Championship tournament, the winner, Charley Hull, had scores of -5 , -2 , -6 , and -6 . What was her total score for the tournament? (*Source:* LPGA)

- 40.** A diver starts out at 127 feet below the surface and then swims downward another 23 feet. Use a signed number to represent the diver's current depth.

- 42.** The Ryder Cup, a biennial (every two years) pro men's golf tournament between an American team and a European team, scores holes won. During the 2016 Ryder Cup, the winners, the American team, had a score of 17. The losing team, the Europeans, had a score 6 less than the Americans' score. What was the European team's score? (*Source:* Ryder Cup 2016)

(2.3) *Subtract.*

43. $12 - 4$

44. $-12 - 4$

45. $-7 - 17$

46. $7 - 17$

47. $7 - (-13)$

48. $-6 - (-14)$

49. $16 - 16$

50. $-16 - 16$

51. $-12 - (-12)$

52. $-5 - (-12)$

53. $-(-5) - 12 + (-3)$

54. $-8 + (-12) - 10 - (-3)$

Solve.

- 55.** If the elevation of Lake Superior is 600 feet above sea level and the elevation of the Caspian Sea is 92 feet below sea level, find the difference of the elevations.

- 56.** Josh Weidner has \$142 in his checking account. He writes a check for \$125, makes a deposit of \$43, and then writes another check for \$85. Represent the balance in his account by an integer.

- 57.** Some roller coasters travel above and below ground. One such roller coaster is Tremors, located in Silverwood Theme Park, Athol, Idaho. If this coaster rises to a height of 85 feet above ground, then drops 99 feet, how many feet below ground are you at the end of the drop? (*Source:* ultimaterollercoaster.com)

- 58.** Go to the bar graph for Review Exercises **25** and **26** and find the total length of the elevator shaft for Elevator C.



Answer true or false for each statement.

59. $|-5| - |-6| = 5 - 6$

60. $|-5 - (-6)| = 5 + 6$

(2.4) Multiply.

61. $-3(-7)$

62. $-6(3)$

63. $-4(16)$

64. $-5(-12)$

65. $(-5)^2$

66. $(-1)^5$

67. $12(-3)(0)$

68. $-1(6)(2)(-2)$

Divide.

69. $-15 \div 3$

70. $\frac{-24}{-8}$

71. $\frac{0}{-3}$

72. $\frac{-46}{0}$

73. $\frac{100}{-5}$

74. $\frac{-72}{8}$

75. $\frac{-38}{-1}$

76. $\frac{45}{-9}$

Solve.

77. A football team lost 5 yards on each of two consecutive plays. Represent the total loss by a product of integers, and find the product.

78. A horse race bettor lost \$50 on each of four consecutive races. Represent the total loss by a product of integers, and find the product.

79. A person has a debt of \$1024 and is ordered to pay it back in four equal payments. Represent the amount of each payment by a quotient of integers, and find the quotient.

80. Overnight, the temperature dropped 45 degrees Fahrenheit. If this took place over a time period of nine hours, represent the average temperature drop each hour by a quotient of integers. Then find the quotient.

(2.5) Simplify.

81. $(-7)^2$

82. -7^2

83. $5 - 8 + 3$

84. $-3 + 12 + (-7) - 10$

85. $-10 + 3 \cdot (-2)$

86. $5 - 10 \cdot (-3)$

87. $16 \div (-2) \cdot 4$

88. $-20 \div 5 \cdot 2$

89. $16 + (-3) \cdot 12 \div 4$

90. $-12 + 10 \div (-5)$

91. $4^3 - (8 - 3)^2$

92. $(-3)^3 - 90$

93. $\frac{(-4)(-3) - (-2)(-1)}{-10 + 5}$

94. $\frac{4(12 - 18)}{-10 \div (-2 - 3)}$

Find the average of each list of numbers.

95. $-18, 25, -30, 7, 0, -2$

96. $-45, -40, -30, -25$

Evaluate each expression for $x = -2$ and $y = 1$.

97. $2x - y$

98. $y^2 + x^2$

99. $\frac{3x}{6}$

100. $\frac{5y - x}{-y}$

(2.6) For Exercises 101 and 102, answer “yes” or “no.”

101. Is -5 a solution of $2n - 6 = 16$?

102. Is -2 a solution of $2(c - 8) = -20$?

Solve.

103. $n - 7 = -20$

104. $-5 = n + 15$

105. $10x = -30$

106. $-8x = 72$

107. $-20 + 7 = y$

108. $x - 31 = -62$

109. $\frac{n}{-4} = -11$

110. $\frac{x}{-2} = 13$

111. $n + 12 = -7$

112. $n - 40 = -2$

113. $-36 = -6x$

114. $-40 = 8y$

Mixed Review

Perform the indicated operations.

115. $-6 + (-9)$

116. $-16 - 3$

117. $-4(-12)$

118. $\frac{84}{-4}$

119. $-76 - (-97)$

120. $-9 + 4$

Solve.

121. Wednesday's lowest temperature was -18°C . The cold weather continued and by Friday, it had dropped another 9°C . What was the temperature on Friday?

122. The temperature at noon on a Monday in December was -11°C . By noon on Tuesday, it had warmed by 17°C . What was the temperature at noon on Tuesday?

123. The top of a mountain has an altitude of 12,923 feet. The bottom of a valley is 195 feet below sea level. Find the difference between these two elevations.

124. Joe owed his mother \$32. He gave her \$23. Write his financial situation as a signed number.

Simplify.

125. $(3 - 7)^2 \div (6 - 4)^3$

126. $3(4 + 2) + (-6) - 3^2$

127. $2 - 4 \cdot 3 + 5$

128. $4 - 6 \cdot 5 + 1$

129. $\frac{-|-14| - 6}{7 + 2(-3)}$

130. $5(7 - 6)^3 - 4(2 - 3)^2 + 2^4$

Solve.

131. $n - 9 = -30$

132. $n + 18 = 1$

133. $-4x = -48$

134. $9x = -81$

135. $\frac{n}{-2} = 100$

136. $\frac{y}{-1} = -3$

MULTIPLE CHOICE All the exercises below are **Multiple Choice**. Choose the correct letter. Not all letters may be used and some may be used more than once.

For Exercises 1 through 4, identify each answer as

- A.** 2 **B.** -2 **C.** 0

- ▶ **1.** -2; choose the opposite
- ▶ **2.** -2; choose the absolute value
- ▶ **3.** 2; choose the absolute value
- ▶ **4.** 2; choose the opposite

For Exercises 5 through 10, identify each answer as

- A.** addition **B.** subtraction **C.** multiplication **D.** division

- ▶ **5.** For xy , the operation is _____.
- ▶ **6.** For $-12(+3)$, the operation is _____.
- ▶ **7.** For $-12 + 3$, the operation is _____.
- ▶ **8.** For $\frac{-12}{+3}$, the operation is _____.
- ▶ **9.** For $4 + 6 \cdot 2$, which operation is performed first?
- ▶ **10.** For $4 + 6 \div 2$, which operation is performed first?

For Exercises 11 through 14, choose the solution of each equation as

- A.** 2 **B.** -2 **C.** 18 **D.** -18

- ▶ **11.** $-3x = 6$
- ▶ **12.** $x - 2 = -4$
- ▶ **13.** $\frac{x}{-3} = 6$
- ▶ **14.** $x + 2 = 4$

Answers

Simplify each expression.

▶ 1. $-5 + 8$

▶ 2. $18 - 24$

▶ 3. $5 \cdot (-20)$

▶ 4. $-16 \div (-4)$

▶ 5. $-18 + (-12)$

▶ 6. $-7 - (-19)$

▶ 7. $-5 \cdot (-13)$

▶ 8. $\frac{-25}{-5}$

▶ 9. $|-25| + (-13)$

▶ 10. $14 - |-20|$

▶ 11. $|5| \cdot |-10|$

▶ 12. $\frac{|-10|}{-|-5|}$

▶ 13. $-8 + 9 \div (-3)$

▶ 14. $-7 + (-32) - 12 + 5$

▶ 15. $(-5)^3 - 24 \div (-3)$

▶ 16. $(5 - 9)^2 \cdot (8 - 2)^3$

▶ 17. $-(-7)^2 \div 7 \cdot (-4)$

▶ 18. $3 - (8 - 2)^3$

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

12.

13.

14.

15.

16.

17.

18.

▶ 19. $\frac{4}{2} - \frac{8^2}{16}$

▶ 20. $\frac{-3(-2) + 12}{-1(-4 - 5)}$

▶ 21. $\frac{|25 - 30|^2}{2(-6) + 7}$

▶ 22. $5(-8) - [6 - (2 - 4)] + (12 - 16)^2$

Evaluate each expression for $x = 0$, $y = -3$, and $z = 2$.

▶ 23. $7x + 3y - 4z$

▶ 24. $10 - y^2$

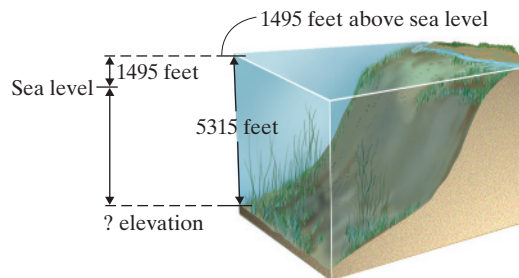
▶ 25. $\frac{3z}{2y}$

- ▶ 26. Mary Dunstan, a diver, starts at sea level and then makes 4 successive descents of 22 feet. After the descents, what is her elevation?

- ▶ 27. Aaron Hawn has \$129 in his checking account. He writes a check for \$79, withdraws \$40 from an ATM, and then deposits \$35. Represent the new balance in his account by an integer.

- ▶ 28. Mt. Washington in New Hampshire has an elevation of 6288 feet above sea level. The Romanche Gap in the Atlantic Ocean has an elevation of 25,354 feet below sea level. Represent the difference in elevation between these two points by an integer. (Source: National Geographic Society and Defense Mapping Agency)

- ▶ 29. Lake Baykal in Siberian Russia is the deepest lake in the world, with a maximum depth of 5315 feet. The elevation of the lake's surface is 1495 feet above sea level. What is the elevation (with respect to sea level) of the deepest point in the lake? (Source: U.S. Geological Survey)



- ▶ 30. Find the average of -12 , -13 , 0 , 9 .

- ▶ 31. Translate the following phrases into mathematical expressions. Use x to represent "a number."

a. The product of a number and 17

b. A number subtracted from 20

Solve.

▶ 32. $-9n = -45$

▶ 33. $\frac{n}{-7} = 4$

▶ 34. $x - 16 = -36$

▶ 35. $-20 + 8 + 8 = x$

19. _____

20. _____

21. _____

22. _____

23. _____

24. _____

25. _____

26. _____

27. _____

28. _____

29. _____

30. _____

31. a. _____

b. _____

32. _____

33. _____

34. _____

35. _____

Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. a. _____
b. _____
c. _____
8. a. _____
b. _____
c. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. a. _____
b. _____
c. _____
20. a. _____
b. _____
c. _____
21. _____
22. _____
23. a. _____
b. _____
c. _____
24. a. _____
b. _____
c. _____

Find the place value of the digit 3 in each whole number.

1. 396,418
2. 4308
3. 93,192
4. 693,298
5. 534,275,866
6. 267,301,818
7. Insert $<$ or $>$ to make a true statement.
a. -7 7
b. 0 -4
c. -9 -11
8. Insert $<$ or $>$ to make a true statement
a. 12 -4
b. -13 -31
c. -82 79
9. Add:
 $13 + 2 + 7 + 8 + 9$
10. Add: $11 + 3 + 9 + 16$
11. Subtract: $7826 - 505$
Check by adding.
12. Subtract: $3285 - 272$
Check by adding.
13. The radius of Jupiter is 43,441 miles. The radius of Saturn is 7257 miles less than the radius of Jupiter. Find the radius of Saturn. (Source: National Space Science Data Center)
14. C. J. Dufour wants to buy a digital camera. She has \$762 in her savings account. If the camera costs \$237, how much money will she have in her account after buying the camera?
15. Round 568 to the nearest ten.
16. Round 568 to the nearest hundred.
17. Round each number to the nearest hundred to find an estimated difference.
 4725
 -2879
18. Round each number to the nearest thousand to find an estimated difference.
 8394
 -2913
19. Rewrite each using the distributive property.
a. $5(6 + 5)$
b. $20(4 + 7)$
c. $2(7 + 9)$
20. Rewrite each using the distributive property.
a. $5(2 + 12)$
b. $9(3 + 6)$
c. $4(8 + 1)$
21. Multiply: 631×125
22. Multiply: 299×104
23. Find each quotient. Check by multiplying.
a. $42 \div 7$
b. $\frac{64}{8}$
c. $3 \overline{)21}$
24. Find each quotient. Check by multiplying.
a. $\frac{35}{5}$
b. $64 \div 8$
c. $4 \overline{)48}$

25. Divide: $3705 \div 5$. Check by multiplying. **26.** Divide: $3648 \div 8$. Check by multiplying.

27. As part of a promotion, an executive receives 238 cards, each good for one free song download. If she wants to share them evenly with 19 friends, how many download cards will each friend receive? How many will be left over?

28. Mrs. Mallory's first-grade class is going to the zoo. She pays a total of \$324 for 36 admission tickets. How much does each ticket cost?

Evaluate.

29. 9^2

30. 5^3

31. 6^1

32. 4^1

33. $5 \cdot 6^2$

34. $2^3 \cdot 7$

35. Simplify: $\frac{7 - 2 \cdot 3 + 3^2}{5(2 - 1)}$

36. Simplify: $\frac{6^2 + 4 \cdot 4 + 2^3}{37 - 5^2}$

37. Evaluate $x + 6$ if x is 8.

38. Evaluate $5 + x$ if x is 9.

39. Simplify:

a. $|-9|$

b. $|8|$

c. $|0|$

40. Simplify:

a. $|4|$

b. $|-7|$

41. Add: $-2 + 25$

42. Add: $8 + (-3)$

43. Evaluate $2a - b$ for $a = 8$ and $b = -6$.

44. Evaluate $x - y$ for $x = -2$ and $y = -7$.

45. Multiply: $-7 \cdot 3$

46. Multiply: $5(-2)$

47. Multiply: $0 \cdot (-4)$

48. Multiply: $-6 \cdot 9$

49. Simplify: $3(4 - 7) + (-2) - 5$

50. Simplify: $4 - 8(7 - 3) - (-1)$

25. _____

26. _____

27. _____

28. _____

29. _____

30. _____

31. _____

32. _____

33. _____

34. _____

35. _____

36. _____

37. _____

38. _____

39. a. _____

b. _____

c. _____

40. a. _____

b. _____

41. _____

42. _____

43. _____

44. _____

45. _____

46. _____

47. _____

48. _____

49. _____

50. _____

3

Solving Equations and Problem Solving

In this chapter, we continue making the transition from arithmetic to algebra. Recall that in algebra, letters (called variables) represent unknown quantities.

Using variables is a very powerful method for solving problems that cannot be solved with arithmetic alone. This chapter introduces operations on algebraic expressions, and we continue solving variable equations.

Sections

- 3.1** Simplifying Algebraic Expressions
- 3.2** Solving Equations: Review of the Addition and Multiplication Properties
Integrated Review—Expressions and Equations
- 3.3** Solving Linear Equations in One Variable
- 3.4** Linear Equations in One Variable and Problem Solving

Check Your Progress

Vocabulary Check
Chapter Highlights
Chapter Review
Getting Ready for the Test
Chapter Test
Cumulative Review



Soccer, or Football as It Is Known Outside the United States, Is the Most Popular Sport in the World.

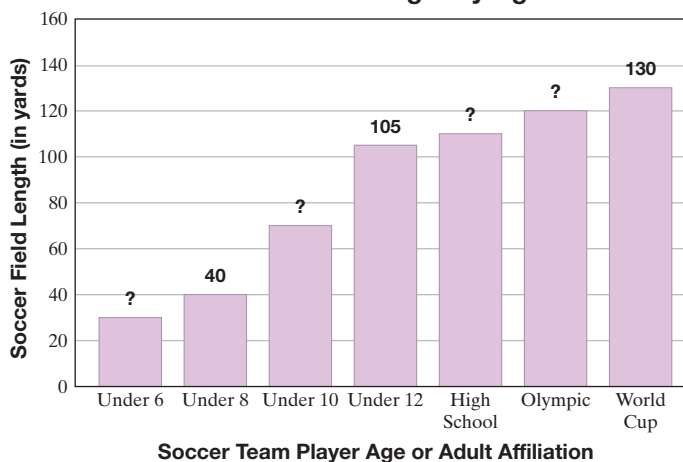
The 2016 FIFA World Cup was held in Japan and attracted an audience of 110 million viewers in Europe and more than 26 million in the United States watching the final.

FIFA, the international governing body of soccer, has strict regulations for the dimensions of the field. There is a minimum and maximum length allowed as well as a minimum and maximum width.

Soccer is a game that can be played by anyone who can walk, from toddler to old age. To accommodate the various skill levels as well as developmental levels, the standard size of the fields changes with skill and age. The bar graph below demonstrates how the maximum length, in yards, of a soccer field varies based on these criteria.

In Section 3.4, Exercises 45 and 46, we will solve applications that give us the unknown heights in the bar graph. (Source: FIFA and Trumark Athletics)

Soccer Field Recommended Maximum Length by Age



3.1 Simplifying Algebraic Expressions

Recall from Section 1.8 that a combination of numbers, letters (variables), and operation symbols is called an **algebraic expression** or simply an **expression**. Examples of expressions are below.

Algebraic Expressions

$$4 \cdot x, \quad n + 7, \quad \text{and} \quad 3y - 5 - x$$

Recall that if two variables or a number and a variable are next to each other, with no operation sign between them, the indicated operation is multiplication. For example,

$$3y \text{ means } 3 \cdot y$$

and

$$xy \text{ or } x(y) \text{ means } x \cdot y$$

Also, the meaning of an exponent remains the same when the base is a variable. For example,

$$y^2 = \underbrace{y \cdot y}_{\text{2 factors of } y} \quad \text{and} \quad x^4 = \underbrace{x \cdot x \cdot x \cdot x}_{\text{4 factors of } x}$$

Just as we can add, subtract, multiply, and divide numbers, we can add, subtract, multiply, and divide algebraic expressions. In previous sections we evaluated algebraic expressions like $x + 3$, $4x$, and $x + 2y$ for particular values of the variables. In this section, we explore working with variable expressions without evaluating them. We begin with a definition of a term.

Objective A Combining Like Terms

The addends of an algebraic expression are called the **terms** of the expression.

$$\begin{array}{c} x + 3 \\ \uparrow \quad \uparrow \\ \text{2 terms} \end{array}$$

$$\begin{array}{c} 3y^2 + (-6y) + 4 \\ \uparrow \quad \uparrow \quad \uparrow \\ \text{3 terms} \end{array}$$

A term that is only a number has a special name. It is called a **constant term**, or simply a **constant**. A term that contains a variable is called a **variable term**.

$$\begin{array}{c} x \quad + \quad 3 \\ \uparrow \quad \uparrow \\ \text{variable} \quad \text{constant} \\ \text{term} \quad \text{term} \end{array} \quad \text{and} \quad \begin{array}{c} 3y^2 + (-6y) + 4 \\ \uparrow \quad \uparrow \quad \uparrow \\ \text{variable} \quad \text{variable} \quad \text{constant} \\ \text{terms} \quad \text{term} \end{array}$$

The number factor of a variable term is called the **numerical coefficient**. A numerical coefficient of 1 is usually not written.

$$\begin{array}{c} 5x \\ \uparrow \\ \text{Numerical} \\ \text{coefficient} \\ \text{is } 5. \end{array} \quad \begin{array}{c} x \text{ or } 1x \\ \uparrow \\ \text{Understood} \\ \text{numerical} \\ \text{coefficient is } 1. \end{array} \quad \begin{array}{c} 3y^2 \\ \uparrow \\ \text{Numerical} \\ \text{coefficient} \\ \text{is } 3. \end{array} \quad \begin{array}{c} -6y \\ \uparrow \\ \text{Numerical} \\ \text{coefficient} \\ \text{is } -6. \end{array}$$

Objectives

- A** Use Properties of Numbers to Combine Like Terms. 
- B** Use Properties of Numbers to Multiply Expressions. 
- C** Simplify Expressions by Multiplying and Then Combining Like Terms. 
- D** Find the Perimeter and Area of Figures. 

Helpful Hint

Recall that $1 \cdot \text{any number} = \text{that number}$. This means that

$$1 \cdot x = x \quad \text{or that} \quad 1x = x$$

Thus x can always be replaced by $1x$ or $1 \cdot x$.

Terms with the same variable factors, except that they may have different numerical coefficients, are called **like terms**.

Like Terms	Unlike Terms
$3x, -4x$	$5x, x^2$
$-6y, 2y, y$	$7x, 7y$

 **Concept Check** True or false? The terms $-7xz^2$ and $3z^2x$ are like terms. Explain.

A sum or difference of like terms can be simplified using the **distributive property**. Recall from Section 1.5 that the distributive property says that multiplication distributes over addition (and subtraction). Using variables, we can write the distributive property as follows:

$$(a + b)c = ac + bc.$$

If we write the right side of the equation first, then the left side, we have the following:

Distributive Property

If a , b , and c are numbers, then

$$ac + bc = (a + b)c$$

Also,

$$ac - bc = (a - b)c$$

The distributive property guarantees that, no matter what number x is, $7x + 2x$ (for example) has the same value as $(7 + 2)x$, or $9x$. We then have that

$$7x + 2x = (7 + 2)x = 9x$$

This is an example of **combining like terms**. An algebraic expression is **simplified** when all like terms have been combined.

Practice 1

Simplify each expression by combining like terms.

a. $8m - 14m$

b. $6a + a$

c. $-y^2 + 3y^2 + 7$

Answers

1. a. $-6m$ b. $7a$ c. $2y^2 + 7$

 **Concept Check Answer**
true

Example 1

Simplify each expression by combining like terms.

a. $4x + 6x$

b. $y - 5y$

c. $3x^2 + 5x^2 - 2$

Solution: Add or subtract like terms.

a. $4x + 6x = (4 + 6)x$
 $= 10x$

b. $y - 5y = 1y - 5y$
 $= (1 - 5)y$
 $= -4y$

c. $3x^2 + 5x^2 - 2 = (3 + 5)x^2 - 2$
 $= 8x^2 - 2$

Work Practice 1

Helpful Hint

In this section, we are simplifying expressions. Try not to confuse the two processes below.

Expression: $5y - 8y$

Simplify the expression:

$$\begin{aligned} 5y - 8y &= (5 - 8)y \\ &= -3y \end{aligned}$$

Equation: $8n = -40$

Solve the equation:

$$\begin{aligned} 8n &= -40 \\ \frac{8n}{8} &= \frac{-40}{8} && \text{Divide both sides by 8.} \\ n &= -5 && \text{The solution is } -5. \end{aligned}$$

The commutative and associative properties of addition and multiplication can also help us simplify expressions. We presented these properties in Sections 1.3 and 1.5 and state them again using variables.

Properties of Addition and Multiplication

If a , b , and c are numbers, then

$$a + b = b + a \quad \text{Commutative property of addition}$$

$$a \cdot b = b \cdot a \quad \text{Commutative property of multiplication}$$

That is, the **order** of adding or multiplying two numbers can be changed without changing their sum or product.

$$(a + b) + c = a + (b + c) \quad \text{Associative property of addition}$$

$$(a \cdot b) \cdot c = a \cdot (b \cdot c) \quad \text{Associative property of multiplication}$$

That is, the **grouping** of numbers in addition or multiplication can be changed without changing their sum or product.

Helpful Hint

- Examples of these properties are

$$2 + 3 = 3 + 2 \quad \text{Commutative property of addition}$$

$$7 \cdot 9 = 9 \cdot 7 \quad \text{Commutative property of multiplication}$$

$$(1 + 8) + 10 = 1 + (8 + 10) \quad \text{Associative property of addition}$$

$$(4 \cdot 2) \cdot 3 = 4 \cdot (2 \cdot 3) \quad \text{Associative property of multiplication}$$

- These properties are not true for subtraction or division.

Example 2 Simplify: $2y - 6 + 4y + 8$

Solution: We begin by writing subtraction as the opposite of addition.

$$\begin{aligned} 2y - 6 + 4y + 8 &= 2y + (-6) + 4y + 8 && \text{Apply the commutative property of addition.} \\ &= 2y + 4y + (-6) + 8 \\ &= (2 + 4)y + (-6) + 8 && \text{Apply the distributive property.} \\ &= 6y + 2 && \text{Simplify.} \end{aligned}$$

Work Practice 2**Practice 2**

Simplify: $6z + 5 + z - 4$

Answer

2. $7z + 1$

Practice 3–5

Simplify each expression by combining like terms.

3. $6y + 12y - 6$

4. $7y - 5 + y + 8$

5. $-7y + 2 - 2y - 9x + 12 - x$

Examples Simplify each expression by combining like terms.

3. $6x + 2x - 5$
 $= 8x - 5$

4. $4x + 2 - 5x + 3$
 $= 4x + 2 + (-5x) + 3$
 $= 4x + (-5x) + 2 + 3$
 $= -1x + 5 \quad \text{or} \quad -x + 5$

5. $2x - 5 + 3y + 4x - 10y + 11$
 $= 2x + (-5) + 3y + 4x + (-10y) + 11$
 $= 2x + 4x + 3y + (-10y) + (-5) + 11$
 $= 6x - 7y + 6$

Work Practice 3–5

As we practice combining like terms, keep in mind that some of the steps may be performed mentally.

Objective B **Multiplying Expressions**

We can also use properties of numbers to multiply expressions such as $3(2x)$. By the associative property of multiplication, we can write the product $3(2x)$ as $(3 \cdot 2)x$, which simplifies to $6x$.

Practice 6–7

Multiply.

6. $6(4a)$

7. $-8(9x)$

Examples Multiply.

6. $5(3y) = (5 \cdot 3)y$ *Apply the associative property of multiplication.*
 $= 15y$ *Multiply.*

7. $-2(4x) = (-2 \cdot 4)x$ *Apply the associative property of multiplication.*
 $= -8x$ *Multiply.*

Work Practice 6–7

We can use the distributive property to combine like terms, which we have done, and also to multiply expressions such as $2(3 + x)$. By the distributive property, we have

$$\begin{array}{l} 2(3 + x) = 2 \cdot 3 + 2 \cdot x \quad \text{Apply the distributive property.} \\ = 6 + 2x \quad \text{Multiply.} \end{array}$$

Practice 8

Use the distributive property to multiply: $8(y + 2)$

Answers

3. $18y - 6$ 4. $8y + 3$

5. $-9y - 10x + 14$ 6. $24a$

7. $-72x$ 8. $8y + 16$

 Concept Check Answer

did not distribute the 8;
 $8(a - b) = 8a - 8b$

Example 8 Use the distributive property to multiply: $5(x + 4)$

Solution: By the distributive property,

$$\begin{array}{l} 5(x + 4) = 5 \cdot x + 5 \cdot 4 \quad \text{Apply the distributive property.} \\ = 5x + 20 \quad \text{Multiply.} \end{array}$$

Work Practice 8

 **Concept Check** What's wrong with the following?

$$8(a - b) = 8a - b$$

Example 9 Multiply: $-3(5a + 2)$ **Solution:** By the distributive property,

$$\begin{aligned}
 -3(5a + 2) &= -3(5a) + (-3)(2) && \text{Apply the distributive property.} \\
 &= (-3 \cdot 5)a + (-6) && \text{Use the associative property and multiply.} \\
 &= -15a - 6 && \text{Multiply.}
 \end{aligned}$$

Work Practice 9**Example 10** Multiply: $8(x - 4)$ **Solution:**

$$\begin{aligned}
 8(x - 4) &= 8 \cdot x - 8 \cdot 4 \\
 &= 8x - 32
 \end{aligned}$$

Work Practice 10**Objective C** Simplifying Expressions

Next we will **simplify** expressions by first using the distributive property to multiply and then **combining** any like terms.

Example 11 Simplify: $2(3 + 7x) - 15$ **Solution:** First we use the distributive property to remove parentheses.

$$\begin{aligned}
 2(3 + 7x) - 15 &= 2(3) + 2(7x) - 15 && \text{Apply the distributive property.} \\
 &= 6 + 14x - 15 && \text{Multiply.} \\
 &= 14x + (-9) \quad \text{or} \quad 14x - 9 && \text{Combine like terms.}
 \end{aligned}$$

Work Practice 11**Example 12** Simplify: $-2(x - 5) + 4(2x + 2)$ **Solution:** First we use the distributive property to remove parentheses.

$$\begin{aligned}
 -2(x - 5) + 4(2x + 2) &= -2(x) - (-2)(5) + 4(2x) + 4(2) && \text{Apply the distributive property.} \\
 &= -2x + 10 + 8x + 8 && \text{Multiply.} \\
 &= 6x + 18 && \text{Combine like terms.}
 \end{aligned}$$

Work Practice 12**Example 13** Simplify: $-(x + 4) + 5x + 16$ **Solution:** The expression $-(x + 4)$ means $-1(x + 4)$.

$$\begin{aligned}
 -(x + 4) + 5x + 16 &= -1(x + 4) + 5x + 16 \\
 &= -1 \cdot x + (-1)(4) + 5x + 16 && \text{Apply the distributive property.} \\
 &= -x + (-4) + 5x + 16 && \text{Multiply.} \\
 &= 4x + 12 && \text{Combine like terms.}
 \end{aligned}$$

Work Practice 13**Practice 9**Multiply: $3(7a - 5)$ **Practice 10**Multiply: $6(5 - y)$ **Practice 11**Simplify: $5(2y - 3) - 8$ **Helpful Hint**

2 is *not* distributed to the -15 since it is not within the parentheses.

Practice 12

Simplify:

$$-7(x - 1) + 5(2x + 3)$$

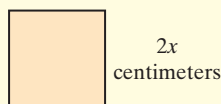
Practice 13Simplify: $-(y + 1) + 3y - 12$ **Answers**

9. $21a - 15$ 10. $30 - 6y$
 11. $10y - 23$ 12. $3x + 22$
 13. $2y - 13$

Objective D Finding Perimeter and Area

Practice 14

Find the perimeter of the square.

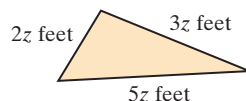


Practice 15

Find the area of the rectangular garden.



Example 14 Find the perimeter of the triangle.



Solution: Recall that the perimeter of a figure is the distance around the figure. To find the perimeter, then, we find the sum of the lengths of the sides. We use the letter P to represent perimeter.

$$\begin{aligned} P &= 2z + 3z + 5z \\ &= 10z \end{aligned}$$

The perimeter is $10z$ feet.

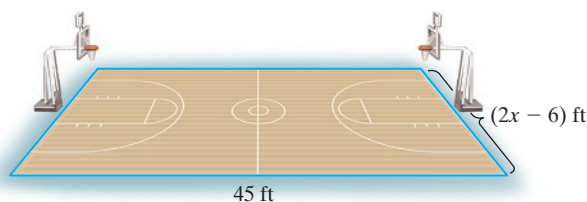
Helpful Hint

Don't forget to insert proper units.

Work Practice 14

Example 15 Finding the Area of a Basketball Court

Find the area of this YMCA basketball court.



Solution: Recall how to find the area of a rectangle. **Area = Length · Width**, or if A represents area, l represents length, and w represents width, we have $A = l \cdot w$.

$$\begin{aligned} A &= l \cdot w \\ &= 45(2x - 6) \quad \text{Let length} = 45 \text{ and width} = (2x - 6). \\ &= 90x - 270 \quad \text{Multiply.} \end{aligned}$$

The area is $(90x - 270)$ square feet.

Work Practice 15

Helpful Hint

Don't forget...

Area:

- surface enclosed
- measured in square units

Perimeter:

- distance around
- measured in units

Answers

14. $8x$ cm 15. $(36y + 27)$ sq yd

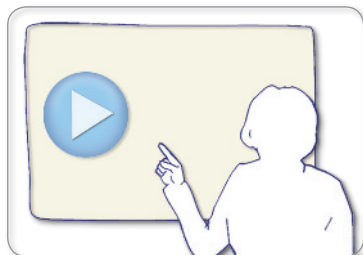
Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

numerical coefficient combine like terms like term variable associative
constant expression unlike distributive commutative

- $14y^2 + 2x - 23$ is called a(n) _____ while $14y^2$, $2x$, and -23 are each called a(n) _____.
- To multiply $3(-7x + 1)$, we use the _____ property.
- To simplify an expression like $y + 7y$, we _____.
- By the _____ properties, the *order* of adding or multiplying two numbers can be changed without changing their sum or product.
- The term $5x$ is called a(n) _____ term while the term 7 is called a(n) _____ term.
- The term z has an understood _____ of 1 .
- By the _____ properties, the *grouping* of adding or multiplying numbers can be changed without changing their sum or product.
- The terms $-x$ and $5x$ are _____ terms and the terms $5x$ and $5y$ are _____ terms.
- For the term $-3x^2y$, -3 is called the _____.

Martin-Gay Interactive Videos



See Video 3.1

Watch the section lecture video and answer the following questions.

- Objective A** 10. In Example 2, why can't the expression be simplified after the first step?
- Objective B** 11. In Example 5, what property is used to multiply?
- Objective C** 12. In Example 6, why is the 20 not multiplied by the 2?
- Objective D** 13. In Example 8, what operation is used to find P ? What operation is used to find A ? What do P and A stand for?

3.1 Exercise Set MyLab Math

Objective A Simplify each expression by combining like terms. See Examples 1 through 5.

- $3x + 5x$
- $8y + 3y$
- $2n - 3n$
- $7z - 10z$
- $4c + c - 7c$
- $5b - 8b - b$
- $4x - 6x + x - 5x$
- $8y + y - 2y - 8y$
- $3a + 2a + 7a - 5$
- $5b - 4b + b - 15$

Objective B Multiply. See Examples 6 and 7.

- $6(7x)$
- $4(4x)$
- $-3(11y)$
- $-3(21z)$
- $12(6a)$
- $13(5b)$

Multiply. See Examples 8 through 10.

17. $2(y + 3)$

18. $3(x + 1)$

▶ 19. $3(a - 6)$

20. $4(y - 6)$

21. $-4(3x + 7)$

22. $-8(8y + 10)$

Objective C Simplify each expression. First use the distributive property to multiply and remove parentheses. See Examples 11 through 13.

23. $2(x + 4) - 7$

24. $5(6 - y) - 2$

25. $8 + 5(3c - 1)$

26. $10 + 4(6d - 2)$

27. $-4(6n - 5) + 3n$

28. $-3(5 - 2b) - 4b$

29. $3 + 6(w + 2) + w$

30. $8z + 5(6 + z) + 20$

31. $2(3x + 1) + 5(x - 2)$

32. $3(5x - 2) + 2(3x + 1)$

33. $-(2y - 6) + 10$

34. $-(5x - 1) - 10$

Objectives A B C Mixed Practice Simplify each expression. See Examples 1 through 13.

35. $18y - 20y$

36. $x + 12x$

37. $z - 8z$

38. $-12x + 8x$

39. $9d - 3c - d$

40. $8r + s - 7s$

41. $2y - 6 + 4y - 8$

42. $a + 4 - 7a - 5$

43. $5q + p - 6q - p$

44. $m - 8n + m + 8n$

▶ 45. $2(x + 1) + 20$

46. $5(x - 1) + 18$

47. $5(x - 7) - 8x$

48. $3(x + 2) - 11x$

49. $-5(z + 3) + 2z$

50. $-8(1 + v) + 6v$

51. $8 - x + 4x - 2 - 9x$

52. $5y - 4 + 9y - y + 15$

▶ 53. $-7(x + 5) + 5(2x + 1)$

54. $-2(x + 4) + 8(3x - 1)$

55. $3r - 5r + 8 + r$

▶ 56. $6x - 4 + 2x - x + 3$

57. $-3(n - 1) - 4n$

58. $5(c + 2) + 7c$

59. $4(z - 3) + 5z - 2$

60. $8(m + 3) - 20 + m$

61. $6(2x - 1) - 12x$

62. $5(2a + 3) - 10a$

63. $-(4x - 5) + 5$

64. $-(7y - 2) + 6$

65. $-(4x - 10) + 2(3x + 5)$

66. $-(12b - 20) + 5(3b - 2)$

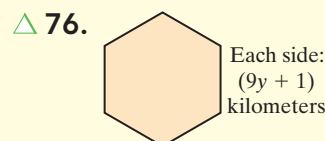
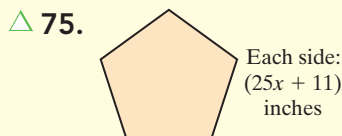
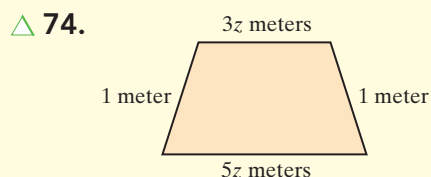
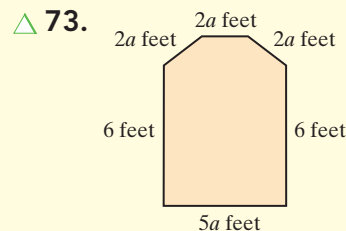
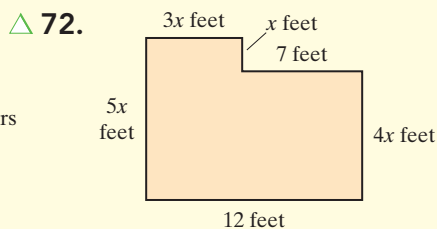
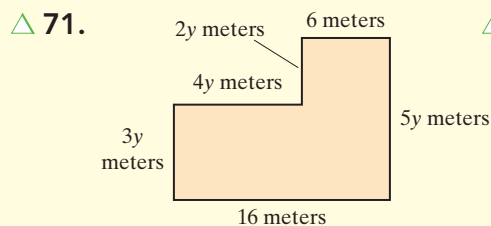
67. $3a + 4(a + 3)$

68. $b + 2(b - 5)$

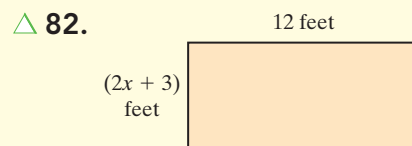
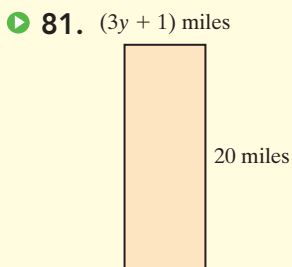
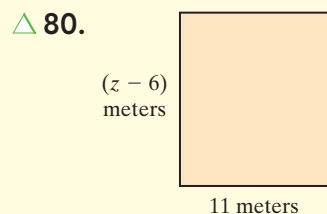
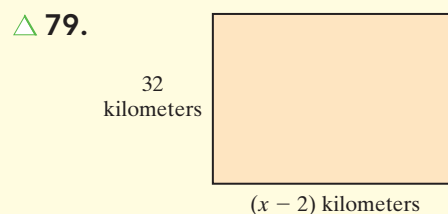
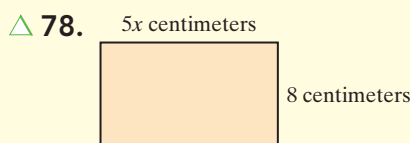
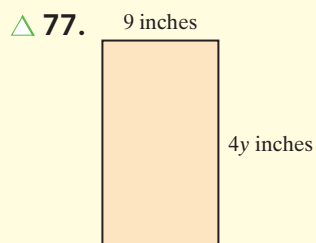
69. $5y - 2(y - 1) + 3$

70. $3x - 4(x + 2) + 1$

Objective D Find the perimeter of each figure. See Example 14.



Find the area of each rectangle. See Example 15.



Objectives A B C D Mixed Practice Solve. See Examples 1 through 15.

83. Find the area of a regulation NCAA basketball court that is 94 feet long and 50 feet wide.
84. Find the area of a rectangular movie screen that is 50 feet long and 40 feet high.
85. A decorator wishes to put a wallpaper border around a rectangular room that measures 14 feet by 18 feet. Find the room's perimeter.
86. How much fencing will a rancher need for a rectangular cattle lot that measures 80 feet by 120 feet?
87. Find the perimeter of a triangular garden that measures 5 feet by x feet by $(2x + 1)$ feet.
88. Find the perimeter of a triangular picture frame that measures x inches by x inches by $(x - 14)$ inches.

Review

Perform each indicated operation. See Sections 2.2 and 2.3.

89. $-13 + 10$

90. $-15 + 23$

91. $-4 - (-12)$

92. $-7 - (-4)$

93. $-4 + 4$

94. $8 + (-8)$

Concept Extensions

If the expression on the left side of the equal sign is equivalent to the right side, write “correct.” If not, write “incorrect” and then write an expression that is equivalent to the left side. See the second Concept Check in this section.

95. $5(3x - 2) \stackrel{?}{=} 15x - 2$

96. $-2(4x - 1) \stackrel{?}{=} -8x - 2$

97. $2(xy) \stackrel{?}{=} 2x \cdot 2y$

98. $-8(ab) \stackrel{?}{=} -8a \cdot (-8b)$

99. $7x - (x + 2) \stackrel{?}{=} 7x - x - 2$

100. $12y - (3y - 1) \stackrel{?}{=} 12y - 3y + 1$

101. $4(y - 3) + 11 \stackrel{?}{=} 4y - 7 + 11$

102. $6(x + 5) + 2 \stackrel{?}{=} 6x + 30 + 12$

Review commutative, associative, and distributive properties. Then identify which property allows us to write the equivalent expression on the right side of the equal sign.

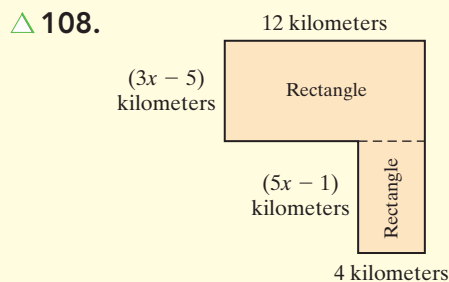
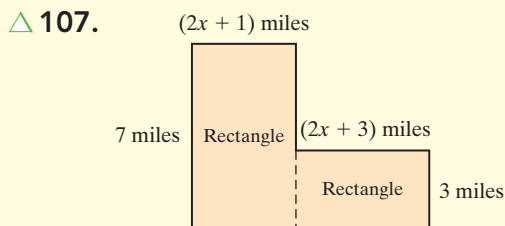
103. $6(2x - 3) + 5 = 12x - 18 + 5$

104. $9 + 7x + (-2) = 7x + 9 + (-2)$

105. $-7 + (4 + y) = (-7 + 4) + y$

106. $(x + y) + 11 = 11 + (x + y)$

Write the expression that represents the area of each composite figure. Then simplify to find the total area.



Simplify.

🧮 109. $9684q - 686 - 4860q + 12,960$

🧮 110. $76(268x + 592) - 2960$

✏ 111. If x is a whole number, which expression is greater: $2x$ or $5x$? Explain your answer.

✏ 112. If x is a whole number, which expression is greater: $-2x$ or $-5x$? Explain your answer.

✏ 113. Explain what makes two terms “like terms.”

✏ 114. Explain how to combine like terms.

3.2 Solving Equations: Review of the Addition and Multiplication Properties

Objective A Using the Addition Property or the Multiplication Property

In this section, we continue solving equations using the properties first introduced in Section 2.6.

First, let's recall the difference between an **expression** and an **equation**. Remember—an equation contains an equal sign and an expression does not.

	Equations	Expressions	
equal signs	$7x = 6x + 4$	$7x - 6x + 4$	no equal signs
	$3(3y - 5) = 10y$	$y - 1 + 11y - 21$	

Thus far in this text, we have

Solved some equations (Section 2.6)

and

Simplified some expressions (Section 3.1)

As we will see in this section, the ability to simplify expressions will help us as we solve more equations.

The addition and multiplication properties are reviewed below.

Addition Property of Equality

Let a , b , and c represent numbers. Then

$a = b$	Also, $a = b$
and $a + c = b + c$	and $a - c = b - c$
are equivalent equations.	are equivalent equations.

Multiplication Property of Equality

Let a , b , and c represent numbers and let $c \neq 0$. Then

$a = b$	Also, $a = b$
and $a \cdot c = b \cdot c$	and $\frac{a}{c} = \frac{b}{c}$
are equivalent equations.	are equivalent equations.

In other words, the same number may be added to or subtracted from *both* sides of an equation without changing the solution of the equation. Also, *both* sides of an equation may be multiplied or divided by the same nonzero number without changing the solution of the equation.

Many equations in this section will contain expressions that can be simplified.

If one or both sides of an equation can be simplified, do that first.

Example 1 Solve: $y - 5 = -2 - 6$

Solution: First we simplify the right side of the equation.

$$y - 5 = -2 - 6$$

$$y - 5 = -8 \quad \text{Combine like terms.}$$

Objectives

- A** Use the Addition Property or the Multiplication Property to Solve Equations.
- B** Use Both Properties to Solve Equations.
- C** Translate Word Phrases to Mathematical Expressions.

Practice 1

Solve: $x + 6 = 1 - 3$

Answer

1. -8

(Continued on next page)

Next we get y alone by using the addition property of equality. We add 5 to both sides of the equation.

$$y - 5 + 5 = -8 + 5 \quad \text{Add 5 to both sides.}$$

$$y = -3 \quad \text{Simplify.}$$

Check: To see that -3 is the solution, replace y with -3 in the original equation.

$$y - 5 = -2 - 6$$

$$-3 - 5 \stackrel{?}{=} -2 - 6 \quad \text{Replace } y \text{ with } -3.$$

$$-8 = -8 \quad \text{True}$$

Since $-8 = -8$ is true, the solution is -3 .

Work Practice 1

Practice 2

Solve: $10 = 2m - 4m$

Example 2 Solve: $3y - 7y = 12$

Solution: First, simplify the left side of the equation by combining like terms.

$$3y - 7y = 12$$

$$-4y = 12 \quad \text{Combine like terms.}$$

Next, we get y alone by using the multiplication property of equality and dividing both sides by -4 .

$$\frac{-4y}{-4} = \frac{12}{-4} \quad \text{Divide both sides by } -4.$$

$$y = -3 \quad \text{Simplify.}$$

Check: Replace y with -3 in the original equation.

$$3y - 7y = 12$$

$$3(-3) - 7(-3) \stackrel{?}{=} 12$$

$$-9 + 21 \stackrel{?}{=} 12$$

$$12 = 12 \quad \text{True}$$

The solution is -3 .

Work Practice 2

✓Concept Check What's wrong with the following solution?

$$\begin{aligned} 4x - 6x &= 10 \\ 2x &= 10 \\ \frac{2x}{2} &= \frac{10}{2} \\ x &= 5 \end{aligned}$$

Practice 3

Solve: $-8 + 6 = \frac{a}{3}$

Answers

2. -5 3. -6

✓Concept Check answer

On the left side of the equation, $4x - 6x$ simplifies to $-2x$ not $2x$.

Example 3 Solve: $\frac{z}{-4} = 11 - 5$

Solution: Simplify the right side of the equation first.

$$\frac{z}{-4} = 11 - 5$$

$$\frac{z}{-4} = 6$$

Next, to get z alone, multiply both sides by -4 .

$$-4 \cdot \frac{z}{-4} = -4 \cdot 6 \quad \text{Multiply both sides by } -4.$$

$$\frac{-4}{-4} \cdot z = -4 \cdot 6$$

$$1z = -24 \quad \text{or} \quad z = -24$$

Check to see that -24 is the solution.

Work Practice 3

Example 4 Solve: $5x + 2 - 4x = 7 - 19$

Solution: First we simplify each side of the equation separately.

$$5x + 2 - 4x = 7 - 19$$

$$\underbrace{5x - 4x} + 2 = \underbrace{7 - 19}$$

$$1x + 2 = -12 \quad \text{Combine like terms.}$$

To get x alone on the left side, we subtract 2 from both sides.

$$1x + 2 - 2 = -12 - 2 \quad \text{Subtract 2 from both sides.}$$

$$1x = -14 \quad \text{or} \quad x = -14 \quad \text{Simplify.}$$

Check to see that -14 is the solution.

Work Practice 4

Example 5 Solve: $8x - 9x = 12 - 17$

Solution: First combine like terms on each side of the equation.

$$8x - 9x = 12 - 17$$

$$-x = -5$$

Recall that $-x$ means $-1x$ and divide both sides by -1 .

$$\frac{-1x}{-1} = \frac{-5}{-1} \quad \text{Divide both sides by } -1.$$

$$x = 5 \quad \text{Simplify.}$$

Check to see that the solution is 5.

Work Practice 5

Example 6 Solve: $3(3x - 5) = 10x$

Solution: First we multiply on the left side to remove the parentheses.

$$3(3x - 5) = 10x$$

$$3 \cdot 3x - 3 \cdot 5 = 10x \quad \text{Use the distributive property.}$$

$$9x - 15 = 10x$$

Now we subtract $9x$ from both sides.

$$9x - 15 - 9x = 10x - 9x \quad \text{Subtract } 9x \text{ from both sides.}$$

$$-15 = 1x \quad \text{or} \quad x = -15 \quad \text{Simplify.}$$

Work Practice 6

Practice 4

Solve:

$$-6y - 1 + 7y = 17 + 2$$

Practice 5

Solve: $-4 - 10 = 4y - 5y$

Practice 6

Solve: $13x = 4(3x - 1)$

Answers

4. 20 5. 14 6. -4

Objective B Using Both Properties to Solve Equations

We now solve equations in one variable using more than one property of equality. To solve an equation such as $2x - 6 = 18$, we first get the variable term $2x$ alone on one side of the equation.

Practice 7

Solve: $5y + 2 = 17$

Example 7 Solve: $2x - 6 = 18$

Solution: We start by adding 6 to both sides to get the variable term $2x$ alone.

$$\begin{aligned} 2x - 6 &= 18 \\ 2x - 6 + 6 &= 18 + 6 && \text{Add 6 to both sides.} \\ 2x &= 24 && \text{Simplify.} \end{aligned}$$

To finish solving, we divide both sides by 2.

$$\begin{aligned} \frac{2x}{2} &= \frac{24}{2} && \text{Divide both sides by 2.} \\ x &= 12 && \text{Simplify.} \end{aligned}$$

Check:

$$\begin{aligned} 2x - 6 &= 18 \\ 2(12) - 6 &\stackrel{?}{=} 18 && \text{Replace } x \text{ with 12 and simplify.} \\ 24 - 6 &\stackrel{?}{=} 18 \\ 18 &= 18 && \text{True} \end{aligned}$$

Helpful Hint

Don't forget to check the proposed solution in the *original* equation.

The solution is 12.

Work Practice 7

Don't forget, if one or both sides of an equation can be simplified, do that first.

Practice 8

Solve:
 $-4(x + 2) - 60 = 2 - 10$

Example 8 Solve: $2 - 6 = -5(x + 4) - 39$

Solution: First, simplify each side of the equation.

$$\begin{aligned} 2 - 6 &= -5(x + 4) - 39 \\ 2 - 6 &= -5x - 20 - 39 && \text{Use the distributive property.} \\ -4 &= -5x - 59 && \text{Combine like terms on each side.} \\ -4 + 59 &= -5x - 59 + 59 && \text{Add 59 to both sides to get the variable term alone.} \\ 55 &= -5x && \text{Simplify.} \\ \frac{55}{-5} &= \frac{-5x}{-5} && \text{Divide both sides by } -5. \\ -11 &= x \text{ or } x = -11 && \text{Simplify.} \end{aligned}$$

Check to see that -11 is the solution.

Work Practice 8**Answers**

7. 3 8. -15

Objective C Translating Word Phrases into Expressions

Section 3.4 in this chapter contains a formal introduction to problem solving. To prepare for this section, let's once again review writing phrases as algebraic expressions using the following key words and phrases as a guide:

Addition	Subtraction	Multiplication	Division
sum	difference	product	quotient
plus	minus	times	divided
added to	subtracted from	multiply	shared equally among
more than	less than	twice	per
increased by	decreased by	of	divided by
total	less	double/triple	divided into

Helpful Hint

Don't forget: Order matters for subtraction and division, so be careful when translating.

Example 9 Write each phrase as an algebraic expression. Use x to represent "a number."

- a. a number increased by -5 b. the product of -7 and a number
 c. a number less 20 d. the quotient of -18 and a number
 e. a number subtracted from -2

Solution:

a. In words: a number increased by -5
 Translate: $x + (-5)$ or $x - 5$

b. In words: the product of -7 and a number
 Translate: $-7 \cdot x$ or $-7x$

c. In words: a number less 20
 Translate: $x - 20$

d. In words: the quotient of -18 and a number
 Translate: $-18 \div x$ or $-\frac{18}{x}$

e. In words: a number subtracted from -2
 Translate: $-2 - x$

Practice 9

Write each phrase as an algebraic expression. Use x to represent "a number."

- a. the sum of -3 and a number
 b. -5 decreased by a number
 c. three times a number
 d. a number subtracted from 83
 e. the quotient of a number and -4

Answers

9. a. $-3 + x$ b. $-5 - x$ c. $3x$
 d. $83 - x$ e. $\frac{x}{-4}$ or $-\frac{x}{4}$

Work Practice 9

Helpful Hint

As we mentioned in Chapter 1, don't forget that order is important when subtracting. Notice the translation order of numbers and variables below.

Phrase	Translation
a number “less” 9	$x - 9$
a number “subtracted from” 9	$9 - x$

Practice 10

Translate each phrase into an algebraic expression. Let x be the unknown number.

- The product of 5 and a number, decreased by 25
- Twice the sum of a number and 3
- The quotient of 39 and twice a number

Example 10

Write each phrase as an algebraic expression. Let x be the unknown number.

- Twice a number, increased by -9
- Three times the difference of a number and 11
- The quotient of 5 times a number and 17

Solution:

- a.** In words: twice a number increased by -9
- Translate: $2x + (-9)$ or $2x - 9$
- b.** In words: three times the difference of a number and 11
- Translate: $3(x - 11)$ or $3(x - 11)$
- c.** In words: the quotient of 5 times a number and 17
- Translate: $5x \div 17$ or $\frac{5x}{17}$

Work Practice 10

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

equation
solving

multiplication
addition

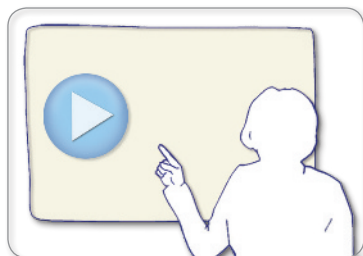
equivalent
simplifying

expression

1. The equations $-3x = 51$ and $\frac{-3x}{-3} = \frac{51}{-3}$ are called _____ equations.
2. The difference between an equation and an expression is that a(n) _____ contains an equal sign, while a(n) _____ does not.
3. The process of writing $-3x + 10x$ as $7x$ is called _____ the expression.
4. For the equation $-5x - 1 = -21$, the process of finding that 4 is the solution is called _____ the equation.

5. By the _____ property of equality, $x = -2$ and $x + 7 = -2 + 7$ are equivalent equations.
 6. By the _____ property of equality, $y = 8$ and $3 \cdot y = 3 \cdot 8$ are equivalent equations.

Martin-Gay Interactive Videos



See Video 3.2

Watch the section lecture video and answer the following questions.

- Objective A** 7. When solving Example 1, what must be done before applying the multiplication property of equality?
 8. When solving Example 4, what must be done before applying the addition property of equality?
Objective B 9. In Examples 5 and 6, both the addition and the multiplication properties of equality are used. Which property is used first both times?
Objective C 10. In Example 7, why are we told to be careful when translating a subtraction phrase?

3.2 Exercise Set MyLab Math

Objective A Solve each equation. First combine any like terms on each side of the equation. See Examples 1 through 5.

- | | | |
|-----------------------------|-----------------------------|------------------------------|
| 1. $x - 3 = -1 + 4$ | 2. $x + 7 = 2 + 3$ | 3. $-7 + 10 = m - 5$ |
| 4. $1 - 8 = n + 2$ | 5. $2w - 12w = 40$ | 6. $10y - y = 45$ |
| 7. $24 = t + 3t$ | 8. $100 = 15y + 5y$ | 9. $2z = 12 - 14$ |
| 10. $-3x = 11 - 2$ | 11. $4 - 10 = \frac{z}{-3}$ | 12. $20 - 22 = \frac{z}{-4}$ |
| 13. $-3x - 3x = 50 - 2$ | 14. $5y - 9y = -14 + (-14)$ | 15. $\frac{x}{5} = -26 + 16$ |
| 16. $\frac{y}{3} = 32 - 52$ | 17. $7x + 7 - 6x = 10$ | 18. $-3 + 5x - 4x = 13$ |
| 19. $-8 - 9 = 3x + 5 - 2x$ | 20. $-7 + 10 = 4x - 6 - 3x$ | |

Solve. First multiply to remove parentheses. See Example 6.

- | | | |
|------------------------|------------------------|-----------------------|
| 21. $2(5x - 3) = 11x$ | 22. $6(3x + 1) = 19x$ | 23. $3y = 2(y + 12)$ |
| 24. $17x = 4(4x - 6)$ | 25. $21y = 5(4y - 6)$ | 26. $28z = 9(3z - 2)$ |
| 27. $-3(-4 - 2z) = 7z$ | 28. $-2(-1 - 3y) = 7y$ | |

Objective B Solve each equation. See Examples 7 and 8.

- | | | | |
|------------------------------|---------------------------------|-------------------------------|-----------------------|
| 29. $2x - 8 = 0$ | 30. $3y - 12 = 0$ | 31. $7y + 3 = 24$ | 32. $5m + 1 = 46$ |
| 33. $-7 = 2x - 1$ | 34. $-11 = 3t - 2$ | 35. $6(6 - 4y) = 12y$ | 36. $4(3y - 5) = 14y$ |
| 37. $11(x - 6) = -4 - 7$ | 38. $5(x - 6) = -2 - 8$ | 39. $-3(x + 1) - 10 = 12 + 8$ | |
| 40. $-2(x + 5) - 2 = -8 - 4$ | 41. $y - 20 = 6y$ | 42. $x - 63 = 10x$ | |
| 43. $22 - 42 = 4(x - 1) - 4$ | 44. $35 - (-3) = 3(x - 2) + 17$ | | |

Objectives A B Mixed Practice Solve each equation. See Examples 1 through 8.

45. $-2 - 3 = -4 + x$

46. $7 - (-10) = x - 5$

47. $y + 1 = -3 + 4$

48. $y - 8 = -5 - 1$

▶ 49. $3w - 12w = -27$

50. $y - 6y = 20$

51. $-4x = 20 - (-4)$

52. $6x = 5 - 35$

53. $18 - 11 = \frac{x}{-5}$

54. $9 - 14 = \frac{x}{-12}$

55. $9x - 12 = 78$

56. $8x - 8 = 32$

57. $10 = 7t - 12t$

58. $-30 = t + 9t$

59. $5 - 5 = 3x + 2x$

60. $-42 + 20 = -2x + 13x$

61. $50y = 7(7y + 4)$

62. $65y = 8(8y - 9)$

63. $8x = 2(6x + 10)$

64. $10x = 6(2x - 3)$

65. $7x + 14 - 6x = -4 - 10$

66. $-10x + 11x + 5 = 9 - 5$

67. $\frac{x}{-4} = -1 - (-8)$

68. $\frac{y}{-6} = 6 - (-1)$

69. $23x + 8 - 25x = 7 - 9$

70. $8x - 4 - 6x = 12 - 22$

▶ 71. $-3(x + 9) - 41 = 4 - 60$

72. $-4(x + 7) - 30 = 3 - 37$

Objective C Translating Write each phrase as a variable expression. Use x to represent “a number.” See Examples 9 and 10.73. The sum of -7 and a number

74. Negative eight plus a number

▶ 75. Eleven subtracted from a number

76. A number subtracted from twelve

77. The product of -13 and a number

78. Twice a number

79. A number divided by -12

80. The quotient of negative six and a number

▶ 81. The product of -11 and a number, increased by 5

82. Negative four times a number, increased by 18

83. Negative ten decreased by 7 times a number

84. Twice a number, decreased by thirty

85. Seven added to the product of 4 and a number

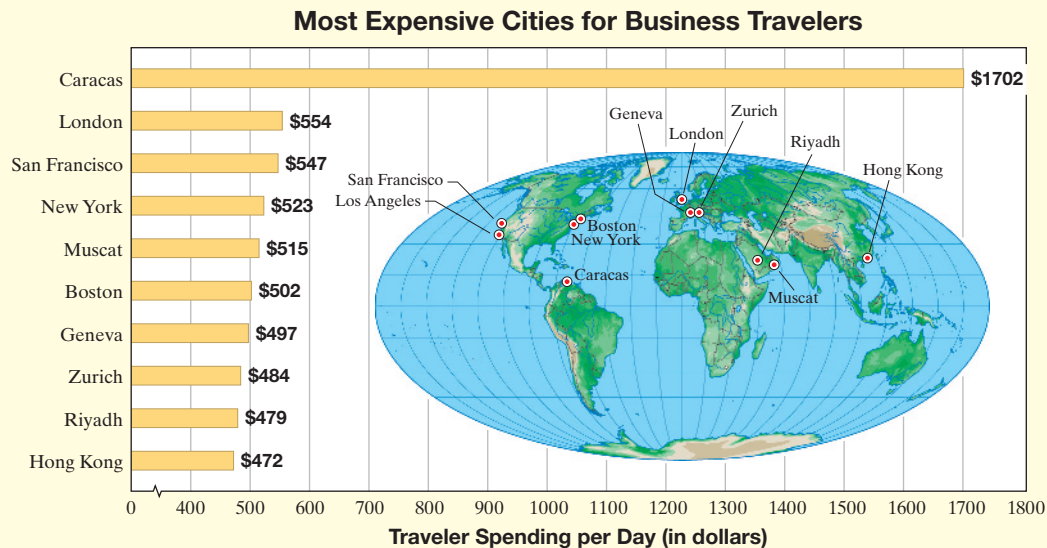
86. The product of 7 and a number, added to 100

87. Twice a number, decreased by 17

88. The difference of -9 times a number, and 189. The product of -6 and the sum of a number and 1590. Twice the sum of a number and -5 91. The quotient of 45 and the product of a number and -5 92. The quotient of ten times a number, and -4 93. The quotient of seventeen and a number, increased by -15 94. The quotient of -20 and a number, decreased by three

Review

This horizontal bar graph shows the top ten expensive cities for business travelers in 2016. Use this graph to answer Exercises 95 through 98. See Sections 1.2 and 1.3.



95. For which city is the daily cost of business travel the highest?
96. For the cities shown, which cities have daily costs less than \$510?
97. What is the combined daily cost for a business traveler going to London, Geneva, and Zurich?
98. What is the combined daily cost for a business traveler going to San Francisco, New York, and Boston?

Concept Extensions

99. In your own words, explain the addition property of equality.
100. Write an equation that can be solved using the addition property of equality.

For exercises 101 and 102, see the Concept Check in this section.

101. Are the equations below equivalent? Why or why not?
- $$x + 7 = 4 + (-9)$$
- $$x + 7 \stackrel{?}{=} 5$$
102. Are the equations below equivalent? Why or why not?
- $$3x - 6x = 12$$
- $$3x \stackrel{?}{=} 12$$
103. Why does the multiplication property of equality not allow us to divide both sides of an equation by zero?
104. Is the equation $-x = 6$ solved for the variable? Explain why or why not.

Solve.

105. $\frac{y}{72} = -86 - (-1029)$
106. $\frac{x}{-13} = 4^6 - 5^7$
107. $\frac{x}{-2} = 5^2 - |-10| - (-9)$
108. $\frac{y}{10} = (-8)^2 - |20| + (-2)^2$
109. $|-13| + 3^2 = 100y - |-20| - 99y$
110. $4(x - 11) + |90| - |-86| + 2^5 = 5x$

Expressions and Equations

Answers

For the table below, identify each as an expression or an equation.

	Expression or Equation	
1.	$7x - 5y + 14$	
2.	$7x = 35 + 14$	
3.	$3(x - 2) = 5(x + 1) - 17$	
4.	$-9(2x + 1) - 4(x - 2) + 14$	

Fill in each blank with “simplify” or “solve.”

5. To _____ an expression, we combine any like terms.

6. To _____ an equation, we use the properties of equality to find any value of the variable that makes the equation a true statement.

Simplify each expression by combining like terms.

7. $7x + x$

8. $6y - 10y$

9. $2a + 5a - 9a - 2$

10. $6a - 12 - a - 14$

Multiply and simplify if possible.

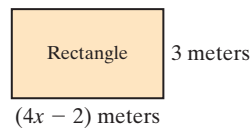
11. $-2(4x + 7)$

12. $-3(2x - 10)$

13. $5(y + 2) - 20$

14. $12x + 3(x - 6) - 13$

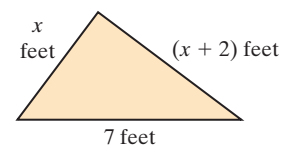
△ 15. Find the area of the rectangle.



Solve and check.

17. $12 = 11x - 14x$

△ 16. Find the perimeter of the triangle.



18. $8y + 7y = -45$

19. $x - 12 = -45 + 23$

20. $6 - (-5) = x + 5$

Solve and check.

21. $\frac{x}{3} = -14 + 9$

22. $\frac{z}{4} = -23 - 7$

23. $-6 + 2 = 4x + 1 - 3x$

24. $5 - 8 = 5x + 10 - 4x$

25. $6(3x - 4) = 19x$

26. $25x = 6(4x - 9)$

27. $-36x - 10 + 37x = -12 - (-14)$

28. $-8 + (-14) = -80y + 20 + 81y$

29. $3x - 16 = -10$

30. $4x - 21 = -13$

31. $-8z - 2z = 26 - (-4)$

32. $-12 + (-13) = 5x - 10x$

33. $-4(x + 8) - 11 = 3 - 26$

34. $-6(x - 2) + 10 = -4 - 10$

Translating Write each phrase as an algebraic expression. Use x to represent “a number.”

35. The difference of a number and 10

36. The sum of -20 and a number

37. The product of 10 and a number

38. The quotient of 10 and a number

39. Five added to the product of -2 and a number

40. The product of -4 and the difference of a number and 1

21. _____

22. _____

23. _____

24. _____

25. _____

26. _____

27. _____

28. _____

29. _____

30. _____

31. _____

32. _____

33. _____

34. _____

35. _____

36. _____

37. _____

38. _____

39. _____

40. _____

3.3 Solving Linear Equations in One Variable

Objectives

- A** Solve Linear Equations Using the Addition and Multiplication Properties. 
- B** Solve Linear Equations Containing Parentheses. 
- C** Write Numerical Sentences as Equations. 

Practice 1

Solve: $7x + 12 = 3x - 4$

In this chapter, the equations we are solving are called **linear equations in one variable** or **first-degree equations in one variable**. For example, an equation such as $5x - 2 = 6x$ is a linear equation in one variable. It is called linear or first degree because the exponent on each x is 1 and there is no variable below a fraction bar. It is an equation in one variable because it contains one variable, x .

Let's continue solving linear equations in one variable.

Objective A Solving Equations Using the Addition and Multiplication Properties

If an equation contains variable terms on both sides, we use the addition property of equality to get all the variable terms on one side and all the constants or numbers on the other side.

Example 1 Solve: $3a - 6 = a + 4$

Solution: Although it makes no difference which side we choose, let's move variable terms to the left side and constants to the right side.

$$\begin{aligned}
 3a - 6 &= a + 4 \\
 3a - 6 + 6 &= a + 4 + 6 && \text{Add 6 to both sides.} \\
 3a &= a + 10 && \text{Simplify.} \\
 3a - a &= a + 10 - a && \text{Subtract } a \text{ from both sides.} \\
 2a &= 10 && \text{Simplify.} \\
 \frac{2a}{2} &= \frac{10}{2} && \text{Divide both sides by 2.} \\
 a &= 5 && \text{Simplify.}
 \end{aligned}$$

Check:

$$\begin{aligned}
 3a - 6 &= a + 4 && \text{Original equation} \\
 3 \cdot 5 - 6 &\stackrel{?}{=} 5 + 4 && \text{Replace } a \text{ with 5.} \\
 15 - 6 &\stackrel{?}{=} 9 && \text{Simplify.} \\
 9 &= 9 && \text{True}
 \end{aligned}$$

The solution is 5.

Work Practice 1

Helpful Hint

Make sure you understand which property to use to solve an equation.

Addition

$$x + 2 = 10$$

To undo addition of 2, we subtract 2 from both sides.

$$\begin{aligned}
 x + 2 - 2 &= 10 - 2 && \text{Use addition property of equality.} \\
 x &= 8
 \end{aligned}$$

Check:

$$\begin{aligned}
 x + 2 &= 10 \\
 8 + 2 &\stackrel{?}{=} 10 \\
 10 &= 10 && \text{True}
 \end{aligned}$$

Understood multiplication

$$2x = 10$$

To undo multiplication of 2, we divide both sides by 2.

$$\begin{aligned}
 \frac{2x}{2} &= \frac{10}{2} && \text{Use multiplication property of equality.} \\
 x &= 5
 \end{aligned}$$

Check:

$$\begin{aligned}
 2x &= 10 \\
 2 \cdot 5 &\stackrel{?}{=} 10 \\
 10 &= 10 && \text{True}
 \end{aligned}$$

Answer

1. -4

Example 2 Solve: $17 - 7x + 3 = -3x + 21 - 3x$

Solution: First, simplify both sides of the equation.

$$17 - 7x + 3 = -3x + 21 - 3x$$

$$20 - 7x = -6x + 21 \quad \text{Simplify.}$$

Next, move variable terms to one side of the equation and constants, or numbers, to the other side. To begin, let's add $6x$ to both sides.

$$20 - 7x + 6x = -6x + 21 + 6x \quad \text{Add } 6x \text{ to each side.}$$

$$20 - x = 21 \quad \text{Simplify.}$$

$$20 - x - 20 = 21 - 20 \quad \text{Subtract 20 from both sides.}$$

$$-1x = 1 \quad \text{Simplify. Recall that } -x \text{ means } -1x.$$

$$\frac{-1x}{-1} = \frac{1}{-1} \quad \text{Divide both sides by } -1.$$

$$x = -1 \quad \text{Simplify.}$$

Check:

$$\begin{aligned} 17 - 7x + 3 &= -3x + 21 - 3x \\ 17 - 7(-1) + 3 &\stackrel{?}{=} -3(-1) + 21 - 3(-1) \\ 17 + 7 + 3 &\stackrel{?}{=} 3 + 21 + 3 \\ 27 &= 27 \end{aligned}$$

True

The solution is -1 .

Work Practice 2

Objective B Solving Equations Containing Parentheses

Recall from the previous section that if an equation contains parentheses, we will first use the distributive property to remove them.

Example 3 Solve: $7(x - 2) = 9x - 6$

Solution: First we apply the distributive property.

$$7(x - 2) = 9x - 6$$

$$7x - 14 = 9x - 6 \quad \text{Apply the distributive property.}$$

Next we move variable terms to one side of the equation and constants to the other side.

$$7x - 14 - 9x = 9x - 6 - 9x \quad \text{Subtract } 9x \text{ from both sides.}$$

$$-2x - 14 = -6 \quad \text{Simplify.}$$

$$-2x - 14 + 14 = -6 + 14 \quad \text{Add 14 to both sides.}$$

$$-2x = 8 \quad \text{Simplify.}$$

$$\frac{-2x}{-2} = \frac{8}{-2} \quad \text{Divide both sides by } -2.$$

$$x = -4 \quad \text{Simplify.}$$

Check to see that -4 is the solution.

Work Practice 3

✓Concept Check In Example 3, the solution is -4 . To check this solution, what equation should we use?

Practice 2

Solve:

$$40 - 5y + 5 = -2y - 10 - 4y$$

Practice 3

$$\text{Solve: } 6(a - 5) = 4a + 4$$

Answers

2. -55 3. 17

✓Concept Check Answer

$$7(x - 2) = 9x - 6$$

You may want to use the following steps to solve equations.

Steps for Solving an Equation

- Step 1:** If parentheses are present, use the distributive property.
- Step 2:** Combine any like terms on each side of the equation.
- Step 3:** Use the addition property of equality to rewrite the equation so that variable terms are on one side of the equation and constant terms are on the other side.
- Step 4:** Use the multiplication property of equality to divide both sides by the numerical coefficient of the variable to solve.
- Step 5:** Check the solution in the *original equation*.

Practice 4

Solve: $4(x + 3) + 1 = 13$

Example 4 Solve: $3(2x - 6) + 6 = 0$

Solution: $3(2x - 6) + 6 = 0$

Step 1: $6x - 18 + 6 = 0$ Apply the distributive property.

Step 2: $6x - 12 = 0$ Combine like terms on the left side of the equation.

Step 3: $6x - 12 + 12 = 0 + 12$ Add 12 to both sides.

$6x = 12$ Simplify.

Step 4: $\frac{6x}{6} = \frac{12}{6}$ Divide both sides by 6.

$x = 2$ Simplify.

Check:

Step 5: $3(2x - 6) + 6 = 0$

$3(2 \cdot 2 - 6) + 6 \stackrel{?}{=} 0$

$3(4 - 6) + 6 \stackrel{?}{=} 0$

$3(-2) + 6 \stackrel{?}{=} 0$

$-6 + 6 \stackrel{?}{=} 0$

$0 = 0$ True

The solution is 2.

Work Practice 4

Objective C Writing Numerical Sentences as Equations

Next, we practice translating sentences into equations. Below are key words and phrases that translate to an equal sign. (Note: For a review of key words and phrases that translate to addition, subtraction, multiplication, and division, see Sections 1.8 and 3.2.)

Key Words or Phrases	Examples	Symbols
equals	3 equals 2 plus 1	$3 = 2 + 1$
gives	the quotient of 10 and -5 gives -2	$\frac{10}{-5} = -2$
is/was	17 minus 12 is 5	$17 - 12 = 5$
yields	11 plus 2 yields 13	$11 + 2 = 13$
amounts to	twice -15 amounts to -30	$2(-15) = -30$
is equal to	-24 is equal to 2 times -12	$-24 = 2(-12)$

Example 5 Translate each sentence into an equation.

- The product of 7 and 6 is 42.
- Twice the sum of 3 and 5 is equal to 16.
- The quotient of -45 and 5 yields -9 .

Solution:

- a.** In words: $\begin{array}{ccc} \text{the product} & & \\ \text{of 7 and 6} & \text{is} & 42 \\ \downarrow & \downarrow & \downarrow \\ 7 \cdot 6 & = & 42 \end{array}$
- Translate: $7 \cdot 6 = 42$
- b.** In words: $\begin{array}{cccc} & \text{the sum} & & \\ & \text{of 3 and 5} & \text{is equal to} & 16 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \text{twice} & (3 + 5) & = & 16 \end{array}$
- Translate: $2(3 + 5) = 16$
- c.** In words: $\begin{array}{ccc} \text{the quotient} & & \\ \text{of } -45 \text{ and } 5 & \text{yields} & -9 \\ \downarrow & \downarrow & \downarrow \\ -45 & = & -9 \\ \downarrow & & \\ 5 & & \end{array}$
- Translate: $\frac{-45}{5} = -9$

Work Practice 5**Practice 5**

Translate each sentence into an equation.

- The difference of 110 and 80 is 30.
- The product of 3 and the sum of -9 and 11 amounts to 6.
- The quotient of 24 and -6 yields -4 .

Answers

- 5 a.** $110 - 80 = 30$
b. $3(-9 + 11) = 6$ **c.** $\frac{24}{-6} = -4$

**Calculator Explorations Checking Equations**

A calculator can be used to check possible solutions of equations. To do this, replace the variable by the possible solution and evaluate each side of the equation separately. For example, to see whether 7 is a solution of the equation $52x = 15x + 259$, replace x with 7 and use your calculator to evaluate each side separately.

Equation: $52x = 15x + 259$

$52 \cdot 7 \stackrel{?}{=} 15 \cdot 7 + 259$ Replace x with 7.

Evaluate left side: 52×7 then $=$ or **ENTER**.

Display: 364 .

Evaluate right side: $15 \times 7 + 259$ then $=$ or **ENTER**.

Display: 364 .

Since the left side equals the right side, 7 is a solution of the equation $52x = 15x + 259$.

Use a calculator to determine whether the numbers given are solutions of each equation.

- $76(x - 25) = -988$; 12
- $-47x + 862 = -783$; 35
- $x + 562 = 3x + 900$; -170
- $55(x + 10) = 75x + 910$; -18
- $29x - 1034 = 61x - 362$; -21
- $-38x + 205 = 25x + 120$; 25

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

addition

multiplication

combine like terms

$5(2x + 6) - 1 = 39$

$3x - 9 + x - 16$

distributive

- An example of an expression is _____ while an example of an equation is _____.
- To solve $\frac{x}{-7} = -10$, we use the _____ property of equality.
- To solve $x - 7 = -10$, we use the _____ property of equality.

Use the order of the Steps for Solving an Equation in this section to answer Exercises 4 through 6.

4. To solve $9x - 6x = 10 + 6$, first _____.
5. To solve $5(x - 1) = 25$, first use the _____ property.
6. To solve $4x + 3 = 19$, first use the _____ property of equality.

Martin-Gay Interactive Videos Watch the section lecture video and answer the following questions.



See Video 3.3

- Objective A** 7. In Example 1, the number 1 is subtracted from the left side of the equation. What property tells us we must also subtract 1 from the right side? Why is it important to do the same thing to both sides?
- Objective B** 8. From Example 3, what is the first step when solving an equation that contains parentheses? What property do we use to perform this step?
- Objective C** 9. What word or phrase translates to “equals” in Example 5? In Example 6?

3.3 Exercise Set MyLab Math

Objective A Solve each equation. See Examples 1 and 2.

- | | | |
|-----------------------------------|------------------------------------|-----------------------------------|
| 1. $3x - 7 = 4x + 5$ | 2. $7x - 1 = 8x + 4$ | 3. $10x + 15 = 6x + 3$ |
| 4. $5x - 3 = 2x - 18$ | 5. $19 - 3x = 14 + 2x$ | 6. $4 - 7m = -3m + 4$ |
| 7. $-14x - 20 = -12x + 70$ | 8. $57y + 140 = 54y - 100$ | 9. $x + 20 + 2x = -10 - 2x - 15$ |
| 10. $2x + 10 + 3x = -12 - x - 20$ | 11. $40 + 4y - 16 = 13y - 12 - 3y$ | 12. $19x - 2 - 7x = 31 + 6x - 15$ |

Objective B Solve each equation. See Examples 3 and 4.

- | | | |
|--------------------------|--------------------------------|-------------------------------|
| 13. $35 - 17 = 3(x - 2)$ | 14. $22 - 42 = 4(x - 1)$ | 15. $3(x - 1) - 12 = 0$ |
| 16. $2(x + 5) + 8 = 0$ | 17. $2(y - 3) = y - 6$ | 18. $3(z + 2) = 5z + 6$ |
| 19. $-2(y + 4) = 2$ | 20. $-1(y + 3) = 10$ | 21. $2t - 1 = 3(t + 7)$ |
| 22. $-4 + 3c = 4(c + 2)$ | 23. $3(5c + 1) - 12 = 13c + 3$ | 24. $4(3t + 4) - 20 = 3 + 5t$ |

Mixed Practice (Sections 2.6, 3.2, and 3.3) Solve each equation.

- | | | |
|-------------------|------------------|------------------|
| 25. $-4x = 44$ | 26. $-3x = 51$ | 27. $x + 9 = 2$ |
| 28. $y - 6 = -11$ | 29. $8 - b = 13$ | 30. $7 - z = 15$ |

31. $-20 - (-50) = \frac{x}{9}$

32. $-2 - 10 = \frac{z}{10}$

33. $3r + 4 = 19$

34. $7y + 3 = 38$

▶ 35. $-7c + 1 = -20$

36. $-2b + 5 = -7$

37. $8y - 13y = -20 - 25$

38. $4x - 11x = -14 - 14$

39. $6(7x - 1) = 43x$

40. $5(3y - 2) = 16y$

41. $-4 + 12 = 16x - 3 - 15x$

42. $-9 + 20 = 19x - 4 - 18x$

43. $-10(x + 3) + 28 = -16 - 16$

44. $-9(x + 2) + 25 = -19 - 19$

45. $4x + 3 = 2x + 11$

46. $6y - 8 = 3y + 7$

47. $-2y - 10 = 5y + 18$

48. $7n + 5 = 12n - 10$

49. $-8n + 1 = -6n - 5$

50. $10w + 8 = w - 10$

51. $9 - 3x = 14 + 2x$

52. $4 - 7m = -3m$

53. $9a + 29 + 7 = 0$

54. $10 + 4v + 6 = 0$

55. $7(y - 2) = 4y - 29$

56. $2(z - 2) = 5z + 17$

57. $12 + 5t = 6(t + 2)$

58. $4 + 3c = 2(c + 2)$

▶ 59. $3(5c - 1) - 2 = 13c + 3$

60. $4(2t + 5) - 21 = 7t - 6$

61. $10 + 5(z - 2) = 4z + 1$

62. $14 + 4(w - 5) = 6 - 2w$

63. $7(6 + w) = 6(2 + w)$

64. $6(5 + c) = 5(c - 4)$

Objective C Translating Write each sentence as an equation. See Example 5.

65. The sum of -42 and 16 is -26 .

66. The difference of -30 and 10 equals -40 .

▶ 67. The product of -5 and -29 gives 145 .

68. The quotient of -16 and 2 yields -8 .

▶ 69. Three times the difference of -14 and 2 amounts to -48 .

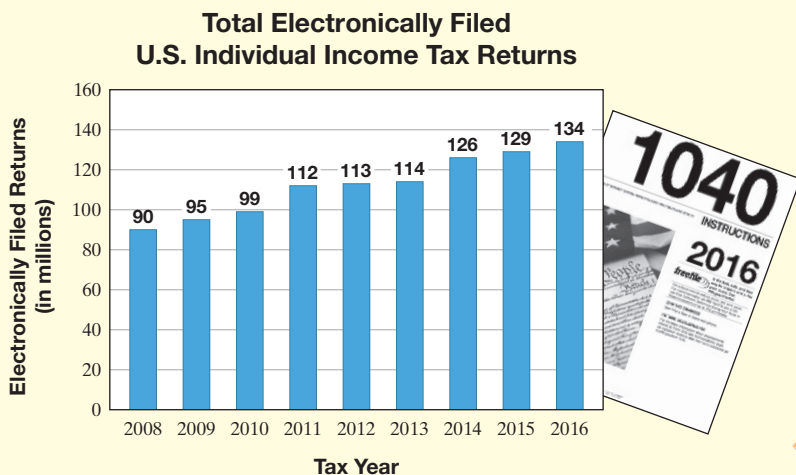
70. Negative 2 times the sum of 3 and 12 is -30 .

71. The quotient of 100 and twice 50 is equal to 1 .

72. Seventeen subtracted from -12 equals -29 .

Review

The following bar graph shows the number of U.S. federal individual income tax returns that are filed electronically during the years shown using efile. Use this graph to answer Exercises 73 through 76. Write number answers in standard form. See Sections 1.2 and 1.3.



Source: efile

Evaluate each expression for $x = 3$, $y = -1$, and $z = 0$. See Section 2.5.

77. $x^3 - 2xy$

78. $y^3 + 3xyz$

79. $y^5 - 4x^2$

80. $(-y)^3 + 3xyz$

Concept Extensions

Using the Steps for Solving an Equation, choose the next operation for solving the given equation.

81. $2x - 5 = -7$

- Add 7 to both sides.
- Add 5 to both sides.
- Divide both sides by 2.

83. $-3x = -12$

- Divide both sides by -3 .
- Add 12 to both sides.
- Add $3x$ to both sides.

82. $3x + 2x = -x - 4$

- Add 4 to both sides.
- Subtract $2x$ from both sides.
- Add $3x$ and $2x$.

84. $9 - 5x = 15$

- Divide both sides by -5 .
- Subtract 15 from both sides.
- Subtract 9 from both sides.

A classmate shows you steps for solving an equation. The solution does not check, but the classmate is unable to find the error. For each set of steps, check the solution, find the error, and correct it.

85. $2(3x - 5) = 5x - 7$
 $6x - 5 = 5x - 7$
 $6x - 5 + 5 = 5x - 7 + 5$
 $6x = 5x - 2$
 $6x - 5x = 5x - 2 - 5x$
 $x = -2$

86. $37x + 1 = 9(4x - 7)$
 $37x + 1 = 36x - 7$
 $37x + 1 - 1 = 36x - 7 - 1$
 $37x = 36x - 8$
 $37x - 36x = 36x - 8 - 36x$
 $x = -8$

Solve.

87. $(-8)^2 + 3x = 5x + 4^3$

88. $3^2 \cdot x = (-9)^3$

89. $2^3(x + 4) = 3^2(x + 4)$

90. $x + 45^2 = 54^2$

91. A classmate tries to solve $3x = 39$ by subtracting 3 from both sides of the equation. Will this step solve the equation for x ? Why or why not?

92. A classmate tries to solve $2 + x = 20$ by dividing both sides by 2. Will this step solve the equation for x ? Why or why not?

3.4 Linear Equations in One Variable and Problem Solving

Objective A Writing Sentences as Equations

Now that we have practiced solving equations for a variable, we can extend our problem-solving skills considerably. We begin by writing sentences as equations using the following key words and phrases as a guide:

Addition	Subtraction	Multiplication	Division	Equal Sign
sum	difference	product	quotient	equals
plus	minus	times	divide	gives
added to	subtracted from	multiply	shared equally among	is/was
more than	less than	twice	per	yields
increased by	decreased by	of	divided by	amounts to
total	less	double/triple	divided into	is equal to

Example 1 Write each sentence as an equation. Use x to represent “a number.”

- Twenty increased by a number is 5.
- Twice a number equals -10 .
- A number minus 11 amounts to 168.
- Three times the sum of a number and 5 is -30 .
- The quotient of twice a number and 8 is equal to 2.

Solution:

a. In words: twenty increased by a number is 5

Translate: $20 + x = 5$

b. In words: twice a number equals -10

Translate: $2x = -10$

c. In words: a number minus 11 amounts to 168

Translate: $x - 11 = 168$

d. In words: three times the sum of a number and 5 is -30

Translate: $3(x + 5) = -30$

(Continued on next page)

Objectives

- Write Sentences as Equations.
- Use Problem-Solving Steps to Solve Problems.

Practice 1

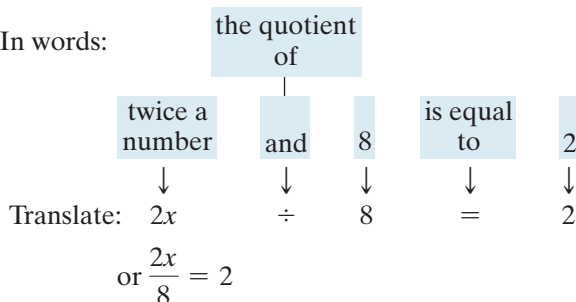
Write each sentence as an equation. Use x to represent “a number.”

- Four times a number is 20.
- The sum of a number and -5 yields 32.
- Fifteen subtracted from a number amounts to -23 .
- Five times the difference of a number and 7 is equal to -8 .
- The quotient of triple a number and 5 gives 1.

Answers

- $4x = 20$
 - $x + (-5) = 32$
 - $x - 15 = -23$
 - $5(x - 7) = -8$
 - $\frac{3x}{5} = 1$

e. In words:

**Work Practice 1****Objective B** Using Problem-Solving Steps to Solve Problems

Our main purpose for studying arithmetic and algebra is to solve problems. In previous sections, we have prepared for problem solving by writing phrases as algebraic expressions and sentences as equations. We now draw upon this experience as we solve problems. The following problem-solving steps will be used throughout this text.

Problem-Solving Steps

1. UNDERSTAND the problem. During this step, become comfortable with the problem. Some ways of doing this are as follows:
 - Read and reread the problem.
 - Construct a drawing.
 - Propose a solution and check. Pay careful attention to how you check your proposed solution. This will help when writing an equation to model the problem.
 - Choose a variable to represent the unknown. Use this variable to represent any other unknowns.
2. TRANSLATE the problem into an equation.
3. SOLVE the equation.
4. INTERPRET the results: *Check* the proposed solution in the stated problem and *state* your conclusion.

The first problem that we solve consists of finding an unknown number.

Practice 2

Translate “the sum of a number and 2 equals 6 added to three times the number” into an equation and solve.

Answer2. -2 **Example 2** Finding an Unknown Number

Twice a number plus 3 is the same as the number minus 6. Find the unknown number.

Solution:

1. UNDERSTAND the problem. To do so, we read and reread the problem.

Let’s propose a solution to help us understand. Suppose the unknown number is 5. Twice this number plus 3 is $2 \cdot 5 + 3$ or 13. Is this the same as the number minus 6, or $5 - 6$, or -1 ? Since 13 is not the same as -1 , we know that 5 is not the solution. However, remember that the purpose of proposing a solution is not to guess correctly, but to better understand the problem.

Now let’s choose a variable to represent the unknown. Let’s let

$$x = \text{unknown number}$$

2. TRANSLATE the problem into an equation.

In words: twice a number plus 3 is the same as the number minus 6

↓ ↓ ↓ ↓

Translate: $2x$ $+3$ $=$ $x - 6$

3. SOLVE the equation. To solve the equation, we first subtract x from both sides.

$$\begin{aligned} 2x + 3 &= x - 6 \\ 2x + 3 - x &= x - 6 - x \\ x + 3 &= -6 && \text{Simplify.} \\ x + 3 - 3 &= -6 - 3 && \text{Subtract 3 from both sides.} \\ x &= -9 && \text{Simplify.} \end{aligned}$$

4. INTERPRET the results. First, *check* the proposed solution in the stated problem. Twice “ -9 ” is -18 and $-18 + 3$ is -15 . This is equal to the number minus 6, or “ -9 ” $- 6$, or -15 . Then *state* your conclusion: The unknown number is -9 .

Work Practice 2

Concept Check Suppose you have solved an equation involving perimeter to find the length of a rectangular table. Explain why you would want to recheck your math if you obtain the result of -5 .

Example 3 Determining Distances

The distance by road from Chicago, Illinois, to Los Angeles, California, is 1091 miles *more* than the distance from Chicago to Boston, Massachusetts. If the total of these two distances is 3017 miles, find the distance from Chicago to Boston. (Source: *World Almanac*)

Solution:

1. UNDERSTAND the problem. We read and reread the problem.

Let's propose and check a solution to help us better understand the problem. Suppose the distance from Chicago to Boston is 600 miles. Since the distance from Chicago to Los Angeles is 1091 miles *more*, then this distance is $600 + 1091 = 1691$ miles. With these numbers, the total of the distances is $600 + 1691 = 2291$ miles. This is less than the given total of 3017 miles, so we are incorrect. But not only do we have a better understanding of this exercise, we also know that the distance from Boston to Chicago is greater than 600 miles since this proposed solution led to a total too small. Now let's choose a variable to represent an unknown. Then we'll use this variable to represent any other unknown quantities. Let

x = distance from Chicago to Boston

Then

$x + 1091$ = distance from Chicago to Los Angeles

Since that distance is 1091 miles more.

2. TRANSLATE the problem into an equation.

In words: Chicago to Boston distance $+$ Chicago to Los Angeles distance $=$ total miles

↓ ↓ ↓

Translate: x $+$ $x + 1091$ $=$ 3017

(Continued on next page)

Practice 3

The distance by road from Cincinnati, Ohio, to Denver, Colorado, is 71 miles *less* than the distance from Denver to San Francisco, California. If the total of these two distances is 2399 miles, find the distance from Denver to San Francisco.



Answer

3. 1235 miles

Concept Check Answer
Length cannot be negative.

3. SOLVE the equation:

$$x + x + 1091 = 3017$$

$$2x + 1091 = 3017$$

Combine like terms.

$$2x + 1091 - 1091 = 3017 - 1091$$

Subtract 1091 from both sides.

$$2x = 1926$$

Simplify.

$$\frac{2x}{2} = \frac{1926}{2}$$

Divide both sides by 2.

$$x = 963$$

Simplify.

4. INTERPRET the results. First *check* the proposed solution in the stated problem. Since x represents the distance from Chicago to Boston, this is 963 miles. The distance from Chicago to Los Angeles is $x + 1091 = 963 + 1091 = 2054$ miles. To check, notice that the total number of miles is $963 + 2054 = 3017$ miles, the given total of miles. Also, 2054 is 1091 more miles than 963, so the solution checks. Then, *state* your conclusion: The distance from Chicago to Boston is 963 miles.

Work Practice 3

Practice 4

A woman's \$57,000 estate is to be divided so that her husband receives twice as much as her son. How much will each receive?

Example 4 Calculating Separate Costs

A salesperson at an electronics store sold a computer system and software for \$2100, receiving four times as much money for the computer system as for the software. Find the price of each.



Solution:

1. UNDERSTAND the problem. We read and re-read the problem. Then we choose a variable to represent an unknown. We use this variable to represent any other unknown quantities. We let

x = the software price

Then

$4x$ = the computer system price

2. TRANSLATE the problem into an equation.

In words:	software price	and	computer price	is	2100
	↓	↓	↓	↓	↓
Translate:	x	+	$4x$	=	2100

3. SOLVE the equation:

$$x + 4x = 2100$$

$$5x = 2100$$

Combine like terms.

$$\frac{5x}{5} = \frac{2100}{5}$$

Divide both sides by 5.

$$x = 420$$

Simplify.

4. INTERPRET the results. *Check* the proposed solution in the stated problem. The software sold for \$420. The computer system sold for $4x = 4(\$420) = \$1680$. Since $\$420 + \$1680 = \$2100$, the total price, and \$1680 is four times \$420, the solution checks. *State* your conclusion: The software sold for \$420, and the computer system sold for \$1680.

Work Practice 4

Answer

4. husband: \$38,000; son: \$19,000

Vocabulary, Readiness & Video Check

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 3.4

Objective A 1. In Example 2, why does the left side of the equation translate to $-20 - x$ and not $x - (-20)$? ▶

Objective B 2. Why are parentheses used in the translation of the left side of the equation in Example 4? ▶

3. In Example 5, the solution to the equation is $x = 37$. Why is this not the solution to the application? ▶

3.4 Exercise Set MyLab Math ▶

Objective A Translating Write each sentence as an equation. Use x to represent “a number.” See Example 1.

1. A number added to -5 is -7 .
2. Five subtracted from a number equals 10.
3. Three times a number yields 27.
4. The quotient of 8 and a number is -2 .
- ▶ 5. A number subtracted from -20 amounts to 104.
6. Two added to twice a number gives -14 .
- ▶ 7. Twice a number gives 108.
8. Five times a number is equal to -75 .
9. The product of 5 and the sum of -3 and a number is -20 .
10. Twice the sum of -17 and a number is -14 .

Objective B Translating Translate each sentence into an equation. Then solve the equation. See Example 2.

- ▶ 11. Three times a number, added to 9, is 33. Find the number.
12. Twice a number, subtracted from 60, is 20. Find the number.
13. The sum of 3, 4, and a number amounts to 16. Find the number.
14. The sum of 7, 9, and a number is 40. Find the number.
15. The difference of a number and 3 is equal to the quotient of 10 and 5. Find the number.
16. Eight decreased by a number equals the quotient of 15 and 5. Find the number.
17. Thirty less a number is equal to the product of 3 and the sum of the number and 6. Find the number.
18. The product of a number and 3 is twice the sum of that number and 5. Find the number.
19. 40 subtracted from five times a number is 8 more than the number. Find the number.
20. Five times the sum of a number and 2 is 11 less than the number times 8. Find the number.

- ▶ 21. Three times the difference of some number and 5 amounts to the quotient of 108 and 12. Find the number.

23. The product of 4 and a number is the same as 30 less twice that same number. Find the number.

22. Seven times the difference of some number and 1 gives the quotient of 70 and 10. Find the number.

24. Twice a number equals 25 less triple that same number. Find the number.

Solve. For Exercises 25 and 26, the solutions have been started for you. See Examples 3 and 4.

25. Florida has 26 fewer electoral votes for president than California. If the total number of electoral votes for these two states is 84, find the number for each state. (Source: state.1Keydata.com)

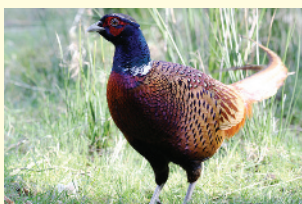
Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed. Let's let
 x = number of electoral votes for California
 Then
 $x - 26$ = number of electoral votes for Florida
2. TRANSLATE into an equation. (Fill in the blanks below.)

$$\begin{array}{ccc} \text{votes for} & + & \text{votes for} \\ \text{California} & & \text{Florida} \\ \downarrow & & \downarrow \\ \underline{\hspace{2cm}} & + & \underline{\hspace{2cm}} = 84 \end{array}$$

Now, you finish with

- ▶ 27. A falcon, when diving, can travel five times as fast as a pheasant's top speed. If the total speed for these two birds is 222 miles per hour, find the fastest speed of the falcon and the fastest speed of the pheasant.
 (Source: *Fantastic Book of Comparisons*)



26. Ohio has twice the number of electoral votes for president as Alabama. If the total number of electoral votes for these two states is 27, find the number for each state. (Source: state.1Keydata.com)

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed. Let's let
 x = number of electoral votes for Alabama
 Then
 $2x$ = number of electoral votes for Ohio
2. TRANSLATE into an equation. (Fill in the blanks below.)

$$\begin{array}{ccc} \text{votes for} & + & \text{votes for} \\ \text{Alabama} & & \text{Ohio} \\ \downarrow & & \downarrow \\ \underline{\hspace{2cm}} & + & \underline{\hspace{2cm}} = 27 \end{array}$$

Now, you finish with

3. SOLVE the equation.
4. INTERPRET the results.

28. Norway has had three times as many rulers as Liechtenstein. If the total number of rulers for both countries is 56, find the number of rulers for Norway and the number for Liechtenstein.



29. The U.S. newspaper with the greatest circulation is *USA Today*, followed by *The Wall Street Journal*. If the average daily circulation for *USA Today* is 226 thousand more than the average daily circulation of *The Wall Street Journal* and their combined circulation is 4340 thousand, find the circulation for each paper. (Source: *USA Today*)
30. The average life expectancy for an elephant is 24 years longer than the life expectancy for a chimpanzee. If the total of these life expectancies is 130 years, find the life expectancy of each.
31. Apple was recently named the Most Valuable Global Brand by *Forbes*. It has a recognition value worth \$71 billion more than Google's. If the total value of these two global brands is \$237 billion, find the value of each of the brands.
32. The global brand Microsoft has a recognition value worth \$16 billion more than Coca-Cola's. If the total value of these two global brands is \$134 billion, find the value of each of the brands.



33. An Xbox 360 game system and several games are sold for \$440. The cost of the Xbox 360 is 3 times as much as the cost of the games. Find the cost of the Xbox 360 and the cost of the games.
34. The two top-selling Xbox 360 games are *Kinect Adventures* and *Grand Theft Auto V*. A price for *Grand Theft Auto V* is \$24 more than a price for *Kinect Adventures*. If the total of these two prices is \$36, find the price of each game. (Source: Gamestop.com)



35. By air, the distance from New York City to London is 2001 miles *less* than the distance from Los Angeles to Tokyo. If the total of these two distances is 8939 miles, find the distance from Los Angeles to Tokyo.
36. By air, the distance from Melbourne, Australia, to Cairo, Egypt, is 2338 miles *more* than the distance from Madrid, Spain, to Bangkok, Thailand. If the total of these distances is 15,012 miles, find the distance from Madrid to Bangkok.



37. The two NCAA stadiums with the largest capacities are Beaver Stadium (Penn State) and Michigan Stadium (Univ. of Michigan). Beaver Stadium has a capacity of 1081 more than Michigan Stadium. If the combined capacity for the two stadiums is 213,483, find the capacity for each stadium. (Source: National Collegiate Athletic Association)



38. A National Hot Rod Association (NHRA) top fuel dragster has a top speed of 95 mph faster than an Indy Racing League car. If the top speed for these two cars combined is 565 mph, find the top speed of each car. (Source: USA Today)



39. In 2020, China is projected to be the country with the greatest number of visiting tourists. This number is twice the number of tourists projected for Spain. If the total number of tourists for these two countries is projected to be 210 million, find the number projected for each. (Source: *The State of the World Atlas* by Dan Smith)
40. As of 2015, California contained the largest state population of Native Americans. This population was five times the Native American population of Washington State. If the total of these two populations is 792,000, find the Native American population in each of these two states. (Source: U.S. Census Bureau)
41. During 2015, twice as many motor vehicles were manufactured in the United States as in Germany. If the total number of vehicles manufactured in these two countries was approximately 18,300,000, find the number manufactured in the United States and the number manufactured in Germany. (Source: Organization of Motor Vehicle Manufacturers)
42. A Toyota Camry is traveling twice as fast as a Dodge truck. If their combined speed is 105 miles per hour, find the speed of the car and find the speed of the truck.
43. A biker sold his used mountain bike and accessories for \$270. If he received five times as much money for the bike as he did for the accessories, find how much money he received for the bike.
44. A tractor and a plow attachment are worth \$1200. The tractor is worth seven times as much money as the plow. Find the value of the tractor and the value of the plow.
45. The maximum length of a soccer field for Olympic play is 50 yards longer than the maximum length of a soccer field for play by under-10-year-olds. If the total length of these two categories of soccer fields is 190 yards, what is the maximum length for each field? (See the Chapter 3 Opener.)
46. The maximum length recommended for a high school soccer field is 80 yards longer than that for an under-6-year-old team. If the total length of these two categories of soccer fields is 140 yards, what is the maximum length for each field? (See the Chapter 3 Opener.)

- 47.** During the 2016 men's NCAA Division 1 basketball championship game, the North Carolina Tar Heels scored 3 points fewer than the Villanova Wildcats. Together, both teams scored 151 points. How many points did the 2016 champion Villanova Wildcats score during the game? (*Source: National Collegiate Athletic Association*)
- 48.** During the 2016 women's NCAA Division 1 basketball championship game, the Connecticut Huskies scored 31 points more than the Syracuse Orange. Together, both teams scored a total of 133 points. How many points did the 2016 champion Connecticut Huskies score during this game?

Advertisers in the United States use many media: newspapers, magazines, mailings, Internet, and TV. See Exercises 49 and 50.

- 49.** By 2017, the money spent on Internet advertising in the United States is projected to exceed the amount spent on TV advertising. If it is estimated that 0.6 billion dollars more will be spent on Internet advertising in 2017 and the total amount spent on Internet and TV advertising is expected to be \$150 billion, find the amount of money projected to be spent on each medium. (*Source: USA Today*)
- 50.** By 2020, the money spent on Internet advertising in the United States is projected to continue to exceed the amount spent on TV advertising. If it is estimated that 11.8 billion dollars more will be spent on Internet advertising in 2020 and the total amount spent on Internet and TV advertising is expected to be about \$175 billion, find the amount of money projected to be spent on each medium. (*Source: USA Today*)

Review

Round each number to the given place value. See Section 1.4.

- 51.** 586 to the nearest ten
- 52.** 82 to the nearest ten
- 53.** 1026 to the nearest hundred
- 54.** 52,333 to the nearest thousand
- 55.** 2986 to the nearest thousand
- 56.** 101,552 to the nearest hundred

Concept Extensions

-  **57.** Solve Example 3 again, but this time let x be the distance from Chicago to Los Angeles. Did you get the same results? Explain why or why not.
-  **58.** Solve Exercise 25 again, but this time let x be the number of electoral votes for Florida. Did you get the same results? Explain why or why not.

In real estate, a house's selling price P is found by adding the real estate agent's commission C to the amount A that the seller of the house receives: $P = A + C$.

- 59.** A house sold for \$230,000. The owner's real estate agent received a commission of \$13,800. How much did the seller receive? (*Hint: Substitute the known values into the equation, then solve the equation for the remaining unknown.*)
- 60.** A homeowner plans to use a real estate agent to sell his house. He hopes to sell the house for \$165,000 and keep \$156,750 of that. If everything goes as he has planned, how much will his real estate agent receive as a commission?

In retailing, the retail price P of an item can be computed using the equation $P = C + M$, where C is the wholesale cost of the item and M is the amount of markup.

- 61.** The retail price of a computer system is \$999 after a markup of \$450. What is the wholesale cost of the computer system? (*Hint: Substitute the known values into the equation, then solve the equation for the remaining unknown.*)
- 62.** Slidell Feed and Seed sells a bag of cat food for \$12. If the store paid \$7 for the cat food, what is the markup on the cat food?

Chapter 3 Group Activity

Modeling Equation Solving with Addition and Subtraction

Sections 3.1–3.4

We can use positive counters ● and negative counters ● to help us model the equation-solving process. We also need to use an object that represents a variable. We use small slips of paper with the variable name written on them.

Recall that taking a ● and ● together creates a neutral or zero pair. After a neutral pair has been formed, it can be removed from or added to an equation model without changing the overall value. We also need to remember that we can add or remove the same number of positive or negative counters from both sides of an equation without changing the overall value.

We can represent the equation $x + 5 = 2$ as follows:

To get the variable by itself, we must remove 5 black counters from both sides of the model. Because there are only 2 counters on the right side, we must add 5 negative counters to both sides of the model. Then we can remove neutral pairs: 5 from the left side and 2 from the right side.

We are left with the following model, which represents the solution, $x = -3$.

Similarly, we can represent the equation $x - 4 = -6$ as follows:

To get the variable by itself, we must remove 4 red counters from both sides of the model.

We are left with the following model, which represents the solution, $x = -2$.

Use the counter model to solve each equation.

- | | |
|-----------------|------------------|
| 1. $x - 3 = -7$ | 2. $x - 1 = -9$ |
| 3. $x + 2 = 8$ | 4. $x + 4 = 5$ |
| 5. $x + 8 = 3$ | 6. $x - 5 = -1$ |
| 7. $x - 2 = 1$ | 8. $x - 5 = 10$ |
| 9. $x + 3 = -7$ | 10. $x + 8 = -2$ |

Chapter 3 Vocabulary Check

Fill in each blank with one of the words or phrases listed below.

variable	addition	constant
terms	simplified	multiplication
like	combined	numerical coefficient

algebraic expression	equation
evaluating the expression	solution
distributive	

- An algebraic expression is _____ when all like terms have been _____.
- Terms that are exactly the same, except that they may have different numerical coefficients, are called _____ terms.
- A letter used to represent a number is called a(n) _____.
- A combination of operations on variables and numbers is called a(n) _____.
- The addends of an algebraic expression are called the _____ of the expression.
- The number factor of a variable term is called the _____.
- Replacing a variable in an expression by a number and then finding the value of the expression is called _____ for the variable.
- A term that is a number only is called a(n) _____.
- A(n) _____ is of the form $\text{expression} = \text{expression}$.
- A(n) _____ of an equation is a value for the variable that makes the equation a true statement.

11. To multiply $-3(2x + 1)$, we use the _____ property.
12. By the _____ property of equality, we may multiply or divide both sides of an equation by any nonzero number without changing the solution of the equation.
13. By the _____ property of equality, the same number may be added to or subtracted from both sides of an equation without changing the solution of the equation.

Helpful Hint

▶ Are you preparing for your test?

To help, don't forget to take these:

- Chapter 3 Getting Ready for the Test on page 209
- Chapter 3 Test on page 210

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

3

Chapter Highlights

Definitions and Concepts	Examples								
Section 3.1 Simplifying Algebraic Expressions									
The addends of an algebraic expression are called the terms of the expression. The number factor of a variable term is called the numerical coefficient. Terms that are exactly the same, except that they may have different numerical coefficients, are called like terms. An algebraic expression is simplified when all like terms have been combined. Use the distributive property to multiply an algebraic expression by a term.	$5x^2 + (-4x) + (-2)$ 3 terms <table border="1"> <thead> <tr> <th>Term</th><th>Numerical Coefficient</th></tr> </thead> <tbody> <tr> <td>$7x$</td><td>7</td></tr> <tr> <td>$-6y$</td><td>-6</td></tr> <tr> <td>x or $1x$</td><td>1</td></tr> </tbody> </table> $5x + 11x = (5 + 11)x = 16x$ like terms $y - 6y = (1 - 6)y = -5y$ Simplify: $-4(x + 2) + 3(5x - 7)$ $= -4 \cdot x + (-4) \cdot 2 + 3 \cdot 5x - 3 \cdot 7$ $= -4x + (-8) + 15x + (-21)$ $= 11x + (-29) \text{ or } 11x - 29$	Term	Numerical Coefficient	$7x$	7	$-6y$	-6	x or $1x$	1
Term	Numerical Coefficient								
$7x$	7								
$-6y$	-6								
x or $1x$	1								
Section 3.2 Solving Equations: Review of the Addition and Multiplication Properties									
Addition Property of Equality Let a, b, and c represent numbers. Then <table border="0"> <tr> <td>$a = b$</td><td>Also, $a = b$</td></tr> <tr> <td>and $a + c = b + c$</td><td>and $a - c = b - c$</td></tr> </table> are equivalent equations. are equivalent equations. In other words, the same number may be added to or subtracted from both sides of an equation without changing the solution of the equation.	$a = b$	Also, $a = b$	and $a + c = b + c$	and $a - c = b - c$	Solve for x: $x + 8 = 2 + (-1)$ $x + 8 = 1$ Combine like terms. $x + 8 - 8 = 1 - 8$ Subtract 8 from both sides. $x = -7$ Simplify. The solution is -7.				
$a = b$	Also, $a = b$								
and $a + c = b + c$	and $a - c = b - c$								

(continued)

Definitions and Concepts	Examples
Section 3.2 Solving Equations: Review of the Addition and Multiplication Properties (continued)	
Multiplication Property of Equality Let a , b , and c represent numbers and let $c \neq 0$. Then $a = b$ Also, $a = b$ and $a \cdot c = b \cdot c$ and $\frac{a}{c} = \frac{b}{c}$ are equivalent equations. are equivalent equations. In other words, both sides of an equation may be multiplied or divided by the same nonzero number without changing the solution of the equation.	Solve: $y - 7y = 30$ $-6y = 30$ Combine like terms. $\frac{-6y}{-6} = \frac{30}{-6}$ Divide both sides by -6 . $y = -5$ Simplify. The solution is -5 .

Section 3.3 Solving Linear Equations in One Variable	
Steps for Solving an Equation Step 1: If parentheses are present, use the distributive property. Step 2: Combine any like terms on each side of the equation. Step 3: Use the addition property of equality to rewrite the equation so that variable terms are on one side of the equation and constant terms are on the other side. Step 4: Use the multiplication property of equality to divide both sides by the numerical coefficient of the variable to solve. Step 5: Check the solution in the <i>original equation</i> .	 Solve for x : $5(3x - 1) + 15 = -5$ Step 1: $15x - 5 + 15 = -5$ Apply the distributive property. Step 2: $15x + 10 = -5$ Combine like terms. Step 3: $15x + 10 - 10 = -5 - 10$ Subtract 10 from both sides. $15x = -15$ Step 4: $\frac{15x}{15} = \frac{-15}{15}$ Divide both sides by 15. $x = -1$ Step 5: Check to see that -1 is the solution.

Section 3.4 Linear Equations in One Variable and Problem Solving																
Problem-Solving Steps 1. UNDERSTAND the problem. Some ways of doing this are Read and reread the problem. Construct a drawing. Choose a variable to represent an unknown in the problem. 2. TRANSLATE the problem into an equation.	The incubation period for a golden eagle is three times the incubation period for a hummingbird. If the total of their incubation periods is 60 days, find the incubation period for each bird. (<i>Source: Wildlife Fact File, International Masters Publishers</i>) 1. UNDERSTAND the problem. Then choose a variable to represent an unknown. Let x = incubation period of a hummingbird Then $3x$ = incubation period of a golden eagle 2. TRANSLATE. <table><tr><td>incubation of hummingbird</td><td>+</td><td>incubation of golden eagle</td><td>is</td><td>60</td></tr><tr><td>↓</td><td></td><td>↓</td><td>↓</td><td>↓</td></tr><tr><td>x</td><td>+</td><td>$3x$</td><td>=</td><td>60</td></tr></table>	incubation of hummingbird	+	incubation of golden eagle	is	60	↓		↓	↓	↓	x	+	$3x$	=	60
incubation of hummingbird	+	incubation of golden eagle	is	60												
↓		↓	↓	↓												
x	+	$3x$	=	60												

Definitions and Concepts	Examples
Section 3.4 Linear Equations in One Variable and Problem Solving (continued)	
3. SOLVE the equation. 4. INTERPRET the results. <i>Check</i> the proposed solution in the stated problem and <i>state</i> your conclusion.	3. SOLVE: $x + 3x = 60$ $4x = 60$ $\frac{4x}{4} = \frac{60}{4}$ $x = 15$ 4. INTERPRET the solution in the stated problem. The incubation period for a hummingbird is 15 days. The incubation period for a golden eagle is $3x = 3 \cdot 15 = 45 \text{ days.}$ Since 15 days + 45 days = 60 days and 45 is 3(15), the solution checks. <i>State</i> your conclusion: The incubation period for a hummingbird is 15 days. The incubation period for a golden eagle is 45 days.

Chapter 3 Review

(3.1) Simplify each expression by combining like terms.

1. $3y + 7y - 15$

2. $2y - 10 - 8y$

3. $8a + a - 7 - 15a$

4. $y + 3 - 9y - 1$

Multiply.

5. $2(x + 5)$

6. $-3(y + 8)$

Simplify.

7. $7x + 3(x - 4) + x$

8. $-(3m + 2) - m - 10$

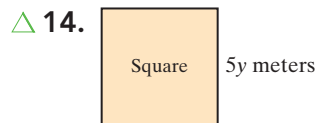
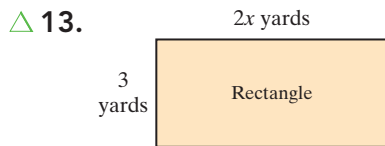
9. $3(5a - 2) - 20a + 10$

10. $6y + 3 + 2(3y - 6)$

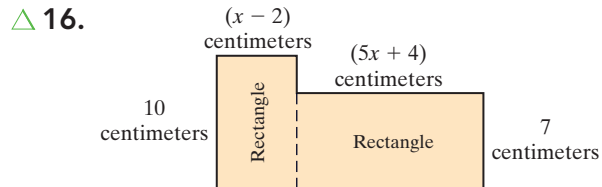
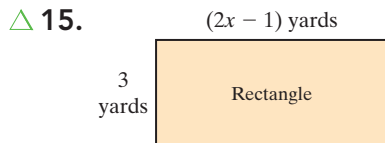
11. $6y - 7 + 11y - y + 2$

12. $10 - x + 5x - 12 - 3x$

Find the perimeter of each figure.



Find the area of each figure.



(3.2) Solve each equation.

17. $z - 5 = -7$

18. $3x + 10 = 4x$

19. $3y = -21$

20. $-3a = -15$

21. $\frac{x}{-6} = 2$

22. $\frac{y}{-15} = -3$

23. $n + 18 = 10 - (-2)$

24. $c - 5 = -13 + 7$

25. $7x + 5 - 6x = -20$

26. $17x = 2(8x - 4)$

27. $5x + 7 = -3$

28. $-14 = 9y + 4$

29. $\frac{z}{4} = -8 - (-6)$

30. $-1 + (-8) = \frac{x}{5}$

31. $6y - 7y = 100 - 105$

32. $19x - 16x = 45 - 60$

33. $9(2x - 7) = 19x$

34. $-5(3x + 3) = -14x$

35. $3x - 4 = 11$

36. $6y + 1 = 73$

37. $2(x + 4) - 10 = -2(7)$

38. $-3(x - 6) + 13 = 20 - 1$

Translating Translate each phrase into an algebraic expression. Let x represent “a number.”

39. The product of -5 and a number

40. Three subtracted from a number

41. The sum of -5 and a number

42. The quotient of -2 and a number

43. The product of -5 and a number, decreased by 50

44. Eleven added to twice a number

45. The quotient of 70 and the sum of a number and 6

46. Twice the difference of a number and 13

(3.3) Solve each equation.

47. $2x + 5 = 7x - 100$

48. $-6x - 4 = x + 66$

49. $2x + 7 = 6x - 1$

50. $5x - 18 = -4x$

51. $5(n - 3) = 7 + 3n$

52. $7(2 + x) = 4x - 1$

53. $6x + 3 - (-x) = -20 + 5x - 7$

54. $x - 25 + 2x = -5 + 2x - 10$

55. $3(x - 4) = 5x - 8$

56. $4(x - 3) = -2x - 48$

57. $6(2n - 1) + 18 = 0$

58. $7(3y - 2) - 7 = 0$

59. $95x - 14 = 20x - 10 + 10x - 4$

60. $32z + 11 - 28z = 50 + 2z - (-1)$

Translating Write each sentence as an equation.

61. The difference of 20 and -8 is 28.

62. Nineteen subtracted from -2 amounts to -21 .

63. The quotient of -75 and the sum of 5 and 20 is equal to -3 .

64. Five times the sum of 2 and -6 yields -20 .

(3.4) Translating Write each sentence as an equation using x as the variable.

65. Twice a number minus 8 is 40.

66. The product of a number and 6 is equal to the sum of the number and 20.

67. Twelve subtracted from the quotient of a number and 2 is 10.

68. The difference of a number and 3 is the quotient of 8 and 4.

Solve.

69. Five times a number subtracted from 40 is the same as three times the number. Find the number.

70. The product of a number and 3 is twice the difference of that number and 8. Find the number.

71. In an election, the incumbent received 14,000 votes of the 18,500 votes cast. Of the remaining votes, the Democratic candidate received 272 more than the Independent candidate. Find how many votes the Democratic candidate received.

72. Rajiv Puri has twice as many movies on DVD as he has on Blu-ray Disc. Find the number of DVDs if he has a total of 126 movie recordings.

Mixed Review

Simplify.

73. $9x - 20x$

74. $-5(7x)$

75. $12x + 5(2x - 3) - 4$

76. $-7(x + 6) - 2(x - 5)$

Solve.

77. $c - 5 = -13 + 7$

78. $7x + 5 - 6x = -20$

79. $-7x + 3x = -50 - 2$

80. $-x + 8x = -38 - 4$

81. $9x + 12 - 8x = -6 + (-4)$

82. $-17x + 14 + 20x - 2x = 5 - (-3)$

83. $5(2x - 3) = 11x$

84. $\frac{y}{-3} = -1 - 5$

85. $12y - 10 = -70$

86. $4n - 8 = 2n + 14$

87. $-6(x - 3) = x + 4$

88. $9(3x - 4) + 63 = 0$

89. $-5z + 3z - 7 = 8z - 1 - 6$

90. $4x - 3 + 6x = 5x - 3 - 30$

91. Three times a number added to twelve is 27. Find the number.

92. Twice the sum of a number and four is ten. Find the number.

93. Out of 50 states and the District of Columbia, Vermont ranks 51st in the amount of energy consumption, followed by the District of Columbia. If the District of Columbia used 24.3 trillion BTU more than Vermont and the total number of BTU used by both entities was 240.5 trillion BTU, find the amount of energy consumption for each. (Source: Bureau of Transportation Statistics)

94. North and South Dakota both have over 80,000 roadway miles. North Dakota has 4489 more miles and the total number of roadway miles for both states is 169,197. Find the number of roadway miles for North Dakota and for South Dakota. (Source: U.S. Federal Highway Administration)

MULTIPLE CHOICE Simplify each expression in Exercises 1 through 6. Then choose the simplified form: A, B, C, D, or E. Choices may be used more than once or not at all.

A. $-6x$

B. $6x$

C. $4x + 2$

D. $4x + 1$

E. $4x + 6$

▶ 1. $x - 7x$

▶ 2. $-2x + 3 + 8x - 3$

▶ 3. $-3(2x)$

▶ 4. $2(2x + 1)$

▶ 5. $-(x + 2) + 5x + 4$

▶ 6. $3(1 + 2x) - 3$

MULTIPLE CHOICE

▶ 7. To solve $5x - 10 = 0$, we will first add 10 to each side of the equation. Once this is done, the equivalent equation is

A. $5x = 0$

B. $5x = 10$

C. $5x = -10$

▶ 8. To solve $4(2x + 1) - 8 = 14x - 10$, we will first simplify the left side of the equation. Once this is done, the equivalent equation is

A. $8x - 4 = 14x - 10$

B. $8x - 7 = 14x - 10$

C. $2x - 7 = 14x - 10$

MATCHING Let x represent “a number.” Match each phrase in the first column with its translated variable expression in the second column.

▶ 9. the sum of -3 and a number

A. $-3x$

▶ 10. the product of -3 and a number

B. $3 + x$

▶ 11. 5 subtracted from a number

C. $-3 + x$

▶ 12. 5 minus a number

D. $x - 5$

E. $5 - x$

F. $5x$

MATCHING Let x represent “a number.” Match each sentence in the first column with its translated equation in the second or third column.

▶ 13. Twice a number gives 14.

A. $6x = 14$

E. $x - 2 = 14$

▶ 14. A number divided by 6 yields 14.

B. $2x = 14$

F. $2 + 6x = 14$

▶ 15. Two subtracted from a number equals 14.

C. $\frac{x}{6} = 14$

G. $6(2 + x) = 14$

▶ 16. Six times a number, added to 2 is 14.

D. $2 - x = 14$

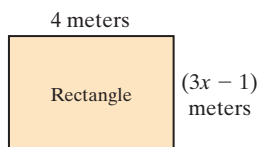
Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____

▶ 1. Simplify $7x - 5 - 12x + 10$ by combining like terms.

▶ 3. Simplify: $-(3z + 2) - 5z - 18$

▶ 5. Write an expression that represents the area of the rectangle. Simplify the expression.



Solve each equation.

▶ 6. $12 = y - 3y$

▶ 8. $5x + 12 - 4x - 14 = 22$

▶ 10. $2(x - 6) = 0$

▶ 12. $5x - 2 = x - 10$

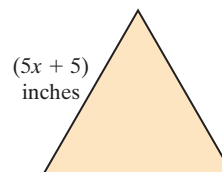
▶ 14. $6 + 2(3n - 1) = 28$

Translate the following phrases into mathematical expressions. If needed, use x to represent "a number."

▶ 15. The sum of -23 and a number

▶ 2. Multiply: $-2(3y + 7)$

▶ 4. Write an expression that represents the perimeter of the equilateral triangle (a triangle with three sides of equal length). Simplify the expression.



▶ 7. $\frac{x}{2} = -5 - (-2)$

▶ 9. $-4x + 7 = 15$

▶ 11. $-4(x - 11) - 34 = 10 - 12$

▶ 13. $4(5x + 3) = 2(7x + 6)$

▶ 16. Three times a number, subtracted from -2

Translate each sentence into an equation. If needed, use x to represent “a number.”

- ▶ 17. The sum of twice 5 and -15 is -5 . ▶ 18. Six added to three times a number equals -30 .

Solve.

- ▶ 19. The difference of three times a number and five times the same number is 4. Find the number.
- ▶ 20. In a championship basketball game, Paula made twice as many free throws as Maria. If the total number of free throws made by both women was 12, find how many free throws Paula made.
- ▶ 21. In a 10-kilometer race, there are 112 more men entered than women. Find the number of female runners if the total number of runners in the race is 600.

17. _____

18. _____

19. _____

20. _____

21. _____



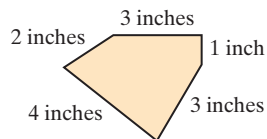
Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. a. _____
b. _____
19. _____
20. _____

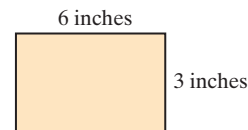
1. Write 308,063,557 in words.

2. Write 276,004 in words.

△ 3. Find the perimeter of the polygon shown.



△ 4. Find the perimeter of the rectangle shown.



5. Subtract: $900 - 174$. Check by adding.

6. Subtract: $17,801 - 8216$. Check by adding.

7. Round 248,982 to the nearest hundred.

8. Round 844,497 to the nearest thousand.

9. Multiply: 25×8

10. Multiply: 395×74

11. Divide and check: $1872 \div 9$

12. Divide and check: $3956 \div 46$

13. Simplify: $2 \cdot 4 - 3 \div 3$

14. Simplify: $8 \cdot 4 + 9 \div 3$

15. Evaluate $x^2 + z - 3$ for $x = 5$ and $z = 4$.

16. Evaluate $2a^2 + 5 - c$ for $a = 2$ and $c = 3$.

17. Determine which numbers in the set $\{26, 40, 20\}$ are solutions of the equation $2n - 30 = 10$.

18. Insert $<$ or $>$ to make a true statement.
a. -14 0
b. $-(-7)$ -8

19. Add using a number line: $5 + (-2)$

20. Add using a number line: $-3 + (-4)$

Add.

21. $-15 + (-10)$

22. $3 + (-7)$

23. $-2 + 25$

24. $21 + 15 + (-19)$

Subtract.

25. $-4 - 10$

26. $-2 - 3$

27. $6 - (-5)$

28. $19 - (-10)$

29. $-11 - (-7)$

30. $-16 - (-13)$

Divide.

31. $\frac{-12}{6}$

32. $\frac{-30}{-5}$

33. $-20 \div (-4)$

34. $26 \div (-2)$

35. $\frac{48}{-3}$

36. $\frac{-120}{12}$

Find the value of each expression.

37. $(-3)^2$

38. -2^5

39. -3^2

40. $(-5)^2$

41. Simplify: $2y - 6 + 4y + 8$

42. Simplify: $6x + 2 - 3x + 7$

43. Determine whether -1 is a solution of the equation $3y + 1 = 3$.

44. Determine whether 2 is a solution of $5x - 3 = 7$.

45. Solve: $-12x = -36$

46. Solve: $-3y = 15$

47. Solve: $2x - 6 = 18$

48. Solve: $3a + 5 = -1$

49. A salesperson at an electronics store sold a computer system and software for \$2100, receiving four times as much money for the computer system as for the software. Find the price of each.

50. Rose Daunis is thinking of a number. Two times the number plus four is the same amount as three times the number minus seven. Find Rose's number.

21. _____
22. _____
23. _____
24. _____
25. _____
26. _____
27. _____
28. _____
29. _____
30. _____
31. _____
32. _____
33. _____
34. _____
35. _____
36. _____
37. _____
38. _____
39. _____
40. _____
41. _____
42. _____
43. _____
44. _____
45. _____
46. _____
47. _____
48. _____
49. _____
50. _____

4

Fractions and Mixed Numbers

Fractions are numbers and, like whole numbers and integers, they can be added, subtracted, multiplied, and divided. Fractions are very useful and appear frequently in everyday language, in common phrases such as “half an hour,” “quarter of a pound,” and “third of a cup.” This chapter reviews the concepts of fractions and mixed numbers and demonstrates how to add, subtract, multiply, and divide these numbers.



Pinnacles National Park in California was signed into law on January 10, 2013, as the United States' 59th and newest national park. (The former Pinnacles National Monument was established in 1908 by U.S. President Theodore Roosevelt.)

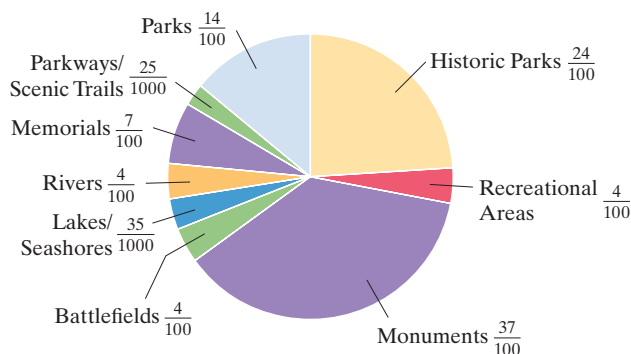
The National Park Service Turned 100 on August 25, 2016.

The National Park Service is well known for its stewardship of national parks and protecting natural resources for future generations. Many of us recognize the National Park ranger by the “Smokey Bear” hat, but the National Park Service also employs archeologists, architects, curators, historians, and other professionals to help manage and preserve the vastly different properties under its oversight.

The following graph is called a circle graph or pie chart. Each sector (shaped like a piece of a pie) shows the fraction of the areas managed by the National Park System by type of national designation.

In Section 4.2, Exercises 97–100 we show this same circle graph but in 3-D design. We simplify some of the fractions in it and also study sector size versus fraction value.

Areas Maintained by the National Park Service



Source: National Park Service

Sections

- 4.1 Introduction to Fractions and Mixed Numbers
- 4.2 Factors and Simplest Form
- 4.3 Multiplying and Dividing Fractions
- 4.4 Adding and Subtracting Like Fractions, Least Common Denominator, and Equivalent Fractions
- 4.5 Adding and Subtracting Unlike Fractions
- Integrated Review**—
Summary on Fractions and Operations on Fractions
- 4.6 Complex Fractions and Review of Order of Operations
- 4.7 Operations on Mixed Numbers
- 4.8 Solving Equations Containing Fractions

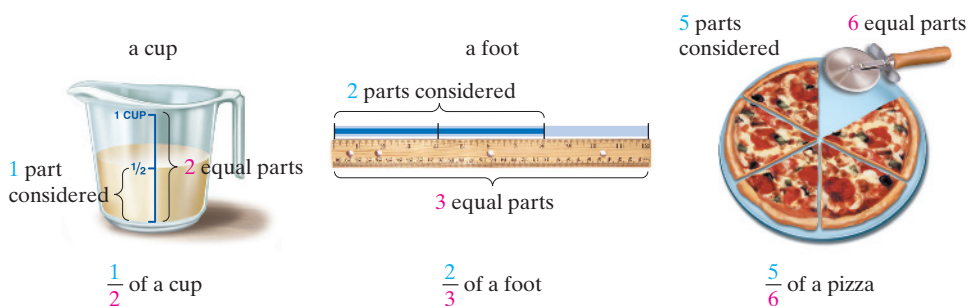
Check Your Progress

- Vocabulary Check
- Chapter Highlights
- Chapter Review
- Getting Ready for the Test
- Chapter Test
- Cumulative Review

4.1 Introduction to Fractions and Mixed Numbers

Objective A Identifying Numerators and Denominators

Whole numbers are used to count whole things or units, such as cars, horses, dollars, and people. To refer to a part of a whole, fractions can be used. Here are some examples of **fractions**. Study these examples for a moment.



In a fraction, the top number is called the **numerator** and the bottom number is called the **denominator**. The bar between the numbers is called the **fraction bar**.

Name	Fraction	Meaning
numerator	→ $\frac{5}{6}$ ←	number of parts being considered
denominator	→ $\frac{5}{6}$ ←	number of equal parts in the whole

Examples

Identify the numerator and the denominator of each fraction.

- $\frac{3}{7}$
← numerator
← denominator
- $\frac{13}{5x}$
← numerator
← denominator

Work Practice 1–2

Helpful Hint

$\frac{3}{7}$ ← Remember that the bar in a fraction means division. Since division by 0 is undefined, a fraction with a denominator of 0 is undefined. For example, $\frac{3}{0}$ is undefined.

Objective B Writing Fractions to Represent Parts of Figures or Real-Life Data

One way to become familiar with the concept of fractions is to visualize fractions with shaded figures. We can then write a fraction to represent the shaded area of the figure (or diagram).

Objectives

- Identify the Numerator and the Denominator of a Fraction.
- Write a Fraction to Represent Parts of Figures or Real-Life Data.
- Graph Fractions on a Number Line.
- Review Division Properties of 0 and 1.
- Write Mixed Numbers as Improper Fractions.
- Write Improper Fractions as Mixed Numbers or Whole Numbers.

Practice 1–2

Identify the numerator and the denominator of each fraction.

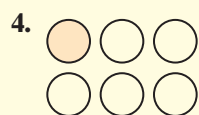
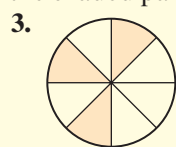
- $\frac{11}{2}$
- $\frac{10y}{17}$

Answers

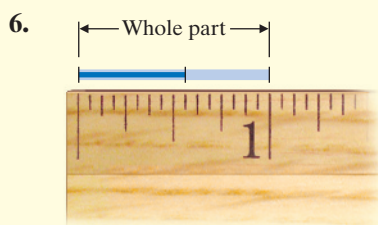
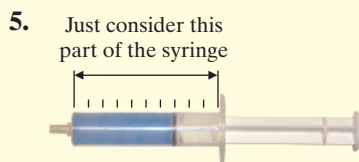
- numerator = 11, denominator = 2
- numerator = 10y, denominator = 17

Practice 3–4

Write a fraction to represent the shaded part of each figure.

**Practice 5–6**

Write a fraction to represent the part of the whole shown.

**Practice 7**

Draw and shade a part of a figure to represent the fraction.

7. $\frac{2}{3}$ of a figure

Answers

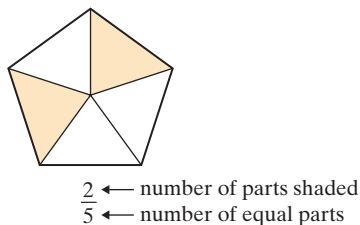
3. $\frac{3}{8}$ 4. $\frac{1}{6}$ 5. $\frac{7}{10}$ 6. $\frac{9}{16}$

7. answers may vary; for example,

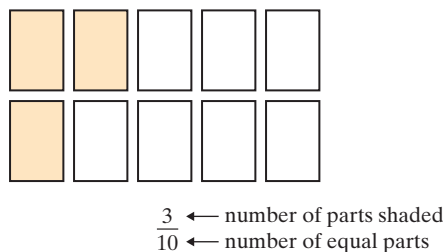
**Examples**

Write a fraction to represent the shaded part of each figure.

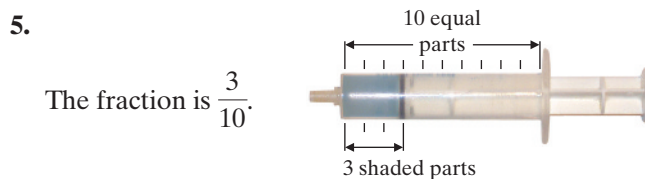
3. In this figure, 2 of the 5 equal parts are shaded. Thus, the fraction is $\frac{2}{5}$.



4. In this figure, 3 of the 10 rectangles are shaded. Thus, the fraction is $\frac{3}{10}$.

**Work Practice 3–4****Examples**

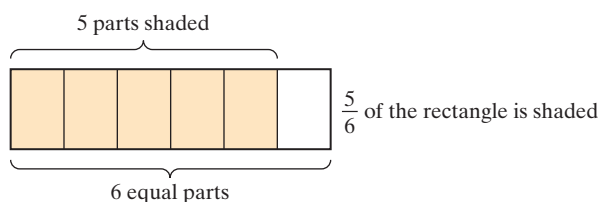
Write a fraction to represent the shaded part of the diagram.

**Work Practice 5–6****Examples**

Draw a figure and then shade a part of it to represent each fraction.

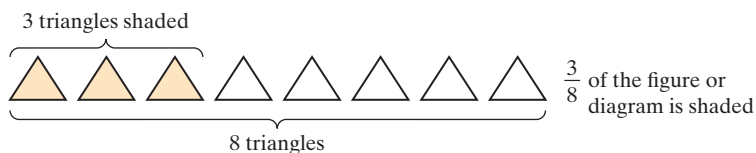
7. $\frac{5}{6}$ of a figure

We will use a geometric figure such as a rectangle. Since the denominator is 6, we divide it into 6 equal parts. Then we shade 5 of the equal parts.



8. $\frac{3}{8}$ of a figure

If you'd like, our figure can consist of 8 triangles of the same size. We will shade 3 of the triangles.

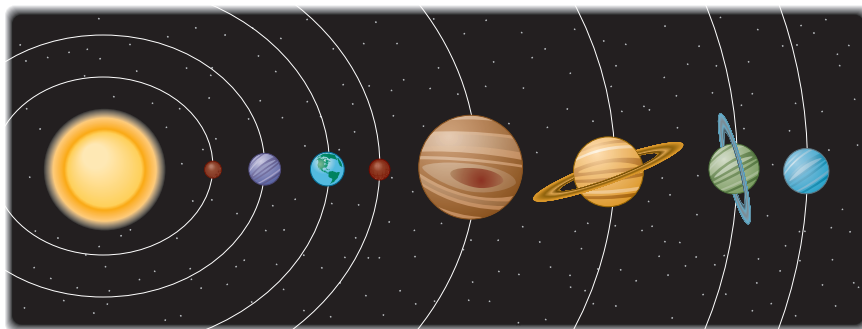


Work Practice 7–8

✓ **Concept Check** If  represents $\frac{6}{7}$ of a whole diagram, sketch the whole diagram.

Example 9 Writing Fractions from Real-Life Data

Of the eight planets in our solar system (Pluto is now a dwarf planet), three are closer to the Sun than Mars. What fraction of the planets are closer to the Sun than Mars?



Solution: The fraction of planets closer to the Sun than Mars is:

$$\frac{3}{8} \quad \begin{array}{l} \leftarrow \text{number of planets closer} \\ \leftarrow \text{number of planets in our solar system} \end{array}$$

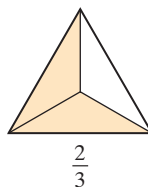
Thus, $\frac{3}{8}$ of the planets in our solar system are closer to the Sun than Mars.

Work Practice 9

The definitions and statements below apply to positive fractions.

A **proper fraction** is a fraction whose numerator is less than its denominator. Proper fractions are less than 1. For example, the shaded portion of the triangle is represented by $\frac{2}{3}$.

An **improper fraction** is a fraction whose numerator is greater than or equal to its denominator. Improper fractions are greater than or equal to 1.



Practice 8

Draw and shade a part of a figure to represent the fraction.

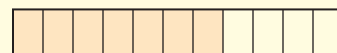
8. $\frac{7}{11}$ of a figure

Practice 9

Of the eight planets in our solar system, five are farther from the Sun than Earth is. What fraction of the planets are farther from the Sun than Earth is?

Answers

8. answers may vary; for example,

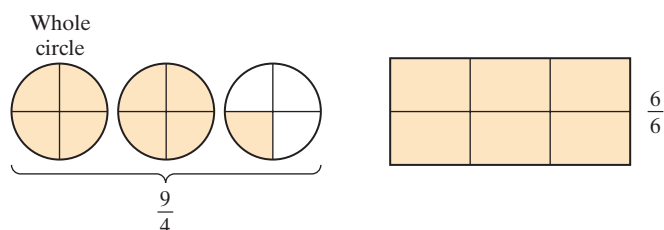


9. $\frac{5}{8}$

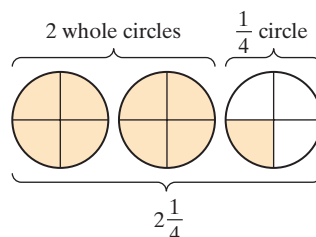
✓ **Concept Check Answer**



The shaded part of the group of circles below is $\frac{9}{4}$. The shaded part of the rectangle is $\frac{6}{6}$. Recall that $\frac{6}{6}$ simplifies to 1 and notice that the entire rectangle (1 whole figure) is shaded below.



A **mixed number** contains a whole number and a fraction. Mixed numbers are greater than 1. Above, we wrote the shaded part of the group of circles as the improper fraction $\frac{9}{4}$. Now let's write the shaded part as a mixed number. The shaded part of the group of circles' area is $2\frac{1}{4}$. Read this as "two and one-fourth."

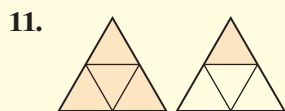
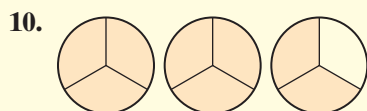


Note: The mixed number $2\frac{1}{4}$, diagrammed above, represents $2 + \frac{1}{4}$.

The mixed number $-3\frac{1}{5}$ represents $-(3 + \frac{1}{5})$ or $-3 - \frac{1}{5}$. We review this later in this chapter.

Practice 10–11

Represent the shaded part of each figure group as both an improper fraction and a mixed number.



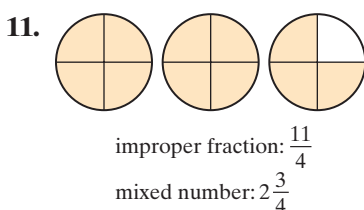
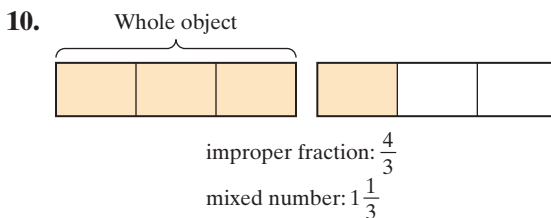
Answers

10. $\frac{8}{3}, 2\frac{2}{3}$ 11. $\frac{5}{4}, 1\frac{1}{4}$

✓ **Concept Check Answer**
3; answers may vary

Examples

Represent the shaded part of each figure group as both an improper fraction and a mixed number.

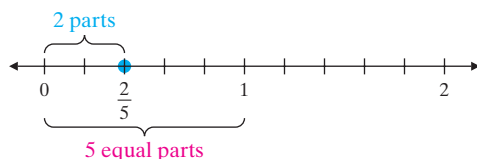


Work Practice 10–11

✓ **Concept Check** If you were to round $2\frac{3}{4}$, shown in Example 11 above, to the nearest whole number, would you choose 2 or 3? Why?

Objective C Graphing Fractions on a Number Line

Another way to visualize fractions is to graph them on a number line. To do this, think of 1 unit on the number line as a whole. To graph $\frac{2}{5}$, for example, divide the distance from 0 to 1 into **5 equal parts**. Then start at 0 and count **2 parts** to the right.



Notice that the graph of $\frac{2}{5}$ lies between 0 and 1. This means

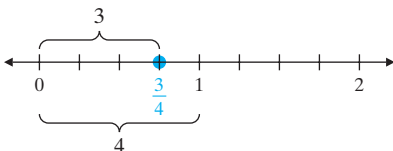
$$0 < \frac{2}{5} \left(\text{or } \frac{2}{5} > 0 \right) \text{ and also } \frac{2}{5} < 1$$

Example 12 Graph each proper fraction on a number line.

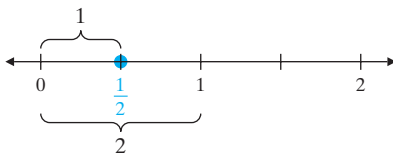
- a. $\frac{3}{4}$ b. $\frac{1}{2}$ c. $\frac{3}{6}$

Solution:

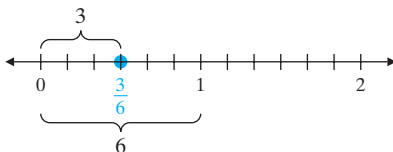
- a. To graph $\frac{3}{4}$, divide the distance from 0 to 1 into 4 parts. Then start at 0 and count over 3 parts.



b.



c.



Work Practice 12

The statements below apply to positive fractions.

The fractions in Example 12 are all proper fractions. Notice that the value of each is less than 1. This is always true for proper fractions since the numerator of a proper fraction is less than the denominator.

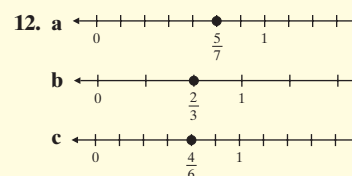
On the next page, we graph improper fractions. Notice that improper fractions are greater than or equal to 1. This is always true since the numerator of an improper fraction is greater than or equal to the denominator.

Practice 12

Graph each proper fraction on a number line.

- a. $\frac{5}{7}$ b. $\frac{2}{3}$ c. $\frac{4}{6}$

Answers



Practice 13

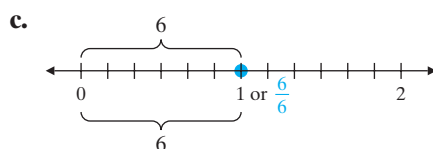
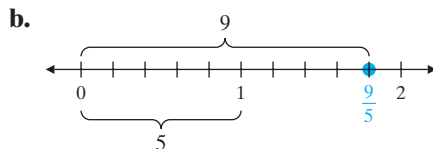
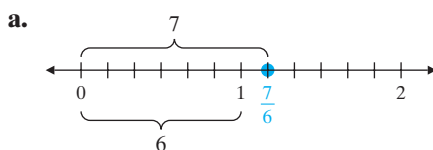
Graph each improper fraction on a number line.

a. $\frac{8}{3}$ b. $\frac{5}{4}$ c. $\frac{7}{7}$

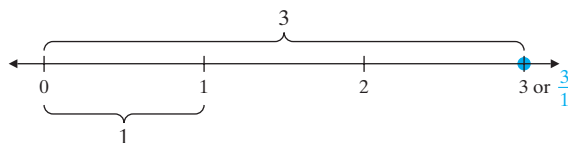
Example 13 Graph each improper fraction on a number line.

a. $\frac{7}{6}$ b. $\frac{9}{5}$ c. $\frac{6}{6}$ d. $\frac{3}{1}$

Solution:



d. Each 1-unit distance has 1 equal part. Count over 3 parts.



Work Practice 13

Note: We will graph mixed numbers at the end of this chapter.

Objective D Reviewing Division Properties of 0 and 1

Before we continue further, don't forget from Section 1.6 that a fraction bar indicates division. Let's review some division properties of 1 and 0.

$$\frac{9}{9} = 1 \text{ because } 1 \cdot 9 = 9 \quad \frac{-11}{1} = -11 \text{ because } -11 \cdot 1 = -11$$

$$\frac{0}{6} = 0 \text{ because } 0 \cdot 6 = 0 \quad \frac{6}{0} \text{ is undefined because there is no number that when multiplied by 0 gives 6.}$$

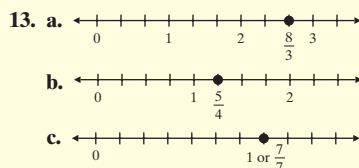
In general, we can say the following.

Let n be any integer except 0.

$$\frac{n}{n} = 1 \quad \frac{0}{n} = 0$$

$$\frac{n}{1} = n \quad \frac{n}{0} \text{ is undefined.}$$

Answers



Examples Simplify.

14. $\frac{5}{5} = 1$

15. $\frac{-2}{-2} = 1$

16. $\frac{0}{-5} = 0$

17. $\frac{-5}{1} = -5$

18. $\frac{41}{1} = 41$

19. $\frac{19}{0}$ is undefined

Work Practice 14–19

Notice from Example 17 that we can have negative fractions. In fact,

$$\frac{-5}{1} = -5, \quad \frac{5}{-1} = -5, \quad \text{and} \quad \frac{-5}{-1} = 5$$

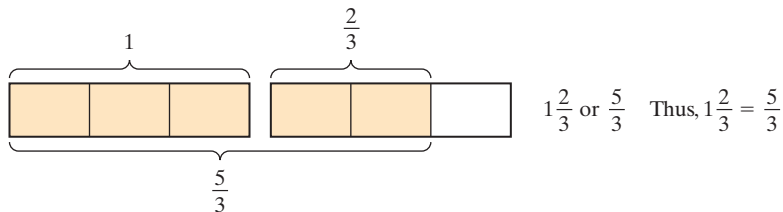
Because all of the fractions equal -5 , we have

$$\frac{-5}{1} = \frac{5}{-1} = -\frac{5}{1}$$

This means that the negative sign in a fraction can be written in the numerator, in the denominator, or in front of the fraction. Remember this as we work with negative fractions.

Objective E Writing Mixed Numbers as Improper Fractions

Earlier in this section, mixed numbers and improper fractions were both used to represent the shaded part of figure groups. For example,



The following steps may be used to write a mixed number as an improper fraction:

Writing a Mixed Number as an Improper Fraction

To write a mixed number as an improper fraction:

Step 1: Multiply the denominator of the fraction by the whole number.

Step 2: Add the numerator of the fraction to the product from Step 1.

Step 3: Write the sum from Step 2 as the numerator of the improper fraction over the original denominator.

For example,

$$1\frac{2}{3} = \frac{\overset{\text{Step 1}}{3 \cdot 1} + \overset{\text{Step 2}}{2}}{\underset{\text{Step 3}}{3}} = \frac{3 + 2}{3} = \frac{5}{3} \quad \text{or} \quad 1\frac{2}{3} = \frac{5}{3}, \text{ as stated above.}$$

Example 20 Write each as an improper fraction.

a. $4\frac{2}{9} = \frac{9 \cdot 4 + 2}{9} = \frac{36 + 2}{9} = \frac{38}{9}$

b. $1\frac{8}{11} = \frac{11 \cdot 1 + 8}{11} = \frac{11 + 8}{11} = \frac{19}{11}$

Work Practice 20**Practice 14–19**

Simplify.

14. $\frac{9}{9}$

15. $\frac{-6}{-6}$

16. $\frac{0}{-1}$

17. $\frac{4}{1}$

18. $\frac{-13}{0}$

19. $\frac{-13}{1}$

Helpful Hint

Remember, for example, that

$$-\frac{2}{3} = \frac{-2}{3} = \frac{2}{-3}$$

Practice 20

Write each as an improper fraction.

a. $5\frac{2}{7}$

b. $6\frac{2}{3}$

c. $10\frac{9}{10}$

d. $4\frac{1}{5}$

Answers

14. 1 15. 1 16. 0 17. 4

18. undefined 19. -13

20. a. $\frac{37}{7}$ b. $\frac{20}{3}$ c. $\frac{109}{10}$ d. $\frac{21}{5}$

Objective F Writing Improper Fractions as Mixed Numbers or Whole Numbers

Just as there are times when an improper fraction is preferred, sometimes a mixed or a whole number better suits a situation. To write improper fractions as mixed or whole numbers, we use division. Recall once again from Section 1.6 that the fraction bar means division. This means that the fraction

$$\frac{5}{3} \quad \begin{array}{l} \text{numerator} \\ \text{denominator} \end{array} \quad \text{means} \quad \begin{array}{r} 3 \overline{)5} \\ \uparrow \quad \uparrow \\ \text{numerator} \\ \text{denominator} \end{array}$$

Writing an Improper Fraction as a Mixed Number or a Whole Number

To write an improper fraction as a mixed number or a whole number:

Step 1: Divide the denominator into the numerator.

Step 2: The whole number part of the mixed number is the quotient. The fraction part of the mixed number is the remainder over the original denominator.

$$\text{quotient} \frac{\text{remainder}}{\text{original denominator}}$$

For example,

$$\begin{array}{r} \text{Step 1} \\ 3 \overline{)5} \\ \underline{-3} \\ 2 \end{array} \quad \begin{array}{r} \text{Step 2} \\ 3 \overline{)5} = 1 \frac{2}{3} \\ \uparrow \quad \uparrow \\ \text{quotient} \quad \text{remainder} \\ \text{original denominator} \end{array}$$

Practice 21

Write each as a mixed number or a whole number.

a. $\frac{9}{5}$ b. $\frac{23}{9}$ c. $\frac{48}{4}$
d. $\frac{62}{13}$ e. $\frac{51}{7}$ f. $\frac{21}{20}$

Answers

21. a. $1\frac{4}{5}$ b. $2\frac{5}{9}$ c. 12 d. $4\frac{10}{13}$
e. $7\frac{2}{7}$ f. $1\frac{1}{20}$

Example 21

Write each as a mixed number or a whole number.

a. $\frac{30}{7}$

b. $\frac{16}{15}$

c. $\frac{84}{6}$

Solution:

a. $\frac{30}{7} : 7 \overline{)30}$
 $\underline{-28}$
2

$\frac{30}{7} = 4\frac{2}{7}$

b. $\frac{16}{15} : 15 \overline{)16}$
 $\underline{-15}$
1

$\frac{16}{15} = 1\frac{1}{15}$

c. $\frac{84}{6} : 6 \overline{)84}$
 $\underline{-6}$
24
 $\underline{-24}$
0

$\frac{84}{6} = 14$ Since the remainder is 0, the result is the whole number 14.

Helpful Hint

When the remainder is 0, the improper fraction is a whole number. For example, $\frac{92}{4} = 23$.

$$\begin{array}{r} 23 \\ 4 \overline{)92} \\ \underline{-8} \\ 12 \\ \underline{-12} \\ 0 \end{array}$$

Work Practice 21

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

improper ≥ 1	fraction denominator	proper $= 1$	is undefined < 1	mixed number numerator	$= 0$
----------------------	-------------------------	-----------------	-----------------------	---------------------------	-------

- The number $\frac{17}{31}$ is called a(n) _____. The number 31 is called its _____ and 17 is called its _____.
- If we simplify each fraction, $\frac{-9}{-9}$ _____, $\frac{0}{-4}$ _____, and we say $\frac{-4}{0}$ _____.
- The fraction $\frac{8}{3}$ is called a(n) _____ fraction, the fraction $\frac{3}{8}$ is called a(n) _____ fraction, and $10\frac{3}{8}$ is called a(n) _____.
- The value of an improper fraction is always _____, and the value of a proper fraction is always _____.

Martin-Gay Interactive Videos



See Video 4.1

Watch the section lecture video and answer the following questions.

- Objective A** 5. Complete this statement based on Example 1: When the numerator is greater than or _____ to the denominator, you have a(n) _____ fraction.
- Objective B** 6. In Example 4, there are two shapes in the diagram, so why do the representative fractions have a denominator 3?
- Objective C** 7. From Examples 6 and 7, when graphing a positive fraction on a number line, how does the denominator help? What does the denominator tell you?
- Objective D** 8. From Example 10, what can you conclude about any fraction where the numerator and denominator are the same nonzero number?
- Objective E** 9. Complete this statement based on the lecture before Example 13: The operation of _____ is understood in a mixed number notation; for example, $1\frac{1}{3}$ means 1 _____ $\frac{1}{3}$.
- Objective F** 10. From the lecture before Example 16, what operation is used to write an improper fraction as a mixed number?

4.1 Exercise Set MyLab Math

Objectives A B Identify the numerator and the denominator of each fraction and identify each fraction as proper or improper. See Examples 1, 2, 10, and 11.

1. $\frac{1}{2}$

2. $\frac{1}{4}$

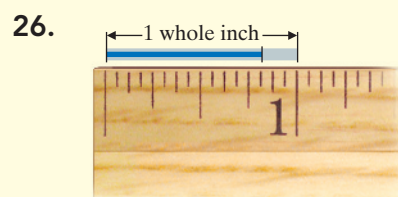
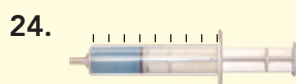
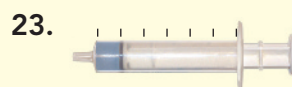
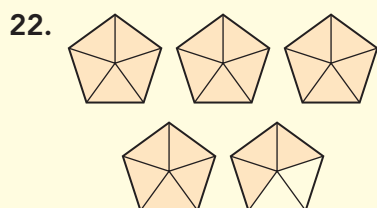
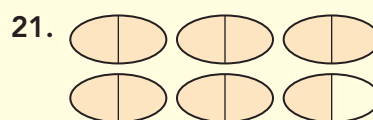
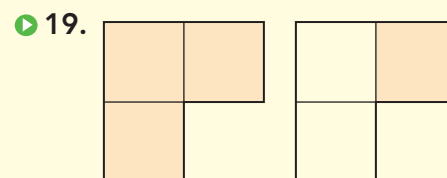
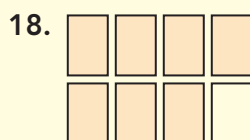
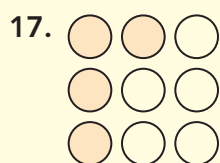
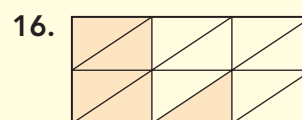
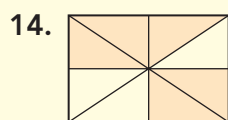
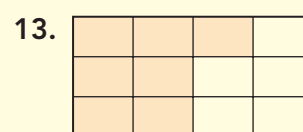
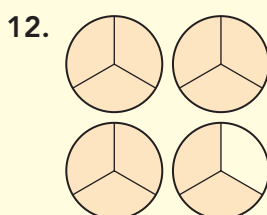
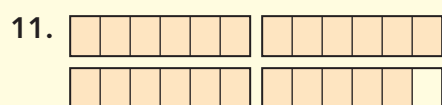
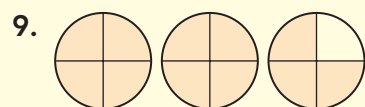
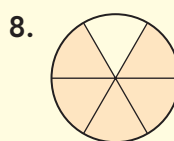
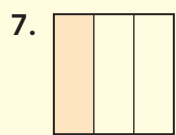
3. $\frac{10}{3}$

4. $\frac{53}{21}$

5. $\frac{15}{15}$

6. $\frac{26}{26}$

Objective B Write a proper or improper fraction to represent the shaded part of each diagram. If an improper fraction is appropriate, write the shaded part of the diagram as (a) an improper fraction and (b) a mixed number. (Note to students: In case you know how to simplify fractions, none of the fractions in this section are simplified.) See Examples 3 through 6, and 10, and 11.



Objective B Draw and shade a part of a diagram to represent each fraction. See Examples 7 and 8.

27. $\frac{1}{5}$ of a diagram

28. $\frac{1}{16}$ of a diagram

29. $\frac{6}{7}$ of a diagram

30. $\frac{7}{9}$ of a diagram

31. $\frac{4}{4}$ of a diagram

32. $\frac{6}{6}$ of a diagram

Write each fraction. (Note to students: In case you know how to simplify fractions, none of the fractions in this section are simplified.) See Example 9.

33. Of the 131 students at a small private school, 42 are freshmen. What fraction of the students are freshmen?

34. Of the 63 employees at a new biomedical engineering firm, 22 are men. What fraction of the employees are men?

35. Use Exercise 33 to answer **a** and **b**.

- a. How many students are *not* freshmen?
- b. What fraction of the students are *not* freshmen?

36. Use Exercise 34 to answer **a** and **b**.

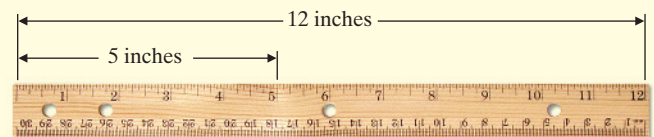
- a. How many of the employees are women?
- b. What fraction of the employees are women?

37. As of 2017, the United States has had 44 different men as presidents. A total of seven U.S. presidents were born in the state of Ohio, second only to the state of Virginia in producing U.S. presidents. What fraction of U.S. presidents were born in Ohio? (Source: World Almanac, 2017; Note: Grover Cleveland was our 22nd and 24th president, thus was counted once.)

38. Of the eight planets in our solar system, four have days that are longer than the 24-hour Earth day. What fraction of the planets have longer days than Earth has? (Source: National Space Science Data Center)

39. The 2016 Atlantic hurricane season was the deadliest since 2005. It was an above-average season, producing 15 named tropical storms, 10 of which turned into hurricanes. What fraction of the 2016 Atlantic tropical storms escalated into hurricanes?

40. There are 12 inches in a foot. What fractional part of a foot do 5 inches represent?



41. There are 31 days in the month of March. What fraction of the month do 11 days represent?

42. There are 60 minutes in an hour. What fraction of an hour do 37 minutes represent?

Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Sun.
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						



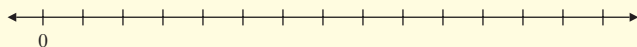
43. In a prealgebra class containing 31 students, there are 18 freshmen, 10 sophomores, and 3 juniors. What fraction of the class is sophomores?

44. In a sports team with 20 children, there are 9 boys and 11 girls. What fraction of the team is boys?

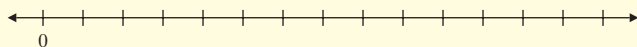
45. Thirty-three out of the fifty total states in the United States contain federal Indian reservations.
- What fraction of the states contain federal Indian reservations?
 - How many states do not contain federal Indian reservations?
 - What fraction of the states do not contain federal Indian reservations? (*Source: Tiller Research, Inc., Albuquerque, NM*)
46. Consumer fireworks are legal in 44 out of the 50 total states in the United States.
- In what fraction of the states are consumer fireworks legal?
 - In how many states are consumer fireworks illegal?
 - In what fraction of the states are consumer fireworks illegal? (*Source: United States Fireworks Safety Council*)
47. A bag contains 50 red or blue marbles. If 21 marbles are blue, answer each question.
- What fraction of the marbles are blue?
 - How many marbles are red?
 - What fraction of the marbles are red?
48. An art dealer is taking inventory. His shop contains a total of 37 pieces, which are all sculptures, watercolor paintings, or oil paintings. If there are 15 watercolor paintings and 17 oil paintings, answer each question.
- What fraction of the inventory is watercolor paintings?
 - What fraction of the inventory is oil paintings?
 - How many sculptures are there?
 - What fraction of the inventory is sculptures?

Objective C Graph each fraction on a number line. See Examples 12 and 13.

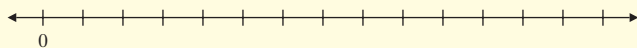
49. $\frac{1}{4}$



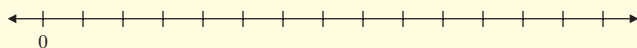
50. $\frac{1}{3}$



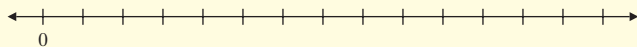
51. $\frac{4}{7}$



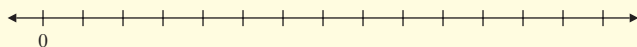
52. $\frac{5}{6}$



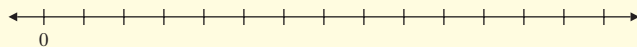
53. $\frac{8}{5}$



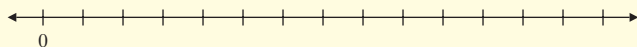
54. $\frac{9}{8}$



55. $\frac{7}{3}$



56. $\frac{13}{7}$



Objective D Simplify by dividing. See Examples 14 through 19.

57. $\frac{12}{12}$

58. $\frac{-3}{-3}$

59. $\frac{-5}{1}$

60. $\frac{-20}{1}$

61. $\frac{0}{-2}$

62. $\frac{0}{-8}$

63. $\frac{-8}{-8}$

64. $\frac{-14}{-14}$

65. $\frac{-9}{0}$

66. $\frac{-7}{0}$

67. $\frac{3}{1}$

68. $\frac{5}{5}$

Objective E Write each mixed number as an improper fraction. See Example 20.

69. $2\frac{1}{3}$

70. $1\frac{13}{17}$

71. $3\frac{3}{5}$

72. $2\frac{5}{9}$

73. $6\frac{5}{8}$

74. $7\frac{3}{8}$

75. $11\frac{6}{7}$

76. $12\frac{2}{5}$

77. $9\frac{7}{20}$

78. $10\frac{14}{27}$

79. $166\frac{2}{3}$

80. $114\frac{2}{7}$

Objective F Write each improper fraction as a mixed number or a whole number. See Example 21.

81. $\frac{17}{5}$

82. $\frac{13}{7}$

83. $\frac{37}{8}$

84. $\frac{64}{9}$

85. $\frac{47}{15}$

86. $\frac{65}{12}$

87. $\frac{225}{15}$

88. $\frac{196}{14}$

89. $\frac{182}{175}$

90. $\frac{149}{143}$

91. $\frac{737}{112}$

92. $\frac{901}{123}$

Review

Simplify. See Section 1.7.

93. 3^2

94. 4^3

95. 5^3

96. 3^4

Concept Extensions

Write each fraction in two other equivalent ways by inserting the negative sign in different places.

97. $-\frac{11}{2} = \quad =$

99. $\frac{-13}{15} = \quad =$

98. $-\frac{21}{4} = \quad =$

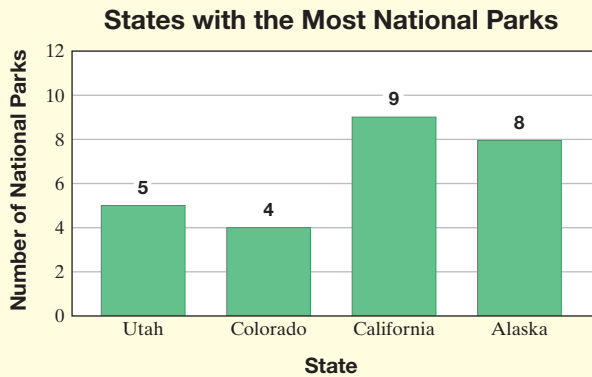
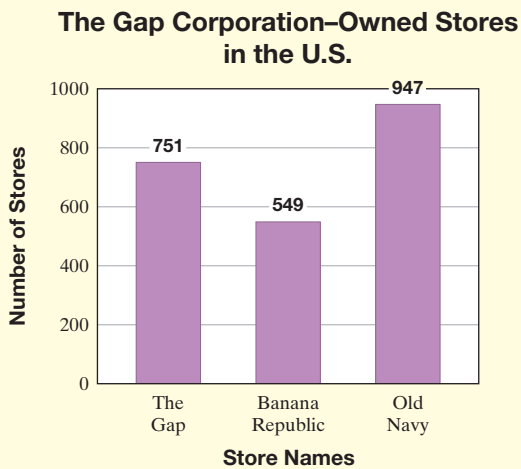
100. $\frac{45}{-57} = \quad =$

101. In your own words, explain why $\frac{0}{10} = 0$ and $\frac{10}{0}$ is undefined.

102. In your own words, explain why $\frac{0}{-3} = 0$ and $\frac{-3}{0}$ is undefined.

Solve. See the Concept Checks in this section.

103. If  represents $\frac{4}{9}$ of a whole diagram, sketch the whole diagram.
104. If  represents $\frac{1}{3}$ of a whole diagram, sketch the whole diagram.
105. Round the mixed number $7\frac{1}{8}$ to the nearest whole number.
106. Round the mixed number $5\frac{11}{12}$ to the nearest whole number.
107. The Gap Corporation owns stores in the United States with five different brand names, with the three most popular shown on the bar graph. What fraction of the total stores shown on the graph are named “Old Navy”?
108. There are currently 59 national parks in the United States. The bar graph below shows the four states with the greatest number of national parks. What fraction of the national parks found in these four states are located in California? (Source: The National Park Service)



109. Habitat for Humanity is a nonprofit organization that helps provide affordable housing to families in need. Habitat for Humanity does its work of building and renovating houses through 1500 local affiliates in the United States and 80 international affiliates. What fraction of the total Habitat for Humanity affiliates are located in the United States? (Hint: First find the total number of affiliates.) (Source: Habitat for Humanity International)
110. The United States Marine Corps (USMC) has five principal training centers in California, three in North Carolina, two in South Carolina, one in Arizona, one in Hawaii, and one in Virginia. What fraction of the total USMC principal training centers are located in California? (Hint: First find the total number of USMC training centers.) (Source: U.S. Department of Defense)

4.2 Factors and Simplest Form

Objective A Writing a Number as a Product of Prime Numbers

Recall from Section 1.5 that since $12 = 2 \cdot 2 \cdot 3$, the numbers 2 and 3 are called *factors* of 12. A **factor** is any number that divides a number evenly (with a remainder of 0).

To perform operations on fractions, it is necessary to be able to factor a number.

Factoring a number means writing a number as a product. We first practice writing a number as a product of prime numbers.

A **prime number** is a natural number greater than 1 whose only factors are 1 and itself. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...

A **composite number** is a natural number greater than 1 that is not prime.

Helpful Hint

The natural number 1 is neither prime nor composite.

When a composite number is written as a product of prime numbers, this product is called the **prime factorization** of the number. For example, the prime factorization of 12 is $2 \cdot 2 \cdot 3$ because

$$12 = 2 \cdot 2 \cdot 3$$

This product is 12 and each number is a prime number.

Because multiplication is commutative, the order of the factors is not important. We can write the factorization $2 \cdot 2 \cdot 3$ as $2 \cdot 3 \cdot 2$ or $3 \cdot 2 \cdot 2$. Any of these is called the prime factorization of 12.

Every whole number greater than 1 has exactly one prime factorization.

One method for finding the prime factorization of a number is by using a factor tree, as shown in the next example.

Example 1 Write the prime factorization of 45.

Solution: We can begin by writing 45 as the product of two numbers, say, 5 and 9.

$$\begin{array}{c} 45 \\ \swarrow \quad \searrow \\ 5 \quad \cdot \quad 9 \end{array}$$

The number 5 is prime but 9 is not, so we write 9 as $3 \cdot 3$.

$$\left. \begin{array}{c} 45 \\ \swarrow \quad \searrow \\ 5 \quad \cdot \quad 9 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 5 \quad \cdot \quad 3 \quad \cdot \quad 3 \end{array} \right\} \text{A factor tree}$$

Each factor is now a prime number, so the prime factorization of 45 is $3 \cdot 3 \cdot 5$ or $3^2 \cdot 5$.

Work Practice 1

Objectives

- A** Write a Number as a Product of Prime Numbers.
- B** Write a Fraction in Simplest Form.
- C** Determine Whether Two Fractions Are Equivalent.
- D** Solve Problems by Writing Fractions in Simplest Form.

Practice 1

Use a factor tree to find the prime factorization of each number.

- a. 30 b. 56 c. 72

Answers

1. a. $2 \cdot 3 \cdot 5$ b. $2^3 \cdot 7$ c. $2^3 \cdot 3^2$

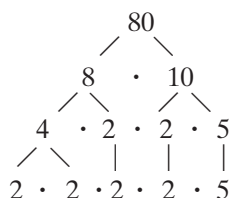
✓ **Concept Check** True or false? Two different numbers can have exactly the same prime factorization. Explain your answer.

Practice 2

Write the prime factorization of 60.

Example 2 Write the prime factorization of 80.

Solution: Write 80 as a product of two numbers. Continue this process until all factors are prime.



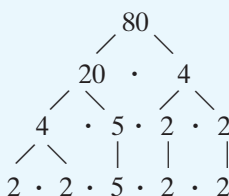
All factors are now prime, so the prime factorization of 80 is

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \quad \text{or} \quad 2^4 \cdot 5.$$

Work Practice 2

Helpful Hint

It makes no difference which factors you start with. The prime factorization of a number will be the same.



Same factors as
in Example 2

There are a few quick **divisibility tests** to determine whether a number is divisible by the primes 2, 3, or 5. (A number is divisible by 2, for example, if 2 divides it evenly so that the remainder is 0.)

Divisibility Tests

A whole number is divisible by:

- 2 if the last digit is 0, 2, 4, 6, or 8.



132 is divisible by 2 since the last digit is a 2.

- 3 if the sum of the digits is divisible by 3.

144 is divisible by 3 since $1 + 4 + 4 = 9$ is divisible by 3.

- 5 if the last digit is 0 or 5.



1115 is divisible by 5 since the last digit is a 5.

Answer

$$2 \cdot 2 \cdot 3 \cdot 5$$

✓ **Concept Check Answer**
false; answers may vary

Helpful Hint

Here are a few other divisibility tests you may want to use. A whole number is divisible by:

- 4 if its last two digits are divisible by 4.
1712 is divisible by 4.
- 6 if it's divisible by 2 and 3.
9858 is divisible by 6.
- 9 if the sum of its digits is divisible by 9.
5238 is divisible by 9 since $5 + 2 + 3 + 8 = 18$ is divisible by 9.

When finding the prime factorization of larger numbers, you may want to use the procedure shown in Example 3.

Example 3 Write the prime factorization of 252.

Solution: For this method, we divide prime numbers into the given number. Since the ones digit of 252 is 2, we know that 252 is divisible by 2.

$$\begin{array}{r} 126 \\ 2 \overline{)252} \end{array}$$

126 is divisible by 2 also.

$$\begin{array}{r} 63 \\ 2 \overline{)126} \\ 2 \overline{)252} \end{array}$$

63 is not divisible by 2 but is divisible by 3. Divide 63 by 3 and continue in this same manner until the quotient is a prime number.

$$\begin{array}{r} 7 \\ 3 \overline{)21} \\ 3 \overline{)63} \\ 2 \overline{)126} \\ 2 \overline{)252} \end{array}$$

Helpful Hint

The order of choosing prime numbers does not matter. For consistency, we use the order 2, 3, 5, 7, ...

The prime factorization of 252 is $2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$ or $2^2 \cdot 3^2 \cdot 7$.

Work Practice 3

In this text, we will write the factorization of a number from the smallest factor to the largest factor.

✓ Concept Check True or false? The prime factorization of 117 is $9 \cdot 13$. Explain your reasoning.

Objective B Writing Fractions in Simplest Form

Fractions that represent the same portion of a whole or the same point on a number line are called **equivalent fractions**. Study the table on the next page to see two ways to visualize equivalent fractions.

Practice 3

Write the prime factorization of 297.

Answer
3, $3^3 \cdot 11$

✓ Concept Check Answer
False; 9 is not prime.

Also, in general, if $\frac{a}{b}$ and $\frac{c}{d}$ are fractions (with b and d not 0), the following is true.

$$\frac{a \cdot c}{b \cdot d} = \frac{a}{b} \cdot \frac{c}{d}$$

These properties allow us to do the following:

$$\frac{4}{6} = \frac{2 \cdot 2}{2 \cdot 3} = \frac{2}{2} \cdot \frac{2}{3} = 1 \cdot \frac{2}{3} = \frac{2}{3}$$

When 1 is multiplied by a number, the result is the same number.
This is 1.

Example 4 Write in simplest form: $\frac{12}{20}$

Solution: Notice that 12 and 20 have a common factor of 4.

$$\frac{12}{20} = \frac{4 \cdot 3}{4 \cdot 5} = \frac{4}{4} \cdot \frac{3}{5} = 1 \cdot \frac{3}{5} = \frac{3}{5}$$

Since 3 and 5 have no common factors (other than 1), $\frac{3}{5}$ is in simplest form.

Work Practice 4

If you have trouble finding common factors, write the prime factorization of the numerator and the denominator.

Example 5 Write in simplest form: $\frac{42x}{66}$

Solution: Let's write the prime factorizations of 42 and 66. Remember that $42x$ means $42 \cdot x$.

$$\frac{42x}{66} = \frac{2 \cdot 3 \cdot 7 \cdot x}{2 \cdot 3 \cdot 11} = \frac{2}{2} \cdot \frac{3}{3} \cdot \frac{7x}{11} = 1 \cdot 1 \cdot \frac{7x}{11} = \frac{7x}{11}$$

Work Practice 5

In the example above, you may have saved time by noticing that 42 and 66 have a common factor of 6.

$$\frac{42x}{66} = \frac{6 \cdot 7x}{6 \cdot 11} = \frac{6}{6} \cdot \frac{7x}{11} = 1 \cdot \frac{7x}{11} = \frac{7x}{11}$$

Helpful Hint

Writing the prime factorizations of the numerator and the denominator is helpful in finding any common factors.

The method for simplifying negative fractions is the same as for positive fractions.

Example 6 Write in simplest form: $-\frac{10}{27}$

Solution:

$$-\frac{10}{27} = -\frac{2 \cdot 5}{3 \cdot 3 \cdot 3}$$

Prime factorizations of 10 and 27

Since 10 and 27 have no common factors, $-\frac{10}{27}$ is already in simplest form.

Work Practice 6

*Note: We will study this concept further in the next section.

Helpful Hint

These two properties together are called the Fundamental Property of Fractions

$$\frac{a \cdot c}{b \cdot c} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a}{b}$$

Practice 4

Write in simplest form: $\frac{30}{45}$

Practice 5

Write in simplest form: $\frac{39x}{51}$

Practice 6

Write in simplest form: $-\frac{9}{50}$

Answers

4. $\frac{2}{3}$ 5. $\frac{13x}{17}$ 6. $-\frac{9}{50}$

Practice 7Write in simplest form: $\frac{49}{112}$ **Example 7**Write in simplest form: $\frac{30}{108}$ **Solution:**

$$\frac{30}{108} = \frac{2 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot 5}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 2 \cdot 3 \cdot \underset{1}{\cancel{3}}} = 1 \cdot 1 \cdot \frac{5}{18} = \frac{5}{18}$$

Work Practice 7

We can use a shortcut procedure with common factors when simplifying.

$$\frac{4}{6} = \frac{\overset{1}{\cancel{2}} \cdot 2}{\underset{1}{\cancel{2}} \cdot 3} = \frac{1 \cdot 2}{1 \cdot 3} = \frac{2}{3} \quad \text{Divide out the common factor of 2 in the numerator and denominator.}$$

This procedure is possible because dividing out a common factor in the numerator and denominator is the same as removing a factor of 1 in the product.

Writing a Fraction in Simplest Form

To write a fraction in simplest form, write the prime factorization of the numerator and the denominator and then divide both by all common factors.

Example 8Write in simplest form: $-\frac{72}{26}$ **Solution:**

$$-\frac{72}{26} = -\frac{\overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 3 \cdot 3}{\underset{1}{\cancel{2}} \cdot 13} = -\frac{1 \cdot 2 \cdot \overset{1}{\cancel{2}} \cdot 3 \cdot 3}{1 \cdot 13} = -\frac{36}{13}$$

Work Practice 8**✓ Concept Check** Which is the correct way to simplify the fraction $\frac{15}{25}$? Or are both correct? Explain.

$$\text{a. } \frac{15}{25} = \frac{3 \cdot \overset{1}{\cancel{5}}}{5 \cdot \underset{1}{\cancel{5}}} = \frac{3}{5} \qquad \text{b. } \frac{1\overset{1}{\cancel{5}}}{2\underset{1}{\cancel{5}}} = \frac{11}{21}$$

In this chapter, we will simplify and perform operations on fractions containing variables. When the denominator of a fraction contains a variable, such as $\frac{6x^2}{60x^3}$, we will assume that the variable does not represent 0. Recall that the denominator of a fraction cannot be 0.

Example 9Write in simplest form: $\frac{6x^2}{60x^3}$ **Solution:** Notice that 6 and 60 have a common factor of 6. Let's also use the definition of an exponent to factor x^2 and x^3 .

$$\frac{6x^2}{60x^3} = \frac{\overset{1}{\cancel{6}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{6}} \cdot 10 \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot x} = \frac{1 \cdot 1 \cdot 1}{1 \cdot 10 \cdot 1 \cdot 1 \cdot x} = \frac{1}{10x}$$

Work Practice 9**Practice 8**Write in simplest form: $-\frac{64}{20}$ **Practice 9**Write in simplest form: $\frac{7a^3}{56a^2}$ **Answers**

$$7. \frac{7}{16} \quad 8. -\frac{16}{5} \quad 9. \frac{a}{8}$$

✓ Concept Check Answers

a. correct b. incorrect

Helpful Hint

Be careful when all factors of the numerator or denominator are divided out. In Example 9, the numerator was $1 \cdot 1 \cdot 1 = 1$, so the final result was $\frac{1}{10x}$.

Objective C Determining Whether Two Fractions Are Equivalent

Recall from Objective B that two fractions are equivalent if they represent the same part of a whole. One way to determine whether two fractions are equivalent is to see whether they simplify to the same fraction.

Example 10 Determine whether $\frac{16}{40}$ and $\frac{10}{25}$ are equivalent.

Solution: Simplify each fraction.

$$\frac{16}{40} = \frac{\overset{1}{\cancel{8}} \cdot 2}{\underset{1}{\cancel{8}} \cdot 5} = \frac{1 \cdot 2}{1 \cdot 5} = \frac{2}{5}$$

$$\frac{10}{25} = \frac{2 \cdot \underset{1}{\cancel{5}}}{5 \cdot \underset{1}{\cancel{5}}} = \frac{2 \cdot 1}{5 \cdot 1} = \frac{2}{5}$$

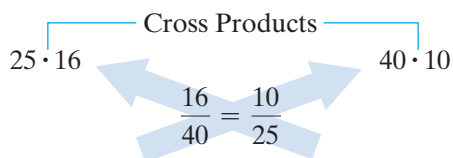
Since these fractions are the same, $\frac{16}{40} = \frac{10}{25}$.

Work Practice 10

There is a shortcut method you may use to check or test whether two fractions are equivalent. In the example above, we learned that the fractions are equivalent, or

$$\frac{16}{40} = \frac{10}{25}$$

In this example above, we call $25 \cdot 16$ and $40 \cdot 10$ **cross products** because they are the products one obtains by multiplying diagonally across the equal sign, as shown below.

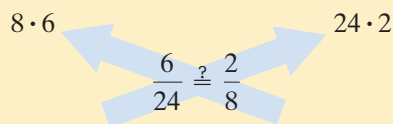


Notice that these cross products are equal:

$$25 \cdot 16 = 400, \quad 40 \cdot 10 = 400$$

In general, this is true for equivalent fractions.

Equality of Fractions



Since the cross products ($8 \cdot 6 = 48$ and $24 \cdot 2 = 48$) are equal, the fractions are equal.

Note: If the cross products are not equal, the fractions are not equal.

Practice 10

Determine whether $\frac{7}{9}$ and $\frac{21}{27}$ are equivalent.

Answer

10. equivalent

Practice 11

Determine whether $\frac{4}{13}$ and $\frac{5}{18}$ are equivalent.

Example 11 Determine whether $\frac{8}{11}$ and $\frac{19}{26}$ are equivalent.

Solution: Let's check cross products.

$$\begin{array}{ccc} 26 \cdot 8 & \begin{array}{c} \nearrow \quad \nwarrow \\ \frac{8}{11} \quad \frac{19}{26} \end{array} & 11 \cdot 19 \\ = 208 & & = 209 \end{array}$$

Since $208 \neq 209$, then $\frac{8}{11} \neq \frac{19}{26}$.

Helpful Hint

symbol

“Not equal to”

Work Practice 11

Objective D Solving Problems by Writing Fractions in Simplest Form

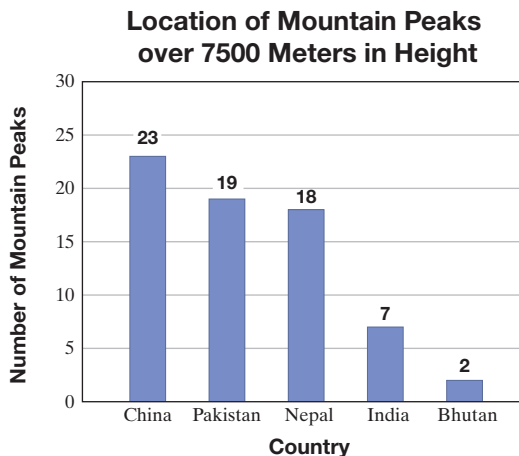
Many real-life problems can be solved by writing fractions. To make the answers clearer, these fractions should be written in simplest form.

Practice 12

There are only 14 mountains over 8000 meters in the world, all located in Asia. If 7 of these mountains are located in Nepal, determine what fraction of the mountains over 8000 meters are located in Nepal. Write the fraction in simplest form.

Example 12 Calculating Fraction of High Mountain Peaks

There are 49 mountain peaks over 7500 meters in the world, all located in Asia. Many of them form the borders between two countries and are thus counted twice on the bar graph below. Use the graph below to determine what fraction of the 49 mountain peaks over 7500 meters are located in India. Write the fraction in simplest form.



Source: Internet Research



Solution: First we determine the fraction of highest mountain peaks found in India.

$$\frac{7}{49} \quad \begin{array}{l} \leftarrow \text{mountain peaks over 7500 meters in India} \\ \leftarrow \text{total mountain peaks over 7500 meters} \end{array}$$

Next, we simplify the fraction.

$$\frac{7}{49} = \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{7}} \cdot 7} = \frac{1}{1 \cdot 7} = \frac{1}{7}$$

Thus, $\frac{1}{7}$ of the world's mountain peaks over 7500 meters in height are located in India.

Work Practice 12

Answers

11. not equivalent 12. $\frac{1}{2}$



Calculator Explorations Simplifying Fractions

Scientific Calculator

Many calculators have a fraction key, such as $\boxed{a^b/c}$, that allows you to simplify a fraction on the calculator. For example, to simplify $\frac{324}{612}$, enter

$\boxed{324} \boxed{a^b/c} \boxed{612} \boxed{=}$

The display will read

$\boxed{9} \boxed{17}$

which represents $\frac{9}{17}$, the original fraction simplified.

Graphing Calculator

Graphing calculators also allow you to simplify fractions. The fraction option on a graphing calculator may be found under the $\boxed{\text{MATH}}$ menu.

To simplify $\frac{324}{612}$, enter

$\boxed{324} \boxed{\div} \boxed{612} \boxed{\text{MATH}} \boxed{\text{ENTER}} \boxed{\text{ENTER}}$

The display will read

$\boxed{324/612 \blacktriangleright \text{Frac } 9/17}$



The Calculator Explorations boxes in this chapter provide only an introduction to fraction keys on calculators. Any time you use a calculator, there are both advantages and limitations to its use. Never rely solely on your calculator. It is very important that you understand how to perform all operations on fractions by hand in order to progress through later topics. For further information, talk to your instructor.

Use your calculator to simplify each fraction.

- | | | |
|-----------------------|----------------------|----------------------|
| 1. $\frac{128}{224}$ | 2. $\frac{231}{396}$ | 3. $\frac{340}{459}$ |
| 4. $\frac{999}{1350}$ | 5. $\frac{432}{810}$ | 6. $\frac{225}{315}$ |
| 7. $\frac{54}{243}$ | 8. $\frac{455}{689}$ | |

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

cross products	equivalent	composite
simplest form	prime factorization	prime

- The number 40 equals $2 \cdot 2 \cdot 2 \cdot 5$. Since each factor is prime, we call $2 \cdot 2 \cdot 2 \cdot 5$ the _____ of 40.
- A natural number, other than 1, that is not prime is called a(n) _____ number.
- A natural number that has exactly two different factors, 1 and itself, is called a(n) _____ number.
- In $\frac{11}{48}$, since 11 and 48 have no common factors other than 1, $\frac{11}{48}$ is in _____.
- Fractions that represent the same portion of a whole are called _____ fractions.
- In the statement $\frac{5}{12} = \frac{15}{36}$, $5 \cdot 36$ and $12 \cdot 15$ are called _____.

Martin-Gay Interactive Videos Watch the section lecture video and answer the following questions.



See Video 4.2 

- Objective A** 7. From  Example 1, what two things should you check to make sure your prime factorization of a number is correct? 
- Objective B** 8. From the lecture before  Example 3, when you have a common factor in the numerator and denominator of a fraction, essentially you have what? 
- Objective C** 9. Describe another way to solve  Example 7 besides using cross products. 
- Objective D** 10. Why isn't $\frac{10}{24}$ the final answer to  Example 8? What is the final answer? 

4.2 Exercise Set MyLab Math

Objective A Write the prime factorization of each number. See Examples 1 through 3.

- | | | | |
|---|---------|---|---------|
| 1. 20 | 2. 12 |  3. 48 | 4. 75 |
| 5. 81 | 6. 64 | 7. 162 | 8. 128 |
| 9. 110 | 10. 130 | 11. 85 | 12. 93 |
|  13. 240 | 14. 836 | 15. 828 | 16. 504 |

Objective B Write each fraction in simplest form. See Examples 4 through 9.

- | | | | | |
|--|-----------------------------|---|-------------------------|--|
| 17. $\frac{3}{12}$ | 18. $\frac{5}{30}$ | 19. $\frac{4x}{42}$ | 20. $\frac{9y}{48}$ | 21. $\frac{14}{16}$ |
| 22. $\frac{22}{34}$ | 23. $\frac{20}{30}$ | 24. $\frac{70}{80}$ | 25. $\frac{35a}{50a}$ | 26. $\frac{25z}{55z}$ |
|  27. $-\frac{63}{81}$ | 28. $-\frac{21}{49}$ |  29. $\frac{30x^2}{36x}$ | 30. $\frac{45b}{80b^2}$ | 31. $\frac{27}{64}$ |
| 32. $\frac{32}{63}$ | 33. $\frac{25xy}{40y}$ | 34. $\frac{36y}{42yz}$ | 35. $-\frac{40}{64}$ | 36. $-\frac{28}{60}$ |
| 37. $\frac{36x^3y^2}{24xy}$ | 38. $\frac{60a^2b}{36ab^3}$ | 39. $\frac{90}{120}$ | 40. $\frac{60}{150}$ |  41. $\frac{40xy}{64xyz}$ |

42. $\frac{28abc}{60ac}$

43. $\frac{66}{308}$

44. $\frac{65}{234}$

45. $-\frac{55}{85y}$

46. $-\frac{78}{90x}$

47. $\frac{189z}{216z}$

48. $\frac{144y}{162y}$

49. $\frac{224a^3b^4c^2}{16ab^4c^2}$

50. $\frac{270x^4y^3z^3}{15x^3y^3z^3}$

Objective C Determine whether each pair of fractions is equivalent. See Examples 10 and 11.

51. $\frac{2}{6}$ and $\frac{4}{12}$

52. $\frac{3}{6}$ and $\frac{5}{10}$

▶ 53. $\frac{7}{11}$ and $\frac{5}{8}$

54. $\frac{2}{5}$ and $\frac{4}{11}$

55. $\frac{10}{15}$ and $\frac{6}{9}$

56. $\frac{4}{10}$ and $\frac{6}{15}$

▶ 57. $\frac{3}{9}$ and $\frac{6}{18}$

58. $\frac{2}{8}$ and $\frac{7}{28}$

59. $\frac{10}{13}$ and $\frac{13}{15}$

60. $\frac{16}{20}$ and $\frac{9}{16}$

61. $\frac{8}{18}$ and $\frac{12}{24}$

62. $\frac{6}{21}$ and $\frac{14}{35}$

Objective D Solve. Write each fraction in simplest form. See Example 12.

63. A work shift for an employee at Starbucks consists of 8 hours. What fraction of the employee's work shift is represented by 2 hours?

64. Two thousand baseball caps were sold one year at the U.S. Open Golf Tournament. What fractional part of this total do 200 caps represent?

65. There are 5280 feet in a mile. What fraction of a mile is represented by 2640 feet?

66. There are 100 centimeters in 1 meter. What fraction of a meter is 20 centimeters?

67. Sixteen out of the total fifty states in the United States have Ritz-Carlton hotels. (Source: Ritz-Carlton Hotel Company, LLC)

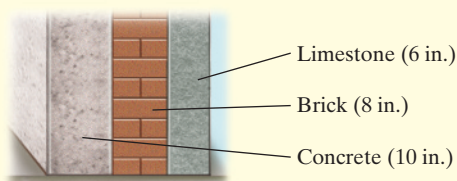
- What fraction of states can claim at least one Ritz-Carlton hotel?
- How many states do not have a Ritz-Carlton hotel?
- Write the fraction of states without a Ritz-Carlton hotel.

68. There are 152 national monuments in the United States. Twelve of these monuments are located in New Mexico. (Source: National Park Service)

- What fraction of the national monuments in the United States can be found in New Mexico?
- How many of the national monuments in the United States are found outside New Mexico?
- Write the fraction of national monuments found in states other than New Mexico.

- ▶ 69. The outer wall of the Pentagon is 24 inches thick. Ten inches is concrete, 8 inches is brick, and 6 inches is limestone. What fraction of the wall is concrete?

70. There are 35 students in a biology class. If 10 students made an A on the first test, what fraction of the students made an A?



71. In the United States, Alaska has no income tax and Hawaii has state and federal income tax that can be filed at the same time. Thirty-six out of the 48 contiguous United States (the “lower” 48) allow residents to file their federal and state income tax electronically at the same time via efile.
- How many of the contiguous states do not have this type of service?
 - What fraction of the contiguous states do not have this type of service? (*Source: efile.com*)
72. Katy Scarpulla bought a brand-new 2017 Toyota Camry Hybrid for \$26,500. Her old car was traded in for \$15,000.
- How much of her purchase price was not covered by her trade-in?
 - What fraction of the purchase price was not covered by the trade-in?
73. As of this writing, a total of 366 individuals from the United States have been active astronauts. Of these, 45 are currently active and available for assignment in space. What fraction of U.S. astronauts are currently active? (*Source: NASA*)
74. Worldwide, Hallmark employs about 27,300 individuals. About 2700 employees work at the Hallmark headquarters in Kansas City, Missouri. What fraction of Hallmark employees work in Kansas City? (*Source: Hallmark Card, Inc.*)

Review

Evaluate each expression using the given replacement numbers. See Section 2.5.

75. $\frac{x^3}{9}$ when $x = -3$

76. $\frac{y^3}{5}$ when $y = -5$

77. $2y$ when $y = -7$

78. $-5a$ when $a = -4$

Concept Extensions

79. In your own words, define equivalent fractions.

80. Given a fraction, say $\frac{15}{40}$, how many fractions are there that are equivalent to it, but in simplest form or lowest terms? Explain your answer.

Write each fraction in simplest form.

81. $\frac{3975}{6625}$

82. $\frac{9506}{12,222}$

There are generally considered to be eight basic blood types. The table shows the number of people with the various blood types in a typical group of 100 blood donors. Use the table to answer Exercises 83 through 86. Write each answer in simplest form.



Distribution of Blood Types in Blood Donors	
Blood Type	Number of People
O Rh-positive	37
O Rh-negative	7
A Rh-positive	36
A Rh-negative	6
B Rh-positive	9
B Rh-negative	1
AB Rh-positive	3
AB Rh-negative	1

(*Source: American Red Cross Biomedical Services*)

83. What fraction of blood donors have blood type A Rh-positive?
84. What fraction of blood donors have an O blood type?
85. What fraction of blood donors have an AB blood type?
86. What fraction of blood donors have a B blood type?

Find the prime factorization of each number.

87. 34,020

88. 131,625

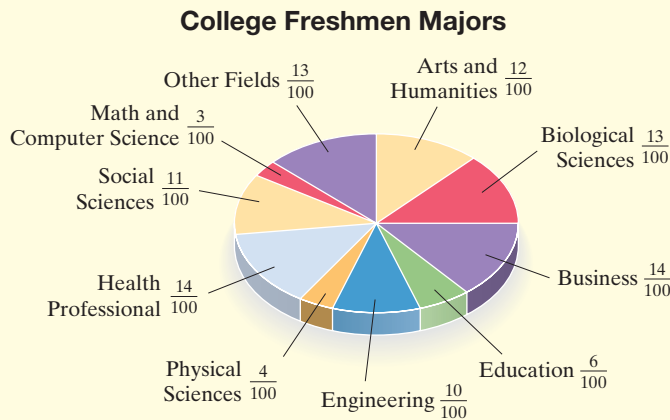
89. In your own words, define a prime number.

90. The number 2 is a prime number. All other even natural numbers are composite numbers. Explain why.

91. Two students have different prime factorizations for the same number. Is this possible? Explain.

92. Two students work to prime factor 120. One student starts by writing 120 as 12×10 . The other student writes 120 as 24×5 . Finish each prime factorization. Are they the same? Why or why not?

The following graph is called a circle graph or pie chart. Each sector (shaped like a piece of pie) shows the fraction of entering college freshmen who choose to major in each discipline shown. The whole circle represents the entire class of college freshmen. Use this graph to answer Exercises 93 through 96. Write each fraction answer in simplest form.



Source: The Higher Education Research Institute

93. What fraction of entering college freshmen plan to major in education?

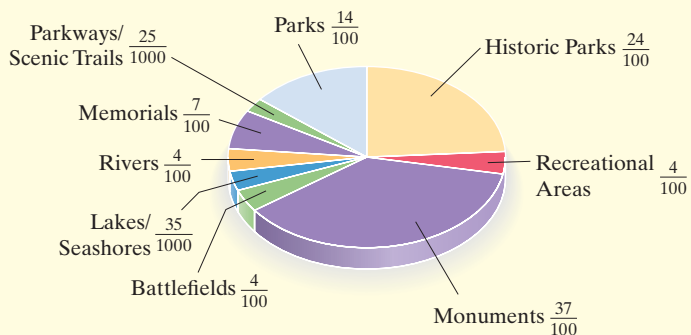
94. What fraction of entering college freshmen plan to major in engineering?

95. Why is the business sector the same size as the health professional sector?

96. Why is the physical sciences sector smaller than the business sector?

Use this circle graph to answer Exercises 97 through 100. Write each fraction answer in simplest form.

Areas Maintained by the National Park Service



Source: National Park Service

97. What fraction of National Park Service areas are historic parks?

98. What fraction of National Park Service areas are parks?

99. Why is the national battlefields sector smaller than the national monuments sector?

100. Why is the recreation areas sector the same size as the battlefields sector?

Use the following numbers for Exercises 101 through 104.

8691 786 1235 2235 85 105 22 222 900 1470

101. List the numbers divisible by both 2 and 3.

102. List the numbers that are divisible by both 3 and 5.

103. The answers to Exercise 101 are also divisible by what number? Tell why.

104. The answers to Exercise 102 are also divisible by what number? Tell why.

4.3 Multiplying and Dividing Fractions

Objectives

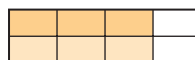
- A** Multiply Fractions
- B** Evaluate Exponential Expressions with Fractional Bases.
- C** Divide Fractions.
- D** Multiply and Divide Given Fractional Replacement Values.
- E** Solve Applications That Require Multiplication of Fractions.

Objective A Multiplying Fractions

Let's use a diagram to discover how fractions are multiplied. For example, to multiply $\frac{1}{2}$ and $\frac{3}{4}$, we find $\frac{1}{2}$ of $\frac{3}{4}$. To do this, we begin with a diagram showing $\frac{3}{4}$ of a rectangle's area shaded.



To find $\frac{1}{2}$ of $\frac{3}{4}$, we heavily shade $\frac{1}{2}$ of the part that is already shaded.



By counting smaller rectangles, we see that $\frac{3}{8}$ of the larger rectangle is now heavily shaded, so that

$$\frac{1}{2} \text{ of } \frac{3}{4} \text{ is } \frac{3}{8}, \text{ or } \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8} \quad \text{Notice that } \frac{1}{2} \cdot \frac{3}{4} = \frac{1 \cdot 3}{2 \cdot 4} = \frac{3}{8}.$$

Multiplying Fractions

To multiply two fractions, multiply the numerators and multiply the denominators. If a , b , c , and d represent numbers, and b and d are not 0, we have

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

Practice 1–2

Multiply.

1. $\frac{3}{7} \cdot \frac{5}{11}$ 2. $\frac{1}{3} \cdot \frac{1}{9}$

Practice 3

Multiply and simplify: $\frac{6}{77} \cdot \frac{7}{8}$

Answers

1. $\frac{15}{77}$ 2. $\frac{1}{27}$ 3. $\frac{3}{44}$

Examples Multiply.

1. $\frac{2}{3} \cdot \frac{5}{11} = \frac{2 \cdot 5}{3 \cdot 11} = \frac{10}{33}$ Multiply numerators.
Multiply denominators.

This fraction is in simplest form since 10 and 33 have no common factors other than 1.

2. $\frac{1}{4} \cdot \frac{1}{2} = \frac{1 \cdot 1}{4 \cdot 2} = \frac{1}{8}$ This fraction is in simplest form.

Work Practice 1–2

Example 3 Multiply and simplify: $\frac{6}{7} \cdot \frac{14}{27}$

Solution:

$$\frac{6}{7} \cdot \frac{14}{27} = \frac{6 \cdot 14}{7 \cdot 27}$$

We can simplify by finding the prime factorizations and using our shortcut procedure of dividing out common factors in the numerator and denominator.

$$\frac{6 \cdot 14}{7 \cdot 27} = \frac{2 \cdot \overset{1}{\cancel{3}} \cdot 2 \cdot \overset{1}{\cancel{7}}}{\overset{1}{\cancel{7}} \cdot \overset{1}{\cancel{3}} \cdot 3 \cdot 3} = \frac{2 \cdot 2}{3 \cdot 3} = \frac{4}{9}$$

Work Practice 3

Helpful Hint

Remember that the shortcut procedure in Example 3 is the same as removing factors of 1 in the product.

$$\frac{6 \cdot 14}{7 \cdot 27} = \frac{2 \cdot 3 \cdot 2 \cdot 7}{7 \cdot 3 \cdot 3 \cdot 3} = \frac{\cancel{7} \cdot \cancel{3} \cdot 2 \cdot 2}{\cancel{7} \cdot \cancel{3} \cdot 3 \cdot 3} = 1 \cdot 1 \cdot \frac{4}{9} = \frac{4}{9}$$

Example 4 Multiply and simplify: $\frac{23}{32} \cdot \frac{4}{7}$

Solution: Notice that 4 and 32 have a common factor of 4.

$$\frac{23}{32} \cdot \frac{4}{7} = \frac{23 \cdot 4}{32 \cdot 7} = \frac{23 \cdot \cancel{4}^1}{\cancel{4}^1 \cdot 8 \cdot 7} = \frac{23}{8 \cdot 7} = \frac{23}{56}$$

Work Practice 4

After multiplying two fractions, always check to see whether the product can be simplified.

Example 5 Multiply: $-\frac{1}{4} \cdot \frac{1}{2}$

Solution: Recall that the product of a negative number and a positive number is a negative number.

$$-\frac{1}{4} \cdot \frac{1}{2} = -\frac{1 \cdot 1}{4 \cdot 2} = -\frac{1}{8}$$

Work Practice 5

Examples Multiply.

$$6. \left(-\frac{6}{13}\right)\left(-\frac{26}{30}\right) = \frac{6 \cdot 26}{13 \cdot 30} = \frac{\cancel{6}^1 \cdot \cancel{13}^1 \cdot 2}{\cancel{13}^1 \cdot \cancel{6}^1 \cdot 5} = \frac{2}{5} \quad \text{The product of two negative numbers is a positive number.}$$

$$7. \frac{1}{3} \cdot \frac{2}{5} \cdot \frac{9}{16} = \frac{1 \cdot 2 \cdot 9}{3 \cdot 5 \cdot 16} = \frac{\cancel{1}^1 \cdot \cancel{2}^1 \cdot \cancel{3}^1 \cdot 3}{\cancel{3}^1 \cdot 5 \cdot \cancel{2}^1 \cdot 8} = \frac{3}{40}$$

Work Practice 6–7

We multiply fractions in the same way if variables are involved.

Example 8 Multiply: $\frac{3x}{4} \cdot \frac{8}{5x}$

Solution: Notice that 8 and 4 have a common factor of 4.

$$\frac{3x}{4} \cdot \frac{8}{5x} = \frac{3 \cdot \cancel{x}^1 \cdot \cancel{4}^1 \cdot 2}{\cancel{4}^1 \cdot 5 \cdot \cancel{x}^1} = \frac{3 \cdot 1 \cdot 1 \cdot 2}{1 \cdot 5 \cdot 1} = \frac{6}{5}$$

Work Practice 8**Helpful Hint**

Recall that when the denominator of a fraction contains a variable, such as $\frac{8}{5x}$, we assume that the variable does not represent 0.

Practice 4

Multiply and simplify: $\frac{4}{27} \cdot \frac{3}{8}$

Helpful Hint

Don't forget that we may identify common factors that are not prime numbers.

Practice 5

Multiply.

$$\frac{1}{2} \cdot \left(-\frac{11}{28}\right)$$

Practice 6–7

Multiply.

$$6. \left(-\frac{4}{11}\right)\left(-\frac{33}{16}\right)$$

$$7. \frac{1}{6} \cdot \frac{3}{10} \cdot \frac{25}{16}$$

Practice 8

Multiply: $\frac{2}{3} \cdot \frac{3y}{2}$

Answers

4. $\frac{1}{18}$ 5. $-\frac{11}{56}$ 6. $\frac{3}{4}$ 7. $\frac{5}{64}$ 8. y

Practice 9

Multiply: $\frac{a^3}{b^2} \cdot \frac{b}{a^2}$

Practice 10

Evaluate.

a. $\left(\frac{3}{4}\right)^3$ b. $\left(-\frac{4}{5}\right)^2$

Answers

9. $\frac{a}{b}$ 10. a. $\frac{27}{64}$ b. $\frac{16}{25}$

Example 9 Multiply: $\frac{x^2}{y} \cdot \frac{y^3}{x}$

Solution:

$$\frac{x^2}{y} \cdot \frac{y^3}{x} = \frac{x^2 \cdot y^3}{y \cdot x} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{y}} \cdot \overset{1}{\cancel{y}} \cdot y}{\overset{1}{\cancel{y}} \cdot \overset{1}{\cancel{x}}} = \frac{x \cdot y \cdot y}{1} = xy^2$$

Work Practice 9

Objective B Evaluating Expressions with Fractional Bases

The base of an exponential expression can also be a fraction.

$$\left(\frac{1}{3}\right)^4 = \underbrace{\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}}_{\frac{1}{3} \text{ is a factor 4 times}} = \frac{1 \cdot 1 \cdot 1 \cdot 1}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{1}{81}$$

Example 10 Evaluate.

a. $\left(\frac{2}{5}\right)^4 = \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} = \frac{2 \cdot 2 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 5 \cdot 5} = \frac{16}{625}$

b. $\left(-\frac{1}{4}\right)^2 = \left(-\frac{1}{4}\right) \cdot \left(-\frac{1}{4}\right) = \frac{1 \cdot 1}{4 \cdot 4} = \frac{1}{16}$ The product of two negative numbers is a positive number.

Work Practice 10

Objective C Dividing Fractions

Before we can divide fractions, we need to know how to find the **reciprocal** of a fraction.

Reciprocal of a Fraction

Two numbers are **reciprocals** of each other if their product is 1. The reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$ because $\frac{a}{b} \cdot \frac{b}{a} = \frac{a \cdot b}{b \cdot a} = 1$.

Helpful Hint

Every number has a reciprocal except 0. The number 0 has no reciprocal because there is no number a such that $0 \cdot a = 1$.

For example,

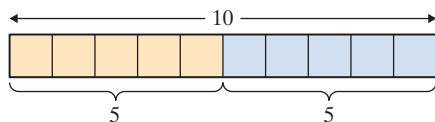
► The reciprocal of $\frac{7}{2}$ is $\frac{2}{7}$ because $\frac{7}{2} \cdot \frac{2}{7} = \frac{14}{14} = 1$.

► The reciprocal of 3 is $\frac{1}{3}$ because $3 \cdot \frac{1}{3} = \frac{3}{1} \cdot \frac{1}{3} = \frac{3}{3} = 1$.

► The reciprocal of $-\frac{5}{6}$ is $-\frac{6}{5}$ because $-\frac{5}{6} \cdot -\frac{6}{5} = \frac{30}{30} = 1$.

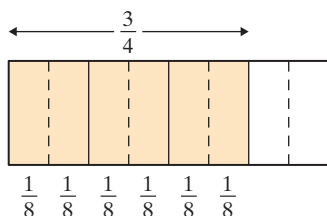
Division of fractions has the same meaning as division of whole numbers. For example,

$10 \div 5$ means: How many 5s are there in 10?



There are two 5s in 10, so
 $10 \div 5 = 2$.

$\frac{3}{4} \div \frac{1}{8}$ means: How many $\frac{1}{8}$ s are there in $\frac{3}{4}$?



There are six $\frac{1}{8}$ s in $\frac{3}{4}$, so $\frac{3}{4} \div \frac{1}{8} = 6$.

We use reciprocals to divide fractions.

Dividing Fractions

To divide two fractions, multiply the first fraction by the reciprocal of the second fraction.

If a , b , c , and d represent numbers, and b , c , and d are not 0, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$$

↑ reciprocal

For example,

$$\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \cdot \frac{8}{1} = \frac{3 \cdot 8}{4 \cdot 1} = \frac{3 \cdot 2 \cdot \cancel{4}}{\cancel{4} \cdot 1} = \frac{6}{1} \text{ or } 6$$

↑ multiply by reciprocal

After dividing fractions, *always* check to see whether the result can be simplified.

Examples

Divide and simplify.

$$11. \frac{5}{16} \div \frac{3}{4} = \frac{5}{16} \cdot \frac{4}{3} = \frac{5 \cdot 4}{16 \cdot 3} = \frac{5 \cdot \cancel{4}}{\cancel{4} \cdot 4 \cdot 3} = \frac{5}{12}$$

$$12. \frac{2}{5} \div \frac{1}{2} = \frac{2}{5} \cdot \frac{2}{1} = \frac{2 \cdot 2}{5 \cdot 1} = \frac{4}{5}$$

Work Practice 11–12

Practice 11–12

Divide and simplify.

$$11. \frac{8}{7} \div \frac{2}{9} \quad 12. \frac{4}{9} \div \frac{1}{2}$$

Answers

$$11. \frac{36}{7} \quad 12. \frac{8}{9}$$

Helpful Hint

When dividing by a fraction, do not look for common factors to divide out until you rewrite the division as multiplication.

Do not try to divide out these two 2s.

$$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$$

Practice 13

Divide: $-\frac{10}{4} \div \frac{2}{9}$

Practice 14

Divide: $\frac{3y}{4} \div 5y^3$

Practice 15

Simplify: $\left(-\frac{2}{3} \cdot \frac{9}{14}\right) \div \frac{7}{15}$

Answers

13. $-\frac{45}{4}$ 14. $\frac{3}{20y^2}$ 15. $-\frac{45}{49}$

✓ Concept Check Answers

a. incorrect b. correct

Example 13

Divide: $-\frac{7}{12} \div -\frac{5}{6}$

Solution: Recall that the quotient (or product) of two negative numbers is a positive number.

$$-\frac{7}{12} \div -\frac{5}{6} = -\frac{7}{12} \cdot -\frac{6}{5} = \frac{7 \cdot \cancel{6}}{2 \cdot \cancel{6} \cdot 5} = \frac{7}{10}$$

Work Practice 13**Example 14**

Divide: $\frac{2x}{3} \div 3x^2$

Solution:

$$\frac{2x}{3} \div 3x^2 = \frac{2x}{3} \div \frac{3x^2}{1} = \frac{2x}{3} \cdot \frac{1}{3x^2} = \frac{2 \cdot \cancel{x} \cdot 1}{3 \cdot 3 \cdot \cancel{x} \cdot x} = \frac{2}{9x}$$

Work Practice 14**Example 15**

Simplify: $\left(\frac{4}{7} \cdot \frac{3}{8}\right) \div -\frac{3}{4}$

Solution: Remember to perform the operations inside the () first.

$$\left(\frac{4}{7} \cdot \frac{3}{8}\right) \div -\frac{3}{4} = \left(\frac{\cancel{4} \cdot 3}{7 \cdot 2 \cdot \cancel{4}}\right) \div -\frac{3}{4} = \frac{3}{14} \div -\frac{3}{4}$$

Now divide.

$$\frac{3}{14} \div -\frac{3}{4} = \frac{3}{14} \cdot -\frac{4}{3} = -\frac{\cancel{3} \cdot 2 \cdot 2}{2 \cdot 7 \cdot \cancel{3}} = -\frac{2}{7}$$

Work Practice 15

✓ Concept Check Which is the correct way to divide $\frac{3}{5}$ by $\frac{5}{12}$? Explain.

a. $\frac{3}{5} \div \frac{5}{12} = \frac{5}{3} \cdot \frac{5}{12}$ b. $\frac{3}{5} \div \frac{5}{12} = \frac{3}{5} \cdot \frac{12}{5}$

Objective D Multiplying and Dividing with Fractional Replacement Values

Recall the difference between an expression and an equation. For example, xy and $x \div y$ are expressions. They contain no equal signs. In Example 16, we practice *simplifying* expressions given fractional replacement values.

Example 16

If $x = \frac{7}{8}$ and $y = -\frac{1}{3}$, evaluate (a) xy and (b) $x \div y$.

Solution: Replace x with $\frac{7}{8}$ and y with $-\frac{1}{3}$.

$$\begin{array}{ll} \text{a. } xy = \frac{7}{8} \cdot -\frac{1}{3} & \text{b. } x \div y = \frac{7}{8} \div -\frac{1}{3} \\ = -\frac{7 \cdot 1}{8 \cdot 3} & = \frac{7}{8} \cdot -\frac{3}{1} \\ = -\frac{7}{24} & = -\frac{7 \cdot 3}{8 \cdot 1} \\ & = -\frac{21}{8} \end{array}$$

Work Practice 16

Example 17

Is $-\frac{2}{3}$ a solution of the equation $-\frac{1}{2}x = \frac{1}{3}$?

Solution: To check whether a number is a solution of an equation, recall that we replace the variable with the given number and see if a true statement results.

$$\begin{array}{ll} -\frac{1}{2} \cdot x = \frac{1}{3} & \text{Recall that } -\frac{1}{2}x \text{ means } -\frac{1}{2} \cdot x. \\ -\frac{1}{2} \cdot -\frac{2}{3} \stackrel{?}{=} \frac{1}{3} & \text{Replace } x \text{ with } -\frac{2}{3}. \\ \frac{1 \cdot 2}{2 \cdot 3} \stackrel{?}{=} \frac{1}{3} & \text{The product of two negative numbers is a positive number.} \\ \frac{1}{1} \stackrel{?}{=} \frac{1}{3} & \text{True} \end{array}$$

Since we have a true statement, $-\frac{2}{3}$ is a solution.

Work Practice 17

Objective E Solving Problems by Multiplying Fractions

To solve real-life problems that involve multiplying fractions, we use our four problem-solving steps from Chapter 3. In Example 18, a new key word that implies multiplication is used. That key word is “of.”

Practice 16

If $x = -\frac{3}{4}$ and $y = \frac{9}{2}$, evaluate (a) xy and (b) $x \div y$.

Practice 17

Is $-\frac{9}{8}$ a solution of the equation $2x = -\frac{9}{4}$?

Helpful Hint

“of” usually translates to multiplication.

Answers

16. a. $-\frac{27}{8}$ b. $-\frac{1}{6}$ 17. yes

Practice 18

Hershey Park is an amusement park in Hershey, Pennsylvania.

Of its 66 rides, $\frac{1}{6}$ of these are roller coasters. How many roller coasters are in Hershey Park?

Example 18 Finding the Number of Roller Coasters in an Amusement Park

Cedar Point is an amusement park located in Sandusky, Ohio. Its collection of 72 rides is the largest in the world. Of the rides, $\frac{2}{9}$ are roller coasters. How many roller coasters are in Cedar Point's collection of rides? (Source: Wikipedia)

**Solution:**

1. UNDERSTAND the problem. To do so, read and reread the problem. We are told that $\frac{2}{9}$ of Cedar Point's rides are roller coasters. The word "of" here means **multiplication**.

2. TRANSLATE.

In words:	number of roller coasters	is	$\frac{2}{9}$	of	total rides at Cedar Point
	↓	↓	↓	↓	↓
Translate:	number of roller coasters	=	$\frac{2}{9}$	·	72

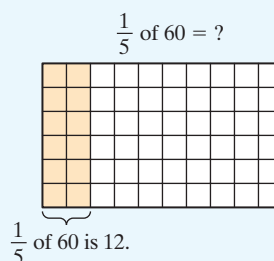
3. SOLVE: Before we solve, let's estimate a reasonable answer. The fraction $\frac{2}{9}$ is less than $\frac{1}{4}$ (draw a diagram, if needed), and $\frac{1}{4}$ of 72 rides is 18 rides, so the number of roller coasters should be less than 18.

$$\frac{2}{9} \cdot 72 = \frac{2}{9} \cdot \frac{72}{1} = \frac{2 \cdot 72}{9 \cdot 1} = \frac{2 \cdot \overset{1}{\cancel{9}} \cdot 8}{\cancel{9} \cdot 1} = \frac{16}{1} \text{ or } 16$$

4. INTERPRET. Check your work. From our estimate, our answer is reasonable. State your conclusion: The number of roller coasters at Cedar Point is 16.

Work Practice 18**Helpful Hint**

To help visualize a fractional part of a whole number, look at the diagram below.



Answer

18. 11 roller coasters

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

multiplication	$\frac{a \cdot d}{b \cdot c}$	$\frac{a \cdot c}{b \cdot d}$	$\frac{2 \cdot 2 \cdot 2}{7}$	$\frac{2}{7} \cdot \frac{2}{7} \cdot \frac{2}{7}$
division	0	reciprocals		

- To multiply two fractions, we write $\frac{a}{b} \cdot \frac{c}{d} =$ _____.
- Two numbers are _____ of each other if their product is 1.
- The expression $\frac{2^3}{7} =$ _____ while $\left(\frac{2}{7}\right)^3 =$ _____.
- Every number has a reciprocal except _____.
- To divide two fractions, we write $\frac{a}{b} \div \frac{c}{d} =$ _____.
- The word “of” indicates _____.

Martin-Gay Interactive Videos



See Video 4.3

Watch the section lecture video and answer the following questions.

- Objective A** 7. In Example 2, how do we know that the answer is negative? ▶
- Objective B** 8. In Example 4, does the exponent apply to the negative sign? Why or why not? ▶
- Objective C** 9. Complete this statement based on Example 5: When writing the reciprocal of a fraction, the denominator becomes the _____, and the numerator becomes the _____. ▶
- Objective D** 10. In Example 11a, why don't we write out the prime factorizations of 4 and 9 in the numerator? ▶
- Objective E** 11. What formula is used to solve Example 12? ▶

4.3 Exercise Set MyLab Math

Objective A Multiply. Write the product in simplest form. See Examples 1 through 9.

- $\frac{6}{11} \cdot \frac{3}{7}$
- $\frac{5}{9} \cdot \frac{7}{4}$
- $-\frac{2}{7} \cdot \frac{5}{8}$
- $\frac{4}{15} \cdot -\frac{1}{20}$
- $-\frac{1}{2} \cdot -\frac{2}{15}$
- $-\frac{3}{11} \cdot -\frac{11}{12}$
- $\frac{18x}{20} \cdot \frac{36}{99}$
- $\frac{5}{32} \cdot \frac{64y}{100}$
- $3a^2 \cdot \frac{1}{4}$
- $-\frac{2}{3} \cdot 6y^3$
- $\frac{x^3}{y^3} \cdot \frac{y^2}{x}$
- $\frac{a}{b^3} \cdot \frac{b}{a^3}$
- $0 \cdot \frac{8}{9}$
- $\frac{11}{12} \cdot 0$
- $-\frac{17y}{20} \cdot \frac{4}{5y}$
- $-\frac{13x}{20} \cdot \frac{5}{6x}$
- $\frac{11}{20} \cdot \frac{1}{7} \cdot \frac{5}{22}$
- $\frac{27}{32} \cdot \frac{10}{13} \cdot \frac{16}{30}$

Objective B Evaluate. See Example 10.

19. $\left(\frac{1}{5}\right)^3$

20. $\left(\frac{8}{9}\right)^2$

21. $\left(-\frac{2}{3}\right)^2$

22. $\left(-\frac{1}{2}\right)^4$

23. $\left(-\frac{2}{3}\right)^3 \cdot \frac{1}{2}$

24. $\left(-\frac{3}{4}\right)^3 \cdot \frac{1}{3}$

Objective C Divide. Write all quotients in simplest form. See Examples 11 through 14.

25. $\frac{2}{3} \div \frac{5}{6}$

26. $\frac{5}{8} \div \frac{3}{4}$

27. $-\frac{6}{15} \div \frac{12}{5}$

28. $-\frac{4}{15} \div -\frac{8}{3}$

29. $-\frac{8}{9} \div \frac{x}{2}$

30. $\frac{10}{11} \div -\frac{4}{5x}$

31. $\frac{11y}{20} \div \frac{3}{11}$

32. $\frac{9z}{20} \div \frac{2}{9}$

33. $-\frac{2}{3} \div 4$

34. $-\frac{5}{6} \div 10$

35. $\frac{1}{5x} \div \frac{5}{x^2}$

36. $\frac{3}{y^2} \div \frac{9}{y^3}$

Objectives A B C Mixed Practice Perform each indicated operation. See Examples 1 through 15.

37. $\frac{2}{3} \cdot \frac{5}{9}$

38. $\frac{8}{15} \cdot \frac{5}{32}$

39. $\frac{3x}{7} \div \frac{5}{6x}$

40. $\frac{2}{5y} \div \frac{5y}{11}$

41. $\frac{16}{27y} \div \frac{8}{15y}$

42. $\frac{12y}{21} \div \frac{4y}{7}$

43. $-\frac{5}{28} \cdot \frac{35}{25}$

44. $\frac{24}{45} \cdot -\frac{5}{8}$

45. $\left(-\frac{3}{4}\right)^2$

46. $\left(-\frac{1}{2}\right)^5$

47. $\frac{x^2}{y} \cdot \frac{y^3}{x}$

48. $\frac{b}{a^2} \cdot \frac{a^3}{b^3}$

49. $7 \div \frac{2}{11}$

50. $-100 \div \frac{1}{2}$

51. $-3x \div \frac{x^2}{12}$

52. $-7x^2 \div \frac{14x}{3}$

53. $\left(\frac{2}{7} \div \frac{7}{2}\right) \cdot \frac{3}{4}$

54. $\frac{1}{2} \cdot \left(\frac{5}{6} \div \frac{1}{12}\right)$

55. $-\frac{19}{63y} \cdot 9y^2$

56. $16a^2 \cdot -\frac{31}{24a}$

57. $-\frac{2}{3} \cdot -\frac{6}{11}$

58. $-\frac{1}{5} \cdot -\frac{6}{7}$

59. $\frac{4}{8} \div \frac{3}{16}$

60. $\frac{9}{2} \div \frac{16}{15}$

61. $\frac{21x^2}{10y} \div \frac{14x}{25y}$

62. $\frac{17y^2}{24x} \div \frac{13y}{18x}$

63. $\left(1 \div \frac{3}{4}\right) \cdot \frac{2}{3}$

64. $\left(33 \div \frac{2}{11}\right) \cdot \frac{5}{9}$

65. $\frac{a^3}{2} \div 30a^3$

66. $15c^3 \div \frac{3c^2}{5}$

67. $\frac{ab^2}{c} \cdot \frac{c}{ab}$

68. $\frac{ac}{b} \cdot \frac{b^3}{a^2c}$

69. $\left(\frac{1}{2} \cdot \frac{2}{3}\right) \div \frac{5}{6}$

70. $\left(\frac{3}{4} \cdot \frac{8}{9}\right) \div \frac{2}{5}$

71. $-\frac{4}{7} \div \left(\frac{4}{5} \cdot \frac{3}{7}\right)$

72. $\frac{5}{8} \div \left(\frac{4}{7} \cdot -\frac{5}{16}\right)$

Objective D Given the following replacement values, evaluate (a) xy and (b) $x \div y$. See Example 16.

73. $x = \frac{2}{5}$ and $y = \frac{5}{6}$

74. $x = \frac{8}{9}$ and $y = \frac{1}{4}$

75. $x = -\frac{4}{5}$ and $y = \frac{9}{11}$

76. $x = \frac{7}{6}$ and $y = -\frac{1}{2}$

Determine whether the given replacement values are solutions of the given equations. See Example 17.

77. Is $-\frac{5}{18}$ a solution to $3x = -\frac{5}{6}$?

78. Is $\frac{9}{11}$ a solution to $\frac{2}{3}y = \frac{6}{11}$?

79. Is $\frac{2}{5}$ a solution to $-\frac{1}{2}z = \frac{1}{10}$?

80. Is $\frac{3}{5}$ a solution to $5x = \frac{1}{3}$?

Objective E Translating Solve. Write each answer in simplest form. For Exercises 81 through 84, recall that “of” translates to multiplication. See Example 18.

81. Find $\frac{1}{4}$ of 200.

82. Find $\frac{1}{5}$ of 200.

83. Find $\frac{5}{6}$ of 24.

84. Find $\frac{5}{8}$ of 24.

Solve. For Exercises 85 and 86, the solutions have been started for you. See Example 18.

85. In the United States, $\frac{7}{50}$ of college freshmen major in business. A community college in Pennsylvania has a freshman enrollment of approximately 800 students. How many of these freshmen might we project are majoring in business?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blank below.)

freshmen majoring in business	is	$\frac{7}{50}$	of	community college freshmen enrollment
↓	↓	↓	↓	↓
freshmen majoring in business	=	$\frac{7}{50}$	·	_____

Finish with:

3. SOLVE

4. INTERPRET

86. A patient was told that, at most, $\frac{1}{5}$ of his calories should come from fat. If his diet consists of 3000 calories a day, find the maximum number of calories that can come from fat.

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blank below.)

patient's fat calories	is	$\frac{1}{5}$	of	his daily calories
↓	↓	↓	↓	↓
patient's fat calories	=	$\frac{1}{5}$	·	_____

Finish with:

3. SOLVE

4. INTERPRET

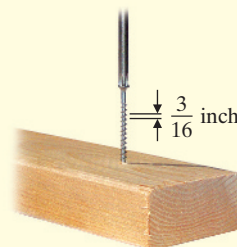
87. In 2015, there were approximately 240 million moviegoers in the United States and Canada. Of these, about $\frac{7}{20}$ viewed at least one 3-D movie. Find the approximate number of people who viewed at least one 3-D movie. (Source: Motion Picture Association of America)

88. In 2015, movie theater owners received a total of \$11,100 million in movie admission tickets. About $\frac{3}{20}$ of this amount was for 3-D movies. Find the amount of money received from 3-D movies. (Source: Motion Picture Association of America)

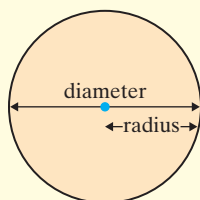
89. The Oregon National Historic Trail is 2170 miles long. It begins in Independence, Missouri, and ends in Oregon City, Oregon. Manfred Coulon has hiked $\frac{2}{5}$ of the trail before. How many miles has he hiked?
(Source: National Park Service)



90. Each turn of a screw sinks it $\frac{3}{16}$ of an inch deeper into a piece of wood. Find how deep the screw is after 8 turns.



91. The radius of a circle is one-half of its diameter, as shown. If the diameter of a circle is $\frac{3}{8}$ of an inch, what is its radius?



92. The diameter of a circle is twice its radius, as shown in the Exercise 91 illustration. If the radius of a circle is $\frac{7}{20}$ of a foot, what is its diameter?

93. A special on a cruise to the Bahamas is advertised to be $\frac{2}{3}$ of the regular price. If the regular price is \$2757, what is the sale price?

94. A family recently sold their house for \$102,000, but $\frac{3}{50}$ of this amount goes to the real estate companies that helped them sell their house. How much money does the family pay to the real estate companies?

95. The state of Mississippi houses $\frac{1}{184}$ of the total U.S. libraries. If there are about 9200 libraries in the United States, how many libraries are in Mississippi?

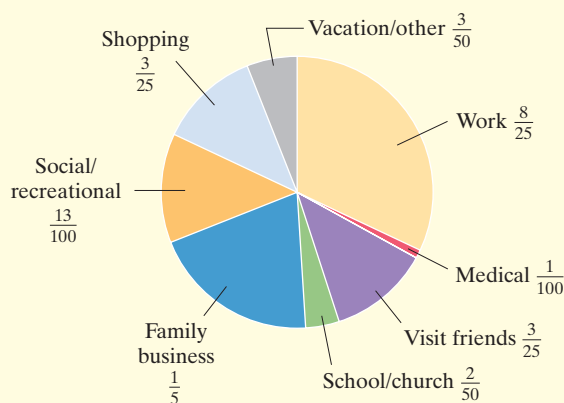
96. There have been about 595 contestants on the reality television show *Survivor* over 33 seasons. Some of these contestants have appeared in multiple seasons. If the number of repeat contestants is $\frac{19}{119}$ of the total number of participants in the first 33 seasons, how many contestants have participated more than once?
(Source: Survivor.com)

Find the area of each rectangle. Recall that $\text{area} = \text{length} \cdot \text{width}$.

97. $\frac{5}{14}$ foot
 $\frac{1}{5}$ foot

98. $\frac{1}{2}$ mile
 $\frac{3}{8}$ mile

Recall from Section 4.2 that the following graph is called a **circle graph** or **pie chart**. Each sector (shaped like a piece of pie) shows the fractional part of a car's total mileage that falls into a particular category. The whole circle represents a car's total mileage.



Source: The American Automobile Manufacturers Association and The National Automobile Dealers Association

In one year, a family drove 12,000 miles in the family car. Use the circle graph to determine how many of these miles might be expected to fall in the categories shown in Exercises 99 through 102.

99. Work

100. Shopping

101. Family business

102. Medical

Review

Perform each indicated operation. See Section 1.3.

$$\begin{array}{r} 103. \quad 27 \\ 76 \\ + 98 \\ \hline \end{array}$$

$$\begin{array}{r} 104. \quad 811 \\ 42 \\ + 69 \\ \hline \end{array}$$

$$\begin{array}{r} 105. \quad 968 \\ - 772 \\ \hline \end{array}$$

$$\begin{array}{r} 106. \quad 882 \\ - 773 \\ \hline \end{array}$$

Concept Extensions

107. In your own words, describe how to divide fractions.

108. In your own words, explain how to multiply fractions.

Simplify.

$$109. \quad \frac{42}{25} \cdot \frac{125}{36} \div \frac{7}{6}$$

$$110. \quad \left(\frac{8}{13} \cdot \frac{39}{16} \cdot \frac{8}{9} \right)^2 \div \frac{1}{2}$$

111. Approximately $\frac{1}{8}$ of the U.S. population lives in the state of California. If the U.S. population is approximately 325,040,000, find the approximate population of California. (Source: U.S. Census Bureau)

112. In 2016, there were approximately 4725 degree-granting postsecondary institutions in the United States. Of these, approximately $\frac{9}{25}$ were two-year colleges. How many two-year colleges were there in 2016? (Round to the nearest whole.) (Source: National Center for Educational Statistics)

113. The National Park Service is charged with maintaining 27,000 historic structures. Monuments and statues make up $\frac{63}{200}$ of these historic structures.

How many monuments and statues is the National Park Service charged with maintaining? (Source: National Park Service)

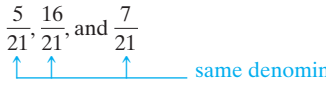
114. If $\frac{3}{4}$ of 36 students on a first bus are girls and $\frac{2}{3}$ of the 30 students on a second bus are boys, how many students on the two buses are girls?

4.4 Adding and Subtracting Like Fractions, Least Common Denominator, and Equivalent Fractions

Objectives

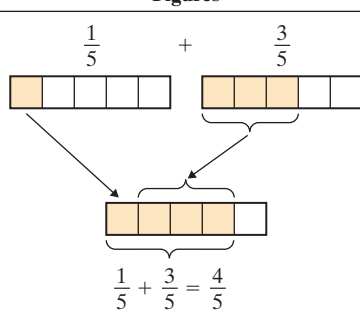
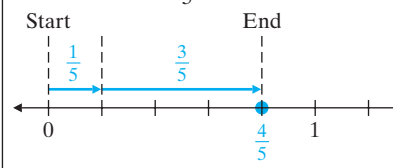
- A** Add or Subtract Like Fractions.
- B** Add or Subtract Given Fractional Replacement Values.
- C** Solve Problems by Adding or Subtracting Like Fractions.
- D** Find the Least Common Denominator of a List of Fractions.
- E** Write Equivalent Fractions.

Fractions with the same denominator are called **like fractions**. Fractions that have different denominators are called **unlike fractions**.

Like Fractions	Unlike Fractions
$\frac{2}{5}$ and $\frac{3}{5}$  $\frac{5}{21}$, $\frac{16}{21}$, and $\frac{7}{21}$ 	$\frac{2}{5}$ and $\frac{3}{4}$  $\frac{5}{7}$ and $\frac{5}{9}$ 

Objective A Adding or Subtracting Like Fractions

To see how we add like fractions (fractions with the same denominator), study one or both illustrations below.

Add: $\frac{1}{5} + \frac{3}{5}$	
Figures	Number Line
 $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$	To add $\frac{1}{5} + \frac{3}{5}$, start at 0 and draw an arrow $\frac{1}{5}$ of a unit long pointing to the right. From the tip of this arrow, draw an arrow $\frac{3}{5}$ of a unit long also pointing to the right. The tip of the second arrow ends at their sum, $\frac{4}{5}$.  Start End $\frac{1}{5}$ $\frac{3}{5}$ $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$
Thus, $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$.	

Notice that the numerator of the sum is the sum of the numerators. Also, the denominator of the sum is the **common denominator**. This is how we add fractions. Similar illustrations can be shown for subtracting fractions.

Adding or Subtracting Like Fractions (Fractions with the Same Denominator)

If a , b , and c are numbers and b is not 0, then

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \text{and also} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

In other words, to add or subtract fractions with the same denominator, add or subtract their numerators and write the sum or difference over the **common denominator**.

For example,

$$\frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{3}{4} \quad \begin{array}{l} \text{Add the numerators.} \\ \text{Keep the denominator.} \end{array}$$

$$\frac{4}{5} - \frac{2}{5} = \frac{4-2}{5} = \frac{2}{5} \quad \begin{array}{l} \text{Subtract the numerators.} \\ \text{Keep the denominator.} \end{array}$$

Helpful Hint

As usual, don't forget to write all answers in simplest form.

Examples

Add and simplify.

$$1. \frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7} \quad \begin{array}{l} \leftarrow \text{Add the numerators.} \\ \leftarrow \text{Keep the common denominator.} \end{array}$$

$$2. \frac{3}{16x} + \frac{7}{16x} = \frac{3+7}{16x} = \frac{10}{16x} = \frac{\overset{1}{\cancel{2}} \cdot 5}{\underset{1}{\cancel{2}} \cdot 8 \cdot x} = \frac{5}{8x}$$

$$3. \frac{7}{8} + \frac{6}{8} + \frac{3}{8} = \frac{7+6+3}{8} = \frac{16}{8} \text{ or } 2$$

Work Practice 1–3

✓ **Concept Check** Find and correct the error in the following:

$$\frac{1}{5} + \frac{1}{5} = \frac{2}{10}$$

Examples

Subtract and simplify.

$$4. \frac{8}{9} - \frac{1}{9} = \frac{8-1}{9} = \frac{7}{9} \quad \begin{array}{l} \leftarrow \text{Subtract the numerators.} \\ \leftarrow \text{Keep the common denominator.} \end{array}$$

$$5. \frac{7}{8} - \frac{5}{8} = \frac{7-5}{8} = \frac{2}{8} = \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \cdot 4} = \frac{1}{4}$$

Work Practice 4–5

From our earlier work, we know that

$$\frac{-12}{6} = \frac{12}{-6} = -\frac{12}{6} \quad \text{since these all simplify to } -2.$$

In general, the following is true:

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b} \quad \text{as long as } b \text{ is not } 0.$$

Example 6

Add: $-\frac{11}{8} + \frac{6}{8}$

$$\begin{aligned} \text{Solution: } -\frac{11}{8} + \frac{6}{8} &= \frac{-11+6}{8} \\ &= \frac{-5}{8} \quad \text{or} \quad -\frac{5}{8} \end{aligned}$$

Work Practice 6

Practice 1–3

Add and simplify.

$$1. \frac{6}{13} + \frac{2}{13}$$

$$2. \frac{5}{8x} + \frac{1}{8x}$$

$$3. \frac{20}{11} + \frac{6}{11} + \frac{7}{11}$$

Practice 4–5

Subtract and simplify.

$$4. \frac{11}{12} - \frac{6}{12}$$

$$5. \frac{7}{15} - \frac{2}{15}$$

Practice 6

$$\text{Add: } -\frac{8}{17} + \frac{4}{17}$$

Answers

$$1. \frac{8}{13} \quad 2. \frac{3}{4x} \quad 3. 3 \quad 4. \frac{5}{12} \quad 5. \frac{1}{3}$$

$$6. -\frac{4}{17}$$

✓ **Concept Check Answer**

We don't add denominators together;

$$\text{correct solution: } \frac{1}{5} + \frac{1}{5} = \frac{2}{5}.$$

Practice 7

Subtract: $\frac{2}{5} - \frac{7y}{5}$

Practice 8

Subtract: $\frac{4}{11} - \frac{6}{11} - \frac{3}{11}$

Practice 9

Evaluate $x + y$ if $x = -\frac{10}{12}$ and $y = \frac{5}{12}$.

Answers

7. $\frac{2-7y}{5}$ 8. $-\frac{5}{11}$ 9. $-\frac{5}{12}$

✓ **Concept Check Answer**
equation; solved; expression; simplified

Example 7

Subtract: $\frac{3x}{4} - \frac{7}{4}$

Solution: $\frac{3x}{4} - \frac{7}{4} = \frac{3x-7}{4}$

Recall from Section 3.1 that the terms in the numerator are unlike terms and cannot be combined.

Work Practice 7**Example 8**

Subtract: $\frac{3}{7} - \frac{6}{7} - \frac{3}{7}$

Solution: $\frac{3}{7} - \frac{6}{7} - \frac{3}{7} = \frac{3-6-3}{7} = \frac{-6}{7}$ or $-\frac{6}{7}$

Work Practice 8**Helpful Hint**

Recall that $\frac{-6}{7} = -\frac{6}{7}$ (Also, $\frac{6}{-7} = -\frac{6}{7}$, if needed.)

Objective B Adding or Subtracting Given Fractional Replacement Values

Example 9

Evaluate $y - x$ if $x = -\frac{3}{10}$ and $y = -\frac{8}{10}$.

Solution: Be very careful when replacing x and y with replacement values.

$$\begin{aligned} y - x &= -\frac{8}{10} - \left(-\frac{3}{10}\right) && \text{Replace } x \text{ with } -\frac{3}{10} \text{ and } y \text{ with } -\frac{8}{10}. \\ &= \frac{-8 - (-3)}{10} \\ &= \frac{-5}{10} = \frac{-1 \cdot 5}{2 \cdot 5} = \frac{-1}{2} \text{ or } -\frac{1}{2} \end{aligned}$$

Work Practice 9

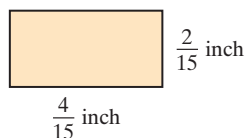
✓ **Concept Check** Fill in each blank with the best choice given.

expression equation simplified solved
A(n) _____ contains an equal sign and may be _____ for the variable.

A(n) _____ does not contain an equal sign but may be _____.

Objective C Solving Problems by Adding or Subtracting Like Fractions

Many real-life problems involve finding the perimeters of square or rectangular-shaped figures such as pastures, swimming pools, and so on. We can use our knowledge of adding fractions to find perimeters.


Example 10 Find the perimeter of the rectangle.


Solution: Recall that perimeter means distance around and that opposite sides of a rectangle are the same length.

$$\begin{array}{l} \frac{4}{15} \text{ inch} \\ \frac{2}{15} \text{ inch} \end{array} \quad \begin{array}{c} \text{Perimeter} = \frac{2}{15} + \frac{4}{15} + \frac{2}{15} + \frac{4}{15} = \frac{2 + 4 + 2 + 4}{15} \\ = \frac{12}{15} = \frac{\cancel{3} \cdot 4}{\cancel{3} \cdot 5} = \frac{4}{5} \end{array}$$

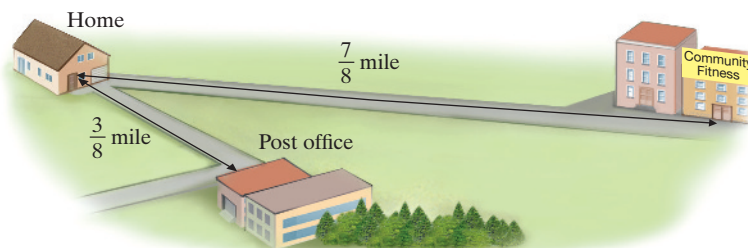
The perimeter of the rectangle is $\frac{4}{5}$ inch.

Work Practice 10

We can combine our skills in adding and subtracting fractions with our four problem-solving steps from Section 3.4 to solve many kinds of real-life problems.

Example 11 Calculating Distance

The distance from home to the Community Fitness center is $\frac{7}{8}$ of a mile and from home to the post office is $\frac{3}{8}$ of a mile. How much farther is it from home to the Community Fitness center than from home to the post office?


Solution:

1. UNDERSTAND. Read and reread the problem. The phrase “How much farther” tells us to subtract distances.
2. TRANSLATE.

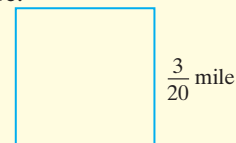
In words:	distance farther	is	home to Community Fitness distance	minus	home to post office distance
Translate:	distance farther	=	$\frac{7}{8}$	−	$\frac{3}{8}$

$$3. \text{ SOLVE: } \frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{\cancel{2} \cdot 2}{\cancel{2} \cdot 4} = \frac{1}{2}$$

4. INTERPRET. Check your work. State your conclusion: The distance from home to Community Fitness is $\frac{1}{2}$ mile farther than from home to the post office.

Work Practice 11
Practice 10

Find the perimeter of the square.


Practice 11

A jogger ran $\frac{13}{4}$ miles on Monday and $\frac{11}{4}$ miles on Wednesday. How much farther did he run on Monday than on Wednesday?

Answers

10. $\frac{3}{5}$ mi 11. $\frac{1}{2}$ mi

Objective D Finding the Least Common Denominator

In the next section, we will add and subtract fractions that have different, or unlike, denominators. To do so, we first write them as equivalent fractions with a common denominator.

Although any common denominator can be used to add or subtract unlike fractions, we will use the **least common denominator (LCD)**. The LCD of a list of fractions is the same as the **least common multiple (LCM)** of the denominators. Why do we use this number as the common denominator? Since the LCD is the *smallest* of all common denominators, operations are usually less tedious with this number.

The **least common denominator (LCD)** of a list of fractions is the smallest positive number divisible by all the denominators in the list. (The least common denominator is also the **least common multiple (LCM) of the denominators**.)

For example, the LCD of $\frac{1}{4}$ and $\frac{3}{10}$ is 20 because 20 is the smallest positive number divisible by both 4 and 10.

Finding the LCD: Method 1

One way to find the LCD is to see whether the larger denominator is divisible by the smaller denominator. If so, the larger number is the LCD. If not, then check consecutive multiples of the larger denominator until the LCD is found.

Method 1: Finding the LCD of a List of Fractions Using Multiples of the Largest Number

Step 1: Write the multiples of the largest denominator (starting with the number itself) until a multiple common to all denominators in the list is found.

Step 2: The multiple found in Step 1 is the LCD.

Practice 12

Find the LCD of $\frac{7}{8}$ and $\frac{11}{16}$.

Practice 13

Find the LCD of $\frac{23}{25}$ and $\frac{1}{30}$.

Example 12 Find the LCD of $\frac{3}{7}$ and $\frac{5}{14}$.

Solution: We write the multiples of 14 until we find one that is also a multiple of 7.

$$14 \cdot 1 = 14 \quad \text{A multiple of 7}$$

The LCD is 14.

Work Practice 12

Example 13 Find the LCD of $\frac{11}{12}$ and $\frac{7}{20}$.

Solution: We write the multiples of 20 until we find one that is also a multiple of 12.

$$20 \cdot 1 = 20 \quad \text{Not a multiple of 12}$$

$$20 \cdot 2 = 40 \quad \text{Not a multiple of 12}$$

$$20 \cdot 3 = 60 \quad \text{A multiple of 12}$$

The LCD is 60.

Work Practice 13

Answers

12. 16 13. 150

Method 1 for finding multiples works fine for smaller numbers, but may get tedious for larger numbers. For this reason, let's study a second method, which uses prime factorization.

Finding the LCD: Method 2

For example, to find the LCD of $\frac{11}{12}$ and $\frac{7}{20}$, such as in Example 13, let's look at the prime factorization of each denominator.

$$12 = 2 \cdot 2 \cdot 3$$

$$20 = 2 \cdot 2 \cdot 5$$

Recall that the LCD must be a multiple of both 12 and 20. Thus, to build the LCD, we will circle the greatest number of factors for each different prime number. The LCD is the product of the circled factors.

Prime Number Factors

$$\begin{array}{l} 12 = \boxed{2 \cdot 2} \boxed{3} \\ 20 = \boxed{2 \cdot 2} \boxed{5} \end{array} \quad \text{Circle either pair of 2s, but not both.}$$

$$\text{LCD} = 2 \cdot 2 \cdot 3 \cdot 5 = 60$$

The number 60 is the smallest number that both 12 and 20 divide into evenly. This method is summarized below:

Method 2: Finding the LCD of a List of Denominators Using Prime Factorization

Step 1: Write the prime factorization of each denominator.

Step 2: For each different prime factor in Step 1, circle the *greatest* number of times that factor occurs in any one factorization.

Step 3: The LCD is the product of the circled factors.

Example 14 Find the LCD of $-\frac{23}{72}$ and $\frac{17}{60}$.

Solution: First we write the prime factorization of each denominator.

$$72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$$

$$60 = 2 \cdot 2 \cdot 3 \cdot 5$$

For the prime factors shown, we circle the greatest number of factors found in either factorization.

$$72 = \boxed{2 \cdot 2 \cdot 2} \cdot \boxed{3 \cdot 3}$$

$$60 = 2 \cdot 2 \cdot 3 \cdot \boxed{5}$$

The LCD is the product of the circled factors.

$$\text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 360$$

The LCD is 360.

Work Practice 14

Practice 14

Find the LCD of $-\frac{3}{40}$ and $\frac{11}{108}$.

Helpful Hint

If you prefer working with exponents, circle the factor with the greatest exponent.

Example 14 :

$$72 = \boxed{2^3} \cdot \boxed{3^2}$$

$$60 = 2^2 \cdot 3 \cdot \boxed{5}$$

$$\text{LCD} = 2^3 \cdot 3^2 \cdot 5 = 360$$

Answer

14. 1080

Helpful Hint

If the number of factors of a prime number is equal, circle either one, but not both. For example,

$$\begin{aligned} 12 &= \underline{2 \cdot 2} \cdot \textcircled{3} \\ 15 &= 3 \cdot \textcircled{5} \end{aligned} \quad \text{Circle either 3 but not both.}$$

The LCD is $2 \cdot 2 \cdot 3 \cdot 5 = 60$.

Practice 15

Find the LCD of $\frac{7}{20}$, $\frac{1}{24}$, and $\frac{13}{45}$.

Practice 16

Find the LCD of $\frac{7}{y}$ and $\frac{6}{11}$.

Example 15

Find the LCD of $\frac{1}{15}$, $\frac{5}{18}$, and $\frac{53}{54}$.

$$\begin{aligned} \text{Solution: } 15 &= 3 \cdot \textcircled{5} \\ 18 &= \textcircled{2} \cdot 3 \cdot 3 \\ 54 &= 2 \cdot \textcircled{3 \cdot 3 \cdot 3} \end{aligned}$$

The LCD is $2 \cdot 3 \cdot 3 \cdot 3 \cdot 5$ or 270.

Work Practice 15**Example 16**

Find the LCD of $\frac{3}{5}$, $\frac{2}{x}$, and $\frac{7}{x^3}$.

$$\begin{aligned} \text{Solution: } 5 &= \textcircled{5} \\ x &= x \\ x^3 &= \textcircled{x \cdot x \cdot x} \end{aligned}$$

LCD = $5 \cdot x \cdot x \cdot x = 5x^3$

Work Practice 16

✓Concept Check True or false? The LCD of the fractions $\frac{1}{6}$ and $\frac{1}{8}$ is 48.

Objective E Writing Equivalent Fractions

To add or subtract unlike fractions in the next section, we first write equivalent fractions with the LCD as the denominator.

To write $\frac{1}{3}$ as an equivalent fraction with a denominator of 6, we multiply by 1 in the form of $\frac{2}{2}$. Why? Because $3 \cdot 2 = 6$, so the new denominator will become 6, as shown below.

$$\frac{1}{3} = \frac{1}{3} \cdot \frac{2}{2} = \frac{1 \cdot 2}{3 \cdot 2} = \frac{2}{6}$$

$\frac{2}{2} = 1$

$$\text{So } \frac{1}{3} = \frac{2}{6}.$$

To write an equivalent fraction,

$$\frac{a}{b} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a \cdot c}{b \cdot c}$$

where a , b , and c are nonzero numbers.

Helpful Hint

Recall from the Helpful Hint on p. 233 that this is also called the Fundamental Property of Fractions.

$$\frac{a}{b} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a \cdot c}{b \cdot c}$$

Answers

15. 360 16. 11y

✓Concept Check Answer

false; it is 24

✓ **Concept Check** Which of the following is *not* equivalent to $\frac{3}{4}$?

- a. $\frac{6}{8}$ b. $\frac{18}{24}$ c. $\frac{9}{14}$ d. $\frac{30}{40}$

Example 17 Write $\frac{3}{4}$ as an equivalent fraction with a denominator of 20.

$$\frac{3}{4} = \frac{\quad}{20}$$

Solution: In the denominators, since $4 \cdot 5 = 20$, we will multiply by 5 in the form of $\frac{5}{5}$.

$$\frac{3}{4} = \frac{3}{4} \cdot \frac{5}{5} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{15}{20}$$

Thus, $\frac{3}{4} = \frac{15}{20}$.

■ **Work Practice 17**

Helpful Hint

To check Example 17, write $\frac{15}{20}$ in simplest form.

$$\frac{15}{20} = \frac{3 \cdot \cancel{5}}{4 \cdot \cancel{5}} = \frac{3}{4}, \text{ the original fraction.}$$

If the original fraction is in lowest terms, we can check our work by writing the new, equivalent fraction in simplest form. This form should be the original fraction.

✓ **Concept Check** True or false? When the fraction $\frac{2}{9}$ is rewritten as an equivalent fraction with 27 as the denominator, the result is $\frac{2}{27}$.

Example 18 Write an equivalent fraction with the given denominator.

$$\frac{2}{5} = \frac{\quad}{15}$$

Solution: Since $5 \cdot 3 = 15$, we multiply by 3 in the form of $\frac{3}{3}$.

$$\frac{2}{5} = \frac{2}{5} \cdot \frac{3}{3} = \frac{2 \cdot 3}{5 \cdot 3} = \frac{6}{15}$$

Then $\frac{2}{5}$ is equivalent to $\frac{6}{15}$. They both represent the same part of a whole.

■ **Work Practice 18**

Practice 17

Write $\frac{7}{8}$ as an equivalent fraction with a denominator of 56.

$$\frac{7}{8} = \frac{\quad}{56}$$

Practice 18

Write an equivalent fraction with the given denominator.

$$\frac{1}{4} = \frac{\quad}{20}$$

Answers

17. $\frac{49}{56}$ 18. $\frac{5}{20}$

✓ **Concept Check Answers**

c
false; the correct result would be $\frac{6}{27}$

Practice 19

Write an equivalent fraction with the given denominator.

$$\frac{3x}{7} = \frac{\quad}{42}$$

Practice 20

Write an equivalent fraction with the given denominator.

$$4 = \frac{\quad}{6}$$

Practice 21

Write an equivalent fraction with the given denominator.

$$\frac{9}{4x} = \frac{\quad}{36x}$$

Answers

19. $\frac{18x}{42}$ 20. $\frac{24}{6}$ 21. $\frac{81}{36x}$

✓ **Concept Check Answer**
answers may vary

Example 19

Write an equivalent fraction with the given denominator.

$$\frac{9x}{11} = \frac{\quad}{44}$$

Solution: Since $11 \cdot 4 = 44$, we multiply by 1 in the form of $\frac{4}{4}$.

$$\frac{9x}{11} = \frac{9x}{11} \cdot \frac{4}{4} = \frac{9x \cdot 4}{11 \cdot 4} = \frac{36x}{44}$$

Then $\frac{9x}{11}$ is equivalent to $\frac{36x}{44}$.

Work Practice 19**Example 20**

Write an equivalent fraction with the given denominator.

$$3 = \frac{\quad}{7}$$

Solution: Recall that $3 = \frac{3}{1}$. Since $1 \cdot 7 = 7$, multiply by 1 in the form $\frac{7}{7}$.

$$\frac{3}{1} = \frac{3}{1} \cdot \frac{7}{7} = \frac{3 \cdot 7}{1 \cdot 7} = \frac{21}{7}$$

Work Practice 20

Don't forget that when the denominator of a fraction contains a variable, such as $\frac{8}{3x}$, we will assume that the variable does not represent 0. Recall that the denominator of a fraction cannot be 0.

Example 21

Write an equivalent fraction with the given denominator.

$$\frac{8}{3x} = \frac{\quad}{24x}$$

Solution: Since $3x \cdot 8 = 24x$, multiply by 1 in the form $\frac{8}{8}$.

$$\frac{8}{3x} = \frac{8}{3x} \cdot \frac{8}{8} = \frac{8 \cdot 8}{3x \cdot 8} = \frac{64}{24x}$$

Work Practice 21

✓ **Concept Check** What is the first step in writing $\frac{3}{10}$ as an equivalent fraction whose denominator is 100?

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

least common denominator (LCD)

like

$-\frac{a}{b}$

$\frac{a-c}{b}$

$\frac{a+c}{b}$

$-\frac{a}{-b}$

perimeter

unlike

equivalent

- The fractions $\frac{9}{11}$ and $\frac{13}{11}$ are called _____ fractions while $\frac{3}{4}$ and $\frac{1}{3}$ are called _____ fractions.
- $\frac{a}{b} + \frac{c}{b} =$ _____ and $\frac{a}{b} - \frac{c}{b} =$ _____.
- As long as b is not 0, $\frac{-a}{b} = \frac{a}{-b} =$ _____.
- The distance around a figure is called its _____.
- The smallest positive number divisible by all the denominators of a list of fractions is called the _____.
- Fractions that represent the same portion of a whole are called _____ fractions.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 4.4

- Objective A** 7. Complete this statement based on the lecture before Example 1: To add like fractions, we add the _____ and keep the same _____.
- Objective B** 8. In Example 6, why are we told to be careful when substituting the replacement value for y ?
- Objective C** 9. What is the perimeter equation used to solve Example 7? What is the final answer?
- Objective D** 10. In Example 8, the LCD is found to be 45. What does this mean in terms of the specific fractions in the problem?
- Objective E** 11. From Example 10, why can we multiply a fraction by a form of 1 to get an equivalent fraction?

4.4 Exercise Set MyLab Math

Objective A Add and simplify. See Examples 1 through 3 and 6.

1. $\frac{5}{11} + \frac{2}{11}$

2. $\frac{9}{17} + \frac{2}{17}$

3. $\frac{2}{9} + \frac{4}{9}$

4. $\frac{3}{10} + \frac{2}{10}$

5. $-\frac{6}{20} + \frac{1}{20}$

6. $-\frac{3}{8} + \frac{1}{8}$

7. $-\frac{3}{14} + \left(-\frac{4}{14}\right)$

8. $-\frac{5}{24} + \left(-\frac{7}{24}\right)$

9. $\frac{2}{9x} + \frac{4}{9x}$

10. $\frac{3}{10y} + \frac{2}{10y}$

11. $-\frac{7x}{18} + \frac{3x}{18} + \frac{2x}{18}$

12. $-\frac{7z}{15} + \frac{3z}{15} + \frac{z}{15}$

Subtract and simplify. See Examples 4, 5, 7 and 8.

▶ 13. $\frac{10}{11} - \frac{4}{11}$

14. $\frac{9}{13} - \frac{5}{13}$

15. $\frac{7}{8} - \frac{1}{8}$

16. $\frac{5}{6} - \frac{1}{6}$

17. $\frac{1}{y} - \frac{4}{y}$

18. $\frac{4}{z} - \frac{7}{z}$

19. $-\frac{27}{33} - \left(-\frac{8}{33}\right)$

20. $-\frac{37}{45} - \left(-\frac{18}{45}\right)$

21. $\frac{20}{21} - \frac{10}{21} - \frac{17}{21}$

22. $\frac{27}{28} - \frac{5}{28} - \frac{28}{28}$

23. $\frac{7a}{4} - \frac{3}{4}$

24. $\frac{18b}{5} - \frac{3}{5}$

Mixed Practice Perform the indicated operation. See Examples 1 through 8.

25. $-\frac{9}{100} + \frac{99}{100}$

26. $-\frac{15}{200} + \frac{85}{200}$

27. $-\frac{13x}{28} - \frac{13x}{28}$

28. $-\frac{15}{26y} - \frac{15}{26y}$

29. $\frac{9x}{15} + \frac{1}{15}$

30. $\frac{2x}{15} + \frac{7}{15}$

▶ 31. $\frac{7x}{16} - \frac{15x}{16}$

32. $\frac{3}{16z} - \frac{15}{16z}$

▶ 33. $\frac{9}{12} - \frac{7}{12} - \frac{10}{12}$

34. $\frac{1}{8} - \frac{15}{8} + \frac{2}{8}$

35. $\frac{x}{4} + \frac{3x}{4} - \frac{2x}{4} + \frac{x}{4}$

36. $\frac{9y}{8} + \frac{2y}{8} + \frac{5y}{8} - \frac{4y}{8}$

Objective B Evaluate each expression for the given replacement values. See Example 9.

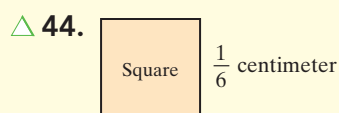
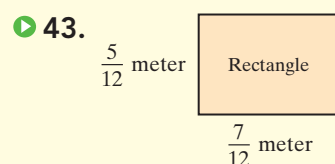
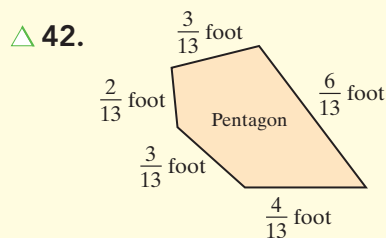
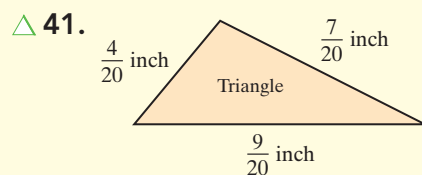
37. $x + y; x = \frac{3}{4}, y = \frac{2}{4}$

38. $x - y; x = \frac{7}{8}, y = \frac{9}{8}$

▶ 39. $x - y; x = -\frac{1}{5}, y = -\frac{3}{5}$

40. $x + y; x = -\frac{1}{6}, y = \frac{5}{6}$

Objective C Find the perimeter of each figure. (Hint: Recall that perimeter means distance around.) See Example 10.



Solve. For Exercises 45 and 46, the solutions have been started for you. Write each answer in simplest form. See Example 11.

45. A railroad inspector must inspect $\frac{19}{20}$ of a mile of railroad track. If she has already inspected $\frac{5}{20}$ of a mile, how much more does she need to inspect?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)

distance left to inspect	is	distance needed to inspect	minus	distance already inspected
↓	↓	↓	↓	↓
distance left to inspect	=	_____	-	_____

Finish with:

3. SOLVE. and 4. INTERPRET.

46. Scott Davis has run $\frac{10}{8}$ miles already and plans to complete $\frac{16}{8}$ miles. To do this, how much farther must he run?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)

distance left to run	is	distance planned to run	minus	distance already run
↓	↓	↓	↓	↓
distance left to run	=	_____	-	_____

Finish with:

3. SOLVE. and 4. INTERPRET.

47. As of early 2017, the fraction of states in the United States with maximum interstate highway speed limits 70 mph or greater was $\frac{41}{50}$. The fraction of states with 70 mph speed limits was $\frac{25}{50}$. What fraction of states had speed limits that were greater than 70 mph? (Source: Insurance Institute for Highway Safety)

48. When people take aspirin, $\frac{31}{50}$ of the time it is used to treat some type of pain. Approximately $\frac{7}{50}$ of all aspirin use is for treating headaches. What fraction of aspirin use is for treating pain other than headaches? (Source: Bayer Market Research)

The map of the world below shows the fraction of the world's surface land area taken up by each continent. In other words, the continent of Africa, for example, makes up $\frac{20}{100}$ of the land in the world. Use this map to solve Exercises 49 through 52. Write answers in simplest form.



49. Find the fractional part of the world's land area within the continents of North America and South America.

50. Find the fractional part of the world's land area within the continents of Asia and Africa.

51. How much greater is the fractional part of the continent of Antarctica than the fractional part of the continent of Europe?

52. How much greater is the fractional part of the continent of Asia than the continent of Australia?

Objective D Find the LCD of each list of fractions. See Examples 12 through 16.

53. $\frac{2}{9}, \frac{6}{15}$

54. $\frac{7}{12}, \frac{3}{20}$

55. $-\frac{1}{36}, \frac{1}{24}$

56. $-\frac{1}{15}, \frac{1}{90}$

57. $\frac{2}{25}, \frac{3}{15}, \frac{5}{6}$

58. $\frac{3}{4}, \frac{1}{14}, \frac{13}{20}$

59. $-\frac{7}{24}, -\frac{5}{x}$

60. $-\frac{11}{y}, -\frac{13}{70}$

61. $\frac{23}{18}, \frac{1}{21}$

62. $\frac{45}{24}, \frac{2}{45}$

63. $\frac{4}{3}, \frac{8}{21}, \frac{3}{56}$

64. $\frac{12}{11}, \frac{20}{33}, \frac{12}{121}$

Objective E Write each fraction as an equivalent fraction with the given denominator. See Examples 17 through 21.

65. $\frac{2}{3} = \frac{\quad}{21}$

66. $\frac{5}{6} = \frac{\quad}{24}$

67. $\frac{4}{7} = \frac{\quad}{35}$

68. $\frac{3}{5} = \frac{\quad}{100}$

69. $\frac{1}{2} = \frac{\quad}{50}$

70. $\frac{1}{5} = \frac{\quad}{50}$

71. $\frac{14x}{17} = \frac{\quad}{68}$

72. $\frac{19z}{21} = \frac{\quad}{126}$

73. $\frac{2y}{3} = \frac{\quad}{12}$

74. $\frac{3x}{2} = \frac{\quad}{12}$

75. $\frac{5}{9} = \frac{\quad}{36a}$

76. $\frac{7}{6} = \frac{\quad}{36a}$

The table on the next page shows the fraction of goods sold online by type of goods in a particular year. Use this table to answer Exercises 77 through 80.

77. Complete the table by writing each fraction as an equivalent fraction with a denominator of 100.

78. Which of these types of goods has the largest fraction sold online?

79. Which of these types of goods has the smallest fraction sold online?

80. Which of the types of goods has **more than** $\frac{3}{5}$ of the goods sold online? (*Hint: Write $\frac{3}{5}$ as an equivalent fraction with a denominator of 100.*)



Type of Goods	Fraction of All Goods That Are Sold Online	Equivalent Fraction with a Denominator of 100
books and magazines	$\frac{27}{50}$	
clothing and accessories	$\frac{1}{2}$	
computer hardware	$\frac{23}{50}$	
computer software	$\frac{1}{2}$	
drugs, health and beauty aids	$\frac{3}{20}$	
electronics and appliances	$\frac{13}{20}$	
food, beer, and wine	$\frac{9}{20}$	
home furnishings	$\frac{13}{25}$	
music and videos	$\frac{3}{5}$	
office equipment and supplies	$\frac{61}{100}$	
sporting goods	$\frac{12}{25}$	
toys, hobbies, and games	$\frac{1}{2}$	

(Source: Fedstats.gov)

Review

Simplify. See Section 1.7.

81. 3^2

82. 4^3

83. 5^3

84. 3^4

85. 7^2

86. 5^4

87. $2^3 \cdot 3$

88. $4^2 \cdot 5$

Concept Extensions

Find and correct the error. See the first Concept Check in this section.

89. $\frac{2}{7} + \frac{9}{7} = \frac{11}{14}$

90. $\frac{3}{4} - \frac{1}{4} = \frac{2}{8} = \frac{1}{4}$

Solve.

 91. In your own words, explain how to add like fractions.

 92. In your own words, explain how to subtract like fractions.

93. Use the map of the world for Exercises 49 through 52 and find the sum of all the continents' fractions. Explain your answer.

94. Mike Cannon jogged $\frac{3}{8}$ of a mile from home and then rested. Then he continued jogging farther from home for another $\frac{3}{8}$ of a mile until he discovered his watch had fallen off. He walked back along the same path for $\frac{4}{8}$ of a mile until he found his watch. Find how far he was from his home.

Write each fraction as an equivalent fraction with the indicated denominator.

95. $\frac{37x}{165} = \frac{\quad}{3630}$

96. $\frac{108}{215y} = \frac{\quad}{4085y}$

97. In your own words, explain how to find the LCD of two fractions.

98. In your own words, explain how to write a fraction as an equivalent fraction with a given denominator.

Solve. See the third and fourth Concept Checks in this section.

99. Which of the following are equivalent to $\frac{2}{3}$?

a. $\frac{10}{15}$

b. $\frac{40}{60}$

c. $\frac{16}{20}$

d. $\frac{200}{300}$

100. True or false? When the fraction $\frac{7}{12}$ is rewritten with a denominator of 48, the result is $\frac{11}{48}$. If false, give the correct fraction.

Objectives

- A** Add or Subtract Unlike Fractions. 
- B** Write Fractions in Order. 
- C** Evaluate Expressions Given Fractional Replacement Values. 
- D** Solve Problems by Adding or Subtracting Unlike Fractions. 

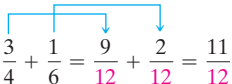
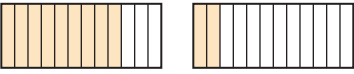
4.5 Adding and Subtracting Unlike Fractions

Objective **A** Adding and Subtracting Unlike Fractions

In this section we add and subtract fractions with unlike denominators. To add or subtract these unlike fractions, we first write the fractions as equivalent fractions with a common denominator and then add or subtract the like fractions. Recall from the previous section that the common denominator we use is called the **least common denominator (LCD)**.

To begin, let's add the unlike fractions $\frac{3}{4} + \frac{1}{6}$.

The LCD of these fractions is 12. So we write each fraction as an equivalent fraction with a denominator of 12, then add as usual. This addition process is shown next and also illustrated by figures.

Add: $\frac{3}{4} + \frac{1}{6}$	
Figures	Algebra
$\frac{3}{4} + \frac{1}{6}$ 	$\frac{3}{4} = \frac{3}{4} \cdot \frac{3}{3} = \frac{9}{12}$ and $\frac{1}{6} = \frac{1}{6} \cdot \frac{2}{2} = \frac{2}{12}$ Remember $\frac{3}{3} = 1$ and $\frac{2}{2} = 1$. Now we can add just as we did in Section 4.4.  $\frac{3}{4} + \frac{1}{6} = \frac{9}{12} + \frac{2}{12} = \frac{11}{12}$
$\frac{9}{12} + \frac{2}{12}$ 	
 $\frac{9}{12} + \frac{2}{12} = \frac{11}{12}$	
Thus, the sum is $\frac{11}{12}$.	

Adding or Subtracting Unlike Fractions

- Step 1:** Find the least common denominator (LCD) of the fractions.
- Step 2:** Write each fraction as an equivalent fraction whose denominator is the LCD.
- Step 3:** Add or subtract the like fractions.
- Step 4:** Write the sum or difference in simplest form.

Example 1 Add: $\frac{2}{5} + \frac{4}{15}$

Solution:

Step 1: The LCD of the fractions is 15. In later examples, we shall simply say, for example, that the LCD of the denominators 5 and 15 is 15.

Step 2: $\frac{2}{5} = \frac{2}{5} \cdot \frac{3}{3} = \frac{6}{15}$, $\frac{4}{15} = \frac{4}{15}$ ← This fraction already has a denominator of 15.
 ↑ Multiply by 1 in the form $\frac{3}{3}$.

Step 3: $\frac{2}{5} + \frac{4}{15} = \frac{6}{15} + \frac{4}{15} = \frac{10}{15}$

Step 4: Write in simplest form.

$$\frac{10}{15} = \frac{2 \cdot \cancel{5}}{3 \cdot \cancel{5}} = \frac{2}{3}$$

Work Practice 1

When the fractions contain variables, we add and subtract the same way.

Practice 1

Add: $\frac{2}{7} + \frac{8}{21}$

Answer

1. $\frac{2}{3}$

Practice 2

Add: $\frac{5y}{6} + \frac{2y}{9}$

Practice 3

Add: $-\frac{1}{5} + \frac{9}{20}$

Practice 4

Subtract: $\frac{5}{7} - \frac{9}{10}$

Answers

2. $\frac{19y}{18}$ 3. $\frac{1}{4}$ 4. $-\frac{13}{70}$

✓ Concept Check Answer

When adding fractions, we don't add the denominators. Correct solution:

$$\frac{2}{9} + \frac{4}{11} = \frac{22}{99} + \frac{36}{99} = \frac{58}{99}$$

Example 2 Add: $\frac{2x}{15} + \frac{3x}{10}$

Solution:

Step 1: The LCD of the denominators 15 and 10 is 30.

Step 2: $\frac{2x}{15} = \frac{2x}{15} \cdot \frac{2}{2} = \frac{4x}{30}$ $\frac{3x}{10} = \frac{3x}{10} \cdot \frac{3}{3} = \frac{9x}{30}$

Step 3: $\frac{2x}{15} + \frac{3x}{10} = \frac{4x}{30} + \frac{9x}{30} = \frac{13x}{30}$

Step 4: $\frac{13x}{30}$ is in simplest form.

Work Practice 2

Example 3 Add: $-\frac{1}{6} + \frac{1}{2}$

Solution: The LCD of the denominators 6 and 2 is 6.

$$\begin{aligned} -\frac{1}{6} + \frac{1}{2} &= \frac{-1}{6} + \frac{1 \cdot 3}{2 \cdot 3} \\ &= \frac{-1}{6} + \frac{3}{6} \\ &= \frac{2}{6} \end{aligned}$$

Next, simplify $\frac{2}{6}$.

$$\frac{2}{6} = \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \cdot 3} = \frac{1}{3}$$

Helpful
Hint

Recall that

$$-\frac{1}{6} = \frac{-1}{6} = \frac{1}{-6}$$

Work Practice 3

✓ **Concept Check** Find and correct the error in the following:

$$\frac{2}{9} + \frac{4}{11} = \frac{6}{20} = \frac{3}{10}$$

Example 4 Subtract: $\frac{2}{3} - \frac{10}{11}$

Solution:

Step 1: The LCD of the denominators 3 and 11 is 33.

Step 2: $\frac{2}{3} = \frac{2}{3} \cdot \frac{11}{11} = \frac{22}{33}$ $\frac{10}{11} = \frac{10}{11} \cdot \frac{3}{3} = \frac{30}{33}$

Step 3: $\frac{2}{3} - \frac{10}{11} = \frac{22}{33} - \frac{30}{33} = \frac{-8}{33}$ or $-\frac{8}{33}$

Step 4: $-\frac{8}{33}$ is in simplest form.

Work Practice 4

Example 5 Find: $-\frac{3}{4} - \frac{1}{14} + \frac{6}{7}$

Solution: The LCD of the denominators 4, 14, and 7 is 28.

$$\begin{aligned} -\frac{3}{4} - \frac{1}{14} + \frac{6}{7} &= -\frac{3 \cdot 7}{4 \cdot 7} - \frac{1 \cdot 2}{14 \cdot 2} + \frac{6 \cdot 4}{7 \cdot 4} \\ &= -\frac{21}{28} - \frac{2}{28} + \frac{24}{28} \\ &= \frac{1}{28} \end{aligned}$$

Work Practice 5

✓Concept Check Find and correct the error in the following:

$$\frac{7}{12} - \frac{3}{4} = \frac{4}{8} = \frac{1}{2}$$

Example 6 Subtract: $2 - \frac{x}{3}$

Solution: Recall that $2 = \frac{2}{1}$. The LCD of the denominators 1 and 3 is 3.

$$\begin{aligned} \frac{2}{1} - \frac{x}{3} &= \frac{2 \cdot 3}{1 \cdot 3} - \frac{x}{3} \\ &= \frac{6}{3} - \frac{x}{3} \\ &= \frac{6 - x}{3} \end{aligned}$$

Helpful Hint

The expression $\frac{6 - x}{3}$ from Example 6 *does not simplify* to $2 - x$. The number 3 must be a factor of both terms in the numerator (not just 6) in order to simplify.

The numerator $6 - x$ cannot be simplified further since 6 and $-x$ are unlike terms.

Work Practice 6

Objective B Writing Fractions in Order

One important application of the least common denominator is to use the LCD to help order or compare fractions.

Example 7 Insert $<$ or $>$ to form a true sentence.

$$\frac{3}{4} \quad \frac{9}{11}$$

Solution: The LCD for these fractions is 44. Let's write each fraction as an equivalent fraction with a denominator of 44.

$$\frac{3}{4} = \frac{3 \cdot 11}{4 \cdot 11} = \frac{33}{44} \qquad \frac{9}{11} = \frac{9 \cdot 4}{11 \cdot 4} = \frac{36}{44}$$

(Continued on next page)

Practice 5

Find: $\frac{5}{8} - \frac{1}{3} - \frac{1}{12}$

Practice 6

Subtract: $5 - \frac{y}{4}$

Practice 7

Insert $<$ or $>$ to form a true sentence.

$$\frac{5}{8} \quad \frac{11}{20}$$

Answers

5. $\frac{5}{24}$ 6. $\frac{20 - y}{4}$ 7. $>$

✓Concept Check Answers

When adding fractions, we don't add the denominators. Correct solutions:

$$\frac{7}{12} - \frac{3}{4} = \frac{7}{12} - \frac{9}{12} = -\frac{2}{12} = -\frac{1}{6}$$

Practice 8

Insert $<$ or $>$ to form a true sentence.

$$-\frac{17}{20} \quad -\frac{4}{5}$$

Practice 9

Evaluate $x - y$ if $x = \frac{5}{11}$ and $y = \frac{4}{9}$.

Since $33 < 36$, then

$$\frac{33}{44} < \frac{36}{44} \text{ or } \frac{3}{4} < \frac{9}{11}$$

Work Practice 7**Example 8**

Insert $<$ or $>$ to form a true sentence.

$$-\frac{2}{7} \quad -\frac{1}{3}$$

Solution: The LCD for these fractions is 21.

$$-\frac{2}{7} = -\frac{2 \cdot 3}{7 \cdot 3} = -\frac{6}{21} \text{ or } \frac{-6}{21} \quad -\frac{1}{3} = -\frac{1 \cdot 7}{3 \cdot 7} = -\frac{7}{21} \text{ or } \frac{-7}{21}$$

Since $-6 > -7$, then

$$-\frac{6}{21} > -\frac{7}{21} \text{ or } -\frac{2}{7} > -\frac{1}{3}$$

Work Practice 8

Objective C Evaluating Expressions Given Fractional Replacement Values

Example 9

Evaluate $x - y$ if $x = \frac{7}{18}$ and $y = \frac{2}{9}$.

Solution: Replace x with $\frac{7}{18}$ and y with $\frac{2}{9}$ in the expression $x - y$.

$$x - y = \frac{7}{18} - \frac{2}{9}$$

The LCD of the denominators 18 and 9 is 18. Then

$$\begin{aligned} \frac{7}{18} - \frac{2}{9} &= \frac{7}{18} - \frac{2 \cdot 2}{9 \cdot 2} \\ &= \frac{7}{18} - \frac{4}{18} \\ &= \frac{3}{18} = \frac{1}{6} \quad \text{Simplified} \end{aligned}$$

Work Practice 9

Objective D Solving Problems by Adding or Subtracting Unlike Fractions

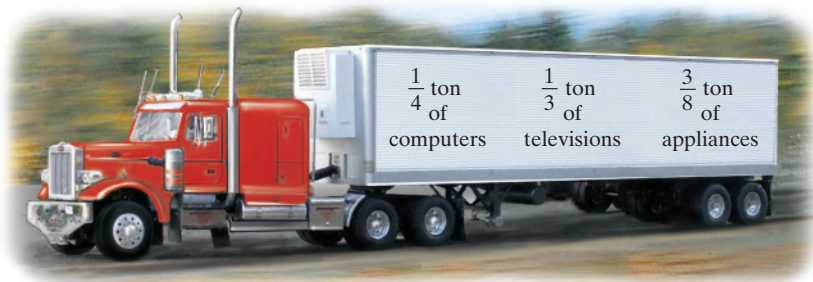
Very often, real-world problems involve adding or subtracting unlike fractions.

Answers

8. $<$ 9. $\frac{1}{99}$

Example 10 Finding Total Weight

A freight truck has $\frac{1}{4}$ of a ton of computers, $\frac{1}{3}$ of a ton of televisions, and $\frac{3}{8}$ of a ton of small appliances. Find the total weight of its load.

**Solution:**

1. **UNDERSTAND.** Read and reread the problem. The phrase “total weight” tells us to add.

2. **TRANSLATE.**

In words:	total weight	is	weight of computers	plus	weight of televisions	plus	weight of appliances
	↓	↓	↓	↓	↓	↓	↓
Translate:	total weight	=	$\frac{1}{4}$	+	$\frac{1}{3}$	+	$\frac{3}{8}$

3. **SOLVE:** The LCD for these fractions is 24.

$$\begin{aligned}\frac{1}{4} + \frac{1}{3} + \frac{3}{8} &= \frac{1}{4} \cdot \frac{6}{6} + \frac{1}{3} \cdot \frac{8}{8} + \frac{3}{8} \cdot \frac{3}{3} \\ &= \frac{6}{24} + \frac{8}{24} + \frac{9}{24} \\ &= \frac{23}{24}\end{aligned}$$

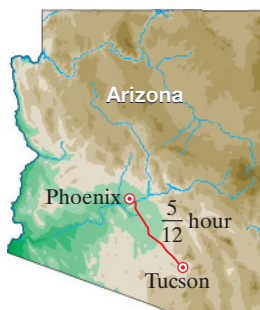
4. **INTERPRET.** Check the solution. State your conclusion: The total weight of the truck’s load is $\frac{23}{24}$ ton.

Work Practice 10**Example 11** Calculating Flight Time

A flight from Tucson to Phoenix, Arizona, requires $\frac{5}{12}$ of an hour. If the plane has been flying $\frac{1}{4}$ of an hour, find how much time remains before landing.

Solution:

1. **UNDERSTAND.** Read and reread the problem. The phrase “how much time remains” tells us to subtract.



(Continued on next page)

Practice 10

To repair her sidewalk, a homeowner must pour cement in three different locations. She needs $\frac{3}{5}$ of a cubic yard, $\frac{3}{10}$ of a cubic yard, and $\frac{1}{15}$ of a cubic yard for these locations. Find the total amount of cement the homeowner needs.

Practice 11

Find the difference in length of two boards if one board is $\frac{3}{4}$ of a foot long and the other is $\frac{2}{3}$ of a foot long.

Answers

10. $\frac{29}{30}$ cu yd 11. $\frac{1}{12}$ ft

2. TRANSLATE.

In words:	time remaining	is	flight time from Tucson to Phoenix	minus	flight time already passed
	↓	↓	↓	↓	↓
Translate:	time remaining	=	$\frac{5}{12}$	—	$\frac{1}{4}$

3. SOLVE: The LCD for these fractions is 12.

$$\begin{aligned}
 \frac{5}{12} - \frac{1}{4} &= \frac{5}{12} - \frac{1}{4} \cdot \frac{3}{3} \\
 &= \frac{5}{12} - \frac{3}{12} \\
 &= \frac{2}{12} = \frac{\cancel{2}}{\cancel{2} \cdot 6} = \frac{1}{6}
 \end{aligned}$$

4. INTERPRET. Check the solution. State your conclusion: The remaining flight time is $\frac{1}{6}$ of an hour.

Work Practice 11



Calculator Explorations Performing Operations on Fractions

Scientific Calculator

Many calculators have a fraction key, such as $\boxed{a^{b/c}}$, that allows you to enter fractions and perform operations on fractions, and gives the result as a fraction. If your calculator has a fraction key, use it to calculate

$$\frac{3}{5} + \frac{4}{7}$$

Enter the keystrokes

$\boxed{3} \boxed{a^{b/c}} \boxed{5} \boxed{+} \boxed{4} \boxed{a^{b/c}} \boxed{7} \boxed{=}$

The display should read $\boxed{1} \boxed{-} \boxed{6} \boxed{35}$

which represents the mixed number $1\frac{6}{35}$. Let's write the result as a fraction. To convert from mixed number notation to fractional notation, press

$\boxed{2^{nd}} \boxed{d/c}$

The display now reads $\boxed{41} \boxed{35}$

which represents $\frac{41}{35}$, the sum in fractional notation.

Graphing Calculator

Graphing calculators also allow you to perform operations on fractions and will give exact fractional results. The fraction option on a graphing calculator may be found under the $\boxed{\text{MATH}}$ menu. To perform the addition in the first column, try the keystrokes

$\boxed{3} \boxed{\div} \boxed{5} \boxed{+} \boxed{4} \boxed{\div} \boxed{7} \boxed{\text{MATH}} \boxed{\text{ENTER}}$
 $\boxed{\text{ENTER}}$

The display should read

$\boxed{3/5 + 4/7 \blacktriangleright \text{Frac } 41/35}$

Use a calculator to add the following fractions. Give each sum as a fraction.

1. $\frac{1}{16} + \frac{2}{5}$

2. $\frac{3}{20} + \frac{2}{25}$

3. $\frac{4}{9} + \frac{7}{8}$

4. $\frac{9}{11} + \frac{5}{12}$

5. $\frac{10}{17} + \frac{12}{19}$

6. $\frac{14}{31} + \frac{15}{21}$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Any numerical answers are not listed. Not all choices may be used.

expression least common denominator >
equation equivalent <

- To add or subtract unlike fractions, we first write the fractions as _____ fractions with a common denominator. The common denominator we use is called the _____.
- The LCD for $\frac{1}{6}$ and $\frac{5}{8}$ is _____.
- $\frac{1}{6} + \frac{5}{8} = \frac{1}{6} \cdot \frac{4}{4} + \frac{5}{8} \cdot \frac{3}{3} = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}.$
- $\frac{1}{6} - \frac{5}{8} = \frac{1}{6} \cdot \frac{4}{4} - \frac{5}{8} \cdot \frac{3}{3} = \underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}.$
- $x - y$ is a(n) _____ while $3x = \frac{1}{5}$ is a(n) _____.
- Since $-10 < -1$, we know that $-\frac{10}{13}$ _____ $-\frac{1}{13}.$

Martin-Gay Interactive Videos



See Video 4.5

Watch the section lecture video and answer the following questions.

- Objective A** 7. In Example 3, why can't we add the two terms in the numerator?
- Objective B** 8. In Example 5, when comparing two fractions, how does writing each fraction with the same denominator help?
- Objective C** 9. In Example 6, if we had chosen to simplify the first fraction before adding, what would our addition problem have become and what would our LCD have been?
- Objective D** 10. What are the two forms of the answer to Example 7?

4.5 Exercise Set MyLab Math

Objective A Add or subtract as indicated. See Examples 1 through 6.

- $\frac{2}{3} + \frac{1}{6}$
- $\frac{5}{6} + \frac{1}{12}$
- $\frac{1}{2} - \frac{1}{3}$
- $\frac{2}{3} - \frac{1}{4}$
- $-\frac{2}{11} + \frac{2}{33}$
- $-\frac{5}{9} + \frac{1}{3}$
- $\frac{3}{14} - \frac{3}{7}$
- $\frac{2}{15} - \frac{2}{5}$
- $\frac{11x}{35} + \frac{2x}{7}$
- $\frac{2y}{5} + \frac{3y}{25}$
- $2 - \frac{y}{12}$
- $5 - \frac{y}{20}$
- $\frac{5}{12} - \frac{1}{9}$
- $\frac{7}{12} - \frac{5}{18}$
- $-7 + \frac{5}{7}$
- $-10 + \frac{7}{10}$

17. $\frac{5a}{11} + \frac{4a}{9}$

18. $\frac{7x}{18} + \frac{2x}{9}$

19. $\frac{2y}{3} - \frac{1}{6}$

20. $\frac{5z}{6} - \frac{1}{12}$

21. $\frac{1}{2} + \frac{3}{x}$

22. $\frac{2}{5} + \frac{3}{x}$

23. $-\frac{2}{11} - \frac{2}{33}$

24. $-\frac{5}{9} - \frac{1}{3}$

25. $\frac{9}{14} - \frac{3}{7}$

26. $\frac{4}{5} - \frac{2}{15}$

27. $\frac{11y}{35} - \frac{2}{7}$

28. $\frac{2b}{5} - \frac{3}{25}$

29. $\frac{1}{9} - \frac{5}{12}$

30. $\frac{5}{18} - \frac{7}{12}$

▶ 31. $\frac{7}{15} - \frac{5}{12}$

32. $\frac{5}{8} - \frac{3}{20}$

33. $\frac{5}{7} - \frac{1}{8}$

34. $\frac{10}{13} - \frac{7}{10}$

▶ 35. $\frac{7}{8} + \frac{3}{16}$

36. $\frac{7}{18} + \frac{2}{9}$

37. $\frac{3}{9} - \frac{5}{9}$

38. $\frac{1}{13} - \frac{4}{13}$

39. $-\frac{2}{5} + \frac{1}{3} - \frac{3}{10}$

40. $-\frac{1}{3} - \frac{1}{4} + \frac{2}{5}$

41. $\frac{5}{11} + \frac{y}{3}$

42. $\frac{5z}{13} + \frac{3}{26}$

43. $-\frac{5}{6} - \frac{3}{7}$

44. $-\frac{1}{2} - \frac{3}{29}$

45. $\frac{x}{2} + \frac{x}{4} + \frac{2x}{16}$

46. $\frac{z}{4} + \frac{z}{8} + \frac{2z}{16}$

47. $\frac{7}{9} - \frac{1}{6}$

48. $\frac{9}{16} - \frac{3}{8}$

49. $\frac{2a}{3} + \frac{6a}{13}$

50. $\frac{3y}{4} + \frac{y}{7}$

51. $\frac{7}{30} - \frac{5}{12}$

52. $\frac{7}{30} - \frac{3}{20}$

▶ 53. $\frac{5}{9} + \frac{1}{y}$

54. $\frac{1}{12} - \frac{5}{x}$

55. $\frac{6}{5} - \frac{3}{4} + \frac{1}{2}$

56. $\frac{6}{5} + \frac{3}{4} - \frac{1}{2}$

57. $\frac{4}{5} + \frac{4}{9}$

58. $\frac{11}{12} - \frac{7}{24}$

59. $\frac{5}{9x} + \frac{1}{8}$

60. $\frac{3}{8} + \frac{5}{12x}$

▶ 61. $-\frac{9}{12} + \frac{17}{24} - \frac{1}{6}$

62. $-\frac{5}{14} + \frac{3}{7} - \frac{1}{2}$

63. $\frac{3x}{8} + \frac{2x}{7} - \frac{5}{14}$

64. $\frac{9x}{10} - \frac{1}{2} + \frac{x}{5}$

Objective B Insert $<$ or $>$ to form a true sentence. See Examples 7 and 8.

▶ 65. $\frac{2}{7} \quad \frac{3}{10}$

66. $\frac{5}{9} \quad \frac{6}{11}$

67. $-\frac{5}{6} \quad -\frac{13}{15}$

68. $-\frac{7}{8} \quad -\frac{5}{6}$

69. $-\frac{3}{4} \quad -\frac{11}{14}$

70. $-\frac{2}{9} \quad -\frac{3}{13}$

Objective C Evaluate each expression if $x = \frac{1}{3}$ and $y = \frac{3}{4}$. See Example 9.

71. $x + y$

72. $x - y$

73. xy

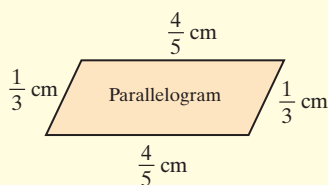
74. $x \div y$

▶ 75. $2y + x$

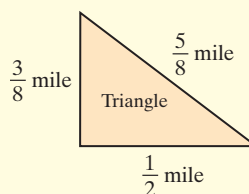
76. $2x + y$

Objective D Find the perimeter of each geometric figure. (Hint: Recall that perimeter means distance around.)

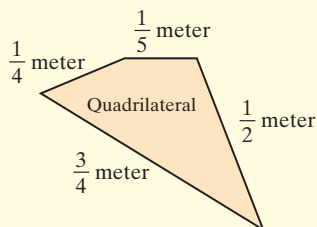
▶ 77.



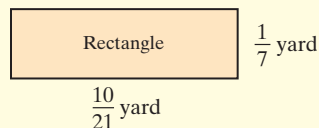
△ 78.



△ 79.



△ 80.



Translating Translate each phrase into an algebraic expression. Use “ x ” to represent “a number.” See Examples 10 and 11.

81. The sum of a number and $\frac{1}{2}$

82. A number increased by $-\frac{2}{5}$

83. A number subtracted from $-\frac{3}{8}$

84. The difference of a number and $\frac{7}{20}$

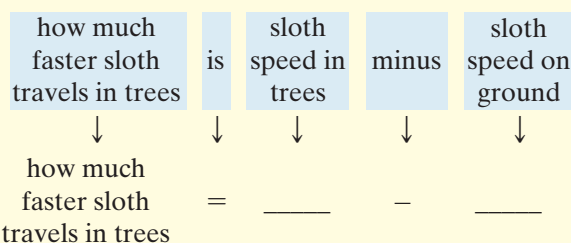
Solve. For Exercises 85 and 86, the solutions have been started for you. See Examples 10 and 11.

85. The slowest mammal is the three-toed sloth from South America. The sloth has an average ground speed of $\frac{1}{10}$ mph. In trees, it can accelerate to $\frac{17}{100}$ mph. How much faster can a sloth travel in trees? (Source: *The Guinness Book of World Records*)

86. Killer bees have been known to chase people for up to $\frac{1}{4}$ of a mile, while domestic European honeybees will normally chase a person for no more than 100 feet, or $\frac{5}{264}$ of a mile. How much farther will a killer bee chase a person than a domestic honeybee? (Source: Coachella Valley Mosquito & Vector Control District)

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)

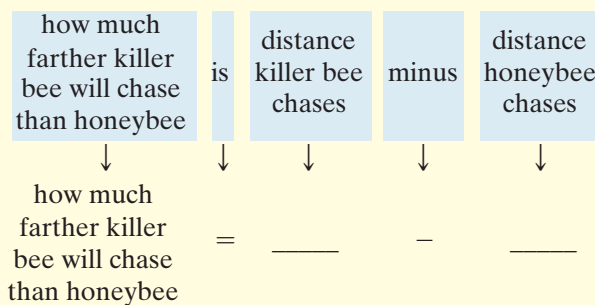


Finish with:

3. SOLVE. and
4. INTERPRET.

Start the solution:

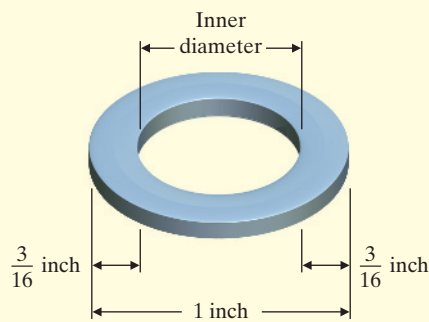
1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)



Finish with

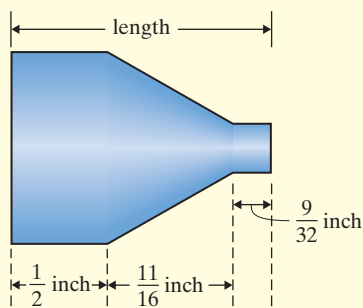
3. SOLVE. and
4. INTERPRET.

87. Find the inner diameter of the washer.



89. About $\frac{13}{20}$ of American students ages 10 to 17 name math, science, or art as their favorite subject in school. Art is the favorite subject for about $\frac{4}{25}$ of the American students ages 10 to 17. For what fraction of students this age is math or science their favorite subject? (Source: Peter D. Hart Research Associates for the National Science Foundation)

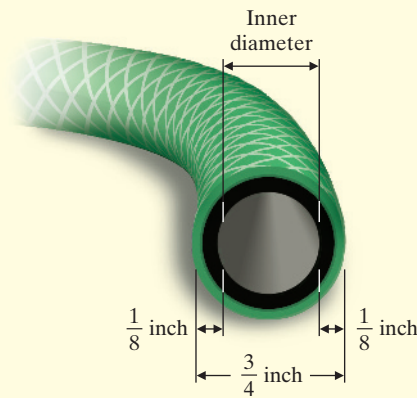
91. Given the following diagram, find its total length.



The table below shows the fraction of the Earth’s water area taken up by each ocean. Use this table for Exercises 93 and 94.

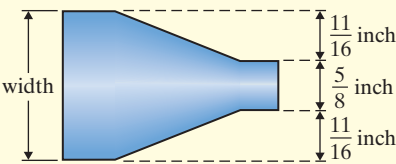
Fraction of Earth’s Water Area per Ocean	
Ocean	Fraction
Arctic	$\frac{1}{25}$
Atlantic	$\frac{13}{50}$
Pacific	$\frac{1}{2}$
Indian	$\frac{1}{5}$

88. Find the inner diameter of the tubing.



90. In the 2016 FIA Formula One World Championship, the Mercedes team won $\frac{19}{21}$ of all races. If Mercedes driver Nico Rosberg won $\frac{3}{7}$ of the races, how many did Mercedes driver Lewis Hamilton win? (Source: formula1.com)

92. Given the following diagram, find its total width.



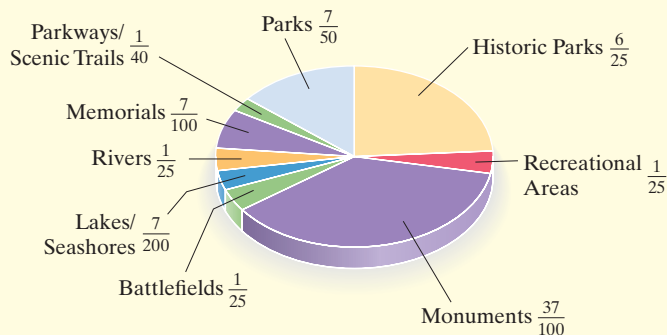
93. What fraction of the world’s water surface area is accounted for by the Pacific and Atlantic Oceans?

94. What fraction of the world’s water surface area is accounted for by the Arctic and Indian Oceans?



Use this circle graph to answer Exercises 95 through 98.

Areas Maintained by the National Park Service



Source: National Park Service

95. What fraction of areas maintained by the National Park Service are designated as rivers or lakes/seashores?
96. What fraction of areas maintained by the National Park Service are designated as recreation areas or memorials?
97. What fraction of areas maintained by the National Park Service are NOT national monuments?
98. What fraction of areas maintained by the National Park Service are NOT national parkways or scenic trails?

Review

Use order of operations to simplify. See Section 2.5.

99. $-50 \div 5 \cdot 2$

100. $8 - 6 \cdot 4 - 7$

101. $(8 - 6) \cdot (4 - 7)$

102. $50 \div (5 \cdot 2)$

Concept Extensions

For Exercises 103 and 104 below, do the following:

- a) Draw three rectangles of the same size and represent each fraction in the sum or difference, one fraction per rectangle, by shading.
- b) Using these rectangles as estimates, determine whether there is an error in the sum or difference.
- c) If there is an error, correctly calculate the sum or difference.

See the Concept Checks in this section.

103. $\frac{3}{5} + \frac{4}{5} \stackrel{?}{=} \frac{7}{10}$

104. $\frac{5}{8} - \frac{3}{4} \stackrel{?}{=} \frac{2}{4}$

Subtract from left to right.

105. $\frac{2}{3} - \frac{1}{4} - \frac{2}{540}$

106. $\frac{9}{10} - \frac{7}{200} - \frac{1}{3}$

Perform each indicated operation.

107. $\frac{30}{55} + \frac{1000}{1760}$

108. $\frac{19}{26} - \frac{968}{1352}$

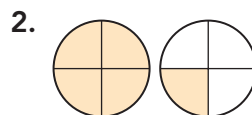
109. In your own words, describe how to add or subtract two fractions with different denominators.
110. Find the sum of the fractions in the circle graph above. Did the sum surprise you? Why or why not?
111. In 2015, about $\frac{2}{5}$ of the total number of pieces of mail delivered by the United States Postal Service was first-class mail. That same year, about $\frac{1}{2}$ of the total number of pieces of mail delivered by the United States Postal Service was standard mail. Which of these two categories account for a greater portion of the mail handled by volume? (Source: United States Postal Service)

Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. a. b. c. _____
24. a. b. c. _____

Summary on Fractions and Operations on Fractions

Use a fraction to represent the shaded area of each figure. If the fraction is improper, also write the fraction as a mixed number.



Solve.

3. In a survey, 73 people out of 85 get fewer than 8 hours of sleep each night. What fraction of people in the survey get fewer than 8 hours of sleep?

4. Sketch a diagram to represent $\frac{9}{13}$.

Simplify.

5. $\frac{11}{-11}$

6. $\frac{17}{1}$

7. $\frac{0}{-3}$

8. $\frac{7}{0}$

Write the prime factorization of each composite number. Write any repeated factors using exponents.

9. 65

10. 70

11. 315

12. 441

Write each fraction in simplest form.

13. $\frac{2}{14}$

14. $\frac{24}{20}$

15. $-\frac{56}{60}$

16. $-\frac{72}{80}$

17. $\frac{54x}{135}$

18. $\frac{90}{240y}$

19. $\frac{165z^3}{210z}$

20. $\frac{245ab}{385a^2b^3}$

Determine whether each pair of fractions is equivalent.

21. $\frac{7}{8}$ and $\frac{9}{10}$

22. $\frac{10}{12}$ and $\frac{15}{18}$

23. Of the 50 states, 2 states are not adjacent to any other states.

- a. What fraction of the states are not adjacent to other states?
- b. How many states are adjacent to other states?
- c. What fraction of the states are adjacent to other states?

24. There are approximately 141,000 digital movie screens in the world. Of these, about 33,000 are in the U.S./Canada. (Source: Motion Picture Association of America)

- a. What fraction of digital movie screens are in the U.S./Canada?
- b. How many digital movie screens are not in the U.S./Canada?
- c. What fraction of digital movie screens are not in the U.S./Canada?

Find the LCM of each list of numbers.

25. 5, 6

26. 2, 14

27. 6, 18, 30

Write each fraction as an equivalent fraction with the indicated denominator.

28. $\frac{7}{9} = \frac{\quad}{36}$

29. $\frac{11}{15} = \frac{\quad}{75}$

30. $\frac{5}{6} = \frac{\quad}{48}$

The following summary will help you with the following review of operations on fractions.

Operations on Fractions

Let a, b, c , and d be integers.

Addition: $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$
 $b \neq 0$ $\uparrow \quad \uparrow$
 common denominator

Multiplication: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$
 $(b \neq 0, d \neq 0)$

Subtraction: $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$
 $(b \neq 0)$ $\uparrow \quad \uparrow$
 common denominator

Division: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$
 $(b \neq 0, d \neq 0, c \neq 0)$

Perform each indicated operation.

31. $\frac{1}{5} + \frac{3}{5}$

32. $\frac{1}{5} - \frac{3}{5}$

33. $\frac{1}{5} \cdot \frac{3}{5}$

34. $\frac{1}{5} \div \frac{3}{5}$

35. $\frac{2}{3} \div \frac{5}{6}$

36. $\frac{2a}{3} \cdot \frac{5}{6a}$

37. $\frac{2}{3y} - \frac{5}{6y}$

38. $\frac{2x}{3} + \frac{5x}{6}$

39. $-\frac{1}{7} \cdot -\frac{7}{18}$

40. $-\frac{4}{9} \cdot -\frac{3}{7}$

41. $-\frac{7z}{8} \div 6z^2$

42. $-\frac{9}{10} \div 5$

43. $\frac{7}{8} + \frac{1}{20}$

44. $\frac{5}{12} - \frac{1}{9}$

45. $\frac{2}{9} + \frac{1}{18} + \frac{1}{3}$

46. $\frac{3y}{10} + \frac{y}{5} + \frac{6}{25}$

Translating Translate each to an expression. Use x to represent “a number.”

47. $\frac{2}{3}$ of a number

48. The quotient of a number and $-\frac{1}{5}$

49. A number subtracted from $-\frac{8}{9}$

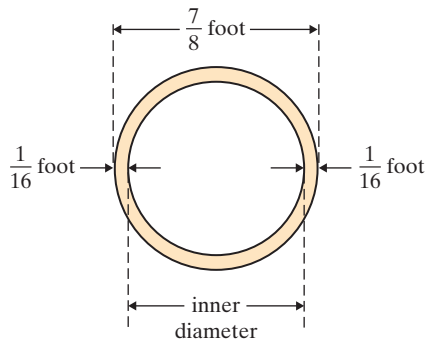
50. $\frac{6}{11}$ increased by a number

Solve.

51. Find $\frac{2}{3}$ of 1530.

52. A contractor is using 18 acres of his land to sell $\frac{3}{4}$ -acre lots. How many lots can he sell?

53. Suppose that the cross-section of a piece of pipe looks like the diagram shown. What is the inner diameter?



25. _____ 26. _____
27. _____ 28. _____
29. _____ 30. _____
31. _____ 32. _____
33. _____ 34. _____
35. _____ 36. _____
37. _____
38. _____
39. _____
40. _____
41. _____
42. _____
43. _____
44. _____
45. _____
46. _____
47. _____
48. _____
49. _____
50. _____
51. _____
52. _____
53. _____

4.6 Complex Fractions and Review of Order of Operations

Objectives

- A** Simplify Complex Fractions. ▶
- B** Review the Order of Operations. ▶
- C** Evaluate Expressions Given Replacement Values. ▶

Objective A Simplifying Complex Fractions ▶

Thus far, we have studied operations on fractions. We now practice simplifying fractions whose numerators or denominators themselves contain fractions. These fractions are called **complex fractions**.

Complex Fraction

A fraction whose numerator or denominator or both numerator and denominator contain fractions is called a **complex fraction**.

Examples of complex fractions are

$$\frac{\frac{x}{4}}{\frac{3}{2}}, \quad \frac{\frac{1}{2} + \frac{3}{8}}{\frac{3}{4} - \frac{1}{6}}, \quad \frac{\frac{y}{5} - 2}{\frac{3}{10}}, \quad \frac{-4z}{\frac{3}{5}}$$

Method 1 for Simplifying Complex Fractions

Two methods are presented to simplify complex fractions. The first method makes use of the fact that a fraction bar means division.

Practice 1

Simplify: $\frac{\frac{7y}{10}}{\frac{1}{5}}$

Practice 2

Simplify: $\frac{\frac{1}{2} + \frac{1}{6}}{\frac{3}{4} - \frac{2}{3}}$

Answers

1. $\frac{7y}{2}$ 2. $\frac{8}{1}$ or 8

Example 1

Simplify: $\frac{\frac{x}{4}}{\frac{3}{2}}$

Solution: Since a fraction bar means division, the complex fraction $\frac{\frac{x}{4}}{\frac{3}{2}}$ can be written as $\frac{x}{4} \div \frac{3}{2}$. Then divide as usual to simplify.

$$\begin{aligned} \frac{x}{4} \div \frac{3}{2} &= \frac{x}{4} \cdot \frac{2}{3} && \text{Multiply by the reciprocal.} \\ &= \frac{x \cdot \overset{1}{2}}{\underset{1}{2} \cdot 2 \cdot 3} \\ &= \frac{x}{6} \end{aligned}$$

Work Practice 1

Example 2

Simplify: $\frac{\frac{1}{2} + \frac{3}{8}}{\frac{3}{4} - \frac{1}{6}}$

Solution: Recall the order of operations. Since the fraction bar is considered a grouping symbol, we simplify the numerator and the denominator of the complex fraction separately. Then we divide.

$$\frac{\frac{1}{2} + \frac{3}{8}}{\frac{3}{4} - \frac{1}{6}} = \frac{\frac{1 \cdot 4}{2 \cdot 4} + \frac{3}{8}}{\frac{3 \cdot 3}{4 \cdot 3} - \frac{1 \cdot 2}{6 \cdot 2}} = \frac{\frac{4}{8} + \frac{3}{8}}{\frac{9}{12} - \frac{2}{12}} = \frac{\frac{7}{8}}{\frac{7}{12}}$$

Thus,

$$\begin{aligned}
 \frac{\frac{1}{2} + \frac{3}{8}}{\frac{3}{4} - \frac{1}{6}} &= \frac{\frac{7}{8}}{\frac{7}{12}} \\
 &= \frac{7}{8} \div \frac{7}{12} && \text{Rewrite the quotient using the } \div \text{ sign.} \\
 &= \frac{7}{8} \cdot \frac{12}{7} && \text{Multiply by the reciprocal.} \\
 &= \frac{\cancel{7} \cdot 3 \cdot \cancel{4}}{2 \cdot \cancel{4} \cdot \cancel{7}} && \text{Multiply.} \\
 &= \frac{3}{2} && \text{Simplify.}
 \end{aligned}$$

Work Practice 2

Method 2 for Simplifying Complex Fractions

The second method for simplifying complex fractions is to multiply the numerator and the denominator of the complex fraction by the LCD of all the fractions in its numerator and its denominator. This has the effect of leaving sums and differences of integers in the numerator and the denominator, as we shall see in the example below.

Let's use this second method to simplify the complex fraction in Example 2 again.

Example 3 Simplify: $\frac{\frac{1}{2} + \frac{3}{8}}{\frac{3}{4} - \frac{1}{6}}$

Solution: The complex fraction contains fractions with denominators 2, 8, 4, and 6. The LCD is 24. By the fundamental property of fractions (See the Helpful Hints on p. 233 and p. 260), we can multiply the numerator and the denominator of the complex fraction by 24. Notice below that by the distributive property, this means that we multiply each term in the numerator and denominator by 24.

$$\begin{aligned}
 \frac{\frac{1}{2} + \frac{3}{8}}{\frac{3}{4} - \frac{1}{6}} &= \frac{24\left(\frac{1}{2} + \frac{3}{8}\right)}{24\left(\frac{3}{4} - \frac{1}{6}\right)} \\
 &= \frac{\left(\overset{12}{\cancel{24}} \cdot \frac{1}{\cancel{2}}\right) + \left(\overset{3}{\cancel{24}} \cdot \frac{3}{\cancel{8}}\right)}{\left(\overset{6}{\cancel{24}} \cdot \frac{3}{\cancel{4}}\right) - \left(\overset{4}{\cancel{24}} \cdot \frac{1}{\cancel{6}}\right)} && \text{Apply the distributive property. Then divide out common factors to aid in multiplying.} \\
 &= \frac{12 + 9}{18 - 4} && \text{Multiply.} \\
 &= \frac{21}{14} \\
 &= \frac{\overset{1}{\cancel{7}} \cdot 3}{\overset{1}{\cancel{7}} \cdot 2} = \frac{3}{2} && \text{Simplify.}
 \end{aligned}$$

Work Practice 3

The simplified result is the same, of course, no matter which method is used.

Practice 3

Use Method 2 to simplify:

$$\frac{\frac{1}{2} + \frac{1}{6}}{\frac{3}{4} - \frac{2}{3}}$$

Answer

3. $\frac{8}{1}$ or 8

Practice 4

Simplify: $\frac{\frac{3}{4}}{\frac{x}{5} - 1}$

Helpful Hint

Don't forget to multiply the numerator and the denominator of the complex fraction by the same number—the LCD.

Example 4

Simplify: $\frac{\frac{y}{5} - 2}{\frac{3}{10}}$

Solution: Use the second method and multiply the numerator and the denominator of the complex fraction by the LCD of all fractions. Recall that $2 = \frac{2}{1}$. The LCD of the denominators 5, 1, and 10 is 10.

$$\frac{\frac{y}{5} - \frac{2}{1}}{\frac{3}{10}} = \frac{10\left(\frac{y}{5} - \frac{2}{1}\right)}{10\left(\frac{3}{10}\right)}$$

Multiply the numerator and denominator by 10.

$$= \frac{\left(\frac{2}{10} \cdot \frac{y}{5}\right) - \left(10 \cdot \frac{2}{1}\right)}{\frac{1}{10} \cdot \frac{3}{10}}$$

Apply the distributive property. Then divide out common factors to aid in multiplying.

$$= \frac{2y - 20}{3}$$

Multiply.

Work Practice 4

Objective B Reviewing the Order of Operations

At this time, it is probably a good idea to review the order of operations on expressions containing fractions. Before we do so, let's review how we perform operations on fractions.

Review of Operations on Fractions		
Operation	Procedure	Example
Multiply	Multiply the numerators and multiply the denominators.	$\frac{5}{9} \cdot \frac{1}{2} = \frac{5 \cdot 1}{9 \cdot 2} = \frac{5}{18}$
Divide	Multiply the first fraction by the reciprocal of the second fraction.	$\frac{2}{3} \div \frac{11}{13} = \frac{2}{3} \cdot \frac{13}{11} = \frac{2 \cdot 13}{3 \cdot 11} = \frac{26}{33}$
Add or Subtract	- Write each fraction as an equivalent fraction whose denominator is the LCD. - Add or subtract numerators and write the result over the common denominator.	$\frac{3}{4} + \frac{1}{8} = \frac{3 \cdot 2}{4 \cdot 2} + \frac{1}{8} = \frac{6}{8} + \frac{1}{8} = \frac{7}{8}$

Now let's review order of operations.

Order of Operations

1. Perform all operations within parentheses (), brackets [], or other grouping symbols such as fraction bars, starting with the innermost set.
2. Evaluate any expressions with exponents.
3. Multiply or divide in order from left to right.
4. Add or subtract in order from left to right.

Practice 5

Simplify: $\left(\frac{2}{3}\right)^3 - 2$

Answers

4. $\frac{15}{4x - 20}$ 5. $-\frac{46}{27}$

Example 5

Simplify: $\left(\frac{4}{5}\right)^2 - 1$

Solution: According to the order of operations, first evaluate $\left(\frac{4}{5}\right)^2$.

$$\left(\frac{4}{5}\right)^2 - 1 = \frac{16}{25} - 1 \quad \text{Write } \left(\frac{4}{5}\right)^2 \text{ as } \frac{16}{25}.$$

Next, combine the fractions. The LCD of the denominators 25 and 1 is 25.

$$\begin{aligned}\frac{16}{25} - 1 &= \frac{16}{25} - \frac{25}{25} && \text{Write 1 as } \frac{25}{25}. \\ &= \frac{-9}{25} \text{ or } -\frac{9}{25} && \text{Subtract.}\end{aligned}$$

Work Practice 5

Example 6 Simplify: $\left(\frac{1}{4} + \frac{2}{3}\right)\left(\frac{11}{12} + \frac{1}{4}\right)$

Solution: First perform operations inside parentheses. Then multiply.

$$\begin{aligned}\left(\frac{1}{4} + \frac{2}{3}\right)\left(\frac{11}{12} + \frac{1}{4}\right) &= \left(\frac{1 \cdot 3}{4 \cdot 3} + \frac{2 \cdot 4}{3 \cdot 4}\right)\left(\frac{11}{12} + \frac{1 \cdot 3}{4 \cdot 3}\right) && \text{Each LCD is 12.} \\ &= \left(\frac{3}{12} + \frac{8}{12}\right)\left(\frac{11}{12} + \frac{3}{12}\right) \\ &= \left(\frac{11}{12}\right)\left(\frac{14}{12}\right) && \text{Add.} \\ &= \frac{11 \cdot \cancel{2} \cdot 7}{\cancel{2} \cdot 6 \cdot 12} && \text{Multiply.} \\ &= \frac{77}{72} && \text{Simplify.}\end{aligned}$$

Work Practice 6

✓ **Concept Check** What should be done first to simplify the expression $\frac{1}{5} \cdot \frac{5}{2} - \left(\frac{2}{3} + \frac{4}{5}\right)^2$?

Objective C Evaluating Algebraic Expressions

Example 7 Evaluate $2x + y^2$ if $x = -\frac{1}{2}$ and $y = \frac{1}{3}$.

Solution: Replace x and y with the given values and simplify.

$$\begin{aligned}2x + y^2 &= 2\left(-\frac{1}{2}\right) + \left(\frac{1}{3}\right)^2 && \text{Replace } x \text{ with } -\frac{1}{2} \text{ and } y \text{ with } \frac{1}{3}. \\ &= 2\left(-\frac{1}{2}\right) + \frac{1}{9} && \text{Write } \left(\frac{1}{3}\right)^2 \text{ as } \frac{1}{9}. \\ &= -1 + \frac{1}{9} && \text{Multiply.} \\ &= -\frac{9}{9} + \frac{1}{9} && \text{The LCD is 9.} \\ &= -\frac{8}{9} && \text{Add.}\end{aligned}$$

Work Practice 7

Practice 6

Simplify: $\left(-\frac{1}{2} + \frac{1}{5}\right)\left(\frac{7}{8} + \frac{1}{8}\right)$

Helpful Hint

If you find it difficult replacing a variable with a number, try the following. First, replace the variable with a set of parentheses, then place the replacement number between the parentheses.

If $x = \frac{4}{5}$, find $2x + x^2$.

$$\begin{aligned}2x + x^2 &= 2(\quad) + (\quad)^2 \\ &= 2\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)^2 \dots\end{aligned}$$

then continue simplifying.

Practice 7

Evaluate $-\frac{3}{5} - xy$ if $x = \frac{3}{10}$ and $y = \frac{2}{3}$.

Answers

6. $-\frac{3}{10}$ 7. $-\frac{4}{5}$

✓ Concept Check Answer

Add inside parentheses.

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

addition
subtraction

multiplication
division

evaluate the exponential expression
complex

1. A fraction whose numerator or denominator or both numerator and denominator contain fractions is called a(n) _____ fraction.
2. To simplify $-\frac{1}{2} + \frac{2}{3} \cdot \frac{7}{8}$, which operation do we perform first? _____
3. To simplify $-\frac{1}{2} \div \frac{2}{3} \cdot \frac{7}{8}$, which operation do we perform first? _____
4. To simplify $\frac{7}{8} \cdot \left(\frac{1}{2} - \frac{2}{3}\right)$, which operation do we perform first? _____
5. To simplify $\frac{1}{3} \div \frac{1}{4} \cdot \left(\frac{9}{11} + \frac{3}{8}\right)^3$, which operation do we perform first? _____
6. To simplify $9 - \left(-\frac{3}{4}\right)^2$, which operation do we perform first? _____

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 4.6

- Objective A** 7. In Example 2, what property is used to simplify the denominator of the complex fraction?
- Objective B** 8. In Example 3, why can we add the fractions in the first set of parentheses right away?
- Objective C** 9. In Example 4, why do we use parentheses when substituting the replacement value for x ? What would happen if we didn't use parentheses?

4.6 Exercise Set MyLab Math

Objective A Simplify each complex fraction. See Examples 1 through 4.

▶ 1. $\frac{\frac{1}{8}}{\frac{3}{4}}$

2. $\frac{\frac{5}{12}}{\frac{15}{12}}$

3. $\frac{\frac{2}{3}}{\frac{2}{7}}$

4. $\frac{\frac{9}{25}}{\frac{6}{25}}$

5. $\frac{\frac{2x}{27}}{\frac{4}{9}}$

6. $\frac{\frac{3y}{11}}{\frac{1}{2}}$

7. $\frac{\frac{\frac{3}{4} + \frac{2}{5}}{1}}{\frac{1}{2} + \frac{3}{5}}$

8. $\frac{\frac{\frac{7}{6} + \frac{2}{3}}{3}}{\frac{3}{2} - \frac{8}{9}}$

▶ 9. $\frac{\frac{\frac{3x}{4}}{5}}{5 - \frac{1}{8}}$

10. $\frac{\frac{\frac{3}{10} + 2}{2}}{5y}$

Objective B Use the order of operations to simplify each expression. See Examples 5 and 6.

11. $\frac{1}{5} + \frac{1}{3} \cdot \frac{1}{4}$

12. $\frac{1}{2} + \frac{1}{6} \cdot \frac{1}{3}$

13. $\frac{5}{6} \div \frac{1}{3} \cdot \frac{1}{4}$

14. $\frac{7}{8} \div \frac{1}{4} \cdot \frac{1}{7}$

15. $2^2 - \left(\frac{1}{3}\right)^2$

16. $3^2 - \left(\frac{1}{2}\right)^2$

▶ 17. $\left(\frac{2}{9} + \frac{4}{9}\right)\left(\frac{1}{3} - \frac{9}{10}\right)$

18. $\left(\frac{1}{5} - \frac{1}{10}\right)\left(\frac{1}{5} + \frac{1}{10}\right)$

19. $\left(\frac{7}{8} - \frac{1}{2}\right) \div \frac{3}{11}$

20. $\left(-\frac{2}{3} - \frac{7}{3}\right) \div \frac{4}{9}$

21. $2 \cdot \left(\frac{1}{4} + \frac{1}{5}\right) + 2$

22. $\frac{2}{5} \cdot \left(5 - \frac{1}{2}\right) - 1$

23. $\left(\frac{3}{4}\right)^2 \div \left(\frac{3}{4} - \frac{1}{12}\right)$

24. $\left(\frac{8}{9}\right)^2 \div \left(2 - \frac{2}{3}\right)$

25. $\left(\frac{2}{5} - \frac{3}{10}\right)^2$

26. $\left(\frac{3}{2} - \frac{4}{3}\right)^3$

27. $\left(\frac{3}{4} + \frac{1}{8}\right)^2 - \left(\frac{1}{2} + \frac{1}{8}\right)$

28. $\left(\frac{1}{6} + \frac{1}{3}\right)^3 + \left(\frac{2}{5} \cdot \frac{3}{4}\right)^2$

Objective C Evaluate each expression if $x = -\frac{1}{3}$, $y = \frac{2}{5}$, and $z = \frac{5}{6}$. See Example 7.

29. $5y - z$

30. $2z - x$

31. $\frac{x}{z}$

32. $\frac{y + x}{z}$

▶ 33. $x^2 - yz$

34. $x^2 - z^2$

35. $(1 + x)(1 + z)$

36. $(1 - x)(1 - z)$

Objectives A B Mixed Practice Simplify the following. See Examples 1 through 6.

37. $\frac{\frac{5a}{24}}{\frac{1}{12}}$

38. $\frac{\frac{7}{10}}{\frac{14z}{25}}$

39. $\left(\frac{3}{2}\right)^3 + \left(\frac{1}{2}\right)^3$

40. $\left(\frac{5}{21} \div \frac{1}{2}\right) + \left(\frac{1}{7} \cdot \frac{1}{3}\right)$

41. $\left(-\frac{1}{2}\right)^2 + \frac{1}{5}$

42. $\left(-\frac{3}{4}\right)^2 + \frac{3}{8}$

$$43. \frac{2 + \frac{1}{6}}{1 - \frac{4}{3}}$$

$$44. \frac{3 - \frac{1}{2}}{4 + \frac{1}{5}}$$

$$45. \left(1 - \frac{2}{5}\right)^2$$

$$46. \left(-\frac{1}{2}\right)^2 - \left(\frac{3}{4}\right)^2$$

$$47. \left(\frac{3}{4} - 1\right)\left(\frac{1}{8} + \frac{1}{2}\right)$$

$$48. \left(\frac{1}{10} + \frac{3}{20}\right)\left(\frac{1}{5} - 1\right)$$

$$49. \left(-\frac{2}{9} - \frac{7}{9}\right)^4$$

$$50. \left(\frac{5}{9} - \frac{2}{3}\right)^2$$

$$51. \frac{\frac{1}{3} - \frac{5}{6}}{\frac{3}{4} + \frac{1}{2}}$$

$$52. \frac{\frac{7}{10} + \frac{1}{2}}{\frac{4}{5} + \frac{3}{4}}$$

$$53. \left(\frac{3}{4} \div \frac{6}{5}\right) - \left(\frac{3}{4} \cdot \frac{6}{5}\right)$$

$$54. \left(\frac{1}{2} \cdot \frac{2}{7}\right) - \left(\frac{1}{2} \div \frac{2}{7}\right)$$

$$55. \frac{\frac{x}{3} + 2}{5 + \frac{1}{3}}$$

$$56. \frac{1 - \frac{x}{4}}{2 + \frac{3}{8}}$$

Review

Perform each indicated operation. If the result is an improper fraction, also write the improper fraction as a mixed number. See Sections 4.1 and 4.5.

$$57. 3 + \frac{1}{2}$$

$$58. 2 + \frac{2}{3}$$

$$59. 9 - \frac{5}{6}$$

$$60. 4 - \frac{1}{5}$$

Concept Extensions

-  **61.** Calculate $\frac{2^3}{3}$ and $\left(\frac{2}{3}\right)^3$. Do both of these expressions simplify to the same number? Explain why or why not.

-  **62.** Calculate $\left(\frac{1}{2}\right)^2 \cdot \left(\frac{3}{4}\right)^2$ and $\left(\frac{1}{2} \cdot \frac{3}{4}\right)^2$. Do both of these expressions simplify to the same number? Explain why or why not.

Recall that to find the average of two numbers, we find their sum and divide by 2. For example, the average of $\frac{1}{2}$ and $\frac{3}{4}$ is $\frac{\frac{1}{2} + \frac{3}{4}}{2}$. Find the average of each pair of numbers.

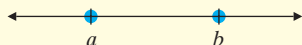
$$63. \frac{1}{2}, \frac{3}{4}$$

$$64. \frac{3}{5}, \frac{9}{10}$$

$$65. \frac{1}{4}, \frac{2}{14}$$

$$66. \frac{5}{6}, \frac{7}{9}$$

67. Two positive numbers, a and b , are graphed below. Where should the graph of their average lie?



68. Study Exercise 67. Without calculating, can $\frac{1}{3}$ be the average of $\frac{1}{2}$ and $\frac{8}{9}$? Explain why or why not.

Answer true or false for each statement.

69. It is possible for the average of two numbers to be greater than both numbers.
70. It is possible for the average of two numbers to be less than both numbers.
71. The sum of two negative fractions is always a negative number.
72. The sum of a negative fraction and a positive fraction is always a positive number.
73. It is possible for the sum of two fractions to be a whole number.
74. It is possible for the difference of two fractions to be a whole number.
75. What operation should be performed first to simplify $\frac{1}{5} \cdot \frac{5}{2} - \left(\frac{2}{3} + \frac{4}{5}\right)^2$? Explain your answer.
76. A student is to evaluate $x - y$ when $x = \frac{1}{5}$ and $y = -\frac{1}{7}$. This student is asking you if he should evaluate $\frac{1}{5} - \frac{1}{7}$. What do you tell this student and why?

Each expression contains one addition, one subtraction, one multiplication, and one division. Write the operations in the order that they should be performed. Do not actually simplify. See the Concept Check in this section.

77. $[9 + 3(4 - 2)] \div \frac{10}{21}$

78. $[30 - 4(3 + 2)] \div \frac{5}{2}$

79. $\frac{1}{3} \div \left(\frac{2}{3}\right)\left(\frac{4}{5}\right) - \frac{1}{4} + \frac{1}{2}$

80. $\left(\frac{5}{6} - \frac{1}{3}\right) \cdot \frac{1}{3} + \frac{1}{2} \div \frac{9}{8}$

Evaluate each expression if $x = \frac{3}{4}$ and $y = -\frac{4}{7}$.

81. $\frac{2 + x}{y}$

82. $4x + y$

83. $x^2 + 7y$

84. $\frac{\frac{9}{14}}{x + y}$

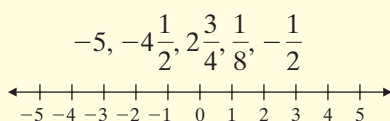
4.7 Operations on Mixed Numbers

Objectives

- A** Graph Positive and Negative Fractions and Mixed Numbers
- B** Multiply or Divide Mixed or Whole Numbers.
- C** Add or Subtract Mixed Numbers.
- D** Solve Problems Containing Mixed Numbers.
- E** Perform Operations on Negative Mixed Numbers.

Practice 1

Graph the numbers on a number line.



Objective A Graphing Fractions and Mixed Numbers

Let's review graphing fractions and practice graphing mixed numbers on a number line. This will help us visualize rounding and estimating operations with mixed numbers.

Recall that $5\frac{2}{3}$ means $5 + \frac{2}{3}$ and

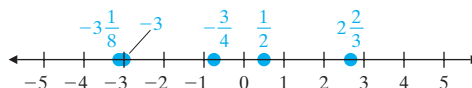
$$-4\frac{1}{6} \text{ means } -\left(4 + \frac{1}{6}\right) \text{ or } -4 - \frac{1}{6} \text{ or } -4 + \left(-\frac{1}{6}\right)$$

Example 1 Graph the numbers on a number line:

$$\frac{1}{2}, -\frac{3}{4}, 2\frac{2}{3}, -3, -3\frac{1}{8}$$

Solution: Remember that $2\frac{2}{3}$ means $2 + \frac{2}{3}$.

Also, $-3\frac{1}{8}$ means $-3 - \frac{1}{8}$, so $-3\frac{1}{8}$ lies to the left of -3 .



Work Practice 1

✓Concept Check Which of the following is/are equivalent to 9?

- a. $7\frac{6}{3}$
- b. $8\frac{4}{4}$
- c. $8\frac{9}{9}$
- d. $\frac{18}{2}$
- e. all of these

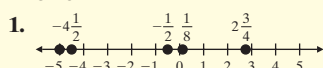
Objective B Multiplying or Dividing with Mixed Numbers or Whole Numbers

When multiplying or dividing a fraction and a mixed or a whole number, remember that mixed and whole numbers can be written as improper fractions.

Multiplying or Dividing Fractions and Mixed Numbers or Whole Numbers

To multiply or divide with mixed numbers or whole numbers, first write any mixed or whole numbers as improper fractions and then multiply or divide as usual.

Answer



✓Concept Check Answer

e

(Note: If an exercise contains a mixed number, we will write the answer as a mixed number, if possible.)

Example 2 Multiply: $3\frac{1}{3} \cdot \frac{7}{8}$

Solution: Recall from Section 4.1 that the mixed number $3\frac{1}{3}$ can be written as the fraction $\frac{10}{3}$. Then

$$3\frac{1}{3} \cdot \frac{7}{8} = \frac{10}{3} \cdot \frac{7}{8} = \frac{\overset{1}{2} \cdot 5 \cdot 7}{\underset{1}{3} \cdot 2 \cdot 4} = \frac{35}{12} \quad \text{or} \quad 2\frac{11}{12}$$

Work Practice 2

Don't forget that a whole number can be written as a fraction by writing the whole number over 1. For example,

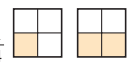
$$20 = \frac{20}{1} \quad \text{and} \quad 7 = \frac{7}{1}$$

Example 3 Multiply: $\frac{3}{4} \cdot 20$

Solution: $\frac{3}{4} \cdot 20 = \frac{3}{4} \cdot \frac{20}{1} = \frac{3 \cdot 20}{4 \cdot 1} = \frac{3 \cdot \overset{1}{4} \cdot 5}{\underset{1}{4} \cdot 1} = \frac{15}{1} \quad \text{or} \quad 15$

Work Practice 3

When both numbers to be multiplied are mixed or whole numbers, it is a good idea to estimate the product to see if your answer is reasonable. To do this, we first practice rounding mixed numbers to the nearest whole. If the fraction part of the mixed number is $\frac{1}{2}$ or greater, we round the whole number part up. If the fraction part of the mixed number is less than $\frac{1}{2}$, then we do not round the whole number part up. Study the table below for examples.

Mixed Number	Rounding
$5\frac{1}{4}$ $\frac{1}{4}$ is less than $\frac{1}{2}$ 	Thus, $5\frac{1}{4}$ rounds to 5.
$3\frac{9}{16}$ $\leftarrow$ 9 is greater than 8. $\rightarrow$ Half of 16 is 8.	Thus, $3\frac{9}{16}$ rounds to 4.
$1\frac{3}{7}$ $\leftarrow$ 3 is less than $3\frac{1}{2}$. $\rightarrow$ Half of 7 is $3\frac{1}{2}$.	Thus, $1\frac{3}{7}$ rounds to 1.

Example 4 Multiply $1\frac{2}{3} \cdot 2\frac{1}{4}$. Check by estimating.

Solution: $1\frac{2}{3} \cdot 2\frac{1}{4} = \frac{5}{3} \cdot \frac{9}{4} = \frac{5 \cdot 9}{3 \cdot 4} = \frac{5 \cdot \overset{1}{3} \cdot 3}{\underset{1}{3} \cdot 4} = \frac{15}{4} \quad \text{or} \quad 3\frac{3}{4} \quad \text{Exact}$

Let's check by estimating.

$$1\frac{2}{3} \text{ rounds to } 2, 2\frac{1}{4} \text{ rounds to } 2, \text{ and } 2 \cdot 2 = 4 \quad \text{Estimate}$$

The estimate is close to the exact value, so our answer is reasonable.

Work Practice 4

Practice 2

Multiply and simplify: $1\frac{2}{3} \cdot \frac{11}{15}$

Practice 3

Multiply: $\frac{5}{6} \cdot 18$

Practice 4

Multiply. Check by estimating.

$$3\frac{1}{5} \cdot 2\frac{3}{4}$$

Answers

2. $1\frac{2}{9}$ 3. 15 4. $8\frac{4}{5}$

Practice 5

Multiply. Check by estimating.

$$3 \cdot 6\frac{7}{15}$$

Practice 6–8

Divide.

$$6. \frac{4}{9} \div 7$$

$$7. \frac{8}{15} \div 3\frac{4}{5}$$

$$8. 3\frac{2}{7} \div 2\frac{3}{14}$$

Practice 9

$$\text{Add: } 2\frac{1}{6} + 4\frac{2}{5}$$

Check by estimating.

Answers

$$5. 19\frac{2}{5} \quad 6. \frac{4}{63} \quad 7. \frac{8}{57} \quad 8. 1\frac{15}{31}$$

$$9. 6\frac{17}{30}$$

✓ Concept Check Answer

forgot to change mixed number to improper fraction

Example 5Multiply: $7 \cdot 2\frac{11}{14}$. Check by estimating.

$$\text{Solution: } 7 \cdot 2\frac{11}{14} = \frac{7}{1} \cdot \frac{39}{14} = \frac{7 \cdot 39}{1 \cdot 14} = \frac{7 \cdot 39}{1 \cdot 2 \cdot 7} = \frac{39}{2} \text{ or } 19\frac{1}{2} \quad \text{Exact}$$

To estimate,

$$2\frac{11}{14} \text{ rounds to } 3 \text{ and } 7 \cdot 3 = 21. \quad \text{Estimate}$$

The estimate is close to the exact value, so our answer is reasonable.

Work Practice 5**✓ Concept Check** Find the error.

$$2\frac{1}{4} \cdot \frac{1}{2} = 2\frac{1 \cdot 1}{4 \cdot 2} = 2\frac{1}{8}$$

Examples

Divide.

$$6. \frac{3}{4} \div 5 = \frac{3}{4} \div \frac{5}{1} = \frac{3}{4} \cdot \frac{1}{5} = \frac{3 \cdot 1}{4 \cdot 5} = \frac{3}{20}$$

$$7. \frac{11}{18} \div 2\frac{5}{6} = \frac{11}{18} \div \frac{17}{6} = \frac{11}{18} \cdot \frac{6}{17} = \frac{11 \cdot 6}{18 \cdot 17} = \frac{11 \cdot \cancel{6}^1}{\cancel{18}_2 \cdot 17} = \frac{11}{51}$$

$$8. 5\frac{2}{3} \div 2\frac{5}{9} = \frac{17}{3} \div \frac{23}{9} = \frac{17}{3} \cdot \frac{9}{23} = \frac{17 \cdot 9}{3 \cdot 23} = \frac{17 \cdot \cancel{9}^1}{\cancel{3}_1 \cdot 23} = \frac{51}{23} \text{ or } 2\frac{5}{23}$$

Work Practice 6–8**Objective C** Adding or Subtracting Mixed Numbers

We can add or subtract mixed numbers, too, by first writing each mixed number as an improper fraction. But it is often easier to add or subtract the whole-number parts and add or subtract the proper-fraction parts vertically.

Adding or Subtracting Mixed Numbers

To add or subtract mixed numbers, add or subtract the fraction parts and then add or subtract the whole number parts.

Example 9Add: $2\frac{1}{3} + 5\frac{3}{8}$. Check by estimating.**Solution:** The LCD of the denominators 3 and 8 is 24.

$$\begin{array}{r} 2\frac{1 \cdot 8}{3 \cdot 8} = 2\frac{8}{24} \\ + 5\frac{3 \cdot 3}{8 \cdot 3} = + 5\frac{9}{24} \\ \hline = 7\frac{17}{24} \end{array} \quad \begin{array}{l} \leftarrow \text{Add the fractions.} \\ \uparrow \text{Add the whole numbers.} \end{array}$$

To check by estimating, we round as usual. The fraction $2\frac{1}{3}$ rounds to 2, $5\frac{3}{8}$ rounds to 5, and $2 + 5 = 7$, our estimate.

Our exact answer is close to 7, so our answer is reasonable.

Work Practice 9

Helpful Hint

When adding or subtracting mixed numbers and whole numbers, it is a good idea to estimate to see if your answer is reasonable.

For the rest of this section, we leave most of the checking by estimating to you.

Example 10 Add: $3\frac{4}{5} + 1\frac{4}{15}$

Solution: The LCD of the denominators 5 and 15 is 15.

$$\begin{array}{r} 3\frac{4}{5} = 3\frac{12}{15} \\ + 1\frac{4}{15} = + 1\frac{4}{15} \\ \hline 4\frac{16}{15} \end{array}$$

Add the fractions; then add the whole numbers.

Notice that the fraction part is improper.

Since $\frac{16}{15}$ is $1\frac{1}{15}$ we can write the sum as

$$4\frac{16}{15} + 4 + 1\frac{1}{15} = 5\frac{1}{15}$$

 Work Practice 10

 **Concept Check** Explain how you could estimate the following sum:

$$5\frac{1}{9} + 14\frac{10}{11}$$

Example 11 Add: $2\frac{4}{5} + 5 + 1\frac{1}{2}$

Solution: The LCD of the denominators 5 and 2 is 10.

$$\begin{array}{r} 2\frac{4}{5} = 2\frac{8}{10} \\ 5 = 5 \\ + 1\frac{1}{2} = + 1\frac{5}{10} \\ \hline 8\frac{13}{10} = 8 + 1\frac{3}{10} = 9\frac{3}{10} \end{array}$$

 Work Practice 11

Example 12 Subtract: $8\frac{3}{7} - 5\frac{2}{21}$. Check by estimating.

Solution: The LCD of the denominators 7 and 21 is 21.

$$\begin{array}{r} 8\frac{3}{7} = 8\frac{9}{21} \quad \leftarrow \text{The LCD of 7 and 21 is 21.} \\ - 5\frac{2}{21} = - 5\frac{2}{21} \\ \hline 3\frac{7}{21} \end{array}$$

$\leftarrow$ Subtract the fractions.

$\uparrow$ Subtract the whole numbers.

Practice 10

Add: $3\frac{5}{14} + 2\frac{6}{7}$

Practice 11

Add: $12 + 3\frac{6}{7} + 2\frac{1}{5}$

Practice 12

Subtract: $32\frac{7}{9} - 16\frac{5}{18}$

Answers

10. $6\frac{3}{14}$ 11. $18\frac{2}{35}$ 12. $16\frac{1}{2}$

 Concept Check Answer

Round each mixed number to the nearest whole number and add. $5\frac{1}{9}$ rounds to 5 and $14\frac{10}{11}$ rounds to 15, and the estimated sum is $5 + 15 = 20$.

(Continued on next page)

Then $3\frac{7}{21}$ simplifies to $3\frac{1}{3}$. The difference is $3\frac{1}{3}$.

To check, $8\frac{3}{7}$ rounds to 8, $5\frac{2}{21}$ rounds to 5, and $8 - 5 = 3$, our estimate.

Our exact answer is close to 3, so our answer is reasonable.

Work Practice 12

When subtracting mixed numbers, borrowing may be needed, as shown in the next example.

Practice 13

Subtract: $9\frac{7}{15} - 4\frac{3}{5}$

Example 13 Subtract: $7\frac{3}{14} - 3\frac{6}{7}$

Solution: The LCD of the denominators 7 and 14 is 14.

$$7\frac{3}{14} = 7\frac{3}{14} \quad \text{Notice that we cannot subtract } \frac{12}{14} \text{ from } \frac{3}{14}, \text{ so we borrow from the whole number 7.}$$

$$- 3\frac{6}{7} = - 3\frac{12}{14}$$

$$\begin{array}{l} \text{borrow 1 from 7} \\ 7\frac{3}{14} = 6 + 1\frac{3}{14} = 6 + \frac{17}{14} \text{ or } 6\frac{17}{14} \end{array}$$

Now subtract.

$$\begin{array}{r} 7\frac{3}{14} = 7\frac{3}{14} = 6\frac{17}{14} \\ - 3\frac{6}{7} = - 3\frac{12}{14} = - 3\frac{12}{14} \\ \hline 3\frac{5}{14} \end{array} \quad \begin{array}{l} \leftarrow \text{Subtract the fractions.} \\ \uparrow \\ \text{Subtract the whole numbers.} \end{array}$$

Work Practice 13

✓Concept Check In the subtraction problem $5\frac{1}{4} - 3\frac{3}{4}$, $5\frac{1}{4}$ must be rewritten because $\frac{3}{4}$ cannot be subtracted from $\frac{1}{4}$. Why is it incorrect to rewrite $5\frac{1}{4}$ as $5\frac{5}{4}$?

Practice 14

Subtract: $25 - 10\frac{2}{9}$

Answers

13. $4\frac{13}{15}$ 14. $14\frac{7}{9}$

✓Concept Check Answer

Rewrite $5\frac{1}{4}$ as $4\frac{5}{4}$ by borrowing from the 5.

Example 14 Subtract: $14 - 8\frac{3}{7}$

Solution: $14 = 13\frac{7}{7}$ Borrow 1 from 14 and write it as $\frac{7}{7}$.

$$- 8\frac{3}{7} = - 8\frac{3}{7}$$

$$\begin{array}{r} 5\frac{4}{7} \end{array} \quad \begin{array}{l} \leftarrow \text{Subtract the fractions.} \\ \uparrow \\ \text{Subtract the whole numbers.} \end{array}$$

Work Practice 14

Objective D Solving Problems Containing Mixed Numbers

Now that we know how to perform operations on mixed numbers, we can solve real-life problems.

Example 15 Finding Legal Lobster Size

Lobster fishermen must measure the upper body shells of the lobsters they catch. Lobsters that are too small are thrown back into the ocean. Each state has its own size standard for lobsters to help control the breeding stock. Massachusetts divides its waters into four Lobster Conservation Management Areas, with a different minimum lobster size permitted in each area. In area three, the legal lobster size increased from $3\frac{13}{32}$ inches to $3\frac{1}{2}$ inches. How much of an increase was this? (Source: Massachusetts Division of Marine Fisheries)



Solution:

1. UNDERSTAND. Read and reread the problem carefully. The word “increase” found in the problem might make you think that we add to solve the problem. But the phrase “how much of an increase” tells us to subtract to find the increase.

2. TRANSLATE.

In words:	increase	is	new lobster size	minus	old lobster size
	↓	↓	↓	↓	↓
Translate:	increase	=	$3\frac{1}{2}$	–	$3\frac{13}{32}$

3. SOLVE. Before we solve, let’s estimate by rounding to the nearest wholes. The fraction $3\frac{1}{2}$ can be rounded up to 4, $3\frac{13}{32}$ rounds to 3, and $4 - 3 = 1$. The increase is not 1, but will be smaller since we rounded $3\frac{1}{2}$ up and rounded $3\frac{13}{32}$ down.

$$\begin{array}{r} 3\frac{1}{2} = 3\frac{16}{32} \\ - 3\frac{13}{32} = -3\frac{13}{32} \\ \hline \frac{3}{32} \end{array}$$

4. INTERPRET. Check your work. Our estimate tells us that the exact increase of $\frac{3}{32}$ is reasonable. State your conclusion: The increase in lobster size was $\frac{3}{32}$ of an inch.

Practice 15

The measurement around the trunk of a tree just below shoulder height is called its girth. The largest known American beech tree in the United States has a girth of $23\frac{1}{4}$ feet. The largest known sugar maple tree in the United States has a girth of $19\frac{5}{12}$ feet. How much larger is the girth of the largest known American beech tree than the girth of the largest known sugar maple tree? (Source: *American Forests*)



Answer

15. $3\frac{5}{6}$ ft

Practice 16

A designer of women's clothing designs a woman's dress that requires $3\frac{1}{7}$ yards of material.

How many dresses can be made from a 44-yard bolt of material?

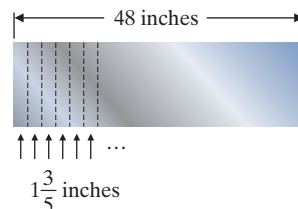
Example 16 Calculating Manufacturing Materials Needed

In a manufacturing process, a metal-cutting machine cuts strips $1\frac{3}{5}$ inches long from a piece of metal stock. How many such strips can be cut from a 48-inch piece of stock?

Solution:

1. UNDERSTAND the problem. To do so, read and reread the problem. Then draw a diagram:

We want to know how many $1\frac{3}{5}$ s there are in 48.



2. TRANSLATE.

In words:	number of strips	is	48	divided by	$1\frac{3}{5}$
	↓	↓	↓	↓	↓
Translate:	number of strips	=	48	÷	$1\frac{3}{5}$

3. SOLVE: Let's estimate a reasonable answer. The mixed number $1\frac{3}{5}$ rounds to 2 and $48 \div 2 = 24$.

$$48 \div 1\frac{3}{5} = 48 \div \frac{8}{5} = \frac{48}{1} \cdot \frac{5}{8} = \frac{48 \cdot 5}{1 \cdot 8} = \frac{8 \cdot 6 \cdot 5}{1 \cdot 8} = \frac{30}{1} \text{ or } 30$$

4. INTERPRET. Check your work. Since the exact answer of 30 is close to our estimate of 24, our answer is reasonable. State your conclusion: Thirty strips can be cut from the 48-inch piece of stock.

Work Practice 16

Objective E Operating on Negative Mixed Numbers

To perform operations on negative mixed numbers, let's first practice writing these numbers as negative fractions and negative fractions as negative mixed numbers.

To understand negative mixed numbers, we simply need to know that, for example,

$$-3\frac{2}{5} \text{ means } -\left(3\frac{2}{5}\right)$$

Thus, to write a negative mixed number as a fraction, we do the following.

$$-3\frac{2}{5} = -\left(3\frac{2}{5}\right) = -\left(\frac{5 \cdot 3 + 2}{5}\right) = -\left(\frac{17}{5}\right) \text{ or } -\frac{17}{5}$$

Practice 17–18

Write each as a fraction.

17. $-9\frac{3}{7}$ 18. $-5\frac{10}{11}$

Answers

16. 14 dresses 17. $-\frac{66}{7}$

18. $-\frac{65}{11}$

Examples

Write each as a fraction.

17. $-1\frac{7}{8} = -\frac{8 \cdot 1 + 7}{8} = -\frac{15}{8}$ Write $1\frac{7}{8}$ as an improper fraction and keep the negative sign.

18. $-23\frac{1}{2} = -\frac{2 \cdot 23 + 1}{2} = -\frac{47}{2}$ Write $23\frac{1}{2}$ as an improper fraction and keep the negative sign.

Work Practice 17–18

To write a negative fraction as a negative mixed number, we use a similar procedure. We simply disregard the negative sign, convert the improper fraction to a mixed number, and then reinsert the negative sign.

Examples

Write each as a mixed number.

$$19. \quad -\frac{22}{5} = -4\frac{2}{5}$$

$$\begin{array}{r} 4 \\ 5 \overline{)22} \\ \underline{-20} \\ 2 \end{array}$$

$$\frac{22}{5} = 4\frac{2}{5}$$

$$20. \quad -\frac{9}{4} = -2\frac{1}{4}$$

$$\begin{array}{r} 2 \\ 4 \overline{)9} \\ \underline{-8} \\ 1 \end{array}$$

$$\frac{9}{4} = 2\frac{1}{4}$$

Work Practice 19–20

We multiply or divide with negative mixed numbers the same way that we multiply or divide with positive mixed numbers. We first write each mixed number as a fraction.

Examples

Perform the indicated operations.

$$21. \quad -4\frac{2}{5} \cdot 1\frac{3}{11} = -\frac{22}{5} \cdot \frac{14}{11} = -\frac{22 \cdot 14}{5 \cdot 11} = -\frac{2 \cdot \cancel{11} \cdot 14}{5 \cdot \cancel{11}} = -\frac{28}{5} \text{ or } -5\frac{3}{5}$$

$$22. \quad -2\frac{1}{3} \div \left(-2\frac{1}{2}\right) = -\frac{7}{3} \div \left(-\frac{5}{2}\right) = -\frac{7}{3} \cdot \left(-\frac{2}{5}\right) = \frac{7 \cdot 2}{3 \cdot 5} = \frac{14}{15}$$

Work Practice 21–22**Helpful Hint**

Recall that

$$(-) \cdot (-) = +$$

To add or subtract with negative mixed numbers, we must be very careful! Problems arise because recall that

$$-3\frac{2}{5} \text{ means } -\left(3\frac{2}{5}\right)$$

This means that

$$-3\frac{2}{5} = -\left(3\frac{2}{5}\right) = -\left(3 + \frac{2}{5}\right) = -3 - \frac{2}{5} \quad \text{This can sometimes be easily overlooked.}$$

To avoid problems, we will add or subtract negative mixed numbers by rewriting as addition and recalling how to add signed numbers.

Practice 19–20

Write each as a mixed number.

$$19. \quad -\frac{37}{8} \quad 20. \quad -\frac{46}{5}$$

Practice 21–22

$$21. \quad 2\frac{3}{4} \cdot \left(-3\frac{3}{5}\right)$$

$$22. \quad -4\frac{2}{7} \div 1\frac{1}{4}$$

Answers

$$19. \quad -4\frac{5}{8} \quad 20. \quad -9\frac{1}{5}$$

$$21. \quad -9\frac{9}{10} \quad 22. \quad -3\frac{3}{7}$$

Practice 23

Add: $6\frac{2}{3} + \left(-12\frac{3}{4}\right)$

Practice 24

Subtract: $-9\frac{2}{7} - 30\frac{11}{14}$

Answers

23. $-6\frac{1}{12}$ 24. $-40\frac{1}{14}$

Example 23

Add: $6\frac{3}{5} + \left(-9\frac{7}{10}\right)$

Solution: Here we are adding two numbers with different signs. Recall that we then subtract the absolute values and keep the sign of the larger absolute value.

Since $-9\frac{7}{10}$ has the larger absolute value, the answer is negative.

First, subtract absolute values:

$$\begin{array}{r} 9\frac{7}{10} \\ - 6\frac{3 \cdot 2}{5 \cdot 2} \\ \hline 3\frac{1}{10} \end{array}$$

Thus,

$$6\frac{3}{5} + \left(-9\frac{7}{10}\right) = -3\frac{1}{10} \quad \text{The result is negative since } -9\frac{7}{10} \text{ has the larger absolute value.}$$

Work Practice 23

Example 24

Subtract: $-11\frac{5}{6} - 20\frac{4}{9}$

Solution: Let's write as an equivalent addition: $-11\frac{5}{6} + \left(-20\frac{4}{9}\right)$. Here, we are adding two numbers with like signs. Recall that we add their absolute values and keep the common negative sign.

First, add absolute values:

$$\begin{array}{r} 11\frac{5 \cdot 3}{6 \cdot 3} = 11\frac{15}{18} \\ + 20\frac{4 \cdot 2}{9 \cdot 2} = 20\frac{8}{18} \\ \hline 31\frac{23}{18} \text{ or } 32\frac{5}{18} \end{array}$$

$\uparrow \quad \uparrow$
 Since $\frac{23}{18} = 1\frac{5}{18}$

Thus,

$$-11\frac{5}{6} - 20\frac{4}{9} = -32\frac{5}{18} \quad \text{Keep the common sign.}$$

Work Practice 24



Calculator Explorations Converting Between Mixed Number and Fraction Notation

If your calculator has a fraction key, such as $\boxed{a^b/c}$, you can use it to convert between mixed number notation and fraction notation.

To write $13\frac{7}{16}$ as an improper fraction, press

$\boxed{13} \boxed{a^b/c} \boxed{7} \boxed{a^b/c} \boxed{16} \boxed{2nd} \boxed{d/c}$

The display will read

$\boxed{215} \boxed{16}$

which represents $\frac{215}{16}$. Thus $13\frac{7}{16} = \frac{215}{16}$.

To convert $\frac{190}{13}$ to a mixed number, press

$\boxed{190} \boxed{a^b/c} \boxed{13} =$

The display will read

$\boxed{14\_8/13}$

which represents $14\frac{8}{13}$. Thus $\frac{190}{13} = 14\frac{8}{13}$.

Write each mixed number as a fraction and each fraction as a mixed number.

1. $25\frac{5}{11}$

2. $67\frac{14}{15}$

3. $107\frac{31}{35}$

4. $186\frac{17}{21}$

5. $\frac{365}{14}$

6. $\frac{290}{13}$

7. $\frac{2769}{30}$

8. $\frac{3941}{17}$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

round

fraction

whole number

improper

mixed number

- The number $5\frac{3}{4}$ is called a(n) _____.
- For $5\frac{3}{4}$, the 5 is called the _____ part and $\frac{3}{4}$ is called the _____ part.
- To estimate operations on mixed numbers, we _____ mixed numbers to the nearest whole number.
- The mixed number $2\frac{5}{8}$ written as a(n) _____ fraction is $\frac{21}{8}$.

Martin-Gay Interactive Videos



See Video 4.7

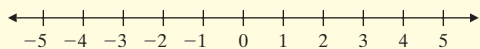
Watch the section lecture video and answer the following questions.

- Objective A** 5. In Example 1, why is the unit distance between -4 and -3 on the number line split into 5 equal parts?
- Objective B** 6. Why do we need to know how to multiply fractions to solve Example 2?
- Objective C** 7. In Example 4, why is the first form of the answer not an appropriate form?
- Objective D** 8. Why do we need to know how to subtract fractions to solve Example 5?
- Objective E** 9. In Example 6, how is it determined whether the answer is positive or negative?

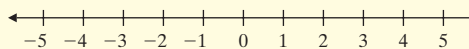
4.7 Exercise Set MyLab Math

Objective A Graph each list of numbers on the given number line. See Example 1.

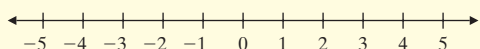
1. $-2, -2\frac{2}{3}, 0, \frac{7}{8}, -\frac{1}{3}$



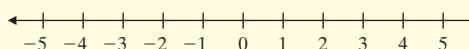
2. $-1, -1\frac{1}{4}, -\frac{1}{4}, 3\frac{1}{4}, 3$



3. $4, \frac{1}{3}, -3, -3\frac{4}{5}, 1\frac{1}{3}$



4. $3, \frac{3}{8}, -4, -4\frac{1}{3}, -\frac{9}{10}$



Objective B Choose the best estimate for each product or quotient. See Examples 4 and 5.

5. $2\frac{11}{12} \cdot 1\frac{1}{4}$

a. 2 b. 3 c. 1 d. 12

6. $5\frac{1}{6} \cdot 3\frac{5}{7}$

a. 9 b. 15 c. 8 d. 20

7. $12\frac{2}{11} \div 3\frac{9}{10}$

a. 3 b. 4 c. 36 d. 9

8. $20\frac{3}{14} \div 4\frac{8}{11}$

a. 5 b. 80 c. 4 d. 16

Multiply or divide. For Exercises 13 through 16, find the exact answer and an estimated answer. See Examples 2 through 8.

9. $2\frac{2}{3} \cdot \frac{1}{7}$

10. $\frac{5}{9} \cdot 4\frac{1}{5}$

11. $7 \div 1\frac{3}{5}$

12. $9 \div 1\frac{2}{3}$

13. $2\frac{1}{5} \cdot 3\frac{1}{2}$

14. $2\frac{1}{4} \cdot 7\frac{1}{8}$

15. $3\frac{4}{5} \cdot 6\frac{2}{7}$

16. $5\frac{5}{6} \cdot 7\frac{3}{5}$

Exact:

Exact:

Exact:

Exact:

Estimate:

Estimate:

Estimate:

Estimate:

17. $5 \cdot 2\frac{1}{2}$

18. $6 \cdot 3\frac{1}{3}$

19. $3\frac{2}{3} \cdot 1\frac{1}{2}$

20. $2\frac{4}{5} \cdot 2\frac{5}{8}$

21. $2\frac{2}{3} \div \frac{1}{7}$

22. $\frac{5}{9} \div 4\frac{1}{5}$

Objective C Choose the best estimate for each sum or difference. See Examples 9 and 12.

23. $3\frac{7}{8} + 2\frac{1}{5}$

a. 6 b. 5 c. 1 d. 2

24. $3\frac{7}{8} - 2\frac{1}{5}$

a. 6 b. 5 c. 1 d. 2

25. $8\frac{1}{3} + 1\frac{1}{2}$

a. 4 b. 10 c. 6 d. 16

26. $8\frac{1}{3} - 1\frac{1}{2}$

a. 4 b. 10 c. 6 d. 16

Add. For Exercises 27 through 30, find the exact sum and an estimated sum. See Examples 9 through 11.

27.
$$\begin{array}{r} 4\frac{7}{12} \\ + 2\frac{1}{12} \\ \hline \end{array}$$

Exact:

Estimate:

28.
$$\begin{array}{r} 7\frac{4}{11} \\ + 3\frac{2}{11} \\ \hline \end{array}$$

Exact:

Estimate:

29.
$$\begin{array}{r} 10\frac{3}{14} \\ + 3\frac{4}{7} \\ \hline \end{array}$$

Exact:

Estimate:

30.
$$\begin{array}{r} 12\frac{5}{12} \\ + 4\frac{1}{6} \\ \hline \end{array}$$

Exact:

Estimate:

31.
$$\begin{array}{r} 9\frac{1}{5} \\ + 8\frac{2}{25} \\ \hline \end{array}$$

32.
$$\begin{array}{r} 6\frac{2}{13} \\ + 8\frac{7}{26} \\ \hline \end{array}$$

33.
$$\begin{array}{r} 12\frac{3}{14} \\ 10 \\ + 25\frac{5}{12} \\ \hline \end{array}$$

34.
$$\begin{array}{r} 8\frac{2}{9} \\ 32 \\ + 9\frac{10}{21} \\ \hline \end{array}$$

▶ 35.
$$\begin{array}{r} 15\frac{4}{7} \\ + 9\frac{11}{14} \\ \hline \end{array}$$

36.
$$\begin{array}{r} 23\frac{3}{5} \\ + 8\frac{8}{15} \\ \hline \end{array}$$

37.
$$\begin{array}{r} 3\frac{5}{8} \\ 2\frac{1}{6} \\ + 7\frac{3}{4} \\ \hline \end{array}$$

38.
$$\begin{array}{r} 4\frac{1}{3} \\ 9\frac{2}{5} \\ + 3\frac{1}{6} \\ \hline \end{array}$$

Subtract. For Exercises 39 through 42, find the exact difference and an estimated difference. See Examples 12 through 14.

39.
$$\begin{array}{r} 4\frac{7}{10} \\ - 2\frac{1}{10} \\ \hline \end{array}$$

Exact:

Estimate:

40.
$$\begin{array}{r} 7\frac{4}{9} \\ - 3\frac{2}{9} \\ \hline \end{array}$$

Exact:

Estimate:

41.
$$\begin{array}{r} 10\frac{13}{14} \\ - 3\frac{4}{7} \\ \hline \end{array}$$

Exact:

Estimate:

42.
$$\begin{array}{r} 12\frac{5}{12} \\ - 4\frac{1}{6} \\ \hline \end{array}$$

Exact:

Estimate:

$$\begin{array}{r} 43. \quad 9\frac{1}{5} \\ - 8\frac{6}{25} \\ \hline \end{array}$$

$$\begin{array}{r} 44. \quad 5\frac{2}{13} \\ - 4\frac{7}{26} \\ \hline \end{array}$$

$$\begin{array}{r} 45. \quad 6 \\ - 2\frac{4}{9} \\ \hline \end{array}$$

$$\begin{array}{r} 46. \quad 8 \\ - 1\frac{7}{10} \\ \hline \end{array}$$

$$\begin{array}{r} 47. \quad 63\frac{1}{6} \\ - 47\frac{5}{12} \\ \hline \end{array}$$

$$\begin{array}{r} 48. \quad 86\frac{2}{15} \\ - 27\frac{3}{10} \\ \hline \end{array}$$

Objectives B C Mixed Practice Perform each indicated operation. See Examples 2 through 14.

$$\begin{array}{r} 49. \quad 2\frac{3}{4} \\ + 1\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 50. \quad 5\frac{5}{8} \\ + 2\frac{3}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 51. \quad 15\frac{4}{7} \\ - 9\frac{11}{14} \\ \hline \end{array}$$

$$\begin{array}{r} 52. \quad 23\frac{8}{15} \\ - 8\frac{3}{5} \\ \hline \end{array}$$

$$53. \quad 3\frac{1}{9} \cdot 2$$

$$54. \quad 4\frac{1}{2} \cdot 3$$

$$55. \quad 1\frac{2}{3} \div 2\frac{1}{5}$$

$$56. \quad 5\frac{1}{5} \div 3\frac{1}{4}$$

$$57. \quad 22\frac{4}{9} + 13\frac{5}{18}$$

$$58. \quad 15\frac{3}{25} + 5\frac{2}{5}$$

$$59. \quad 5\frac{2}{3} - 3\frac{1}{6}$$

$$60. \quad 5\frac{3}{8} - 2\frac{3}{16}$$

$$\begin{array}{r} 61. \quad 15\frac{1}{5} \\ 20\frac{3}{10} \\ + 37\frac{2}{15} \\ \hline \end{array}$$

$$\begin{array}{r} 62. \quad 3\frac{7}{16} \\ 6\frac{1}{2} \\ + 9\frac{3}{8} \\ \hline \end{array}$$

$$63. \quad 6\frac{4}{7} - 5\frac{11}{14}$$

$$64. \quad 47\frac{5}{12} - 23\frac{19}{24}$$

$$65. \quad 4\frac{2}{7} \cdot 1\frac{3}{10}$$

$$66. \quad 6\frac{2}{3} \cdot 2\frac{3}{4}$$

$$\begin{array}{r} 67. \quad 6\frac{2}{11} \\ 3 \\ + 4\frac{10}{33} \\ \hline \end{array}$$

$$\begin{array}{r} 68. \quad 7\frac{3}{7} \\ 15 \\ + 20\frac{1}{2} \\ \hline \end{array}$$

Objective D Translating Translate each phrase into an algebraic expression. Use x to represent “a number.” See Examples 15 and 16.

69. $-5\frac{2}{7}$ decreased by a number

70. The sum of $8\frac{3}{4}$ and a number

71. Multiply $1\frac{9}{10}$ by a number.

72. Divide a number by $-6\frac{1}{11}$.

Solve. For Exercises 73 and 74, the solutions have been started for you. Write each answer in simplest form. See Examples 15 and 16.

73. A heart attack patient in rehabilitation walked on a treadmill $12\frac{3}{4}$ miles over 4 days. How many miles is this per day on average?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)

miles per day	is	total miles	divided by	number of days
↓	↓	↓	↓	↓
miles per day	=	_____	÷	_____

Finish with:

3. SOLVE and 4. INTERPRET

74. A local restaurant is selling hamburgers from a booth on Memorial Day. A total of $27\frac{3}{4}$ pounds of hamburger have been ordered. How many quarter-pound hamburgers can this make?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)

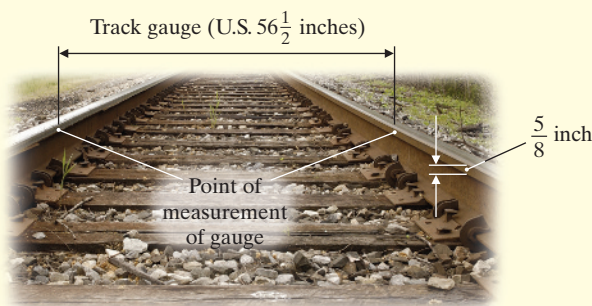
how many quarter-pound hamburgers	is	total pounds of hamburger	divided by	a quarter pound
↓	↓	↓	↓	↓
how many quarter-pound hamburgers	=	_____	÷	_____

Finish with:

3. SOLVE and 4. INTERPRET

75. The Gauge Act of 1846 set the standard gauge for U.S. railroads at $56\frac{1}{2}$ inches. (See figure.) If the standard gauge in Spain is $65\frac{9}{10}$ inches, how much wider is Spain's standard gauge than the U.S. standard gauge? (Source: San Diego Railroad Museum)

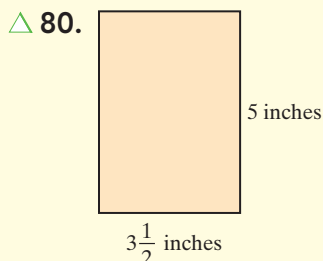
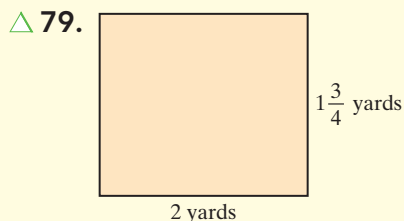
76. The standard railroad track gauge (see figure) in Spain is $65\frac{9}{10}$ inches, while in neighboring Portugal it is $65\frac{11}{20}$ inches. Which gauge is wider and by how much? (Source: San Diego Railroad Museum)



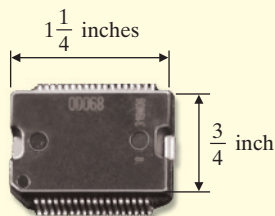
77. If Tucson's average rainfall is $11\frac{1}{4}$ inches and Yuma's is $3\frac{3}{5}$ inches, how much more rain, on the average, does Tucson get than Yuma?

78. A pair of crutches needs adjustment. One crutch is 43 inches and the other is $41\frac{5}{8}$ inches. Find how much the shorter crutch should be lengthened to make both crutches the same length.

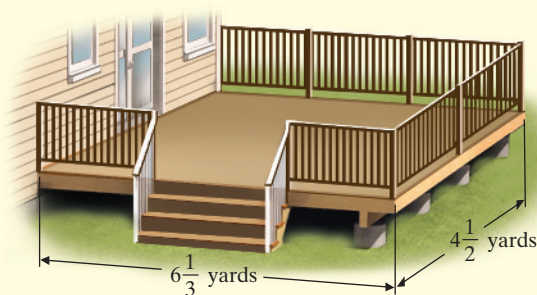
For Exercises 79 and 80, find the area of each figure.



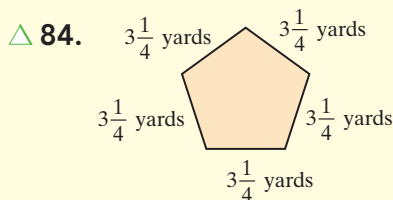
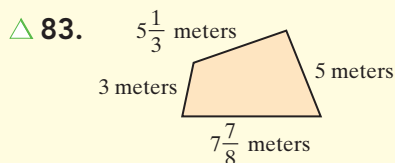
81. A model for a proposed computer chip measures $\frac{3}{4}$ inch by $1\frac{1}{4}$ inches. Find its area.



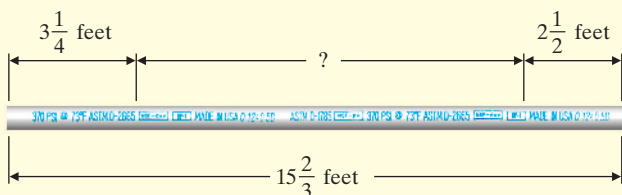
82. The Saltalamachios are planning to build a deck that measures $4\frac{1}{2}$ yards by $6\frac{1}{3}$ yards. Find the area of their proposed deck.



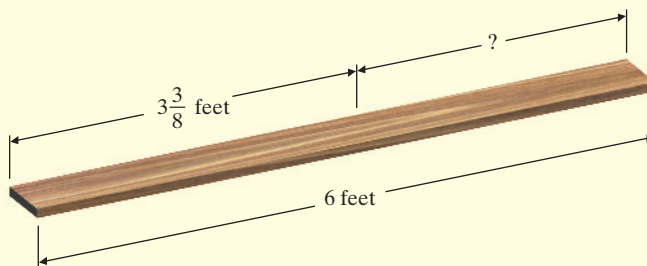
For Exercises 83 and 84, find the perimeter of each figure.



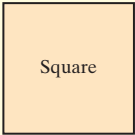
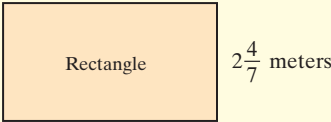
85. A homeowner has $15\frac{2}{3}$ feet of plastic pipe. She cuts off a $2\frac{1}{2}$ -foot length and then a $3\frac{1}{4}$ -foot length. If she now needs a 10-foot piece of pipe, will the remaining piece do? If not, by how much will the piece be short?



86. A trim carpenter cuts a board $3\frac{3}{8}$ feet long from one 6 feet long. How long is the remaining piece?



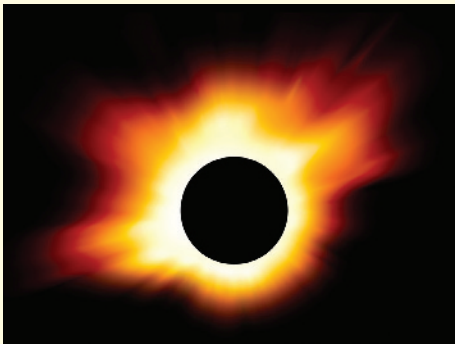
- △ 87. The area of the rectangle below is 12 square meters. If its width is $2\frac{4}{7}$ meters, find its length.
- △ 88. The perimeter of the square below is $23\frac{1}{2}$ feet. Find the length of each side.



The following table lists three upcoming total eclipses of the Sun that will be visible in North America. The duration of each eclipse is listed in the table. Use the table to answer Exercises 89 through 92.

Total Solar Eclipses Visible from North America	
Date of Eclipse	Duration (in minutes)
August 21, 2017	$2\frac{2}{3}$
April 8, 2024	$4\frac{7}{15}$
March 30, 2033	$2\frac{37}{60}$
(Source: NASA/Goddard Space Flight Center)	

89. What is the total duration for the three eclipses?
90. What is the total duration for the two eclipses occurring in odd-numbered years?
91. How much longer will the April 8, 2024, eclipse be than the August 21, 2017, eclipse?
92. How much longer will the April 8, 2024, eclipse be than the March 30, 2033, eclipse?



Objective E Perform the indicated operations. See Examples 17 through 24.

93. $-4\frac{2}{5} \cdot 2\frac{3}{10}$

94. $-3\frac{5}{6} \div \left(-3\frac{2}{3}\right)$

95. $-5\frac{1}{8} - 19\frac{3}{4}$

96. $17\frac{5}{9} + \left(-14\frac{2}{3}\right)$
- ▶ 97. $-31\frac{2}{15} + 17\frac{3}{20}$

98. $-31\frac{7}{8} - \left(-26\frac{5}{12}\right)$

99. $-1\frac{5}{7} \cdot \left(-2\frac{1}{2}\right)$

100. $1\frac{3}{4} \div \left(-3\frac{1}{2}\right)$
101. $11\frac{7}{8} - 13\frac{5}{6}$

102. $-20\frac{2}{5} + \left(-30\frac{3}{10}\right)$

103. $-7\frac{3}{10} \div (-100)$

104. $-4\frac{1}{4} \div 2\frac{3}{8}$

Review

Multiply. See Section 4.3.

105. $\frac{1}{3}(3x)$

106. $\frac{1}{5}(5y)$

107. $\frac{2}{3}\left(\frac{3}{2}a\right)$

108. $-\frac{9}{10}\left(-\frac{10}{9}m\right)$

Concept Extensions

Solve. See the first Concept Check in this section.

109. Which of the following are equivalent to 10?

a. $9\frac{5}{5}$ b. $9\frac{100}{100}$ c. $6\frac{44}{11}$ d. $8\frac{13}{13}$

110. Which of the following are equivalent to $7\frac{3}{4}$?

a. $6\frac{7}{4}$ b. $5\frac{11}{4}$ c. $7\frac{12}{16}$ d. all of them

Solve. See the second Concept Check in this section.

 111. A student asked you to check her work below. Is it correct? If not, where is the error?

$$20\frac{2}{3} \div 10\frac{1}{2} \stackrel{?}{=} 2\frac{1}{3}$$

 112. A student asked you to check his work below. Is it correct? If not, where is the error?

$$3\frac{2}{3} \cdot 1\frac{1}{7} \stackrel{?}{=} 3\frac{2}{21}$$

 113. In your own words, describe how to divide mixed numbers.

 114. In your own words, explain how to multiply

- fractions
- mixed numbers

Solve. See the third Concept Check in this section.

 115. In your own words, explain how to round a mixed number to the nearest whole number.

116. Use rounding to estimate the best sum for

$$11\frac{19}{20} + 9\frac{1}{10}$$

a. 2 b. 3 c. 20 d. 21

Solve.

 117. Explain in your own words why $9\frac{13}{9}$ is equal to $10\frac{4}{9}$.

 118. In your own words, explain

- when to borrow when subtracting mixed numbers, and
- how to borrow when subtracting mixed numbers.

4.8 Solving Equations Containing Fractions

Objective A Solving Equations Containing Fractions

In Chapter 3, we solved linear equations in one variable. In this section, we practice this skill by solving linear equations containing fractions. To help us solve these equations, let's review the properties of equality.

Addition Property of Equality

Let a , b , and c represent numbers. Then

$a = b$	Also, $a = b$
and $a + c = b + c$	and $a - c = b - c$
are equivalent equations.	are equivalent equations.

Multiplication Property of Equality

Let a , b , and c represent numbers and let $c \neq 0$. Then

$a = b$	Also, $a = b$
and $a \cdot c = b \cdot c$	and $\frac{a}{c} = \frac{b}{c}$
are equivalent equations.	are equivalent equations.

In other words, the same number may be added to or subtracted from both sides of an equation without changing the solution of the equation. Also, both sides of an equation may be multiplied or divided by the same nonzero number without changing the solution.

Also, don't forget that to solve an equation in x , our goal is to use properties of equality to write simpler equations, all equivalent to the original equation, until the final equation has the form

$$x = \text{number} \quad \text{or} \quad \text{number} = x$$

Example 1 Solve: $x - \frac{3}{4} = \frac{1}{20}$

Solution: To get x by itself, add $\frac{3}{4}$ to both sides.

$$\begin{aligned}x - \frac{3}{4} &= \frac{1}{20} \\x - \frac{3}{4} + \frac{3}{4} &= \frac{1}{20} + \frac{3}{4} && \text{Add } \frac{3}{4} \text{ to both sides.} \\x &= \frac{1}{20} + \frac{3 \cdot 5}{4 \cdot 5} && \text{The LCD of the denominators 20 and 4 is 20.} \\x &= \frac{1}{20} + \frac{15}{20} \\x &= \frac{16}{20} \\x &= \frac{\overset{1}{\cancel{4}} \cdot 4}{\cancel{4} \cdot 5} = \frac{4}{5} && \text{Write } \frac{16}{20} \text{ in simplest form.}\end{aligned}$$

Objectives

- A** Solve Equations Containing Fractions. ▶
- B** Solve Equations by Multiplying by the LCD. ▶
- C** Review Adding and Subtracting Fractions. ▶

Practice 1

Solve: $y - \frac{2}{3} = \frac{5}{12}$

Answer

1. $\frac{13}{12}$

(Continued on next page)

Check: To check, replace x with $\frac{4}{5}$ in the original equation.

$$x - \frac{3}{4} = \frac{1}{20}$$

$$\frac{4}{5} - \frac{3}{4} \stackrel{?}{=} \frac{1}{20} \quad \text{Replace } x \text{ with } \frac{4}{5}.$$

$$\frac{4 \cdot 4}{5 \cdot 4} - \frac{3 \cdot 5}{4 \cdot 5} \stackrel{?}{=} \frac{1}{20} \quad \text{The LCD of the denominators 5 and 4 is 20.}$$

$$\frac{16}{20} - \frac{15}{20} \stackrel{?}{=} \frac{1}{20}$$

$$\frac{1}{20} = \frac{1}{20} \quad \text{True}$$

Thus $\frac{4}{5}$ is the solution of $x - \frac{3}{4} = \frac{1}{20}$.

Work Practice 1

Practice 2

Solve: $\frac{1}{5}y = 2$

Example 2 Solve: $\frac{1}{3}x = 7$

Solution: Recall that isolating x means that we want the coefficient of x to be 1. To do so, we use the multiplication property of equality and multiply both sides of the equation by the reciprocal of $\frac{1}{3}$, or 3. Since $\frac{1}{3} \cdot 3 = 1$, we will have isolated x .

$$\frac{1}{3}x = 7$$

$$3 \cdot \frac{1}{3}x = 3 \cdot 7 \quad \text{Multiply both sides by 3.}$$

$$1 \cdot x = 21 \text{ or } x = 21 \quad \text{Simplify.}$$

Check: To check, replace x with 21 in the original equation.

$$\frac{1}{3}x = 7 \quad \text{Original equation}$$

$$\frac{1}{3} \cdot 21 \stackrel{?}{=} 7 \quad \text{Replace } x \text{ with 21.}$$

$$7 = 7 \quad \text{True}$$

Since $7 = 7$ is a true statement, 21 is the solution of $\frac{1}{3}x = 7$.

Work Practice 2

Practice 3

Solve: $\frac{5}{7}b = 25$

Example 3 Solve: $\frac{3}{5}a = 9$

Solution: Multiply both sides by $\frac{5}{3}$, the reciprocal of $\frac{3}{5}$, so that the coefficient of a is 1.

$$\frac{3}{5}a = 9$$

$$\frac{5}{3} \cdot \frac{3}{5}a = \frac{5}{3} \cdot 9 \quad \text{Multiply both sides by } \frac{5}{3}.$$

$$1a = \frac{5 \cdot 9}{3} \quad \text{Multiply.}$$

$$a = 15 \quad \text{Simplify.}$$

Answers

2. 10 3. 35

Check: To check, replace a with 15 in the original equation.

$$\frac{3}{5}a = 9$$

$$\frac{3}{5} \cdot 15 \stackrel{?}{=} 9 \quad \text{Replace } a \text{ with 15.}$$

$$\frac{3 \cdot 15}{5} \stackrel{?}{=} 9 \quad \text{Multiply.}$$

$$9 = 9 \quad \text{True}$$

Since $9 = 9$ is true, 15 is the solution of $\frac{3}{5}a = 9$.

Work Practice 3

Example 4 Solve: $\frac{3}{4}x = -\frac{1}{8}$

Solution: Multiply both sides of the equation by $\frac{4}{3}$, the reciprocal of $\frac{3}{4}$.

$$\frac{3}{4}x = -\frac{1}{8}$$

$$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3} \cdot -\frac{1}{8} \quad \text{Multiply both sides by } \frac{4}{3}.$$

$$1x = -\frac{4 \cdot 1}{3 \cdot 8} \quad \text{Multiply.}$$

$$x = -\frac{1}{6} \quad \text{Simplify.}$$

Check: To check, replace x with $-\frac{1}{6}$ in the original equation.

$$\frac{3}{4}x = -\frac{1}{8} \quad \text{Original equation}$$

$$\frac{3}{4} \cdot -\frac{1}{6} \stackrel{?}{=} -\frac{1}{8} \quad \text{Replace } x \text{ with } -\frac{1}{6}.$$

$$-\frac{1}{8} = -\frac{1}{8} \quad \text{True}$$

Since we arrived at a true statement, $-\frac{1}{6}$ is the solution of $\frac{3}{4}x = -\frac{1}{8}$.

Work Practice 4

Example 5 Solve: $3y = -\frac{2}{11}$

Solution: We can either divide both sides by 3 or multiply both sides by the reciprocal of 3, which is $\frac{1}{3}$.

$$3y = -\frac{2}{11}$$

$$\frac{1}{3} \cdot 3y = \frac{1}{3} \cdot -\frac{2}{11} \quad \text{Multiply both sides by } \frac{1}{3}.$$

$$1y = -\frac{1 \cdot 2}{3 \cdot 11} \quad \text{Multiply.}$$

$$y = -\frac{2}{33} \quad \text{Simplify.}$$

Check to see that the solution is $-\frac{2}{33}$.

Work Practice 5

Practice 4

$$\text{Solve: } -\frac{7}{10}x = \frac{2}{5}$$

Practice 5

$$\text{Solve: } 5x = -\frac{3}{4}$$

Answers

$$4. -\frac{4}{7} \quad 5. -\frac{3}{20}$$

Objective B Solving Equations by Multiplying by the LCD

Solving equations with fractions can be tedious. If an equation contains fractions, it is often helpful to first multiply both sides of the equation by the LCD of the fractions. This has the effect of eliminating the fractions in the equation, as shown in the next example.

Let's solve the equation in Example 4 again. This time, we will multiply both sides by the LCD.

Practice 6

Solve: $\frac{11}{15}x = -\frac{3}{5}$

Example 6 Solve: $\frac{3}{4}x = -\frac{1}{8}$ (Example 4 solved an alternate way.)

Solution: First, multiply both sides of the equation by the LCD of the fractions $\frac{3}{4}$ and $-\frac{1}{8}$. The LCD of the denominators is 8.

$$\frac{3}{4}x = -\frac{1}{8}$$
$$8 \cdot \frac{3}{4}x = 8 \cdot -\frac{1}{8} \quad \text{Multiply both sides by 8.}$$
$$\frac{8 \cdot 3}{1 \cdot 4}x = -\frac{8 \cdot 1}{1 \cdot 8} \quad \text{Multiply the fractions.}$$
$$\frac{2 \cdot \cancel{4} \cdot 3}{1 \cdot \cancel{4}}x = -\frac{1 \cdot 1}{1 \cdot 1} \quad \text{Simplify.}$$
$$6x = -1$$
$$\frac{6x}{6} = \frac{-1}{6} \quad \text{Divide both sides by 6.}$$
$$x = \frac{-1}{6} \text{ or } -\frac{1}{6} \quad \text{Simplify.}$$

As seen in Example 4, the solution is $-\frac{1}{6}$.

Work Practice 6

Practice 7

Solve: $\frac{y}{8} + \frac{3}{4} = 2$

Example 7 Solve: $\frac{x}{6} + 1 = \frac{4}{3}$

Solution:

Solve by multiplying by the LCD:	Solve with fractions:
The LCD of the denominators 6 and 3 is 6.	$\frac{x}{6} + 1 = \frac{4}{3}$
$\frac{x}{6} + 1 = \frac{4}{3}$	$\frac{x}{6} + 1 - 1 = \frac{4}{3} - 1 \quad \text{Subtract 1 from both sides.}$
$6\left(\frac{x}{6} + 1\right) = 6\left(\frac{4}{3}\right) \quad \text{Multiply both sides by 6.}$	$\frac{x}{6} = \frac{4}{3} - \frac{3}{3} \quad \text{Write 1 as } \frac{3}{3}.$
$\cancel{6}\left(\frac{x}{\cancel{6}}\right) + \cancel{6}(1) = \cancel{6}\left(\frac{4}{\cancel{3}}\right) \quad \text{Apply the distributive property.}$	$\frac{x}{6} = \frac{1}{3} \quad \text{Subtract.}$
$x + 6 = 8 \quad \text{Simplify.}$	$6 \cdot \frac{x}{6} = 6 \cdot \frac{1}{3} \quad \text{Multiply both sides by 6.}$
$x + 6 + (-6) = 8 + (-6) \quad \text{Add -6 to both sides.}$	$6 \cdot \frac{1}{\cancel{6}} \cdot x = \frac{\cancel{6} \cdot 1}{1 \cdot \cancel{3}} \quad \text{Simplify.}$
$x = 2 \quad \text{Simplify.}$	$1x = 2 \quad \text{or} \quad x = 2$

Check: To check, replace x with 2 in the original equation.

$$\frac{x}{6} + 1 = \frac{4}{3} \quad \text{Original equation}$$

$$\frac{2}{6} + 1 \stackrel{?}{=} \frac{4}{3} \quad \text{Replace } x \text{ with } 2.$$

$$\frac{1}{3} + \frac{3}{3} \stackrel{?}{=} \frac{4}{3} \quad \text{Simplify } \frac{2}{6}. \text{ The LCD of 3 and 1 is 3.}$$

$$\frac{4}{3} = \frac{4}{3} \quad \text{True}$$

Since we arrived at a true statement, 2 is the solution of $\frac{x}{6} + 1 = \frac{4}{3}$.

Work Practice 7

Let's review the steps for solving equations in x . An extra step is now included to handle equations containing fractions.

Solving an Equation in x

Step 1: If fractions are present, multiply both sides of the equation by the LCD of the fractions.

Step 2: If parentheses are present, use the distributive property.

Step 3: Combine any like terms on each side of the equation.

Step 4: Use the addition property of equality to rewrite the equation so that variable terms are on one side of the equation and constant terms are on the other side.

Step 5: Divide both sides of the equation by the numerical coefficient of x to solve.

Step 6: Check the answer in the **original equation**.

Example 8 Solve: $\frac{z}{5} - \frac{z}{3} = 6$

Solution:

$$\frac{z}{5} - \frac{z}{3} = 6$$

$$15\left(\frac{z}{5} - \frac{z}{3}\right) = 15(6) \quad \text{Multiply both sides by the LCD, 15.}$$

$$15\left(\frac{z}{5}\right) - 15\left(\frac{z}{3}\right) = 15(6) \quad \text{Apply the distributive property.}$$

$$3z - 5z = 90 \quad \text{Simplify.}$$

$$-2z = 90 \quad \text{Combine like terms.}$$

$$\frac{-2z}{-2} = \frac{90}{-2} \quad \text{Divide both sides by } -2, \text{ the coefficient of } z.$$

$$z = -45 \quad \text{Simplify.}$$

To check, replace z with -45 in the **original equation** to see that a true statement results.

Work Practice 8

Practice 8

Solve: $\frac{x}{5} - x = \frac{1}{5}$

Helpful Hint

Don't forget to multiply *both* sides of the equation by the LCD.

Answer

8. $-\frac{1}{4}$

Practice 9

Solve: $\frac{y}{2} = \frac{y}{5} + \frac{3}{2}$

Example 9 Solve: $\frac{x}{2} = \frac{x}{3} + \frac{1}{2}$

Solution: First multiply both sides by the LCD, 6.

$$\begin{aligned}\frac{x}{2} &= \frac{x}{3} + \frac{1}{2} \\ 6\left(\frac{x}{2}\right) &= 6\left(\frac{x}{3} + \frac{1}{2}\right) && \text{Multiply both sides by the LCD, 6.} \\ \cancel{6}^3\left(\frac{x}{\cancel{2}_1}\right) &= \cancel{6}^2\left(\frac{x}{\cancel{3}_1}\right) + \cancel{6}^3\left(\frac{1}{\cancel{2}_1}\right) && \text{Apply the distributive property.} \\ 3x &= 2x + 3 && \text{Simplify.} \\ 3x - 2x &= 2x + 3 - 2x && \text{Subtract } 2x \text{ from both sides.} \\ x &= 3 && \text{Simplify.}\end{aligned}$$

To check, replace x with 3 in the original equation to see that a true statement results.

 Work Practice 9

Objective C Review of Adding and Subtracting Fractions

Make sure you understand the difference between **solving an equation** containing fractions and **adding or subtracting two fractions**. To solve an equation containing fractions, we use the multiplication property of equality and multiply both sides by the LCD of the fractions, thus eliminating the fractions. This method does not apply to adding or subtracting fractions. The multiplication property of equality applies only to equations. To add or subtract unlike fractions, we write each fraction as an equivalent fraction using the LCD of the fractions as the denominator. See the next example for a review.

Example 10 Add: $\frac{x}{3} + \frac{2}{5}$

Solution: This is an expression, not an equation. Here, we are adding two unlike fractions. To add unlike fractions, we need to find the LCD. The LCD of the denominators 3 and 5 is 15. Write each fraction as an equivalent fraction with a denominator of 15.

$$\begin{aligned}\frac{x}{3} + \frac{2}{5} &= \frac{x}{3} \cdot \frac{5}{5} + \frac{2}{5} \cdot \frac{3}{3} = \frac{x \cdot 5}{3 \cdot 5} + \frac{2 \cdot 3}{5 \cdot 3} \\ &= \frac{5x}{15} + \frac{6}{15} \\ &= \frac{5x + 6}{15}\end{aligned}$$

 Work Practice 10

 **Concept Check** Which of the following are equations and which are expressions?

- a. $\frac{1}{2} + 3x = 5$ b. $\frac{2}{3}x - \frac{x}{5}$
c. $\frac{x}{12} + \frac{5x}{24}$ d. $\frac{x}{5} = \frac{1}{10}$

Answers

9. 5 10. $\frac{27 - 10y}{30}$

 **Concept Check Answers**
equations: a, d; expressions: b, c

Vocabulary, Readiness & Video Check

Fill in the blank with the least common denominator (LCD). Do not solve these equations.

1. Equation: $\frac{2}{3} + x = \frac{5}{6}$; LCD = ____

2. Equation: $\frac{x}{21} - 1 = \frac{1}{7}$; LCD = ____

3. Equation: $\frac{y}{5} + \frac{1}{3} = 2$; LCD = ____

4. Equation: $\frac{-2n}{11} + \frac{1}{2} = 5$; LCD = ____

Martin-Gay Interactive Videos



See Video 4.8

Watch the section lecture video and answer the following questions.

Objective A 5. In Example 1, what property is used to get x by itself on one side of the equation?

6. Explain how a reciprocal is used to solve Example 2.

Objective B 7. Why are both sides of the equation multiplied by 12 in Example 3? What effect does this have on the fractions in the equation?

Objective C 8. In Example 5, why can't we multiply by the LCD of all fractions?

4.8 Exercise Set MyLab Math

Objective A Solve each equation. Check your proposed solution. See Example 1.

1. $x + \frac{1}{3} = -\frac{1}{3}$

2. $x + \frac{1}{9} = -\frac{7}{9}$

3. $y - \frac{3}{13} = -\frac{2}{13}$

4. $z - \frac{5}{14} = \frac{4}{14}$

5. $3x - \frac{1}{5} - 2x = \frac{1}{5} + \frac{2}{5}$

6. $5x + \frac{1}{11} - 4x = \frac{2}{11} - \frac{5}{11}$

7. $x - \frac{1}{12} = \frac{5}{6}$

8. $y - \frac{8}{9} = \frac{1}{3}$

9. $\frac{2}{5} + y = -\frac{3}{10}$

10. $\frac{1}{2} + a = -\frac{3}{8}$

11. $7z + \frac{1}{16} - 6z = \frac{3}{4}$

12. $9x - \frac{2}{7} - 8x = \frac{11}{14}$

13. $-\frac{2}{9} = x - \frac{5}{6}$

14. $-\frac{1}{4} = y - \frac{7}{10}$

Solve each equation. See Examples 2 through 5.

15. $7x = 2$

16. $-5x = 4$

17. $\frac{1}{4}x = 3$

18. $\frac{1}{3}x = 6$

19. $\frac{2}{9}y = -6$

20. $\frac{4}{7}x = -8$

21. $-\frac{4}{9}z = -\frac{3}{2}$

22. $-\frac{11}{10}x = -\frac{2}{7}$

23. $7a = \frac{1}{3}$

24. $2z = -\frac{5}{12}$

25. $-3x = -\frac{6}{11}$

26. $-4z = -\frac{12}{25}$

Objective B Solve each equation. See Examples 6 through 9.

27. $\frac{5}{9}x = -\frac{3}{18}$

28. $\frac{3}{5}y = -\frac{7}{20}$

29. $\frac{x}{3} + 2 = \frac{7}{3}$

30. $\frac{x}{5} - 1 = \frac{7}{5}$

31. $\frac{x}{5} - x = -8$

32. $\frac{x}{3} - x = -6$

33. $\frac{1}{2} - \frac{3}{5} = \frac{x}{10}$

34. $\frac{2}{3} - \frac{1}{4} = \frac{x}{12}$

35. $\frac{x}{3} = \frac{x}{5} - 2$

36. $\frac{a}{2} = \frac{a}{7} + \frac{5}{2}$

Objective C Add or subtract as indicated. See Example 10.

37. $\frac{x}{7} - \frac{4}{3}$

38. $-\frac{5}{9} + \frac{y}{8}$

39. $\frac{y}{2} + 5$

40. $2 + \frac{7x}{3}$

41. $\frac{3x}{10} + \frac{x}{6}$

42. $\frac{9x}{8} - \frac{5x}{6}$

Objectives A B C Mixed Practice Solve. If no equation is given, perform the indicated operation. See Examples 1 through 10.

43. $\frac{3}{8}x = \frac{1}{2}$

44. $\frac{2}{5}y = \frac{3}{10}$

45. $\frac{2}{3} - \frac{x}{5} = \frac{4}{15}$

46. $\frac{4}{5} + \frac{x}{4} = \frac{21}{20}$

47. $\frac{9}{14}z = \frac{27}{20}$

48. $\frac{5}{16}a = \frac{5}{6}$

49. $-3m - 5m = \frac{4}{7}$

50. $30n - 34n = \frac{3}{20}$

51. $\frac{x}{4} + 1 = \frac{1}{4}$

52. $\frac{y}{7} - 2 = \frac{1}{7}$

53. $\frac{5}{9} - \frac{2}{3}$

54. $\frac{8}{11} - \frac{1}{2}$

55. $\frac{1}{5}y = 10$

56. $\frac{1}{4}x = -2$

57. $\frac{5}{7}y = -\frac{15}{49}$

58. $-\frac{3}{4}x = \frac{9}{2}$

59. $\frac{x}{2} - x = -2$

60. $\frac{y}{3} = -4 + y$

61. $-\frac{5}{8}y = \frac{3}{16} - \frac{9}{16}$

62. $-\frac{7}{9}x = -\frac{5}{18} - \frac{4}{18}$

63. $17x - 25x = \frac{1}{3}$

64. $27x - 30x = \frac{4}{9}$

65. $\frac{7}{6}x = \frac{1}{4} - \frac{2}{3}$

66. $\frac{5}{4}y = \frac{1}{2} - \frac{7}{10}$

67. $\frac{b}{4} = \frac{b}{12} + \frac{2}{3}$

68. $\frac{a}{6} = \frac{a}{3} + \frac{1}{2}$

69. $\frac{x}{3} + 2 = \frac{x}{2} + 8$

70. $\frac{y}{5} - 2 = \frac{y}{3} - 4$

Review

Round each number to the given place value. See Section 1.4.

71. 57,236 to the nearest hundred

72. 576 to the nearest hundred

73. 327 to the nearest ten

74. 2333 to the nearest ten

Concept Extensions

75. Explain why the method for eliminating fractions in an **equation** does not apply to simplifying **expressions** containing fractions.

76. Think about which exercise (part **a** or part **b**) may be completed by multiplying by 6. Now complete each exercise.

a. Solve: $\frac{x}{6} - \frac{5}{3} = 2$

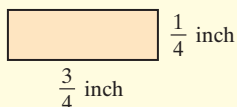
b. Subtract: $\frac{x}{6} - \frac{5}{3}$

Solve.

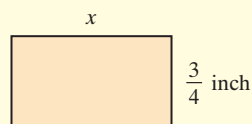
77. $\frac{14}{11} + \frac{3x}{8} = \frac{x}{2}$

78. $\frac{19}{53} = \frac{353x}{1431} + \frac{23}{27}$

79. Find the area and the perimeter of the rectangle. Remember to attach proper units.



80. The area of the rectangle is $\frac{5}{12}$ square inch. Find its length, x .



Chapter 4 Group Activity

Lobster Classification

Sections 4.1, 4.7, 4.8

This activity may be completed by working in groups or individually.

Lobsters are normally classified by weight. Use the weight classification table to answer the questions in this activity.

Classification of Lobsters	
Class	Weight (in pounds)
Chicken	1 to $1\frac{1}{8}$
Eighths	$1\frac{1}{8}$ to $1\frac{1}{4}$
Quarter	$1\frac{1}{4}$ to $1\frac{1}{2}$
Large (or select)	$1\frac{1}{2}$ to $2\frac{1}{2}$
Jumbo	Over $2\frac{1}{2}$

(Source: The Maine Lobster Marketing Collaborative)

1. A lobsterman has kept four lobsters from a lobster trap. Classify each lobster if they have the following weights:
- a. $1\frac{7}{8}$ pounds
 - b. $1\frac{1}{16}$ pounds

- c. $2\frac{3}{4}$ pounds
- d. $1\frac{3}{8}$ pounds

2. A recipe requires 5 pounds of lobster. Using the minimum weight for each class, decide whether a chicken, a quarter, and a jumbo lobster will be enough for the recipe, and explain your reasoning. If not, suggest a better choice of lobsters to meet the recipe requirements.
3. A lobster market customer has selected two chickens, a large and a jumbo. The jumbo lobster weighs $3\frac{1}{4}$ pounds. What is the most that these four lobsters could weigh? What is the least that these four lobsters could weigh?
4. A lobster market customer wishes to buy three quarters, each weighing $1\frac{1}{4}$ pounds. If lobsters sell for \$7 per pound, how much will the customer owe for her purchase?
5. Why do you think there is no classification for lobsters weighing under 1 pound?

Chapter 4 Vocabulary Check

Fill in each blank with one of the words or phrases listed below.

- mixed number

composite number

prime number

reciprocals
- complex fraction

equivalent

improper fraction

proper fraction
- like

cross products

simplest form
- numerator

least common denominator

undefined
- prime factorization

denominator

0

1. Two numbers are _____ of each other if their product is 1.
2. A(n) _____ is a natural number greater than 1 that is not prime.
3. Fractions that represent the same portion of a whole are called _____ fractions.
4. A(n) _____ is a fraction whose numerator is greater than or equal to its denominator.
5. A(n) _____ is a natural number greater than 1 whose only factors are 1 and itself.
6. A fraction is in _____ when the numerator and the denominator have no factors in common other than 1.
7. A(n) _____ is one whose numerator is less than its denominator.
8. A(n) _____ contains a whole number part and a fraction part.
9. In the fraction $\frac{7}{9}$, the 7 is called the _____ and the 9 is called the _____.

10. The _____ of a number is the factorization in which all the factors are prime numbers.
11. The fraction $\frac{3}{0}$ is _____.
12. The fraction $\frac{0}{5} =$ _____.
13. Fractions that have the same denominator are called _____ fractions.
14. The LCM of the denominators in a list of fractions is called the _____.
15. A fraction whose numerator or denominator or both numerator and denominator contain fractions is called a(n) _____.
16. In $\frac{a}{b} = \frac{c}{d}$, $a \cdot d$ and $b \cdot c$ are called _____.

Helpful Hint

▶ Are you preparing for your test?

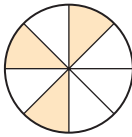
To help, don't forget to take these:

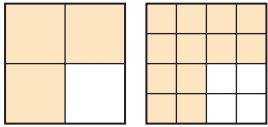
- Chapter 4 Getting Ready for the Test on page 326
- Chapter 4 Test on page 327

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

4

Chapter Highlights

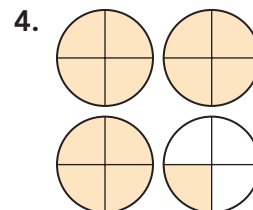
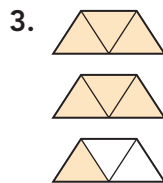
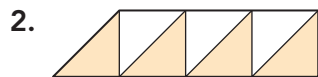
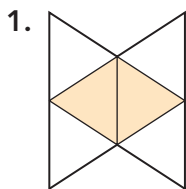
Definitions and Concepts	Examples
Section 4.1 Introduction to Fractions and Mixed Numbers	
A fraction is of the form $\frac{\text{numerator}}{\text{denominator}}$ ← number of parts being considered ← number of equal parts in the whole A fraction is called a proper fraction if its numerator is less than its denominator. A fraction is called an improper fraction if its numerator is greater than or equal to its denominator. A mixed number contains a whole number and a fraction. To Write a Mixed Number as an Improper Fraction 1. Multiply the denominator of the fraction by the whole number. 2. Add the numerator of the fraction to the product from Step 1. 3. Write the sum from Step 2 as the numerator of the improper fraction over the original denominator. To Write an Improper Fraction as a Mixed Number or a Whole Number 1. Divide the denominator into the numerator. 2. The whole number part of the mixed number is the quotient. The fraction is the remainder over the original denominator. $\text{quotient} \frac{\text{remainder}}{\text{original denominator}}$	Write a fraction to represent the shaded part of the figure.  3 ← number of parts shaded 8 ← number of equal parts Proper Fractions: $\frac{1}{3}, \frac{2}{5}, \frac{7}{8}, \frac{100}{101}$ Improper Fractions: $\frac{5}{4}, \frac{2}{2}, \frac{9}{7}, \frac{101}{100}$ Mixed Numbers: $1\frac{1}{2}, 5\frac{7}{8}, 25\frac{9}{10}$ $5\frac{2}{7} = \frac{7 \cdot 5 + 2}{7} = \frac{35 + 2}{7} = \frac{37}{7}$ $\frac{17}{3} = 5\frac{2}{3}$ $\begin{array}{r} 5 \\ 3 \overline{)17} \\ \underline{-15} \\ 2 \end{array}$

Definitions and Concepts	Examples
Section 4.2 Factors and Simplest Form	
A prime number is a natural number that has exactly two different factors, 1 and itself. A composite number is any natural number other than 1 that is not prime. The prime factorization of a number is the factorization in which all the factors are prime numbers. Fractions that represent the same portion of a whole are called equivalent fractions. A fraction is in simplest form or lowest terms when the numerator and the denominator have no common factors other than 1. To write a fraction in simplest form, write the prime factorizations of the numerator and the denominator and then divide both by all common factors. Two fractions are equivalent if Method 1. They simplify to the same fraction. Method 2. Their cross products are equal. $\begin{array}{ccc} 24 \cdot 7 & \begin{array}{c} \nearrow \quad \nwarrow \\ \searrow \quad \nearrow \end{array} & 8 \cdot 21 \\ = 168 & & = 168 \\ & \frac{7}{8} = \frac{21}{24} & \end{array}$ Since $168 = 168$, $\frac{7}{8} = \frac{21}{24}$.	2, 3, 5, 7, 11, 13, 17, ... 4, 6, 8, 9, 10, 12, 14, 15, 16, ... Write the prime factorization of 60. $ \begin{aligned} 60 &= 6 \cdot 10 \\ &= 2 \cdot 3 \cdot 2 \cdot 5 \quad \text{or} \quad 2^2 \cdot 3 \cdot 5 \end{aligned} $  $\frac{3}{4} = \frac{12}{16}$ The fraction $\frac{2}{3}$ is in simplest form. Write in simplest form: $\frac{30}{36}$ $ \begin{aligned} \frac{30}{36} &= \frac{2 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 3} = \frac{2}{2} \cdot \frac{3}{3} \cdot \frac{5}{2 \cdot 3} = 1 \cdot 1 \cdot \frac{5}{6} = \frac{5}{6} \\ \text{or } \frac{30}{36} &= \frac{\overset{1}{2} \cdot \overset{1}{3} \cdot 5}{\underset{1}{2} \cdot 2 \cdot \underset{1}{3} \cdot 3} = \frac{5}{6} \end{aligned} $ Determine whether $\frac{7}{8}$ and $\frac{21}{24}$ are equivalent. $\frac{7}{8}$ is in simplest form. $ \frac{21}{24} = \frac{\overset{1}{3} \cdot 7}{\underset{1}{3} \cdot 8} = \frac{1 \cdot 7}{1 \cdot 8} = \frac{7}{8} $ Since both simplify to $\frac{7}{8}$, then $\frac{7}{8} = \frac{21}{24}$.
Section 4.3 Multiplying and Dividing Fractions	
To multiply two fractions, multiply the numerators and multiply the denominators. To find the reciprocal of a fraction, interchange its numerator and denominator. To divide two fractions, multiply the first fraction by the reciprocal of the second fraction.	Multiply. $ \begin{aligned} \frac{2x}{3} \cdot \frac{5}{7} &= \frac{2x \cdot 5}{3 \cdot 7} = \frac{10x}{21} \\ \frac{3}{4} \cdot \frac{1}{6} &= \frac{3 \cdot 1}{4 \cdot 6} = \frac{\overset{1}{3} \cdot 1}{4 \cdot \underset{1}{3} \cdot 2} = \frac{1}{8} \end{aligned} $ The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$. Divide. $ -\frac{3}{10} \div \frac{7}{9} = -\frac{3}{10} \cdot \frac{9}{7} = -\frac{3 \cdot 9}{10 \cdot 7} = -\frac{27}{70} $

Definitions and Concepts	Examples
Section 4.4 Adding and Subtracting Like Fractions, Least Common Denominator, and Equivalent Fractions	
Fractions that have the same denominator are called like fractions. To add or subtract like fractions, combine the numerators and place the sum or difference over the common denominator. The least common denominator (LCD) of a list of fractions is the smallest positive number divisible by all the denominators in the list. Method 1 for finding the LCD of a list of fractions using multiples Step 1: Write the multiples of the largest denominator (starting with the number itself) until a multiple common to all denominators in the list is found. Step 2: The multiple found in Step 1 is the LCD. Method 2 for finding the LCD of a list of fractions using prime factorization Step 1: Write the prime factorization of each denominator. Step 2: For each different prime factor in Step 1, circle the greatest number of times that factor occurs in any one factorization. Step 3: The LCD is the product of the circled factors. Equivalent fractions represent the same portion of a whole.	$-\frac{1}{3} \text{ and } \frac{2}{3}; \frac{5x}{7} \text{ and } \frac{6}{7}$ $\frac{2}{7} + \frac{3}{7} = \frac{5}{7} \quad \leftarrow \text{Add the numerators.}$ $\frac{7}{8} - \frac{4}{8} = \frac{3}{8} \quad \leftarrow \text{Keep the common denominator.}$ The LCD of $\frac{1}{2}$ and $\frac{5}{6}$ is 6 because 6 is the smallest positive number that is divisible by both 2 and 6. Find the LCD of $\frac{1}{4}$ and $\frac{5}{6}$ using Method 1. $6 \cdot 1 = 6 \quad \text{Not a multiple of 4}$ $6 \cdot 2 = 12 \quad \text{A multiple of 4}$ The LCD is 12. Find the LCD of $\frac{5}{6}$ and $\frac{11}{20}$ using Method 2. $6 = 2 \cdot \textcircled{3}$ $20 = \textcircled{2} \cdot \textcircled{2} \cdot \textcircled{5}$ The LCD is $2 \cdot 2 \cdot 3 \cdot 5 = 60$ Write an equivalent fraction with the indicated denominator. $\frac{2}{8} = \frac{\quad}{16}$ $\frac{2 \cdot 2}{8 \cdot 2} = \frac{4}{16}$
Section 4.5 Adding and Subtracting Unlike Fractions	
To add or subtract fractions with unlike denominators Step 1: Find the LCD. Step 2: Write each fraction as an equivalent fraction whose denominator is the LCD. Step 3: Add or subtract the like fractions. Step 4: Write the sum or difference in simplest form.	Add: $\frac{3}{20} + \frac{2}{5}$ Step 1: The LCD of the denominators 20 and 5 is 20. Step 2: $\frac{3}{20} = \frac{3}{20}; \frac{2}{5} = \frac{2}{5} \cdot \frac{4}{4} = \frac{8}{20}$ Step 3: $\frac{3}{20} + \frac{2}{5} = \frac{3}{20} + \frac{8}{20} = \frac{11}{20}$ Step 4: $\frac{11}{20}$ is in simplest form.

Definitions and Concepts	Examples
Section 4.6 Complex Fractions and Review of Order of Operations	
A fraction whose numerator or denominator or both contain fractions is called a complex fraction. One method for simplifying complex fractions is to multiply the numerator and the denominator of the complex fraction by the LCD of all fractions in its numerator and its denominator.	Complex Fractions: $\frac{\frac{11}{4} \cdot \frac{y}{6} - 11}{\frac{7}{10} \cdot \frac{4}{3}}$ $\frac{\frac{y}{6} - 11}{\frac{4}{3}} = \frac{6\left(\frac{y}{6} - 11\right)}{6\left(\frac{4}{3}\right)} = \frac{6\left(\frac{y}{6}\right) - 6(11)}{6\left(\frac{4}{3}\right)}$ $= \frac{y - 66}{8}$
Section 4.7 Operations on Mixed Numbers	
To multiply with mixed numbers or whole numbers, first write any mixed or whole numbers as improper fractions and then multiply as usual. To divide with mixed numbers or whole numbers, first write any mixed or whole numbers as fractions and then divide as usual. To add or subtract with mixed numbers, add or subtract the fractions and then add or subtract the whole numbers.	$2\frac{1}{3} \cdot \frac{1}{9} = \frac{7}{3} \cdot \frac{1}{9} = \frac{7 \cdot 1}{3 \cdot 9} = \frac{7}{27}$ $2\frac{5}{8} \div 3\frac{7}{16} = \frac{21}{8} \div \frac{55}{16} = \frac{21}{8} \cdot \frac{16}{55} = \frac{21 \cdot 16}{8 \cdot 55}$ $= \frac{21 \cdot 2 \cdot \overset{1}{\cancel{8}}}{\underset{1}{\cancel{8}} \cdot 55} = \frac{42}{55}$ Add: $2\frac{1}{2} + 5\frac{7}{8}$ $\begin{array}{r} 2\frac{1}{2} = 2\frac{4}{8} \\ + 5\frac{7}{8} = + 5\frac{7}{8} \\ \hline 7\frac{11}{8} = 7 + 1\frac{3}{8} = 8\frac{3}{8} \end{array}$
Section 4.8 Solving Equations Containing Fractions	
To Solve an Equation in x Step 1: If fractions are present, multiply both sides of the equation by the LCD of the fractions. Step 2: If parentheses are present, use the distributive property. Step 3: Combine any like terms on each side of the equation. Step 4: Use the addition property of equality to rewrite the equation so that variable terms are on one side of the equation and constant terms are on the other side. Step 5: Divide both sides by the numerical coefficient of x to solve. Step 6: Check the answer in the <i>original equation</i>.	Solve: $\frac{x}{15} + 2 = \frac{7}{3}$ $15\left(\frac{x}{15} + 2\right) = 15\left(\frac{7}{3}\right) \quad \text{Multiply by the LCD 15.}$ $15\left(\frac{x}{15}\right) + 15 \cdot 2 = 15\left(\frac{7}{3}\right)$ $x + 30 = 35$ $x + 30 - 30 = 35 - 30$ $x = 5$ Check to see that 5 is the solution.

(4.1) Write a fraction to represent the shaded area. If the fraction is improper, write the shaded area as a mixed number also. Do not simplify these answers.



Solve.

5. A basketball player made 11 free throws out of 12 tries during a game. What fraction of free throws did the player make?

6. A new car lot contains 23 blue cars out of a total of 131 cars.
a. How many cars on the lot are not blue?
b. What fraction of cars on the lot are not blue?

Simplify by dividing.

7. $-\frac{3}{3}$

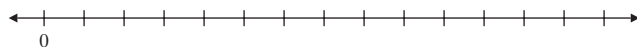
8. $\frac{-20}{-20}$

9. $\frac{0}{-1}$

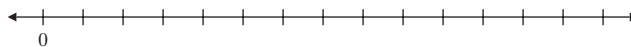
10. $\frac{4}{0}$

Graph each fraction on a number line.

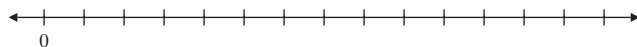
11. $\frac{7}{9}$



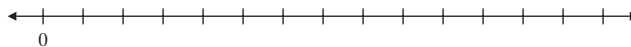
12. $\frac{4}{7}$



13. $\frac{5}{4}$



14. $\frac{7}{5}$



Write each improper fraction as a mixed number or a whole number.

15. $\frac{15}{4}$

16. $\frac{39}{13}$

Write each mixed number as an improper fraction.

17. $2\frac{1}{5}$

18. $3\frac{8}{9}$

(4.2) Write each fraction in simplest form.

19. $\frac{12}{28}$

20. $\frac{15}{27}$

21. $-\frac{25x}{75x^2}$

22. $-\frac{36y^3}{72y}$

23. $\frac{29ab}{32abc}$

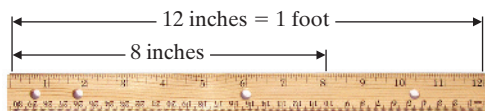
24. $\frac{18xyz}{23xy}$

25. $\frac{45x^2y}{27xy^3}$

26. $\frac{42ab^2c}{30abc^3}$

27. There are 12 inches in a foot. What fractional part of a foot does 8 inches represent?

28. Six out of 15 cars are white. What fraction of the cars are *not* white?



Determine whether each two fractions are equivalent.

29. $\frac{10}{34}$ and $\frac{4}{14}$

30. $\frac{30}{50}$ and $\frac{9}{15}$

(4.3) Multiply.

31. $\frac{3}{5} \cdot \frac{1}{2}$

32. $-\frac{6}{7} \cdot \frac{5}{12}$

33. $-\frac{24x}{5} \cdot -\frac{15}{8x^3}$

34. $\frac{27y^3}{21} \cdot \frac{7}{18y^2}$

35. $\left(-\frac{1}{3}\right)^3$

36. $\left(-\frac{5}{12}\right)^2$

Divide.

37. $-\frac{3}{4} \div \frac{3}{8}$

38. $\frac{21a}{4} \div \frac{7a}{5}$

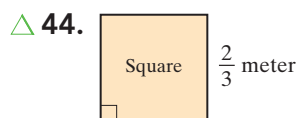
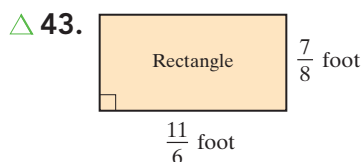
39. $-\frac{9}{2} \div -\frac{1}{3}$

40. $-\frac{5}{3} \div 2y$

41. Evaluate $x \div y$ if $x = \frac{9}{7}$ and $y = \frac{3}{4}$.

42. Evaluate ab if $a = -7$ and $b = \frac{9}{10}$.

Find the area of each figure.



(4.4) Add or subtract as indicated.

45. $\frac{7}{11} + \frac{3}{11}$

46. $\frac{4}{9} + \frac{2}{9}$

47. $\frac{1}{12} - \frac{5}{12}$

48. $\frac{11x}{15} + \frac{x}{15}$

49. $\frac{4y}{21} - \frac{3}{21}$

50. $\frac{4}{15} - \frac{3}{15} - \frac{2}{15}$

Find the LCD of each list of fractions.

51. $\frac{2}{3}, \frac{5}{x}$

52. $\frac{3}{4}, \frac{3}{8}, \frac{7}{12}$

Write each fraction as an equivalent fraction with the given denominator.

53. $\frac{2}{3} = \frac{?}{30}$

54. $\frac{5}{8} = \frac{?}{56}$

55. $\frac{7a}{6} = \frac{?}{42}$

56. $\frac{9b}{4} = \frac{?}{20}$

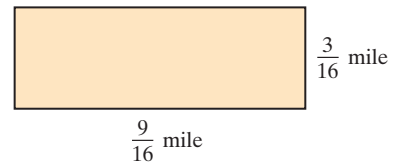
57. $\frac{4}{5x} = \frac{?}{50x}$

58. $\frac{5}{9y} = \frac{?}{18y}$

Solve.

59. One evening Mark Alorezo did $\frac{3}{8}$ of his homework before supper, another $\frac{2}{8}$ of it while his children did their homework, and $\frac{1}{8}$ after his children went to bed. What part of his homework did he do that evening?

- △ 60. The Simpsons will be fencing in their land, which is in the shape of a rectangle. In order to do this, they need to find its perimeter. Find the perimeter of their land.



(4.5) Add or subtract as indicated.

61. $\frac{7}{18} + \frac{2}{9}$

62. $\frac{4}{13} - \frac{1}{26}$

63. $-\frac{1}{3} + \frac{1}{4}$

64. $-\frac{2}{3} + \frac{1}{4}$

65. $\frac{5x}{11} + \frac{2}{55}$

66. $\frac{4}{15} + \frac{b}{5}$

67. $\frac{5y}{12} - \frac{2y}{9}$

68. $\frac{7x}{18} + \frac{2x}{9}$

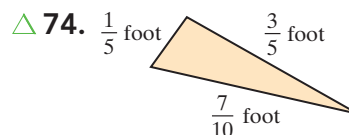
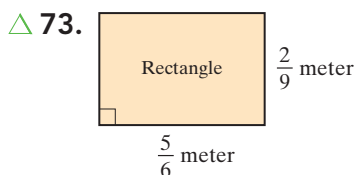
69. $\frac{4}{9} + \frac{5}{y}$

70. $\frac{9}{11} - \frac{3}{y}$

71. $\frac{4}{25} + \frac{23}{75} + \frac{7}{50}$

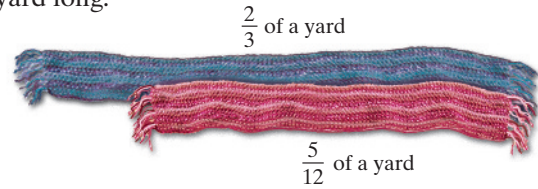
72. $\frac{2}{3} - \frac{2}{9} - \frac{1}{6}$

Find the perimeter of each figure.



75. In a group of 100 blood donors, typically $\frac{9}{25}$ have type A Rh-positive blood and $\frac{3}{50}$ have type A Rh-negative blood. What fraction have type A blood?

76. Find the difference in length of two scarves if one scarf is $\frac{5}{12}$ of a yard long and the other is $\frac{2}{3}$ of a yard long.



(4.6) Simplify each complex fraction.

77. $\frac{\frac{2x}{5}}{\frac{7}{10}}$

78. $\frac{\frac{3y}{7}}{\frac{11}{7}}$

79. $\frac{\frac{2}{5} - \frac{1}{2}}{\frac{3}{4} - \frac{7}{10}}$

80. $\frac{\frac{5}{6} - \frac{1}{4}}{-\frac{1}{12y}}$

Evaluate each expression if $x = \frac{1}{2}$, $y = -\frac{2}{3}$, and $z = \frac{4}{5}$.

81. $\frac{x}{y + z}$

82. $\frac{x + y}{z}$

Evaluate each expression. Use the order of operations to simplify.

83. $\frac{5}{13} \div \frac{1}{2} \cdot \frac{4}{5}$

84. $\frac{2}{27} - \left(\frac{1}{3}\right)^2$

85. $\frac{9}{10} \cdot \frac{1}{3} - \frac{2}{5} \cdot \frac{1}{11}$

86. $-\frac{2}{7} \cdot \left(\frac{1}{5} + \frac{3}{10}\right)$

(4.7) Perform operations as indicated. Simplify your answers. Estimate where noted.

87. $7\frac{3}{8}$
 $9\frac{5}{6}$
 $+ 3\frac{1}{12}$

88. $8\frac{1}{5}$
 $- 5\frac{3}{11}$
Exact:
Estimate:

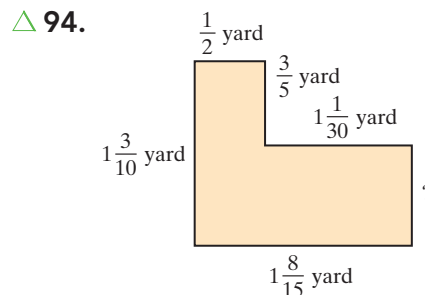
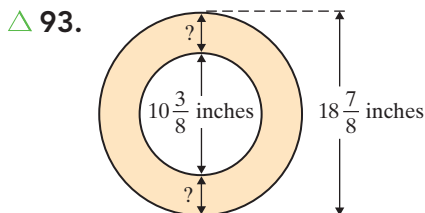
89. $1\frac{5}{8} \cdot 3\frac{1}{5}$
Exact:
Estimate:

90. $6\frac{3}{4} \div 1\frac{2}{7}$

91. A truck traveled 341 miles on $15\frac{1}{2}$ gallons of gas. How many miles might we expect the truck to travel on 1 gallon of gas?

92. There are $7\frac{1}{3}$ grams of fat in each ounce of hamburger. How many grams of fat are in a 5-ounce hamburger patty?

Find the unknown measurements.



Perform the indicated operations.

95. $-12\frac{1}{7} + \left(-15\frac{3}{14}\right)$

96. $23\frac{7}{8} - 24\frac{7}{10}$

97. $-3\frac{1}{5} \div \left(-2\frac{7}{10}\right)$

98. $-2\frac{1}{4} \cdot 1\frac{3}{4}$

(4.8) Solve each equation.

99. $a - \frac{2}{3} = \frac{1}{6}$

100. $9x + \frac{1}{5} - 8x = -\frac{7}{10}$

101. $-\frac{3}{5}x = 6$

102. $\frac{2}{9}y = -\frac{4}{3}$

103. $\frac{x}{7} - 3 = -\frac{6}{7}$

104. $\frac{y}{5} + 2 = \frac{11}{5}$

105. $\frac{1}{6} + \frac{x}{4} = \frac{17}{12}$

106. $\frac{x}{5} - \frac{5}{4} = \frac{x}{2} - \frac{1}{20}$

Mixed Review

Perform the indicated operations. Write each answer in simplest form. Estimate where noted.

107. $\frac{6}{15} \cdot \frac{5}{8}$

108. $\frac{5x^2}{y} \div \frac{10x^3}{y^3}$

109. $\frac{3}{10} - \frac{1}{10}$

110. $\frac{7}{8x} \cdot -\frac{2}{3}$

111. $\frac{2x}{3} + \frac{x}{4}$

112. $-\frac{5}{11} + \frac{2}{55}$

113. $-1\frac{3}{5} \div \frac{1}{4}$

114. $2\frac{7}{8}$
 Exact:
 $+ 9\frac{1}{2}$ Estimate:

115. $12\frac{1}{7}$
 Exact:
 $- 9\frac{3}{5}$ Estimate:

116. Simplify: $\frac{2 + \frac{3}{4}}{1 - \frac{1}{8}}$

117. Evaluate: $-\frac{3}{8} \cdot \left(\frac{2}{3} - \frac{4}{9}\right)$

Solve.

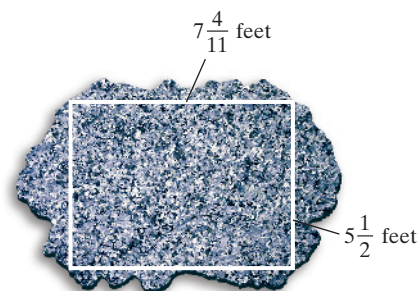
118. $11x - \frac{2}{7} - 10x = -\frac{13}{14}$

119. $-\frac{3}{5}x = \frac{4}{15}$

120. $\frac{x}{12} + \frac{5}{6} = -\frac{3}{4}$

121. A ribbon $5\frac{1}{2}$ yards long is cut from a reel of ribbon with 50 yards on it. Find the length of the piece remaining on the reel.

- △ 122. A slab of natural granite is purchased and a rectangle with length $7\frac{4}{11}$ feet and width $5\frac{1}{2}$ feet is cut from it. Find the area of the rectangle.



MULTIPLE CHOICE Exercises 1–10 are **Multiple Choice**. Choose the correct answer.

For Exercises 1 through 4, choose whether the expression simplifies to

- A. 1 B. -1 C. 0 D. undefined

▶ 1. $\frac{-2}{-2}$ ▶ 2. $\frac{-2}{2}$ ▶ 3. $\frac{2}{0}$ ▶ 4. $\frac{0}{-2}$

▶ 5. The mixed number $4\frac{3}{5}$ written as a fraction is:

- A. $\frac{12}{5}$ B. $\frac{5}{23}$ C. $\frac{23}{5}$ D. $\frac{23}{3}$

▶ 6. The improper fraction $\frac{23}{8}$ written as a mixed number is:

- A. 2.3 B. $2\frac{7}{8}$ C. $8\frac{2}{3}$ D. $2\frac{8}{7}$

For Exercises 7 through 10, the exercise statement and correct answer are given. Choose whether the correct operation in the box should be:

- A. + (addition) B. - (subtraction) C. · (multiplication) D. ÷ (division)

▶ 7. $\frac{8}{11} \square \frac{2}{11}$; Answer: $\frac{16}{121}$

▶ 8. $\frac{8}{11} \square \frac{2}{11}$; Answer: $\frac{8}{2}$ or 4

▶ 9. $\frac{8}{11} \square \frac{2}{11}$; Answer: $\frac{6}{11}$

▶ 10. $\frac{8}{11} \square \frac{2}{11}$; Answer: $\frac{10}{11}$

MATCHING For Exercises 11 through 14, **Match** each operation of fractions in the first column with the correct answer in the second or third column.

▶ 11. $\frac{5}{7} + \frac{1}{7}$

A. $\frac{5}{7}$

E. undefined

▶ 12. $\frac{5}{7} \cdot \frac{1}{7}$

B. $\frac{5}{1}$ or 5

F. $\frac{6}{7}$

▶ 13. $\frac{5}{7} \div \frac{1}{7}$

C. $\frac{6}{14}$ or $\frac{3}{7}$

G. $\frac{6}{49}$

▶ 14. $\frac{5}{7} - \frac{1}{7}$

D. $\frac{4}{7}$

H. $\frac{5}{49}$

MULTIPLE CHOICE Exercises 15 through 21 are **Multiple Choice**. Choose the correct answer.

For each expression in Exercises 15 through 18, use order of operations and decide which operation below should be performed first to simplify.

A. Addition

B. Subtraction

C. Multiplication

D. Division

▶ 15. $\frac{1}{2} + \frac{1}{5} \cdot \frac{1}{4}$

▶ 16. $\left(\frac{1}{2} + \frac{1}{5}\right) \cdot \frac{1}{4}$

▶ 17. $\frac{1}{2} \div \frac{1}{5} \cdot \frac{1}{4}$

▶ 18. $\frac{1}{2} \div \left(\frac{1}{5} \cdot \frac{1}{4}\right)$

▶ 19. To solve $\frac{x}{10} + \frac{3}{5} = 2$, let's multiply both sides of the equation by 10. Once this is done, the equivalent equation is:

A. $x + 6 = 2$

B. $x + 3 = 2$

C. $x + 6 = 12$

D. $x + 6 = 20$

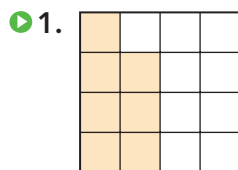
Equations are solved and expressions are simplified. For Exercises 20 through 21, decide whether each exercise is an

A. expression or an B. equation

▶ 20. $\frac{2}{3} - \frac{1}{9}$

▶ 21. $\frac{2}{3} - \frac{1}{9} = \frac{x}{9}$

Write a fraction to represent the shaded area.



Write the mixed number as an improper fraction.

▶ 2. $7\frac{2}{3}$

Write the improper fraction as a mixed number.

▶ 3. $\frac{75}{4}$

Write each fraction in simplest form.

▶ 4. $\frac{24}{210}$

▶ 5. $-\frac{42x}{70}$

Determine whether these fractions are equivalent.

▶ 6. $\frac{5}{7}$ and $\frac{8}{11}$

▶ 7. $\frac{6}{27}$ and $\frac{14}{63}$

Find the prime factorization of each number.

▶ 8. 84

▶ 9. 495

Perform each indicated operation and write the answers in simplest form.

▶ 10. $\frac{4}{4} \div \frac{3}{4}$

▶ 11. $-\frac{4}{3} \cdot \frac{4}{4}$

▶ 12. $\frac{7x}{9} + \frac{x}{9}$

▶ 13. $\frac{1}{7} - \frac{3}{x}$

▶ 14. $\frac{xy^3}{z} \cdot \frac{z}{xy}$

▶ 15. $-\frac{2}{3} \cdot -\frac{8}{15}$

▶ 16. $\frac{9a}{10} + \frac{2}{5}$

▶ 17. $-\frac{8}{15y} - \frac{2}{15y}$

▶ 18. $\frac{3a}{8} \cdot \frac{16}{6a^3}$

▶ 19. $\frac{11}{12} - \frac{3}{8} + \frac{5}{24}$

▶ 20. $3\frac{7}{8}$
 $7\frac{2}{5}$
 $+ 2\frac{3}{4}$

▶ 21. $\frac{19}{-2\frac{3}{11}}$

Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____

- 22. _____
- 23. _____
- 24. _____
- 25. _____
- 26. _____
- 27. _____
- 28. _____
- 29. _____
- 30. _____
- 31. _____
- 32. _____
- 33. _____
- 34. _____
- 35. _____
- 36. _____
- 37. _____
- 38. _____
- 39. _____
- 40. _____

▶ 22. $-\frac{16}{3} \div -\frac{3}{12}$ ▶ 23. $3\frac{1}{3} \cdot 6\frac{3}{4}$ ▶ 24. $-\frac{2}{7} \cdot \left(6 - \frac{1}{6}\right)$ ▶ 25. $\frac{1}{2} \div \frac{2}{3} \cdot \frac{3}{4}$

▶ 26. $\left(-\frac{3}{4}\right)^2 \div \left(\frac{2}{3} + \frac{5}{6}\right)$ ▶ 27. Find the average of $\frac{5}{6}$, $\frac{4}{3}$, and $\frac{7}{12}$.

Simplify each complex fraction.

▶ 28. $\frac{\frac{5x}{7}}{\frac{20x^2}{21}}$ ▶ 29. $\frac{5 + \frac{3}{7}}{2 - \frac{1}{2}}$

Solve.

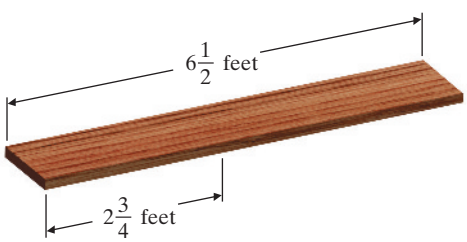
▶ 30. $-\frac{3}{8}x = \frac{3}{4}$ ▶ 31. $\frac{x}{5} + x = -\frac{24}{5}$ ▶ 32. $\frac{2}{3} + \frac{x}{4} = \frac{5}{12} + \frac{x}{2}$

Evaluate each expression for the given replacement values.

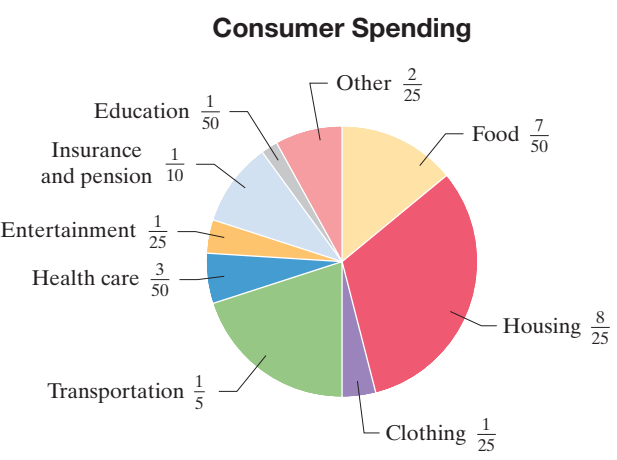
▶ 33. $-5x$; $x = -\frac{1}{2}$ ▶ 34. $x \div y$; $x = \frac{1}{2}$, $y = 3\frac{7}{8}$

Solve.

▶ 35. A carpenter cuts a piece $2\frac{3}{4}$ feet long from a cedar plank that is $6\frac{1}{2}$ feet long. How long is the remaining piece?



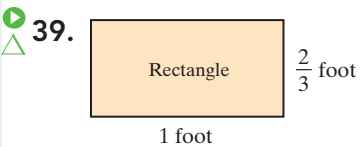
The circle graph below shows us how the average consumer spends money. For example, $\frac{7}{50}$ of spending goes for food. Use this information for Exercises 36 through 38.



Source: U.S. Bureau of Labor Statistics; based on survey

- ▶ 36. What fraction of spending goes for housing and food combined?
- ▶ 37. What fraction of spending goes for education, transportation, and clothing?
- ▶ 38. Suppose your family spent \$47,000 on the items in the graph. How much might we expect was spent on health care?

Find the perimeter and area of the figure.



▶ 40. During a 258-mile trip, a car used $10\frac{3}{4}$ gallons of gas. How many miles would we expect the car to travel on 1 gallon of gas?

Write each number in words.

1. 546
2. 115
3. 27,034
4. 6573
5. Add: $46 + 713$
6. Add: $587 + 44$
7. Subtract: $543 - 29$. Check by adding.
8. Subtract: $995 - 62$. Check by adding.
9. Round 278,362 to the nearest thousand.
10. Round 1436 to the nearest ten.
11. A digital video disc (DVD) can hold about 4800 megabytes (MB) of information. How many megabytes can 12 DVDs hold?
12. On a trip across the country, Daniel Daunis travels 435 miles per day. How many total miles does he travel in 3 days?
13. Divide and check: $56,717 \div 8$
14. Divide and check: $4558 \div 12$

Write using exponential notation.

15. $7 \cdot 7 \cdot 7$
16. $7 \cdot 7$
17. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 9 \cdot 9 \cdot 9$
18. $9 \cdot 9 \cdot 9 \cdot 9 \cdot 5 \cdot 5$
19. Evaluate $2(x - y)$ for $x = 6$ and $y = 3$.
20. Evaluate $8a + 3(b - 5)$ for $a = 5$ and $b = 9$.
21. The world's deepest cave is Krubera (or Voronja), in the country of Georgia, located by the Black Sea in Asia. It has been explored to a depth of 7188 feet below the surface of Earth. Represent this position using an integer. (Source: messagetoeagle.com and Wikipedia)
22. The temperature on a cold day in Minneapolis, MN, was 21°F below zero. Represent this temperature using an integer.

Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____

23. _____

24. _____

25. _____

26. _____

27. _____

28. _____

29. _____

30. _____

31. _____

32. _____

33. _____

34. _____

35. _____

36. _____

37. _____

38. _____

39. a. _____ b. _____

40. _____

41. _____

42. _____

43. _____

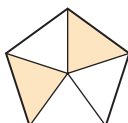
44. _____

45. _____

46. _____

23. Add using a number line: $-7 + 3$ 24. Add using a number line: $-3 + 8$ 25. Simplify: $7 - 8 - (-5) - 1$ 26. Simplify: $6 + (-8) - (-9) + 3$ 27. Evaluate: $(-5)^2$ 28. Evaluate: -2^4 29. Simplify: $3(4 - 7) + (-2) - 5$ 30. Simplify: $(20 - 5^2)^2$ 31. Simplify: $2y - 6 + 4y + 8$ 32. Simplify: $5x - 1 + x + 10$ *Solve.*33. $5x + 2 - 4x = 7 - 19$ 34. $9y + 1 - 8y = 3 - 20$ 35. $17 - 7x + 3 = -3x + 21 - 3x$ 36. $9x - 2 = 7x - 24$

37. Write a fraction to represent the shaded part of the figure.



38. Write the prime factorization of 156.

39. Write each as an improper fraction.

a. $4\frac{2}{9}$ b. $1\frac{8}{11}$ 40. Write $\frac{39}{5}$ as a mixed number.41. Write in simplest form: $\frac{42x}{66}$ 42. Write in simplest form: $\frac{70}{105y}$ 43. Multiply: $3\frac{1}{3} \cdot \frac{7}{8}$ 44. Multiply: $\frac{2}{3} \cdot 4$ 45. Divide and simplify: $\frac{5}{16} \div \frac{3}{4}$ 46. Divide: $1\frac{1}{10} \div 5\frac{3}{5}$

Decimals

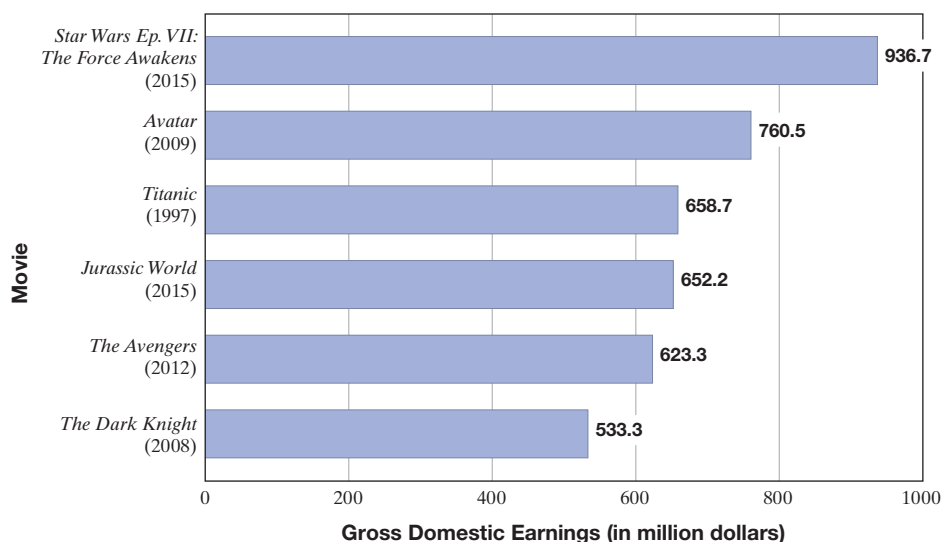


Attendance to U.S. Movie Theaters Is Increasing

Admissions to U.S. movie theaters recently increased by 4%. More than $\frac{2}{3}$ of the U.S. and Canadian population went to the movies in 2016. There were 681 films released in 2016, which is consistent with previous years and appears likely to continue in the future. Not all films are successes, and even fewer are true blockbusters. The chart below lists the top all-time highest-money-grossing films in U.S. movie history.

While we have practiced calculating averages before in this text, an average does not always simplify to a whole number. While fractions are useful, decimals are an important system of numbers that can also be used to show values between whole numbers. Data are easy to round when they are in the form of a decimal. In Section 5.1, Exercise 109, and Section 5.2, Exercise 85, we will continue to study the all-time highest-grossing American movies.

American Movies Earning the Most Money



Source: the-numbers.com; Yearly Box Office

5

Decimal numbers represent parts of a whole, just like fractions. For example, one penny is 0.01 or $\frac{1}{100}$ of a dollar. In this chapter, we learn to perform arithmetic operations on decimals and to analyze the relationship between fractions and decimals. We also learn how decimals are used in the real world.

Sections

- 5.1** Introduction to Decimals
- 5.2** Adding and Subtracting Decimals
- 5.3** Multiplying Decimals and Circumference of a Circle
- 5.4** Dividing Decimals
- Integrated Review**—Operations on Decimals
- 5.5** Fractions, Decimals, and Order of Operations
- 5.6** Solving Equations Containing Decimals
- 5.7** Decimal Applications: Mean, Median, and Mode

Check Your Progress

- Vocabulary Check
- Chapter Highlights
- Chapter Review
- Getting Ready for the Test
- Chapter Test
- Cumulative Review

5.1 Introduction to Decimals

Objectives

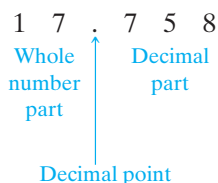
- A** Know the Meaning of Place Value for a Decimal Number and Write Decimals in Words.
- B** Write Decimals in Standard Form.
- C** Write Decimals as Fractions.
- D** Compare Decimals.
- E** Round Decimals to Given Place Values.

Helpful Hint

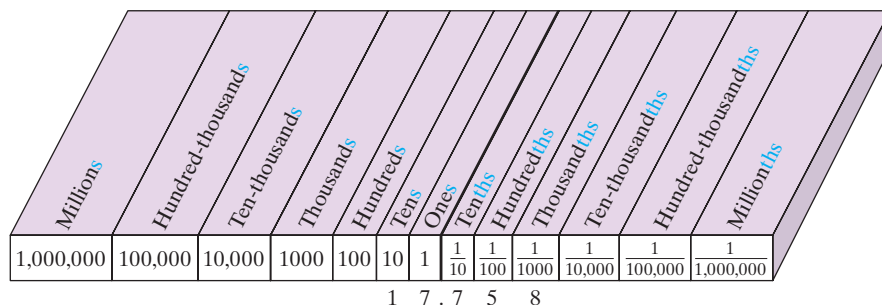
Notice that place values to the left of the decimal point end in “s.” Place values to the right of the decimal point end in “ths.”

Objective A Decimal Notation and Writing Decimals in Words

Like fractional notation, decimal notation is used to denote a part of a whole. Numbers written in decimal notation are called **decimal numbers**, or simply **decimals**. The decimal 17.758 has three parts.



In Section 1.2, we introduced place value for whole numbers. Place names and place values for the whole number part of a decimal number are exactly the same. Place names and place values for the decimal part are shown below.



Notice that the value of each place is $\frac{1}{10}$ of the value of the place to its left. For example,

$$\begin{array}{c}
 1 \cdot \frac{1}{10} = \frac{1}{10} \quad \text{and} \quad \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100} \\
 \uparrow \quad \quad \uparrow \quad \quad \uparrow \quad \quad \uparrow \\
 \text{ones} \quad \text{tenths} \quad \text{tenths} \quad \text{hundredths}
 \end{array}$$

The decimal number 17.758 means

$$\begin{array}{l}
 \begin{array}{ccccccc}
 1 \text{ ten} & + & 7 \text{ ones} & + & 7 \text{ tenths} & + & 5 \text{ hundredths} & + & 8 \text{ thousandths} \\
 \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 \text{or } 1 \cdot 10 & + & 7 \cdot 1 & + & 7 \cdot \frac{1}{10} & + & 5 \cdot \frac{1}{100} & + & 8 \cdot \frac{1}{1000} \\
 \text{or } 10 & + & 7 & + & \frac{7}{10} & + & \frac{5}{100} & + & \frac{8}{1000}
 \end{array}
 \end{array}$$

Writing (or Reading) a Decimal in Words

Step 1: Write the whole number part in words.

Step 2: Write “and” for the decimal point.

Step 3: Write the decimal part in words as though it were a whole number, followed by the place value of the last digit.

Example 1 Write each decimal in words.

- a. 0.7 b. -50.82 c. 21.093

Solution:

- a. seven tenths
b. negative fifty and eighty-two hundredths
c. twenty-one and ninety-three thousandths

Work Practice 1**Example 2** Write the decimal in the following sentence in words: The Golden Jubilee Diamond is a 545.67-carat cut diamond. (Source: *The Guinness Book of Records*)

Solution: five hundred forty-five and sixty-seven hundredths

Work Practice 2**Example 3** Write the decimal in the following sentence in words: The oldest known fragments of the Earth's crust are Zircon crystals; they were discovered in Australia and are thought to be 4.276 billion years old. (Source: *The Guinness Book of Records*)

Solution: four and two hundred seventy-six thousandths

Work Practice 3**Practice 1**

Write each decimal in words.

- a. 0.06
b. -200.073
c. 0.0829

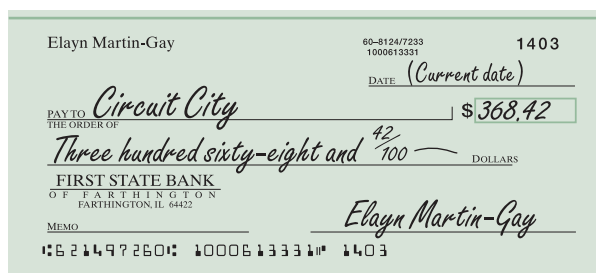
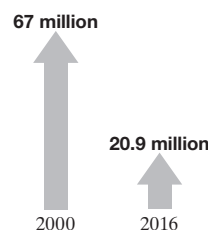
Practice 2

Write the decimal 87.31 in words.

Practice 3

Write the decimal 52.1085 in words.

Suppose that you are paying for a purchase of \$368.42 at Circuit City by writing a check. Checks are usually written using the following format.

**Average Daily Volume of Commercial Checks**

Source: Federal Reserve System

Helpful Hint

Note: Although check-writing has decreased, checks are still used for larger sums such as payroll and rent.

Answers

1. a. six hundredths b. negative two hundred and seventy-three thousandths c. eight hundred twenty-nine ten-thousandths
2. eighty-seven and thirty-one hundredths
3. fifty-two and one thousand eighty-five ten-thousandths

Practice 4

Fill in the check to CLECO (Central Louisiana Electric Company) to pay for your monthly electric bill of \$207.40.

Your Preprinted Name
Your Preprinted Address
DATE
PAY TO THE ORDER OF
FIRST STATE BANK
OF FARTHINGTON, IL 64422
MEMO
⑆ 214972601 1000613331 1406

Example 4

Fill in the check to Camelot Music to pay for your purchase of \$92.98.

Solution:

Your Preprinted Name
Your Preprinted Address
DATE (Current date)
PAY TO THE ORDER OF Camelot Music
Ninety-two and 98/100
FIRST STATE BANK
OF FARTHINGTON, IL 64422
MEMO (Your signature)
⑆ 214972601 1000613331 1406

Work Practice 4

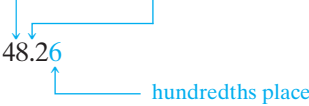
Objective B Writing Decimals in Standard Form

A decimal written in words can be written in standard form by reversing the procedure in Objective A.

Examples

Write each decimal in standard form.

5. Forty-eight and twenty-six hundredths is



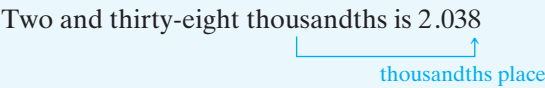
6. Six and ninety-five thousandths is



Work Practice 5–6

Helpful Hint

When converting a decimal from words to decimal notation, make sure the last digit is in the correct place by inserting 0s if necessary. For example,



Objective C Writing Decimals as Fractions

Once you master reading and writing decimals, writing a decimal as a fraction follows naturally.

Decimal	In Words	Fraction
0.7	seven tenths	$\frac{7}{10}$
0.51	fifty-one hundredths	$\frac{51}{100}$
0.009	nine thousandths	$\frac{9}{1000}$
0.05	five hundredths	$\frac{5}{100} = \frac{1}{20}$

Answers

4.

Your Preprinted Name
Your Preprinted Address
DATE (Current date)
PAY TO THE ORDER OF CLECO
Two hundred seven and 40/100
FIRST STATE BANK
OF FARTHINGTON, IL 64422
MEMO (Your signature)
⑆ 214972601 1000613331 1406

- 5. 500.96
- 6. 39.042

Notice that the number of decimal places in a decimal number is the same as the number of zeros in the denominator of the equivalent fraction. We can use this fact to write decimals as fractions.

$$\begin{array}{ccc} 0.31 = \frac{31}{100} & 0.007 = \frac{7}{1000} \\ \uparrow \quad \uparrow & \uparrow \quad \uparrow \\ \text{2 decimal} & \text{3 decimal} \\ \text{places} & \text{places} \\ \uparrow \quad \uparrow & \uparrow \\ \text{2 zeros} & \text{3 zeros} \end{array}$$

Example 7 Write 0.47 as a fraction.

Solution: $0.47 = \frac{47}{100}$

$$\begin{array}{ccc} & \uparrow & \uparrow \\ & 2 & 2 \text{ zeros} \\ & \text{decimal places} & \end{array}$$

Work Practice 7

Example 8 Write 5.9 as a mixed number.

Solution: $5.9 = 5 \frac{9}{10}$

$$\begin{array}{ccc} & \uparrow & \uparrow \\ & 1 & 1 \text{ zero} \\ & \text{decimal place} & \end{array}$$

Work Practice 8

Examples Write each decimal as a fraction or a mixed number. Write your answer in simplest form.

9. $0.125 = \frac{125}{1000} = \frac{\cancel{125}^1}{8 \cdot \cancel{125}_1} = \frac{1}{8}$

10. $43.5 = 43 \frac{5}{10} = 43 \frac{\cancel{5}^1}{2 \cdot \cancel{5}_1} = 43 \frac{1}{2} = 43 \frac{1}{2}$

11. $-105.083 = -105 \frac{83}{1000}$

Work Practice 9–11

Later in the chapter, we write fractions as decimals. If you study Examples 7–11, you already know how to write fractions with denominators of 10, 100, 1000, and so on, as decimals.

Objective D Comparing Decimals

One way to compare positive decimals is by comparing digits in corresponding places.

To see why this works, let's compare 0.5 or $\frac{5}{10}$ and 0.8 or $\frac{8}{10}$. We know

$$\begin{array}{ccc} \frac{5}{10} < \frac{8}{10} & \text{since } 5 < 8, \text{ so} \\ \downarrow & \downarrow \\ 0.5 < 0.8 & \text{since } 5 < 8 \end{array}$$

This leads to the following.

Practice 7

Write 0.051 as a fraction.

Practice 8

Write 29.97 as a mixed number.

Practice 9–11

Write each decimal as a fraction or mixed number. Write your answer in simplest form.

9. 0.12

10. 64.8

11. -209.986

Answers

7. $\frac{51}{1000}$ 8. $29 \frac{97}{100}$ 9. $\frac{3}{25}$
10. $64 \frac{4}{5}$ 11. $-209 \frac{493}{500}$

Comparing Two Positive Decimals

Compare digits in the same places from left to right. When two digits are not equal, the number with the larger digit is the larger decimal. If necessary, insert 0s after the last digit to the right of the decimal point to continue comparing.

Compare hundredths place digits

28.253		28.263
↑		↑
5	<	6
so 28.253	<	28.263

Helpful Hint

For any decimal, writing 0s after the last digit to the right of the decimal point does not change the value of the number.

$7.6 = 7.60 = 7.600$, and so on

When a whole number is written as a decimal, the decimal point is placed to the right of the ones digit.

$25 = 25.0 = 25.00$, and so on

Practice 12

Insert $<$, $>$, or $=$ to form a true statement.

26.208 26.28

Practice 13

Insert $<$, $>$, or $=$ to form a true statement.

0.12 0.026

Example 12

Insert $<$, $>$, or $=$ to form a true statement.

0.378 0.368

Solution:

$\begin{array}{cc} \swarrow & \searrow \\ 0.378 & 0.368 \\ \swarrow & \searrow \\ 0.378 & 0.368 \end{array}$

The tenths places are the same.

The hundredths places are different.

Since $7 > 6$, then $0.378 > 0.368$.

Work Practice 12

Example 13

Insert $<$, $>$, or $=$ to form a true statement.

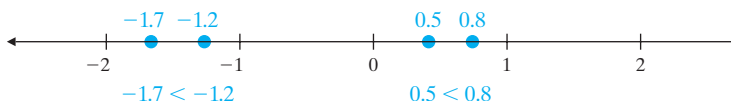
0.052 0.236

$\begin{array}{cc} \uparrow & \uparrow \\ 0.052 & 0.236 \end{array}$

0 is smaller than 2 in the tenths place.

Work Practice 13

We can also use a number line to compare decimals. This is especially helpful when comparing negative decimals. Remember, the number whose graph is to the left is smaller, and the number whose graph is to the right is larger.

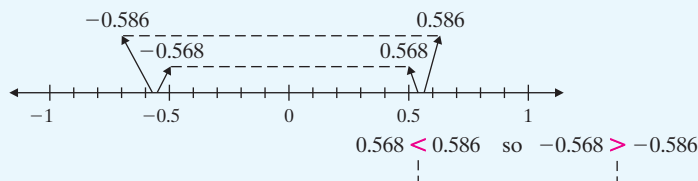


Answers

12. $<$ 13. $>$

Helpful Hint

If you have trouble comparing two negative decimals, try the following: Compare their absolute values. Then to correctly compare the negative decimals, **reverse** the direction of the inequality symbol.



Example 14 Insert $<$, $>$, or $=$ to form a true statement.

$$-0.0101 \quad -0.00109$$

Solution: Since $0.0101 > 0.00109$, then $-0.0101 < -0.00109$.

Work Practice 14

Objective E Rounding Decimals

We **round the decimal part** of a decimal number in nearly the same way as we round whole numbers. The only difference is that we drop digits to the right of the rounding place, instead of replacing these digits with 0s. For example,

36.954 rounded to the nearest hundredth is 36.95.

Rounding Decimals to a Place Value to the Right of the Decimal Point

Step 1: Locate the digit to the right of the given place value.

Step 2: If this digit is 5 or greater, add 1 to the digit in the given place value and drop all digits to its right. If this digit is less than 5, drop all digits to the right of the given place.

Example 15 Round 736.2359 to the nearest tenth.

Solution:

Step 1: We locate the digit to the right of the tenths place.



Step 2: Since the digit to the right is less than 5, we drop it and all digits to its right.

Thus, 736.2359 rounded to the nearest tenth is 736.2.

Work Practice 15

The same steps for rounding can be used when the decimal is negative.

Practice 14

Insert $<$, $>$, or $=$ to form a true statement.

$$-0.039 \quad -0.0309$$

Practice 15

Round 482.7817 to the nearest thousandth.

Answers

14. $<$ 15. 482.782

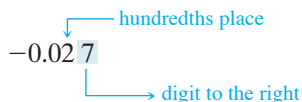
Practice 16

Round -0.032 to the nearest hundredth.

Example 16 Round -0.027 to the nearest hundredth.

Solution:

Step 1: Locate the digit to the right of the hundredths place.

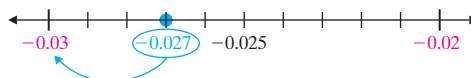


Step 2: Since the digit to the right is 5 or greater, we add 1 to the hundredths digit and drop all digits to its right.

Thus, -0.027 is -0.03 rounded to the nearest hundredth.

Work Practice 16

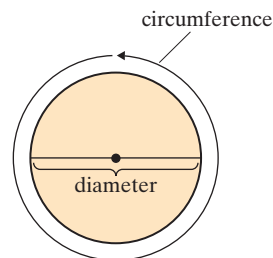
The following number line illustrates the rounding of negative decimals.



In Section 5.3, we will introduce a formula for the distance around a circle. The distance around a circle is given the special name **circumference**.

The symbol π is the Greek letter pi, pronounced “pie.” We use π to denote the following constant:

$$\pi = \frac{\text{circumference of a circle}}{\text{diameter of a circle}}$$

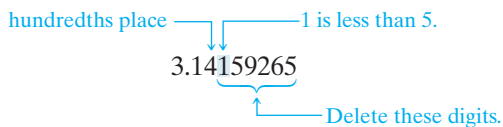


The value π is an **irrational number**. This means if we try to write it as a decimal, it neither ends nor repeats in a pattern.

Practice 17

$\pi \approx 3.14159265$. Round π to the nearest ten-thousandth.

Example 17 $\pi \approx 3.14159265$. Round π to the nearest hundredth.

Solution:

Thus, 3.14159265 rounded to the nearest hundredth is 3.14 . In other words, $\pi \approx 3.14$.

Work Practice 17

Rounding often occurs with money amounts. Since there are 100 cents in a dollar, each cent is $\frac{1}{100}$ of a dollar. This means that if we want to round to the nearest cent, we round to the nearest hundredth of a dollar.

✓Concept Check 1756.0894 rounded to the nearest *ten* is

- a. 1756.1 b. 1760.0894 c. 1760 d. 1750

Answers

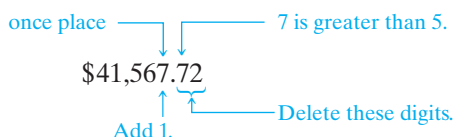
16. -0.03 17. $\pi \approx 3.1416$

✓ **Concept Check Answer**
c

Example 18 Determining State Taxable Income

A high school teacher's taxable income is \$41,567.72. The tax tables in the teacher's state use amounts rounded to the nearest dollar. Round the teacher's income to the nearest whole dollar.

Solution: Rounding to the nearest whole dollar means rounding to the ones place.



Thus, the teacher's income rounded to the nearest dollar is \$41,568.

Work Practice 18**Practice 18**

Water bills in Mexia are always rounded to the nearest dollar. Round a water bill of \$24.62 to the nearest dollar.

Answer

18. \$25

Vocabulary, Readiness & Video Check

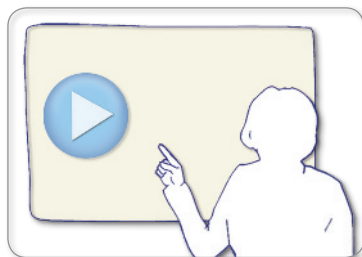
Use the choices below to fill in each blank.

words	decimals	tenths	after
tens	circumference	and	standard form

- The number “twenty and eight hundredths” is written in _____ and “20.08” is written in _____.
- Another name for the distance around a circle is its _____.
- Like fractions, _____ are used to denote part of a whole.
- When writing a decimal number in words, the decimal point is written as _____.
- The place value _____ is to the right of the decimal point while _____ is to the left of the decimal point.
- The decimal point in a whole number is _____ the last digit.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 5.1

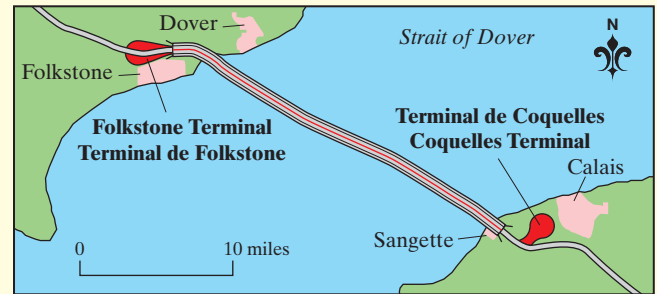
- Objective A** 7. In Example 1, how is the decimal point written?
- Objective B** 8. Why is 9.8 not the correct answer to Example 3? What is the correct answer?
- Objective C** 9. From Example 5, why does reading a decimal number correctly help you write it as an equivalent fraction?
- Objective D** 10. In Example 7, we compare place value by place value in which direction?
- Objective E** 11. Example 8 is being rounded to the nearest tenth, so why is the digit 7, which is not in the tenths place, looked at?

5.1 Exercise Set MyLab Math

Objective A Write each decimal number in words. See Examples 1 through 3.

- | | |
|----------|----------|
| 1. 5.62 | 2. 9.57 |
| 3. 16.23 | 4. 47.65 |

5. -0.205
7. 167009
9. 3000.04
11. 105.6
13. The Akashi Kaikyo Bridge, between Kobe and Awaji-Shima, Japan, is approximately 2.43 miles long.
6. -0.495
8. 233.056
10. 5000.02
12. 410.3
14. The English Channel Tunnel is a 31.04-mile long undersea rail tunnel connecting England and France. (Source: Railway Directory & Year Book)



Fill in each check for the described purchase. See Example 4.

15. Your monthly car loan of \$321.42 to R. W. Financial.

Your Preprinted Name	60-8124/7233 1000613331	1407
Your Preprinted Address		
DATE		
PAY TO THE ORDER OF	\$	
DOLLARS		
FIRST STATE BANK OF FARTHINGTON FARTHINGTON, IL 64422		
MEMO		
⑆ 2 1 4 9 7 2 6 0 ⑆ 1 0 0 0 6 1 3 3 3 1 ⑆ 1 4 0 7		

16. Your part of the monthly apartment rent, which is \$213.70. You pay this to Amanda Dupre.

Your Preprinted Name	60-8124/7233 1000613331	1408
Your Preprinted Address		
DATE		
PAY TO THE ORDER OF	\$	
DOLLARS		
FIRST STATE BANK OF FARTHINGTON FARTHINGTON, IL 64422		
MEMO		
⑆ 2 1 4 9 7 2 6 0 ⑆ 1 0 0 0 6 1 3 3 3 1 ⑆ 1 4 0 8		

17. Your bill of \$91.68 to Verizon wireless.

Your Preprinted Name	60-8124/7233 1000613331	1409
Your Preprinted Address		
DATE		
PAY TO THE ORDER OF	\$	
DOLLARS		
FIRST STATE BANK OF FARTHINGTON FARTHINGTON, IL 64422		
MEMO		
⑆ 2 1 4 9 7 2 6 0 ⑆ 1 0 0 0 6 1 3 3 3 1 ⑆ 1 4 0 9		

18. Your grocery bill of \$387.49 at Kroger.

Your Preprinted Name	60-8124/7233 1000613331	1410
Your Preprinted Address		
DATE		
PAY TO THE ORDER OF	\$	
DOLLARS		
FIRST STATE BANK OF FARTHINGTON FARTHINGTON, IL 64422		
MEMO		
⑆ 2 1 4 9 7 2 6 0 ⑆ 1 0 0 0 6 1 3 3 3 1 ⑆ 1 4 1 0		

Objective B Write each decimal number in standard form. See Examples 5 and 6.

19. Two and eight tenths
21. Nine and eight hundredths
23. Negative seven hundred five and six hundred twenty-five thousandths
25. Forty-six ten-thousandths
20. Five and one tenth
22. Twelve and six hundredths
24. Negative eight hundred four and three hundred ninety-nine thousandths
26. Eighty-three ten-thousandths

Objective C Write each decimal as a fraction or a mixed number. Write your answer in simplest form. See Examples 7 through 11.

27. 0.7

28. 0.9

▶ 29. 0.27

30. 0.39

31. 0.4

32. 0.8

33. 5.4

34. 6.8

35. -0.058

36. -0.024

▶ 37. 7.008

38. 9.005

39. 15.802

40. 11.406

41. 0.3005

42. 0.2006

Objectives A B C Mixed Practice Fill in the chart. The first row is completed for you. See Examples 1 through 11.

	Decimal Number in Standard Form	In Words	Fraction
	0.37	thirty-seven hundredths	$\frac{37}{100}$
43.		eight tenths	
44.		five tenths	
45.	0.077		
46.	0.019		

Objective D Insert $<$, $>$, or $=$ between each pair of numbers to form a true statement. See Examples 12 through 14.

47. 0.15 0.16

48. 0.12 0.15

49. -0.57 -0.54

50. -0.59 -0.52

51. 0.098 0.1

52. 0.0756 0.2

53. 0.54900 0.549

54. 0.98400 0.984

▶ 55. 167908 167980

56. 519.3405 519.3054

57. -1.062 -1.07

58. -18.1 -18.01

59. -7.052 7.0052

60. 0.01 -0.1

61. -0.023 -0.024

62. -0.562 -0.652

Objective E Round each decimal to the given place value. See Examples 15 through 18.

▶ 63. 0.57, nearest tenth

64. 0.64, nearest tenth

65. 98,207.23, nearest ten

66. 68,934.543, nearest ten

67. -0.234, nearest hundredth

68. -0.892, nearest hundredth

▶ 69. 0.5942, nearest thousandth

70. 63.4523, nearest thousandth

Recall that the number π , written as a decimal, neither ends nor repeats in a pattern. Given that $\pi \approx 3.14159265$, round π to the given place values below. (We study π further in Section 5.3.) See Example 17.

71. tenth

72. ones

73. thousandth

74. hundred-thousandth

Round each monetary amount to the nearest cent or dollar as indicated. See Example 18.

75. \$26.95, to the nearest dollar

76. \$14,769.52, to the nearest dollar

77. \$0.1992, to the nearest cent

78. \$0.7633, to the nearest cent

Round each number to the given place value. See Example 18.

79. At the time of this writing, the Apple MacBook Air is the thinnest Mac in production. At its thickest point, it measures 0.68 in. Round this number to the nearest tenth. (Source: Apple, Inc.)



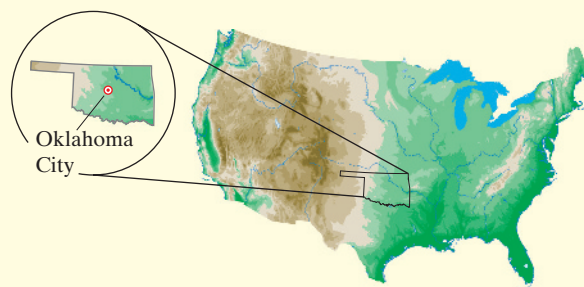
80. A large tropical cockroach of the family Dictyoptera is the fastest-moving insect. This insect was clocked at a speed of 3.36 miles per hour. Round this number to the nearest tenth. (Source: University of California, Berkeley)



81. The most decorated athlete in Olympic history, Michael Phelps of the United States, won his fourth gold medal of the 2016 Rio Summer Olympics in the 200-meter individual medley with a time of 1.918 minutes. Round this time to the nearest hundredth of a minute. (Source: ESPN)



82. The population density of the state of Oklahoma is 56.411 people per square mile. Round this population density to the nearest tenth. (Source: World Population Review)



83. A used biology textbook is priced at \$67.89. Round this price to the nearest dollar.

84. A used office desk is advertised at \$19.95 by Drawley's Office Furniture. Round this price to the nearest dollar.

85. Venus makes a complete orbit around the Sun every 224.695 days. Round this figure to the nearest whole day. (Source: National Space Science Data Center)

86. The length of a day on Mars, a full rotation about its axis, is 24.6229 hours. Round this figure to the nearest thousandth. (Source: National Space Science Data Center)

Review

Perform each indicated operation. See Section 1.3.

87. $3452 + 2314$

88. $8945 + 4536$

89. $82 - 47$

90. $4002 - 3897$

Concept Extensions

Solve. See the Concept Check in this section.

- 91.** 2849.1738 rounded to the nearest hundred is
a. 2849.17 b. 2800 c. 2850 d. 2849.174
- 92.** 146.059 rounded to the nearest ten is
a. 146.0 b. 146.1 c. 140 d. 150
- 93.** 2849.1738 rounded to the nearest hundredth is
a. 2849.17 b. 2800 c. 2850 d. 2849.18
- 94.** 146.059 rounded to the nearest tenth is
a. 146.0 b. 146.1 c. 140 d. 150
-  **95.** In your own words, describe how to write a decimal as a fraction or a mixed number.
- 96.**  Explain how to identify the value of the 9 in the decimal 486.3297.
- 97.** Write $7\frac{12}{100}$ as a decimal.
- 98.** Write $17\frac{268}{1000}$ as a decimal.
- 99.** Write 0.00026849577 as a fraction.
- 100.** Write 0.00026849576 in words.
- 101.** Write a 5-digit number that rounds to 1.7.
- 102.** Write a 4-digit number that rounds to 26.3.
- 103.** Write a decimal number that is greater than 8 but less than 9.
- 104.** Write a decimal number that is greater than 48.1, but less than 48.2.
- 105.** Which number(s) rounds to 0.26?
0.26559 0.26499 0.25786 0.25186
- 106.** Which number(s) rounds to 0.06?
0.0612 0.066 0.0586 0.0506

Write these numbers from smallest to largest.

- 107.** 0.9
0.1038
0.10299
0.1037
- 108.** 0.01
0.0839
0.09
0.1
- 109.** The all-time top six movies (those that have earned the most money in the United States) along with the approximate amount of money they have earned are listed in the table. Estimate the total amount of money that these movies have earned by first rounding each earning to the nearest hundred-million. (*Source:* the-numbers.com)
- 110.** In 2015, there were 1199.1 million singles downloaded at an average price of \$1.20 each. Find an estimate of the total revenue from downloaded singles by answering parts **a–c**. (*Source:* Recording Industry Association of America)
- a.** Round 1199.1 million to the nearest ten million.
- b.** Multiply the rounded value in part **a** by 12.
- c.** Move the decimal point in the product from part **b** one place to the left. This number is the total revenue in millions of dollars.

Top All-Time American Movies	
Movie	Gross Domestic Earnings
<i>Star Wars Ep. VII: The Force Awakens</i> (2015)	\$936.7 million
<i>Avatar</i> (2009)	\$760.5 million
<i>Titanic</i> (1997)	\$658.7 million
<i>Jurassic World</i> (2015)	\$652.2 million
<i>The Avengers</i> (2012)	\$623.3 million
<i>The Dark Knight</i> (2008)	\$533.3 million

5.2 Adding and Subtracting Decimals

Objectives

- A** Add or Subtract Decimals.
- B** Estimate When Adding or Subtracting Decimals.
- C** Evaluate Expressions with Decimal Replacement Values.
- D** Simplify Expressions Containing Decimals.
- E** Solve Problems That Involve Adding or Subtracting Decimals.

Practice 1

Add.

- a. $19.52 + 5.371$
- b. $40.08 + 17.612$
- c. $0.125 + 422.8$

Objective A Adding or Subtracting Decimals

Adding or subtracting decimals is similar to adding or subtracting whole numbers. We add or subtract digits in corresponding place values from right to left, carrying or borrowing if necessary. To make sure that digits in corresponding place values are added or subtracted, we line up the decimal points vertically.

Adding or Subtracting Decimals

Step 1: Write the decimals so that the decimal points line up vertically.

Step 2: Add or subtract as with whole numbers.

Step 3: Place the decimal point in the sum or difference so that it lines up vertically with the decimal points in the problem.

In this section, we will insert zeros in decimal numbers so that place-value digits line up neatly. This is shown in Example 1.

Example 1 Add: $23.85 + 1.604$

Solution: First we line up the decimal points vertically.

$$\begin{array}{r} 23.850 \\ + 1.604 \\ \hline \end{array}$$

Insert one 0 so that digits line up neatly.

Line up decimal points.

Then we add the digits from right to left as for whole numbers.

$$\begin{array}{r} 23.850 \\ + 1.604 \\ \hline 25.454 \end{array}$$

Place the decimal point in the sum so that all decimal points line up.

Work Practice 1

Helpful Hint

Recall that 0s may be placed after the last digit to the right of the decimal point without changing the value of the decimal. This may be used to help line up place values when adding decimals.

$\begin{array}{r} 3.2 \\ 15.567 \\ + 0.11 \\ \hline \end{array}$	becomes	$\begin{array}{r} 3.200 \\ 15.567 \\ + 0.110 \\ \hline 18.877 \end{array}$	Insert two 0s.
			Insert one 0.
			Add.

Answers

1. a. 24.891 b. 57.692 c. 422.925

Example 2 Add: $763.7651 + 22.001 + 43.89$ **Solution:** First we line up the decimal points.

$$\begin{array}{r}
 \overset{\overset{1}{\uparrow}}{7} \overset{\overset{1}{\uparrow}}{6} \overset{\overset{1}{\uparrow}}{3}.7651 \\
 22.0010 \quad \text{Insert one 0.} \\
 + 43.8900 \quad \text{Insert two 0s.} \\
 \hline
 829.6561 \quad \text{Add.}
 \end{array}$$

Work Practice 2**Helpful Hint**

Don't forget that the decimal point in a whole number is positioned after the last digit.

Example 3 Add: $45 + 2.06$ **Solution:**

$$\begin{array}{r}
 45.00 \quad \text{Insert a decimal point and two 0s.} \\
 + 2.06 \quad \text{Line up decimal points.} \\
 \hline
 47.06 \quad \text{Add.}
 \end{array}$$

Work Practice 3

✓ **Concept Check** What is wrong with the following calculation of the sum of 7.03, 2.008, 19.16, and 3.1415?

$$\begin{array}{r}
 7.03 \\
 2.008 \\
 19.16 \\
 + 3.1415 \\
 \hline
 3.6042
 \end{array}$$

Example 4 Add: $3.62 + (-4.78)$

Solution: Recall from Chapter 2 that to add two numbers with different signs, we find the difference of the larger absolute value and the smaller absolute value. The sign of the answer is the same as the sign of the number with the larger absolute value.

$$\begin{array}{r}
 4.78 \\
 - 3.62 \\
 \hline
 1.16 \quad \text{Subtract the absolute values.}
 \end{array}$$

$$\text{Thus, } 3.62 + (-4.78) = -1.16$$

↑ The sign of the number with the larger absolute value;
 -4.78 has the larger absolute value.

Work Practice 4

Subtracting decimals is similar to subtracting whole numbers. We line up digits and subtract from right to left, borrowing when needed.

Practice 2

Add.

a. $34.567 + 129.43 + 2.8903$

b. $11.21 + 46.013 + 362.526$

Practice 3Add: $19 + 26.072$ **Practice 4**Add: $7.12 + (-9.92)$ **Answers**

2. a. 166.8873 b. 419.749

3. 45.072 4. -2.8

✓ **Concept Check Answer**

The decimal places are not lined up properly.

Practice 5

Subtract. Check your answers.

a. $6.7 - 3.92$

b. $9.72 - 4.068$

Practice 6

Subtract. Check your answers.

a. $73 - 29.31$

b. $210 - 68.22$

Practice 7

Subtract 19 from 25.91

Practice 8Subtract: $-5.4 - 9.6$ **Practice 9**Subtract: $-1.05 - (-7.23)$ **Answers**

5. a. 2.78 b. 5.652

6. a. 43.69 b. 141.78

7. 6.91 8. -15 9. 6.18

Example 5Subtract: $3.5 - 0.068$. Check your answer.**Solution:**

$$\begin{array}{r} 4 \cancel{10} 10 \\ 3.5 \cancel{0} 0 \\ - 0.068 \\ \hline 3.432 \end{array}$$

Insert two 0s.

Line up decimal points.

Subtract.

Check: Recall that we can check a subtraction problem by adding.

$$\begin{array}{r} 3.432 \text{ Difference} \\ + 0.068 \text{ Subtrahend} \\ \hline 3.500 \text{ Minuend} \end{array}$$

Work Practice 5**Example 6**Subtract: $85 - 17.31$. Check your answer.**Solution:**

$$\begin{array}{r} 14 \quad 9 \\ 7 \cancel{10} 10 \\ 85 \cancel{0} 0 \\ - 17.31 \\ \hline 67.69 \end{array}$$

Check:

$$\begin{array}{r} 67.69 \text{ Difference} \\ + 17.31 \text{ Subtrahend} \\ \hline 85.00 \text{ Minuend} \end{array}$$

Work Practice 6**Example 7**

Subtract 3 from 6.98.

Solution:

$$\begin{array}{r} 6.98 \\ - 3.00 \text{ Insert two 0s.} \\ \hline 3.98 \end{array}$$

Check:

$$\begin{array}{r} 3.98 \text{ Difference} \\ + 3.00 \text{ Subtrahend} \\ \hline 6.98 \text{ Minuend} \end{array}$$

Work Practice 7**Example 8**Subtract: $-5.8 - 1.7$ **Solution:** Recall from Chapter 2 that to subtract 1.7, we add the opposite of 1.7, or -1.7 . Thus

$$-5.8 - 1.7 = -5.8 + (-1.7) \quad \text{To subtract, add the opposite of 1.7, which is } -1.7.$$

$$= -7.5.$$

Add the absolute values.
 Use the common negative sign.

Work Practice 8**Example 9**Subtract: $-2.56 - (-4.01)$ **Solution:**

$$-2.56 - (-4.01) = -2.56 + 4.01 \quad \text{To subtract, add the opposite of } -4.01, \text{ which is } 4.01.$$

$$= 1.45$$

Subtract the absolute values.
 The answer is positive since 4.01 has the larger absolute value.

Work Practice 9

Objective B Estimating When Adding or Subtracting Decimals

To help avoid errors, we can also estimate to see if our answer is reasonable when adding or subtracting decimals. Although only one estimate is needed per operation, we show two for variety.

Example 10

Add or subtract as indicated. Then estimate to see if the answer is reasonable by rounding the given numbers and adding or subtracting the rounded numbers.

a. $27.6 + 519.25$

Exact		Estimate 1		Estimate 2
$\begin{array}{r} 27.60 \\ + 519.25 \\ \hline 546.85 \end{array}$	$\begin{array}{l} \text{rounds to} \\ \text{rounds to} \end{array}$	$\begin{array}{r} 30 \\ + 500 \\ \hline 530 \end{array}$	or	$\begin{array}{r} 30 \\ + 520 \\ \hline 550 \end{array}$

Since the exact answer is close to either estimate, it is reasonable. (In the first estimate, each number is rounded to the place value of the leftmost digit. In the second estimate, each number is rounded to the nearest ten.)

b. $11.01 - 0.862$

Exact		Estimate 1		Estimate 2
$\begin{array}{r} 11.01 \\ - 0.862 \\ \hline 10.148 \end{array}$	$\begin{array}{l} \text{rounds to} \\ \text{rounds to} \end{array}$	$\begin{array}{r} 10 \\ - 1 \\ \hline 9 \end{array}$	or	$\begin{array}{r} 11 \\ - 1 \\ \hline 10 \end{array}$

In the first estimate, we rounded the first number to the nearest ten and the second number to the nearest one. In the second estimate, we rounded both numbers to the nearest one. Both estimates show us that our answer is reasonable.

Work Practice 10

✓Concept Check Why shouldn't the sum $21.98 + 42.36$ be estimated as $30 + 50 = 80$?

Objective C Using Decimals as Replacement Values

Let's review evaluating expressions with given replacement values. This time the replacement values are decimals.

Example 11

Evaluate $x - y$ for $x = 2.8$ and $y = 0.92$.

Solution: Replace x with 2.8 and y with 0.92 and simplify.

$$\begin{aligned} x - y &= 2.8 - 0.92 && 2.80 \\ &= 1.88 && \underline{-0.92} \\ &&& 1.88 \end{aligned}$$

Work Practice 11

Practice 10

Add or subtract as indicated. Then estimate to see if the answer is reasonable by rounding the given numbers and adding or subtracting the rounded numbers.

a. $58.1 + 326.97$

b. $16.08 - 0.925$

Helpful Hint

Remember that estimates are used for our convenience to quickly check the reasonableness of an answer.

Practice 11

Evaluate $y - z$ for $y = 11.6$ and $z = 10.8$.

Answers

10. a. Exact: 385.07; an estimate: 390
b. Exact: 15.155; an estimate: 15 11. 0.8

✓Concept Check Answer

Each number is rounded incorrectly. The estimate is too high.

Practice 12

Is 12.1 a solution of the equation $y - 4.3 = 7.8$?

Practice 13

Simplify by combining like terms:

$$-4.3y + 7.8 - 20.1y + 14.6$$

Practice 14

Find the total monthly cost of owning and operating a certain automobile given the expenses shown.

Monthly car payment:	\$563.52
Monthly insurance cost:	\$52.68
Average gasoline bill per month:	\$127.50

Answers

12. yes 13. $-24.4y + 22.4$
14. \$743.70

Example 12

Is 2.3 a solution of the equation $6.3 = x + 4$?

Solution: Replace x with 2.3 in the equation $6.3 = x + 4$ to see if the result is a true statement.

$$6.3 = x + 4$$

$$6.3 \stackrel{?}{=} 2.3 + 4 \quad \text{Replace } x \text{ with } 2.3.$$

$$6.3 = 6.3 \quad \text{True}$$

Since $6.3 = 6.3$ is a true statement, 2.3 is a solution of $6.3 = x + 4$.

Work Practice 12**Objective D** Simplifying Expressions Containing Decimals **Example 13**

Simplify by combining like terms:

$$11.1x - 6.3 + 8.9x - 4.6$$

Solution:

$$\begin{aligned} 11.1x - 6.3 + 8.9x - 4.6 &= 11.1x + 8.9x + (-6.3) + (-4.6) \\ &= 20x + (-10.9) \\ &= 20x - 10.9 \end{aligned}$$

Work Practice 13**Objective E** Solving Problems by Adding or Subtracting Decimals

Decimals are very common in real-life problems.

Example 14

Calculating the Cost of Owning an Automobile

Find the total monthly cost of owning and operating a certain automobile given the expenses shown.

Monthly car payment:	\$256.63
Monthly insurance cost:	\$47.52
Average gasoline bill per month:	\$195.33

Solution:

1. UNDERSTAND. Read and reread the problem. The phrase “total monthly cost” tells us to add.
2. TRANSLATE.

In words:	total monthly cost	is	car payment	plus	insurance cost	plus	gasoline bill
	↓	↓	↓	↓	↓	↓	↓
Translate:	total monthly cost	=	\$256.63	+	\$47.52	+	\$195.33

3. SOLVE: Let's also estimate by rounding each number to the nearest ten.

$$\begin{array}{r}
 \overset{111}{256.63} \text{ rounds to } 260 \\
 47.52 \text{ rounds to } 50 \\
 + 195.33 \text{ rounds to } 200 \\
 \hline
 499.48 \text{ Exact } \quad 510 \text{ Estimate}
 \end{array}$$

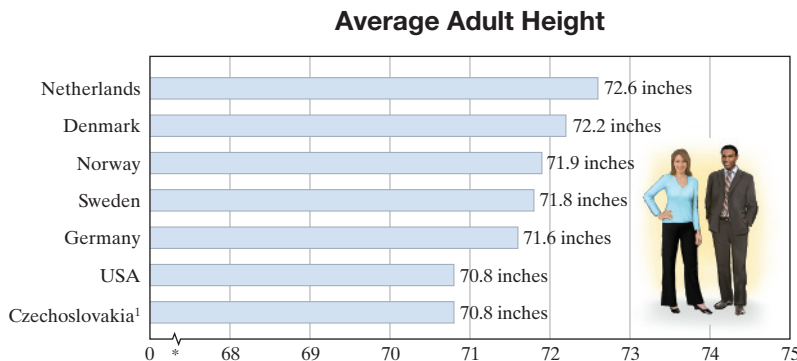
4. INTERPRET. *Check your work.* Since our estimate is close to our exact answer, our answer is reasonable. *State your conclusion:* The total monthly cost is \$499.48.

Work Practice 14

The next bar graph has horizontal bars. To visualize the value represented by a bar, see how far it extends to the right. The value of each bar is labeled, and we will study bar graphs further in a later chapter.

Example 15 Comparing Average Heights

The bar graph shows the current average heights for adults in various countries. How much greater is the average height in Denmark than the average height in the United States?



¹Average for Czech Republic, Slovakia

Source: *USA Today*

* The $\sim$ means that some numbers are purposefully missing on the axis.

Solution:

1. UNDERSTAND. Read and reread the problem. Since we want to know “how much greater,” we subtract.
2. TRANSLATE.

In words:	How much greater	is	Denmark's average height	minus	U.S. average height
	↓	↓	↓	↓	↓
Translate:	How much greater	=	72.2	−	70.8

3. SOLVE: We estimate by rounding each number to the nearest whole.

$$\begin{array}{r}
 \overset{1}{7} \overset{12}{2.2} \text{ rounds to } 72 \\
 - 70.8 \text{ rounds to } -71 \\
 \hline
 1.4 \text{ Exact } \quad 1 \text{ Estimate}
 \end{array}$$

4. INTERPRET. *Check your work.* Since our estimate is close to our exact answer, 1.4 inches is reasonable. *State your conclusion:* The average height in Denmark is 1.4 inches greater than the average U.S. height.

Work Practice 15

Practice 15

Use the bar graph in Example 15. How much greater is the average height in the Netherlands than the average height in Czechoslovakia?

Answer

15. 1.8 in.



Calculator Explorations Decimals

Entering Decimal Numbers

To enter a decimal number, find the key marked $\boxed{\cdot}$.
To enter the number 2.56, for example, press the keys $\boxed{2} \boxed{\cdot} \boxed{5} \boxed{6}$.

The display will read $\boxed{2.56}$.

Operations on Decimal Numbers

Operations on decimal numbers are performed in the same way as operations on whole or signed numbers. For example, to find $8.625 - 4.29$, press the keys $\boxed{8.625} \boxed{-} \boxed{4.29}$ then $\boxed{=}$ or $\boxed{\text{ENTER}}$.

The display will read $\boxed{4.335}$. (Although entering 8.625, for example, requires pressing more than one key, we group numbers together here for easier reading.)

Use a calculator to perform each indicated operation.

- | | |
|--|--|
| 1. $315.782 + 12.96$ | 2. $29.68 + 85.902$ |
| 3. $6.249 - 1.0076$ | 4. $5.238 - 0.682$ |
| 5. $\begin{array}{r} 12.555 \\ 224.987 \\ 5.2 \\ + 622.65 \\ \hline \end{array}$ | 6. $\begin{array}{r} 47.006 \\ 0.17 \\ 313.259 \\ + 139.088 \\ \hline \end{array}$ |

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

minuend	vertically	like	true
difference	subtrahend	last	false

- The decimal point in a whole number is positioned after the _____ digit.
- In $89.2 - 14.9 = 74.3$, the number 74.3 is called the _____, 89.2 is the _____, and 14.9 is the _____.
- To simplify an expression, we combine any _____ terms.
- To add or subtract decimals, we line up the decimal points _____.
- True or false: If we replace x with 11.2 and y with -8.6 in the expression $x - y$, we have $11.2 - 8.6$. _____
- True or false: If we replace x with -9.8 and y with -3.7 in the expression $x + y$, we have $-9.8 + 3.7$. _____

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 5.2

- Objective A** 7. From Examples 1–3, why do you think we line up decimal points?
- Objective B** 8. In Example 4, estimating is used to check whether the answer to the subtraction problem is reasonable, but what is the best way to fully check?
- Objective C** 9. In Example 5, why is the actual subtraction performed to the side?
- Objective D** 10. How many sets of like terms are there in Example 6?
- Objective E** 11. In Example 7, to calculate the amount of border material needed, we are actually calculating the _____ of the triangle.

5.2 Exercise Set MyLab Math

Objectives A B Mixed Practice Add. See Examples 1 through 4 and 10. For those exercises marked, also estimate to see if the answer is reasonable.

1. $5.6 + 2.1$

2. $3.6 + 4.1$

3. $8.2 + 2.15$

4. $5.17 + 3.7$

 5. $24.6 + 2.39 + 0.0678$

6. $32.4 + 1.58 + 0.0934$

7. $-2.6 + (-5.97)$

8. $-18.2 + (-10.8)$

9. $18.56 + (-8.23)$

10. $4.38 + (-6.05)$

11.
$$\begin{array}{r} 234.89 \\ + 230.67 \\ \hline \end{array}$$
 Exact: _____ Estimate: _____

12.
$$\begin{array}{r} 734.89 \\ + 640.56 \\ \hline \end{array}$$
 Exact: _____ Estimate: _____

13.
$$\begin{array}{r} 100.009 \\ 6.08 \\ + 9.034 \\ \hline \end{array}$$
 Exact: _____ Estimate: _____

14.
$$\begin{array}{r} 200.89 \\ 7.49 \\ + 62.83 \\ \hline \end{array}$$
 Exact: _____ Estimate: _____

15. Find the sum of 39, 3.006, and 8.403

16. Find the sum of 65, 5.0903, and 6.9003

Subtract and check. See Examples 5 through 10. For those exercises marked, also estimate to see if the answer is reasonable.

17. $12.6 - 8.2$

18. $8.9 - 3.1$

19. $18 - 2.7$

20. $28 - 3.3$

 21.
$$\begin{array}{r} 654.9 \\ - 56.67 \\ \hline \end{array}$$

22.
$$\begin{array}{r} 863.23 \\ - 39.453 \\ \hline \end{array}$$

23. $5.9 - 4.07$
Exact: _____
Estimate: _____

24. $6.4 - 3.04$
Exact: _____
Estimate: _____

 25.
$$\begin{array}{r} 1000 \\ - 123.4 \\ \hline \end{array}$$

Exact: _____

26.
$$\begin{array}{r} 2000 \\ - 327.47 \\ \hline \end{array}$$

Exact: _____

27. $200 - 5.6$

28. $800 - 8.9$

Estimate: _____

Estimate: _____

 29. $-1.12 - 5.2$

30. $-8.63 - 5.6$

31. $5.21 - 11.36$

32. $8.53 - 17.84$

33. $-2.6 - (-5.7)$

34. $-9.4 - (-10.4)$

35. $3 - 0.0012$

36. $7 - 0.097$

37. Subtract 6.7 from 23.

38. Subtract 9.2 from 45.

Objective A Perform the indicated operation. See Examples 1 through 9.

39. $0.9 + 2.2$
40. $0.7 + 3.4$
41. $-6.06 + 0.44$
42. $-5.05 + 0.88$
43. $500.21 - 136.85$
44. $600.47 - 254.68$
45. $50.2 - 600$
46. $40.3 - 700$
47. Subtract 61.9 from 923.5.
48. Subtract 45.8 from 845.9.
49. Add 200.008 and 706 and 9.046.
50. Add 100.007 and 2.08 and 5.014.
51. $-0.003 + 0.091$
52. $-0.004 + 0.085$
53. $-102.4 - 78.04$
54. $-36.2 - 10.02$
55. $-2.9 - (-1.8)$
56. $-6.5 - (-3.3)$

Objective C Evaluate each expression for $x = 3.6$, $y = 5$, and $z = 0.21$. See Example 11.

57. $x + z$
58. $y + x$
59. $x - z$
60. $y - z$
61. $y - x + z$
62. $x + y + z$

Determine whether the given values are solutions to the given equations. See Example 12.

63. Is 7 a solution to $x + 2.7 = 9.3$?
64. Is 3.7 a solution to $x + 5.9 = 8.6$?
65. Is -11.4 a solution to $27.4 + y = 16$?
66. Is -22.9 a solution to $45.9 + z = 23$?
67. Is 1 a solution to $2.3 + x = 5.3 - x$?
68. Is 0.9 a solution to $1.9 - x = x + 0.1$?

Objective D Simplify by combining like terms. See Example 13.

69. $30.7x + 17.6 - 23.8x - 10.7$
70. $14.2z + 11.9 - 9.6z - 15.2$
71. $-8.61 + 4.23y - 2.36 - 0.76y$
72. $-8.96x - 2.31 - 4.08x + 9.68$

Objective E Solve. For Exercises 73 and 74, the solutions have been started for you. See Examples 14 and 15.

73. Ann-Margaret Tober bought a book for \$32.48. If she paid with two \$20 bills, what was her change?
74. Phillip Guillot bought a car part for \$18.26. If he paid with two \$10 bills, what was his change?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blank.)

change

is

two
\$20 bills

minus

cost of
book

change

=

40

-

Finish with

3. SOLVE and 4. INTERPRET

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blank.)

change

is

two
\$10 bills

minus

cost of
car part

change

=

20

-

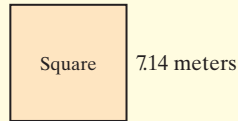
Finish with

3. SOLVE and 4. INTERPRET

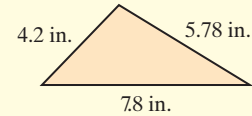
75. Microsoft stock opened the day at \$35.17 per share, and the closing price the same day was \$34.75. By how much did the price of each share change?

76. A pair of eyeglasses costs a total of \$347.89. The frames of the glasses are \$97.23. How much do the lenses of the eyeglasses cost?

- △ 77. Find the perimeter.



- △ 78. Find the perimeter.



79. The Apple iPhone 7 was released in 2016. It measures 5.44 inches by 2.64 inches. Find the perimeter of this phone. (Source: Apple.com)



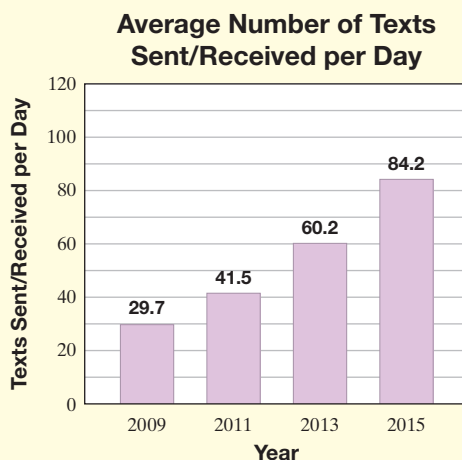
80. The Samsung Galaxy S7 Edge was released in 2016. It measures 5.91 inches by 2.86 inches. Find the perimeter of the phone. (Source: Google.com)



81. The average wind speed at the weather station on Mt. Washington in New Hampshire is 35.2 miles per hour. The highest speed ever recorded at the station is 231.0 miles per hour. How much faster is the highest speed than the average wind speed? (Source: National Centers for Environmental Information)

82. The average annual rainfall in Omaha, Nebraska, is 30.08 inches. The average annual rainfall in New Orleans, Louisiana, is 64.16 inches. On average, how much more rain does New Orleans receive annually than Omaha? (Source: National Centers for Environmental Information)

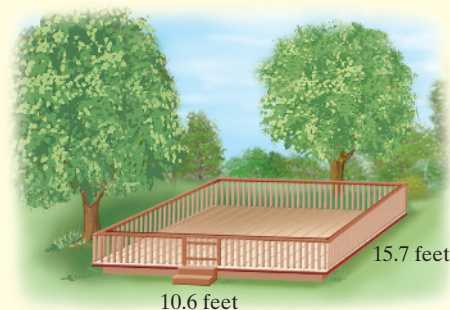
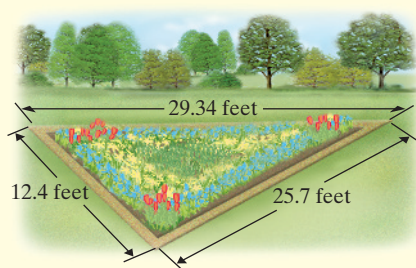
This bar graph shows the increase in the total number of text messages per person per day in the United States. Use this graph for Exercises 83 and 84. (Source: Pew Research Center.)



83. Find the increase in the number of texts sent or received per day from 2011 to 2015.

84. Find the increase in the number of texts sent or received per day from 2013 to 2015.

85. As of this writing, the top three U.S. movies that made the most money through movie ticket sales are *Star Wars Episode VII: The Force Awakens* (2015), \$936.7 million; *Avatar* (2009), \$760.5 million; and *Titanic* (1997), \$658.7 million. What was the total amount of ticket sales for these three movies? Round your answer to the nearest million. (Source: the-numbers.com)
86. In 2011, the average credit card late fee was \$23.15. By 2015, the average credit card late fee had increased by \$3.85. Find the average credit card late fee in 2015. (Source: Consumer Financial Protection Bureau)
87. The snowiest city in the United States is Valdez, AK, which receives an average of 110.5 more inches of snow than the second snowiest city. The second snowiest city in the United States is Crested Butte, CO. Crested Butte receives an average of 215.8 inches annually. How much snow does Valdez receive on average each year? (Source: The Weather Channel)
88. The driest place in the world is the Atacama Desert in Chile, which receives an average of only 0.004 inch of rain per year. Yuma, Arizona, is the driest city in the United States. Yuma receives an average of 3.006 more inches of rain each year than the Atacama Desert. What is the average annual rainfall in Yuma? (Source: National Climatic Data Center)
- ▶ 89. A landscape architect is planning a border for a flower garden shaped like a triangle. The sides of the garden measure 12.4 feet, 29.34 feet, and 25.7 feet. Find the amount of border material needed.
- △ 90. A contractor purchased enough railing to completely enclose the newly built deck shown below. Find the amount of railing purchased.

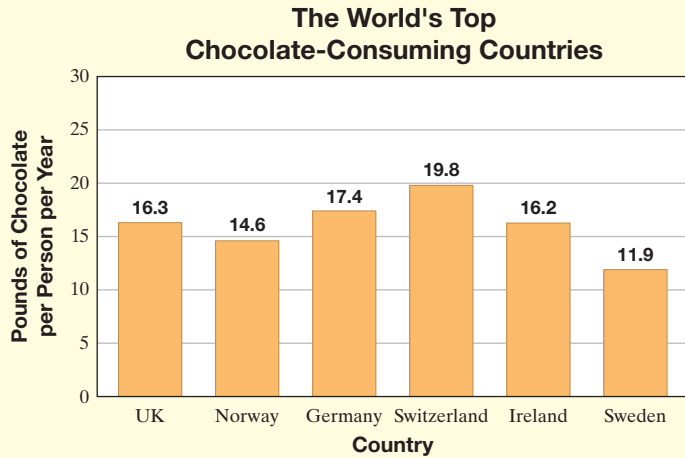


The table shows the average speeds for the Daytona 500 winners for the years shown. Use this table to answer Exercises 91 and 92. (Source: Daytona International Speedway)

Daytona 500 Winners		
Year	Winner	Average Speed
2001	Michael Waltrip	161.783
2004	Dale Earnhardt Jr.	156.341
2007	Kevin Harvick	149.333
2010	Jamie McMurray	137.284
2013	Jimmie Johnson	159.250
2016	Denny Hamlin	157.549

91. How much slower was the average Daytona 500 winning speed in 2016 than in 2001?
92. How much faster was Dale Earnhardt Jr.'s average Daytona 500 winning speed in 2004 than Jamie McMurray's Daytona 500 winning speed in 2010?

The bar graph shows the top five chocolate-consuming nations in the world. Use this table to answer Exercises 93 through 97.



Source: Euromonitor; Forbes statista

93. Which country in the table has the greatest chocolate consumption per person?
94. Which country in the table has the least chocolate consumption per person?
95. How much more is the greatest chocolate consumption than the least chocolate consumption shown in the table?
96. How much more chocolate does the average German consume per year than the average Norwegian?
97. Make a chart listing the countries and their corresponding chocolate consumptions in order from greatest to least.

Review

Multiply. See Sections 1.5 and 4.3.

98. $23 \cdot 2$

99. $46 \cdot 3$

100. $39 \cdot 3$

101. $\left(\frac{2}{3}\right)^2$

102. $\left(\frac{1}{5}\right)^3$

Concept Extensions

A friend asks you to check his calculations for Exercises 103 and 104. Are they correct? If not, explain your friend's errors and correct the calculations. See the first Concept Check in this section.

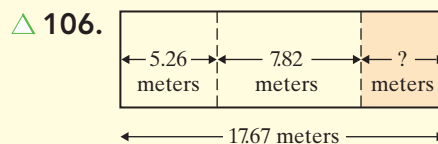
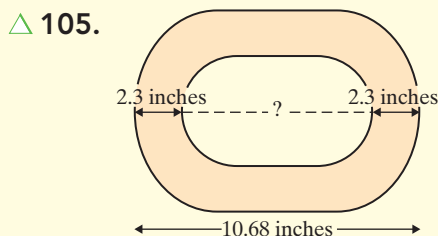
103.

$$\begin{array}{r} 9.2 \\ 8.63 \\ + 4.005 \\ \hline 4.960 \end{array}$$

104.

$$\begin{array}{r} 8999 \\ 900.0 \\ - 96.4 \\ \hline 803.5 \end{array}$$

Find the unknown length in each figure.



Let's review the values of these common U.S. coins in order to answer the following exercises.

Penny	Nickel	Dime	Quarter
			
\$0.01	\$0.05	\$0.10	\$0.25

For Exercises 107 and 108, write the value of each group of coins. To do so, it is usually easiest to start with the coin(s) of greatest value and end with the coin(s) of least value.

107.



108.



109. Name the different ways that coins can have a value of \$0.17 given that you may use no more than 10 coins.

110. Name the different ways that coin(s) can have a value of \$0.25 given that there are no pennies.

111. Why shouldn't the sum

$$82.95 + 51.26$$

be estimated as $90 + 60 = 150$?

See the second Concept Check in this section.

112. Laser beams can be used to measure the distance to the moon. One measurement showed the distance to the moon to be 256,435.235 miles. A later measurement showed that the distance is 256,436.012 miles. Find how much farther away the moon is in the second measurement as compared to the first.

 113. Explain how adding or subtracting decimals is similar to adding or subtracting whole numbers.

 114. Can the sum of two negative decimals ever be a positive decimal? Why or why not?

Combine like terms and simplify.



115. $-8.689 + 4.286x - 14.295 - 12.966x + 30.861x$



116. $14.271 - 8.968x + 1.333 - 201.815x + 101.239x$

5.3 Multiplying Decimals and Circumference of a Circle

Objective A Multiplying Decimals

Multiplying decimals is similar to multiplying whole numbers. The only difference is that we place a decimal point in the product. To discover where a decimal point is placed in a product, let's multiply 0.6×0.03 . We first write each decimal as an equivalent fraction and then multiply.

$$\begin{array}{c} 0.6 \quad \times \quad 0.03 \\ \uparrow \quad \uparrow \\ 1 \text{ decimal place} \quad 2 \text{ decimal places} \end{array} = \frac{6}{10} \times \frac{3}{100} = \frac{18}{1000} = 0.018$$

$\uparrow$
3 decimal places

Notice that $1 + 2 = 3$, the number of decimal places in the product. Now let's multiply 0.03×0.002 .

$$\begin{array}{c} 0.03 \quad \times \quad 0.002 \\ \uparrow \quad \uparrow \\ 2 \text{ decimal places} \quad 3 \text{ decimal places} \end{array} = \frac{3}{100} \times \frac{2}{1000} = \frac{6}{100,000} = 0.00006$$

$\uparrow$
5 decimal places

Again, we see that $2 + 3 = 5$, the number of decimal places in the product.

Instead of writing decimals as fractions each time we want to multiply, we notice a pattern from these examples and state a rule that we can use:

Multiplying Decimals

Step 1: Multiply the decimals as though they are whole numbers.

Step 2: The decimal point in the product is placed so that the number of decimal places in the product is equal to the *sum* of the number of decimal places in the factors.

Example 1 Multiply: 23.6×0.78

Solution:

$$\begin{array}{r} 23.6 \quad 1 \text{ decimal place} \\ \times 0.78 \quad 2 \text{ decimal places} \\ \hline 1888 \\ 16520 \\ \hline 18,408 \end{array}$$

$\uparrow$ Since $1 + 2 = 3$, insert the decimal point in the product so that there are 3 decimal places.

Work Practice 1

Example 2 Multiply: 0.0531×16

Solution:

$$\begin{array}{r} 0.0531 \quad 4 \text{ decimal places} \\ \times 16 \quad 0 \text{ decimal places} \\ \hline 3186 \\ 5310 \\ \hline 0.8496 \end{array}$$

$\uparrow$ 4 decimal places ($4 + 0 = 4$)

Work Practice 2

Objectives

- A** Multiply Decimals. ▶
- B** Estimate When Multiplying Decimals. ▶
- C** Multiply Decimals by Powers of 10. ▶
- D** Evaluate Expressions with Decimal Replacement Values. ▶
- E** Find the Circumference of Circles. ▶
- F** Solve Problems by Multiplying Decimals. ▶

Practice 1

Multiply: 34.8×0.62

Practice 2

Multiply: 0.0641×27

Answers

1. 21.576 2. 1.7307

 **Concept Check** True or false? The number of decimal places in the product of 0.261 and 0.78 is 6. Explain.

Practice 3

Multiply: $(7.3)(-0.9)$

Practice 4

Multiply: 30.26×2.89 . Then estimate to see whether the answer is reasonable.

Example 3 Multiply: $(-2.6)(0.8)$

Solution: Recall that the product of a negative number and a positive number is a negative number.

$$(-2.6)(0.8) = -2.08$$

 **Work Practice 3**

Objective B Estimating When Multiplying Decimals

Just as for addition and subtraction, we can estimate when multiplying decimals to check the reasonableness of our answer.

Example 4 Multiply: 28.06×1.95 . Then estimate to see whether the answer is reasonable by rounding each factor, then multiplying the rounded numbers.

Solution:

Exact	Estimate 1	or	Estimate 2
$\begin{array}{r} 28.06 \\ \times 1.95 \\ \hline 14030 \\ 252540 \\ 280600 \\ \hline 54.7170 \end{array}$	$\begin{array}{r} 28 \\ \times 2 \\ \hline 56 \end{array}$	Rounded to ones	$\begin{array}{r} 30 \\ \times 2 \\ \hline 60 \end{array}$
			Rounded to one nonzero digit

The answer 54.7170 or 54.717 is reasonable.

 **Work Practice 4**

As shown in Example 4, estimated results will vary depending on what estimates are used. Notice that estimating results is a good way to see whether the decimal point has been correctly placed.

Objective C Multiplying Decimals by Powers of 10

There are some patterns that occur when we multiply a number by a power of 10 such as 10, 100, 1000, 10,000, and so on.

$$23.6951 \times 10 = 236.951 \quad \text{Move the decimal point 1 place to the right.}$$

↑
1 zero

$$23.6951 \times 100 = 2369.51 \quad \text{Move the decimal point 2 places to the right.}$$

↑
2 zeros

$$23.6951 \times 100,000 = 2,369,510. \quad \text{Move the decimal point 5 places to the right (insert a 0).}$$

↑
5 zeros

Notice that we move the decimal point the same number of places as there are zeros in the power of 10.

Answers

3. -6.57

4. Exact: 874514; Estimate: $30 \cdot 3 = 90$

Concept Check Answer

false: 3 decimal places and 2 decimal places means 5 decimal places in the product

Multiplying Decimals by Powers of 10 Such as 10, 100, 1000, 10,000

Move the decimal point to the *right* the same number of places as there are *zeros* in the power of 10.

Examples Multiply.

5. $7.68 \times 10 = 76.8$ 
6. $23.702 \times 100 = 2370.2$ 
7. $(-76.3)(1000) = -76,300$ 

Work Practice 5–7

There are also powers of 10 that are less than 1. The decimals 0.1, 0.01, 0.001, 0.0001, and so on, are examples of powers of 10 less than 1. Notice the pattern when we multiply by these powers of 10:

- $$569.2 \times 0.1 = 56.92 \quad \text{Move the decimal point } 1 \text{ place to the left.}$$
- ↑
1 decimal place
- $$569.2 \times 0.01 = 5.692 \quad \text{Move the decimal point } 2 \text{ places to the left.}$$
- ↑
2 decimal places
- $$569.2 \times 0.0001 = 0.05692 \quad \text{Move the decimal point } 4 \text{ places to the left (insert one 0).}$$
- ↑
4 decimal places

Multiplying Decimals by Powers of 10 Such as 0.1, 0.01, 0.001, 0.0001

Move the decimal point to the *left* the same number of places as there are *decimal places* in the power of 10.

Examples Multiply.

8. $42.1 \times 0.1 = 4.21$ 
9. $76,805 \times 0.01 = 768.05$ 
10. $(-9.2)(-0.001) = 0.0092$ 

Work Practice 8–10

Many times we see large numbers written, for example, in the form 2979 million rather than in the longer standard notation. The next example shows us how to interpret these numbers.

Practice 5–7

Multiply.

5. 46.8×10
6. 203.004×100
7. $(-2.33)(1000)$

Practice 8–10

Multiply.

8. 6.94×0.1
9. 3.9×0.01
10. $(-7682)(-0.001)$

Answers

5. 468 6. 20,300.4 7. -2330
8. 0.694 9. 0.039 10. 7.682

Practice 11

In 2016, there were 60.25 million married couples in the United States. Write this number in standard notation. (Source: U.S. Census Bureau)

Practice 12

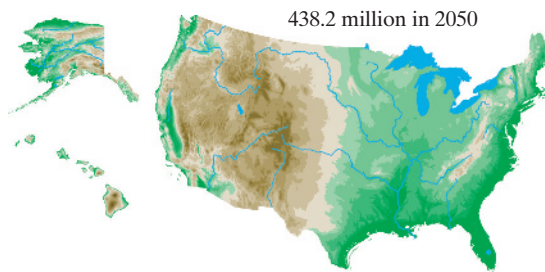
Evaluate $7y$ for $y = -0.028$.

Practice 13

Is -5.5 a solution of the equation $-6x = 33$?

Example 11

In 2050, the population of the United States is projected to be 438.2 million. Write this number in standard notation. (Source: Pew Research Center)



Solution: $438.2 \text{ million} = 438.2 \times 1 \text{ million}$
 $= 438.2 \times 1,000,000 = 438,200,000$

Work Practice 11**Objective D** Using Decimals as Replacement Values

Now let's practice working with variables.

Example 12

Evaluate xy for $x = 2.3$ and $y = 0.44$.

Solution: Recall that xy means $x \cdot y$.

$$\begin{array}{r} xy = (2.3)(0.44) \\ \times 0.44 \\ \hline 92 \\ 920 \\ \hline 1.012 \end{array}$$

$= 1.012$ ←

Work Practice 12**Example 13**

Is -9 a solution of the equation $3.7y = -3.33$?

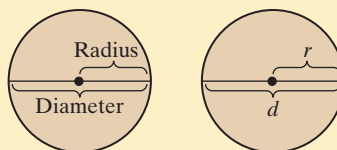
Solution: Replace y with -9 in the equation $3.7y = -3.33$ to see if a true equation results.

$$\begin{array}{l} 3.7y = -3.33 \\ 3.7(-9) \stackrel{?}{=} -3.33 \quad \text{Replace } y \text{ with } -9. \\ -33.3 = -3.33 \quad \text{False} \end{array}$$

Since $-33.3 = -3.33$ is a false statement, -9 is **not** a solution of $3.7y = -3.33$.

Work Practice 13**Objective E** Finding the Circumference of a Circle

Recall from Section 1.3 that the distance around a polygon is called its perimeter. The distance around a circle is given the special name **circumference**, and this distance depends on the radius (or the diameter) of the circle.

Circumference of a Circle

$$\begin{array}{l} \text{Circumference} = 2 \cdot \pi \cdot \text{radius or Circumference} = \pi \cdot \text{diameter} \\ C = 2\pi r \quad \text{or} \quad C = \pi d \end{array}$$

Answers

11. 60,250,000 12. -0.196 13. yes

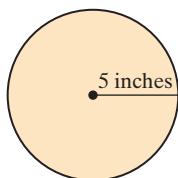
In Section 5.1, we learned about the symbol π as the Greek letter pi, pronounced “pie.” It is a constant between 3 and 4. A decimal approximation for π is 3.14. Also, a fraction approximation for π is $\frac{22}{7}$.

Example 14 Circumference of a Circle

Find the circumference of a circle whose radius is 5 inches. Then use the approximation 3.14 for π to approximate the circumference.

Solution: Let $r = 5$ in the formula $C = 2\pi r$.

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi \cdot 5 \\ &= 10\pi \end{aligned}$$



Next, replace π with the approximation 3.14.

$$\begin{aligned} C &= 10\pi \\ (\text{is approximately}) \longrightarrow &\approx 10(3.14) \\ &\approx 31.4 \end{aligned}$$

The **exact** circumference or distance around the circle is 10π inches, which is **approximately** 31.4 inches.

Work Practice 14

Objective F Solving Problems by Multiplying Decimals

The solutions to many real-life problems are found by multiplying decimals. We continue using our four problem-solving steps to solve such problems.

Example 15 Finding the Total Cost of Materials for a Job

A college student is hired to paint a billboard with paint costing \$2.49 per quart. If the job requires 3 quarts of paint, what is the total cost of the paint?

Solution:

1. UNDERSTAND. Read and reread the problem. The phrase “total cost” might make us think addition, but since this problem requires repeated addition, let’s multiply.

2. TRANSLATE.

In words:	total cost	is	cost per quart of paint	times	number of quarts
	↓	↓	↓	↓	↓
Translate:	total cost	=	2.49	×	3

3. SOLVE. We can estimate to check our calculations. The number 2.49 rounds to 2 and $2 \times 3 = 6$.

$$\begin{array}{r} 2.49 \\ \times \quad 3 \\ \hline 7.47 \end{array}$$

4. INTERPRET. Check your work. Since 7.47 is close to our estimate of 6, our answer is reasonable. State your conclusion: The total cost of the paint is \$7.47.

Work Practice 15

Practice 14

Find the circumference of a circle whose radius is 11 meters. Then use the approximation 3.14 for π to approximate this circumference.

Practice 15

A biology major is fertilizing her personal garden. She uses 5.6 ounces of fertilizer per square yard. The garden measures 60.5 square yards. How much fertilizer does she need?

Answers

14. $22\pi \text{ m} \approx 69.08 \text{ m}$ 15. 338.8 oz

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

circumference	left	sum	zeros
decimal places	right	product	factor

- When multiplying decimals, the number of decimal places in the product is equal to the _____ of the number of decimal places in the factors.
- In $8.6 \times 5 = 43$, the number 43 is called the _____, while 8.6 and 5 are each called a _____.
- When multiplying a decimal number by powers of 10 such as 10, 100, 1000, and so on, we move the decimal point in the number to the _____ the same number of places as there are _____ in the power of 10.
- When multiplying a decimal number by powers of 10 such as 0.1, 0.01, and so on, we move the decimal point in the number to the _____ the same number of places as there are _____ in the power of 10.
- The distance around a circle is called its _____.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 5.3

Objective A 6. From the lecture before Example 1, what's the main difference between multiplying whole numbers and multiplying decimal numbers?

Objective B 7. From Example 3, what does estimating especially help us with?

Objective C 8. Why don't we do any actual multiplying in Example 5?

Objective D 9. In Example 8, once all replacement values are inserted in the variable expression, what is the resulting expression to evaluate?

Objective E 10. Why is 31.4 cm not the exact answer to Example 9?

Objective F 11. In Example 10, why is 24.8 not the complete answer? What is the complete answer?

5.3 Exercise Set MyLab Math

Objectives A B Mixed Practice Multiply. See Examples 1 through 4. For those exercises marked, also estimate to see if the answer is reasonable.

1. 0.17×8

2. 0.23×9

3.
$$\begin{array}{r} 1.2 \\ \times 0.5 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 6.8 \\ \times 0.3 \\ \hline \end{array}$$

5. $(-2.3)(7.65)$

6. $(4.7)(-9.02)$

7. $(-5.73)(-9.6)$

8. $(-7.84)(-3.5)$

9. 6.8×4.2
Exact:
Estimate:

10. 8.3×2.7
Exact:
Estimate:

11.
$$\begin{array}{r} 0.347 \\ \times 0.3 \\ \hline \end{array}$$

12.
$$\begin{array}{r} 0.864 \\ \times 0.4 \\ \hline \end{array}$$

$$\begin{array}{r} 13. \quad 1.0047 \\ \times \quad 8.2 \\ \hline \end{array}$$

Exact:

Estimate:

$$\begin{array}{r} 14. \quad 2.0005 \\ \times \quad 5.5 \\ \hline \end{array}$$

Exact:

Estimate:

$$\begin{array}{r} 15. \quad 490.2 \\ \times 0.023 \\ \hline \end{array}$$

$$\begin{array}{r} 16. \quad 300.9 \\ \times 0.032 \\ \hline \end{array}$$

Objective C Multiply. See Examples 5 through 10.

▶ 17. 6.5×10

18. 7.2×100

▶ 19. 8.3×0.1

20. 23.4×0.1

21. $(-7.093)(1000)$

22. $(-1.123)(1000)$

23. 0.7×100

24. 0.5×100

▶ 25. $(-9.83)(-0.01)$

26. $(-4.72)(-0.01)$

27. 25.23×0.001

28. 36.41×0.001

Objectives A B C Mixed Practice Multiply. See Examples 1 through 10.

29. 0.123×0.4

30. 0.216×0.3

▶ 31. $(1479)(100)$

32. $(345.2)(100)$

33. 8.6×0.15

34. 0.42×5.7

35. $(937.62)(-0.01)$

36. $(-0.001)(562.01)$

37. 562.3×0.001

38. 993.5×0.001

$$\begin{array}{r} 39. \quad 6.32 \\ \times 5.7 \\ \hline \end{array}$$

$$\begin{array}{r} 40. \quad 9.21 \\ \times 3.8 \\ \hline \end{array}$$

Write each number in standard notation. See Example 11.

41. The cost of the Hubble Space Telescope at launch was \$1.5 billion. (Source: NASA)

42. About 56.7 million American households own at least one dog. (Source: American Pet Products Manufacturers Association)

43. The Blue Streak is the oldest roller coaster at Cedar Point, an amusement park in Sandusky, Ohio. Since 1964, it has given more than 49.8 million rides. (Source: Cedar Fair, L.P.)

44. In 2017, the restaurant industry had projected sales of \$798.7 billion. (Source: National Restaurant Association)

Objective D Evaluate each expression for $x = 3$, $y = -0.2$, and $z = 5.7$. See Example 12.

45. xy

46. yz

▶ 47. $xz - y$

48. $-5y + z$

Determine whether the given value is a solution of each given equation. See Example 13.

49. Is 14.2 a solution of $0.6x = 4.92$?

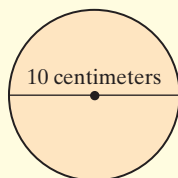
50. Is 1414 a solution of $100z = 14.14$?

51. Is -4 a solution of $3.5y = -14$?

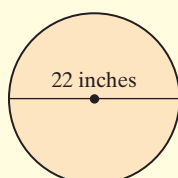
52. Is -3.6 a solution of $0.7x = -2.52$?

Objective E Find the circumference of each circle. Then use the approximation 3.14 for π and approximate each circumference. See Example 14.

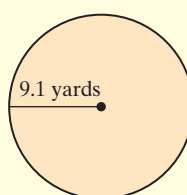
▶ 53.



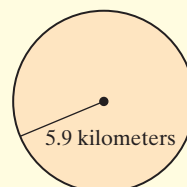
△ 54.



△ 55.



△ 56.

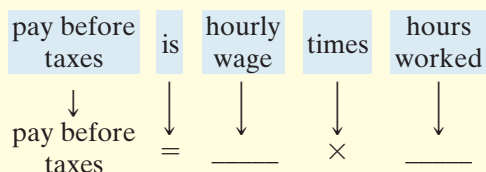


Objectives E F Mixed Practice Solve. For Exercises 57 and 58, the solutions have been started for you. For circumference applications find the exact circumference and then use 3.14 for π to approximate the circumference. See Examples 14 and 15.

- 57.** An electrician for Central Power and Light worked 40 hours last week. Calculate his pay before taxes for last week if his hourly wage is \$17.88.

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)



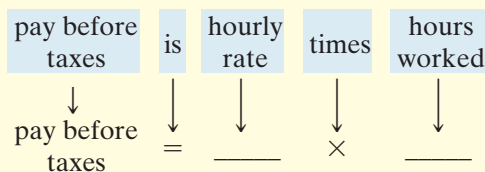
Finish with:

3. SOLVE and 4. INTERPRET.

- 58.** An assembly line worker worked 20 hours last week. Her hourly rate is \$19.52 per hour. Calculate her pay before taxes.

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)



Finish with:

3. SOLVE and 4. INTERPRET.

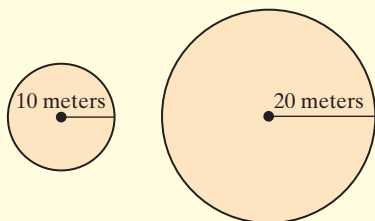
- 59.** A 1-ounce serving of cream cheese contains 6.2 grams of saturated fat. How much saturated fat is in 4 ounces of cream cheese? (*Source: Home and Garden Bulletin No. 72; U.S. Department of Agriculture*)
- 61.** Recall that the face of the Apple iPhone 7 (see Section 5.2) measures approximately 5.44 inches by 2.6 inches. Find the area of the face of the Apple iPhone 7.
- 63.** In 1893, the first ride called a Ferris wheel was constructed by Washington Gale Ferris. Its diameter was 250 feet. Find its circumference. Give the exact answer and an approximation using 3.14 for π . (*Source: The Handy Science Answer Book, Visible Ink Press, 1994*)
- 65.** The London Eye, built for the millennium celebration in London, resembles a gigantic Ferris wheel with a diameter of 135 meters. If Adam Hawn rides the Eye for one revolution, find how far he travels. Give the exact answer and an approximation using 3.14 for π . (*Source: Londoneye.com*)
- 60.** A 3.5-ounce serving of lobster meat contains 0.1 gram of saturated fat. How much saturated fat do 3 servings of lobster meat contain? (*Source: The National Institutes of Health*)
- 62.** Recall that the face of the Samsung Galaxy S7 Edge (see Section 5.2) measures approximately 5.91 inches by 2.9 inches. Find the area of the face of the Samsung Galaxy S7 Edge.
- 64.** The radius of Earth is approximately 3950 miles. Find the distance around Earth at the equator. Give the exact answer and an approximation using 3.14 for π . (*Hint: Find the circumference of a circle with radius 3950 miles.*)
- 66.** The world's longest suspension bridge is the Akashi Kaikyo Bridge in Japan. This bridge has two circular caissons, which are underwater foundations. If the diameter of a caisson is 80 meters, find its circumference. Give the exact answer and an approximation using 3.14 for π . (*Source: Scientific American; How Things Work Today*)



67. A meter is a unit of length in the metric system that is approximately equal to 39.37 inches. Sophia Wagner is 1.65 meters tall. Find her approximate height in inches.

68. The doorway to a room is 2.15 meters tall. Approximate this height in inches. (*Hint: See Exercise 67.*)

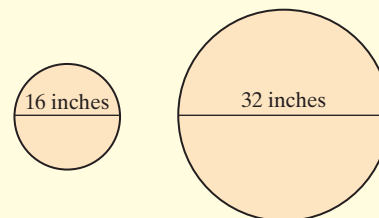
- △ 69. a. Approximate the circumference of each circle.



- b. If the radius of a circle is doubled, is its corresponding circumference doubled?

71. In December 2016, the price of wheat was \$4.0425 per bushel. How much would 100 bushels of wheat cost at this price? (*Source: ndwheat.com*)

- △ 70. a. Approximate the circumference of each circle.



- b. If the diameter of a circle is doubled, is its corresponding circumference doubled?

72. In December 2016, the price of corn was \$3.47 per bushel. How much would a company pay for 10,000 bushels of corn? (*Source: quotecorn.com*)

The table shows currency exchange rates for various countries on February 1, 2017. To find the amount of foreign currency equivalent to an amount of U.S. dollars, multiply the U.S. dollar amount by the exchange rate listed in the table. Use this table to answer Exercises 73 through 76. If needed, round answers to the nearest hundredth.

Foreign Currency Exchange Rates	
Country	Exchange Rate
Canadian dollar	1.3032
European Union euro	0.9292
Mexican peso	20.5077
Chinese yuan	6.88
Japanese yen	112.73
Swiss franc	0.9927

73. How many Canadian dollars are equivalent to \$750 U.S.?
74. Suppose you wish to exchange 300 American dollars for Chinese yuan. How much money, in Chinese yuan, would you receive?
75. The Scarpulla family is traveling to Mexico. How many Mexican pesos can they “buy” with 800 U.S. dollars?
76. A French tourist to the United States spent \$130 for souvenirs at the *Head of the Charles Regatta* in Boston. How much money did he spend in euros? Round to the nearest hundredth.

Review

Divide. See Sections 1.6 and 4.3.

77. $2916 \div 6$

78. $2920 \div 365$

79. $-\frac{24}{7} \div \frac{8}{21}$

80. $\frac{162}{25} \div -\frac{9}{75}$

Concept Extensions

Mixed Practice (Sections 5.2, 5.3) Perform the indicated operations.

81. $3.6 + 0.04$

82. $7.2 + 0.14 + 98.6$

83. $3.6 - 0.04$

84. $100 - 48.6$

85. -0.221×0.5

86. -3.6×0.04

-  **87.** Find how far radio waves travel in 20.6 seconds. (Radio waves travel at a speed of 186,000 miles per second.)
-  **89.** In your own words, explain how to find the number of decimal places in a product of decimal numbers.
-  **91.** Write down two decimal numbers whose product will contain 5 decimal places. Without multiplying, explain how you know your answer is correct.
-  **88.** If it takes radio waves approximately 8.3 minutes to travel from the sun to the earth, find approximately how far it is from the sun to the earth. (*Hint:* See Exercise **87.**)
-  **90.** In your own words, explain how to multiply by a power of 10.

Objectives

- A** Divide Decimals. 
- B** Estimate When Dividing Decimals. 
- C** Divide Decimals by Powers of 10. 
- D** Evaluate Expressions with Decimal Replacement Values. 
- E** Solve Problems by Dividing Decimals. 

Practice 1

Divide: $370.4 \div 8$. Check your answer.

Practice 2

Divide: $48 \overline{)34.08}$. Check your answer.

Answers

1. 46.3 2. 0.71

5.4 Dividing Decimals

Objective A Dividing Decimals

Dividing decimal numbers is similar to dividing whole numbers. The only difference is that we place a decimal point in the quotient. If the divisor is a whole number, we place the decimal point in the quotient directly above the decimal point in the dividend, and then divide as with whole numbers. Recall that division can be checked by multiplication.

Example 1 Divide: $270.2 \div 7$. Check your answer.

Solution: We divide as usual. The decimal point in the quotient is directly above the decimal point in the dividend.

$$\begin{array}{r}
 \text{divisor} \rightarrow 7 \overline{)270.2} \\
 \underline{-21} \\
 60 \\
 \underline{-56} \\
 42 \\
 \underline{-42} \\
 0
 \end{array}$$

← quotient
 ← dividend

Write the decimal point

Check:

$$\begin{array}{r}
 64 \\
 \times 38.6 \\
 \hline
 270.2
 \end{array}$$

The quotient is 38.6.

Work Practice 1

Example 2 Divide: $32 \overline{)8.32}$

Solution: We divide as usual. The decimal point in the quotient is directly above the decimal point in the dividend.

$$\begin{array}{r}
 \text{dividend} \rightarrow 32 \overline{)8.32} \\
 \underline{-64} \\
 192 \\
 \underline{-192} \\
 0
 \end{array}$$

← quotient
 ← divisor

Check:

$$\begin{array}{r}
 0.26 \text{ quotient} \\
 \times 32 \text{ divisor} \\
 \hline
 780 \\
 52 \\
 \hline
 8.32 \text{ dividend}
 \end{array}$$

Work Practice 2

Sometimes to continue dividing we need to insert zeros after the last digit in the dividend.

Example 3 Divide: $-5.98 \div 115$

Solution: Recall that a negative number divided by a positive number gives a negative quotient.

$$\begin{array}{r} 0.052 \\ 115 \overline{)5.980} \quad \leftarrow \text{Insert one 0.} \\ \underline{-575} \\ 230 \\ \underline{-230} \\ 0 \end{array}$$

Thus $-5.98 \div 115 = -0.052$.

Work Practice 3

If the divisor is not a whole number, before we divide we need to move the decimal point to the right until the divisor is a whole number.

$$\begin{array}{c} 1.5 \overline{)64.85} \\ \text{divisor} \quad \uparrow \quad \uparrow \quad \text{dividend} \end{array}$$

To understand how this works, let's rewrite

$$1.5 \overline{)64.85} \quad \text{as} \quad \frac{64.85}{1.5}$$

and then multiply by 1 in the form of $\frac{10}{10}$. We use the form $\frac{10}{10}$ so that the denominator (divisor) becomes a whole number.

$$\frac{64.85}{1.5} = \frac{64.85}{1.5} \cdot 1 = \frac{64.85}{1.5} \cdot \frac{10}{10} = \frac{64.85 \cdot 10}{1.5 \cdot 10} = \frac{648.5}{15},$$

which can be written as $15 \overline{)648.5}$. Notice that

$$1.5 \overline{)64.85} \text{ is equivalent to } 15 \overline{)648.5}$$

The decimal points in the dividend and the divisor were both moved one place to the right, and the divisor is now a whole number. This procedure is summarized next:

Dividing by a Decimal

- Step 1:** Move the decimal point in the divisor to the right until the divisor is a whole number.
- Step 2:** Move the decimal point in the dividend to the right the *same number of places* as the decimal point was moved in Step 1.
- Step 3:** Divide. Place the decimal point in the quotient directly over the moved decimal point in the dividend.

Practice 3

Divide and check.

- a. $-15.89 \div 14$
b. $-2.808 \div (-104)$

Answers

3. a. -1.135 b. 0.027

Practice 4Divide: $166.88 \div 5.6$ **Practice 5**Divide: $1.976 \div 0.16$ **Practice 6**Divide: $23.4 \div 0.57$. Round the quotient to the nearest hundredth.**Answers**

4. 29.8 5. 12.35 6. 41.05

✓ **Concept Check Answer**
no**Example 4** Divide: $10.764 \div 2.3$ **Solution:** We move the decimal points in the divisor and the dividend one place to the right so that the divisor is a whole number.

$$2.3 \overline{)10.764}$$

becomes

$$\begin{array}{r} 4.68 \\ 23 \overline{)107.64} \\ \underline{-92} \\ 156 \\ \underline{-138} \\ 184 \\ \underline{-184} \\ 0 \end{array}$$

Work Practice 4**Example 5** Divide: $5.264 \div 0.32$ **Solution:**

$$0.32 \overline{)5.264}$$

becomes

$$\begin{array}{r} 16.45 \\ 32 \overline{)526.40} \\ \underline{-32} \\ 206 \\ \underline{-192} \\ 144 \\ \underline{-128} \\ 160 \\ \underline{-160} \\ 0 \end{array} \quad \text{Insert one 0.}$$

Work Practice 5✓ **Concept Check** Is it always true that the number of decimal places in a quotient equals the sum of the decimal places in the dividend and divisor?**Example 6** Divide: $17.5 \div 0.48$. Round the quotient to the nearest hundredth.**Solution:** First we move the decimal points in the divisor and the dividend two places. Then we divide and round the quotient to the nearest hundredth.

$$\begin{array}{r} 36.458 \approx 36.46 \\ 48 \overline{)1750.000} \\ \underline{-144} \\ 310 \\ \underline{-288} \\ 220 \\ \underline{-192} \\ 280 \\ \underline{-240} \\ 400 \\ \underline{-384} \\ 16 \end{array}$$

hundredths place
↑
“is approximately”

When rounding to the nearest hundredth, carry the division process out to one more decimal place, the thousandths place.

Work Practice 6

✓ Concept Check If a quotient is to be rounded to the nearest thousandth, to what place should the division be carried out? (Assume that the division carries out to your answer.)

Objective B Estimating When Dividing Decimals

Just as for addition, subtraction, and multiplication of decimals, we can estimate when dividing decimals to check the reasonableness of our answer.

Example 7 Divide: $272.356 \div 28.4$. Then estimate to see whether the proposed result is reasonable.

Solution:

Exact:	Estimate 1	Estimate 2
$\begin{array}{r} 9.59 \\ 284 \overline{)2723.56} \\ \underline{-2556} \\ 1675 \\ \underline{-1420} \\ 2556 \\ \underline{-2556} \\ 0 \end{array}$	$\begin{array}{r} 9 \\ 30 \overline{)270} \end{array}$	$\begin{array}{r} 10 \\ 30 \overline{)300} \end{array}$

The estimate is 9 or 10, so 9.59 is reasonable.

Work Practice 7

Objective C Dividing Decimals by Powers of 10

As with multiplication, there are patterns that occur when we divide decimals by powers of 10 such as 10, 100, 1000, and so on.

$$\frac{569.2}{10} = 56.92 \quad \text{Move the decimal point 1 place to the left.}$$

↑ 1 zero

$$\frac{569.2}{10,000} = 0.05692 \quad \text{Move the decimal point 4 places to the left.}$$

↑ 4 zeros

This pattern suggests the following rule:

Dividing Decimals by Powers of 10 Such as 10, 100, or 1000

Move the decimal point of the dividend to the *left* the same number of places as there are *zeros* in the power of 10.

Examples Divide.

8. $\frac{786.1}{1000} = 0.7861$ Move the decimal point 3 places to the left.

↑ 3 zeros

9. $\frac{0.12}{10} = 0.012$ Move the decimal point 1 place to the left.

↑ 1 zero

Work Practice 8–9

Practice 7

Divide: $722.85 \div 91.5$. Then estimate to see whether the proposed answer is reasonable.

Practice 8–9

Divide.

8. $\frac{362.1}{1000}$ 9. $-\frac{0.49}{10}$

Answers

7. Exact: 7.9; an estimate: 8
8. 0.3621 9. -0.049

✓ Concept Check Answer
ten-thousandths place

Practice 10

Evaluate $x \div y$ for $x = 0.035$ and $y = 0.02$.

Practice 11

Is 39 a solution of the equation

$$\frac{x}{100} = 3.9?$$

Practice 12

A bag of fertilizer covers 1250 square feet of lawn. Tim Parker's lawn measures 14,800 square feet. How many bags of fertilizer does he need? If he can buy only whole bags of fertilizer, how many whole bags does he need?

Answers

10. 1.75 11. no 12. 11.84 bags; 12 bags

Objective D Using Decimals as Replacement Values

Example 10 Evaluate $x \div y$ for $x = 2.5$ and $y = 0.05$.

Solution: Replace x with 2.5 and y with 0.05.

$$x \div y = 2.5 \div 0.05 \quad \begin{array}{r} 0.05 \overline{)2.5} \\ \underline{0.00} \\ 2.50 \\ \underline{0.00} \\ 2.50 \\ \underline{0.00} \\ 2.50 \\ \underline{0.00} \\ 2.50 \end{array} \text{ becomes } \begin{array}{r} 50 \\ 5 \overline{)250} \\ \underline{0} \\ 250 \\ \underline{0} \\ 250 \\ \underline{0} \\ 250 \end{array}$$

$$= 50$$

Work Practice 10

Example 11 Is 720 a solution of the equation $\frac{y}{100} = 7.2$?

Solution: Replace y with 720 to see if a true statement results.

$$\frac{y}{100} = 7.2 \quad \text{Original equation}$$

$$\frac{720}{100} \stackrel{?}{=} 7.2 \quad \text{Replace } y \text{ with } 720.$$

$$7.2 = 7.2 \quad \text{True}$$

Since $7.2 = 7.2$ is a true statement, 720 is a solution of the equation.

Work Practice 11**Objective E** Solving Problems by Dividing Decimals

Many real-life problems involve dividing decimals.

Example 12 Calculating Materials Needed for a Job

A gallon of paint covers a 250-square-foot area. If Betty Adkins wishes to paint a wall that measures 1450 square feet, how many gallons of paint does she need? If she can buy only gallon containers of paint, how many gallon containers does she need?

Solution:

1. UNDERSTAND. Read and reread the problem. We need to know how many 250s are in 1450, so we divide.

2. TRANSLATE.

In words:	number of gallons	is	square feet	divided by	square feet per gallon
	↓	↓	↓	↓	↓
Translate:	number of gallons	=	1450	÷	250

3. SOLVE. Let's see if our answer is reasonable by estimating. The dividend 1450 rounds to 1500 and the divisor 250 rounds to 300. Then $1500 \div 300 = 5$.

$$\begin{array}{r} 5.8 \\ 250 \overline{)1450.0} \\ \underline{-1250} \\ 200 \\ \underline{-200 } \\ 0 \end{array}$$

4. **INTERPRET.** Check your work. Since our estimate is close to our answer of 5, our answer is reasonable. State your conclusion: Betty needs 5.8 gallons of paint. If she can buy only gallon containers of paint, she needs 6 gallon containers of paint to complete the job.

Work Practice 12



Calculator Explorations Estimation

Calculator errors can easily be made by pressing an incorrect key, holding a key too long, or by not pressing a correct key hard enough. Estimation is a valuable tool that can be used to check calculator results.

Example

Use estimation to determine whether the calculator result is reasonable or not. (For example, a result that is not reasonable can occur if proper keys are not pressed.)

Divide: $82.064 \div 23$

Calculator display: 35.68

Solution: Round each number to the nearest 10. Since $80 \div 20 = 4$, the calculator display 35.68 is not reasonable.

Use estimation to determine whether each result is reasonable or not.

1. 102.62×41.8 ; Result: 428.9516
2. $174.835 \div 47.9$; Result: 3.65
3. $1025.68 - 125.42$; Result: 900.26
4. $562.781 + 2.96$; Result: 858.781

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once, and some not used at all.

dividend	divisor	quotient	true
zeros	left	right	false

1. In $6.5 \div 5 = 1.3$, the number 1.3 is called the _____, 5 is the _____, and 6.5 is the _____.
2. To check a division exercise, we can perform the following multiplication: quotient $\cdot$ _____ = _____.
3. To divide a decimal number by a power of 10 such as 10, 100, 1000, and so on, we move the decimal point in the number to the _____ the same number of places as there are _____ in the power of 10.
4. True or false: If we replace x with -12.6 and y with 0.3 in the expression $y \div x$, we have $0.3 \div (-12.6)$.

Martin-Gay Interactive Videos



See Video 5.4

Watch the section lecture video and answer the following questions.

- Objective A** 5. From the lecture before Example 1, what must we make sure the divisor is before dividing decimals?
- Objective B** 6. From Example 4, what does estimating especially help us with?
- Objective C** 7. Why don't we do any actual dividing in Example 5?
- Objective D** 8. In Example 6, 4 does not divide into 1. How does this affect the quotient?
- Objective E** 9. In Example 7, why is the division carried to the hundredths place?

5.4 Exercise Set MyLab Math

Objectives A B Mixed Practice Divide. See Examples 1 through 5 and 7. For those exercises marked, also estimate to see if the answer is reasonable.

1. $6\overline{)27.6}$

2. $4\overline{)23.6}$

3. $5\overline{)0.47}$

4. $6\overline{)0.51}$

5. $0.06\overline{)18}$

6. $0.04\overline{)20}$

7. $0.42\overline{)3.066}$

8. $0.36\overline{)1.764}$

 9. $5.5\overline{)36.3}$
Exact:
Estimate:

10. $2.2\overline{)21.78}$
Exact:
Estimate:

11. $7.434 \div 18$

12. $8.304 \div 16$

13. $36 \div (-0.06)$

14. $36 \div (-0.04)$

15. Divide -4.2 by -0.6 .

16. Divide -3.6 by -0.9 .

17. $0.27\overline{)1.296}$

18. $0.34\overline{)2.176}$

19. $0.02\overline{)42}$

20. $0.03\overline{)24}$

 21. $4.756 \div 0.82$

22. $3.312 \div 0.92$

23. $-36.3 \div (-6.6)$

24. $-21.78 \div (-9.9)$

25. $7.2\overline{)70.56}$
Exact:
Estimate:

26. $6.3\overline{)54.18}$
Exact:
Estimate:

27. $5.4\overline{)51.84}$

28. $7.7\overline{)33.88}$

29. $\frac{1.215}{0.027}$

30. $\frac{3.213}{0.051}$

31. $0.25\overline{)13.648}$

32. $0.75\overline{)49.866}$

33. $3.78\overline{)0.02079}$

34. $2.96\overline{)0.01332}$

Divide. Round the quotients as indicated. See Example 6.

35. Divide: $0.549 \div 0.023$. Round the quotient to the nearest hundredth.

36. Divide: $0.0453 \div 0.98$. Round the quotient to the nearest thousandth.

 37. Divide: $68.39 \div 0.6$. Round the quotient to the nearest tenth.

38. Divide: $98.83 \div 3.5$. Round the quotient to the nearest tenth.

Objective C Divide. See Examples 8 and 9.

39. $\frac{83.397}{100}$

40. $\frac{64.423}{100}$

41. $\frac{26.87}{10}$

42. $\frac{13.49}{10}$

 43. $12.9 \div (-1000)$

44. $13.49 \div (-10,000)$

Objectives A C Mixed Practice Divide. See Examples 1 through 5, 8, and 9.

45. $7\overline{)88.2}$

46. $9\overline{)130.5}$

47. $\frac{13.1}{10}$

48. $\frac{17.7}{10}$

49. $\frac{456.25}{10,000}$

50. $\frac{986.11}{10,000}$

51. $1.239 \div 3$

52. $0.54 \div 12$

53. Divide 4.8 by -0.6 . 54. Divide 4.9 by -0.7 . 55. $-1.224 \div 0.17$ 56. $-1.344 \div 0.42$
57. Divide 42 by 0.03. 58. Divide 27 by 0.03. 59. Divide -18 by -0.6 . 60. Divide 20 by 0.4.
61. Divide 87 by -0.0015 . 62. Divide 35 by -0.0007 . 63. $-1.104 \div 1.6$ 64. $-2.156 \div 0.98$
65. $-2.4 \div (-100)$ 66. $-86.79 \div (-1000)$ 67. $\frac{4.615}{0.071}$ 68. $\frac{23.8}{0.035}$

Objective D Evaluate each expression for $x = 5.65$, $y = -0.8$, and $z = 4.52$. See Example 10.

69. $z \div y$ 70. $z \div x$ 71. $x \div y$ 72. $y \div 2$

Determine whether the given values are solutions of the given equations. See Example 11.

73. $\frac{x}{4} = 3.04$; $x = 12.16$ 74. $\frac{y}{8} = 0.89$; $y = 7.12$ 75. $\frac{z}{100} = 0.8$; $z = 8$ 76. $\frac{x}{10} = 0.23$; $x = 23$

Objective E Solve. For Exercises 77 and 78, the solutions have been started for you. See Example 12.

77. A new homeowner is painting the walls of a room. The walls have a total area of 546 square feet. A quart of paint covers 52 square feet. If the paint is sold in whole quarts only, how many quarts are needed?

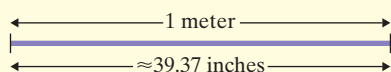
Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)

number of quarts	is	square feet	divided by	square feet per quart
↓	↓	↓	↓	↓
number of quarts	=	_____	÷	_____

3. SOLVE. Don't forget to round up your quotient.
4. INTERPRET.

79. There are approximately 39.37 inches in 1 meter. How many meters, to the nearest tenth of a meter, are there in 200 inches?



78. A shipping box can hold 36 books. If 486 books must be shipped, how many boxes are needed?

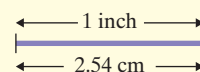
Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)

number of boxes	is	number of books	divided by	books per box
↓	↓	↓	↓	↓
number of boxes	=	_____	÷	_____

3. SOLVE. Don't forget to round up your quotient.
4. INTERPRET.

80. There are 2.54 centimeters in 1 inch. How many inches are there in 50 centimeters? Round to the nearest tenth.



81. In the United States, an average child will wear down 730 crayons by his or her tenth birthday. Find the number of boxes of 64 crayons this is equivalent to. Round to the nearest tenth. (Source: Binney & Smith Inc.)
82. In 2016, American farmers received an average of \$84.20 per hundred pounds of chicken. What was the average price per pound for chickens? Round to the nearest cent. (Source: National Agricultural Statistics Service)

A child is to receive a dose of 0.5 teaspoon of cough medicine every 4 to 6 hours. If the bottle contains 4 fluid ounces, answer Exercises 83 through 86.



87. Americans ages 16–19 drive, on average, 7624 miles per year. About how many miles each week is that? Round to the nearest tenth. (*Note:* There are 52 weeks in a year.) (*Source:* Federal Highway Administration)
88. The book *Harry Potter and the Deathly Hallows* was released to the public on July 21, 2007. Booksellers in the United States sold approximately 8292 thousand copies in the first 24 hours after release. If the same number of books was sold each hour, calculate the number of books sold each hour in the United States for that first day.
83. A fluid ounce equals 6 teaspoons. How many teaspoons are in 4 fluid ounces?
84. The bottle of medicine contains how many doses for the child? (*Hint:* See Exercise 83.)
85. If the child takes a dose every four hours, how many days will the medicine last?
86. If the child takes a dose every six hours, how many days will the medicine last?
88. Drake Saucier was interested in the gas mileage on his “new” used car. He filled the tank, drove 423.8 miles, and filled the tank again. When he refilled the tank, it took 19.35 gallons of gas. Calculate the miles per gallon for Drake’s car. Round to the nearest tenth.
90. The greatest money winner in men’s professional golf from January 2016 through November 2016 was Hideki Matsuyama. He earned approximately \$2,524,750. Suppose he had earned this working 40 hours each week for a year. Determine his hourly wage to the nearest cent. (*Note:* There are 52 weeks in a year) (*Source:* Professional Golfers’ Association)

Review

Perform the indicated operation. See Sections 4.3 and 4.5.

91. $\frac{3}{5} \cdot \frac{7}{10}$

92. $\frac{3}{5} \div \frac{7}{10}$

93. $\frac{3}{5} - \frac{7}{10}$

94. $-\frac{3}{4} - \frac{1}{14}$

Concept Extensions

Mixed Practice (Sections 5.2, 5.3, 5.4) Perform the indicated operation.

95. $1.278 \div 0.3$

96. 1.278×0.3

97. $1.278 + 0.3$

98. $1.278 - 0.3$

99. $(-8.6)(3.1)$

100. $7.2 + 0.05 + 49.1$

101.
$$\begin{array}{r} 1000 \\ - 95.71 \\ \hline \end{array}$$

102.
$$\begin{array}{r} 87.2 \\ -10,000 \\ \hline \end{array}$$

Choose the best estimate.

103. 8.62×41.7

- a. 36
- b. 32
- c. 360
- d. 3.6

104. $1.437 + 20.69$

- a. 34
- b. 22
- c. 3.4
- d. 2.2

105. $78.6 \div 97$

- a. 7.86
- b. 0.786
- c. 786
- d. 7860

106. $302.729 - 28.697$

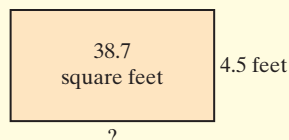
- a. 270
- b. 20
- c. 27
- d. 300

Recall from Section 1.6 that the average of a list of numbers is their total divided by how many numbers there are in the list. Use this procedure to find the average of the test scores listed in Exercises 107 and 108. If necessary, round to the nearest tenth.

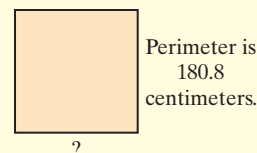
107. 86, 78, 91, 87

108. 56, 75, 80

△ **109.** The area of a rectangle is 38.7 square feet. If its width is 4.5 feet, find its length.



△ **110.** The perimeter of a square is 180.8 centimeters. Find the length of a side.



- ✎ **111.** When dividing decimals, describe the process you use to place the decimal point in the quotient.
- ✎ **112.** In your own words, describe how to quickly divide a number by a power of 10 such as 10, 100, 1000, etc.

To convert wind speeds in miles per hour to knots, divide by 1.15. Use this information and the Saffir-Simpson Hurricane Intensity chart below to answer Exercises 113 and 114. Round to the nearest tenth.

Saffir-Simpson Hurricane Intensity Scale				
Category	Wind Speed	Barometric Pressure [inches of mercury (Hg)]	Storm Surge	Damage Potential
1 (Weak)	75–95 mph	≥ 28.94 in.	4–5 ft	Minimal damage to vegetation
2 (Moderate)	96–110 mph	28.50–28.93 in.	6–8 ft	Moderate damage to houses
3 (Strong)	111–130 mph	27.91–28.49 in.	9–12 ft	Extensive damage to small buildings
4 (Very Strong)	131–155 mph	27.17–27.90 in.	13–18 ft	Extreme structural damage
5 (Devastating)	>155 mph	<27.17 in.	>18 ft	Catastrophic building failures possible

- 113.** The chart gives wind speeds in miles per hour. What is the range of wind speeds for a Category 1 hurricane in knots?
- △ **115.** A rancher is building a horse corral that's shaped like a rectangle with a width of 24.3 meters. He plans to make a four-wire fence; that is, he will string four wires around the corral. If he plans to use all of his 412.8 meters of wire, find the length of the corral he can construct.
- 114.** What is the range of wind speeds for a Category 4 hurricane in knots?
- 116.** A college student signed up for a new credit card that guarantees her no interest charges on transferred balances for a year. She transferred over a \$2523.86 balance from her old credit card. Her minimum payment is \$185.35 per month. If she pays only the minimum, will she pay off her balance before interest charges start again?

Operations on Decimals

Answers

Perform the indicated operation.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. _____
24. _____

1. $1.6 + 0.97$
2. $3.2 + 0.85$
3. $9.8 - 0.9$
4. $10.2 - 6.7$
5.
$$\begin{array}{r} 0.8 \\ \times 0.2 \\ \hline \end{array}$$
6.
$$\begin{array}{r} 0.6 \\ \times 0.4 \\ \hline \end{array}$$
7.
$$\begin{array}{r} 8 \overline{)2.16} \\ \hline \end{array}$$
8.
$$\begin{array}{r} 6 \overline{)3.12} \\ \hline \end{array}$$
9. $(9.6)(-0.5)$
10. $(-8.7)(-0.7)$
11.
$$\begin{array}{r} 123.6 \\ - 48.04 \\ \hline \end{array}$$
12.
$$\begin{array}{r} 325.2 \\ - 36.08 \\ \hline \end{array}$$
13. $-25 + 0.026$
14. $0.125 + (-44)$
15. $29.24 \div (-3.4)$
16. $-10.26 \div (-1.9)$
17. -2.8×100
18. 1.6×1000
19.
$$\begin{array}{r} 96.21 \\ 7.028 \\ + 121.7 \\ \hline \end{array}$$
20.
$$\begin{array}{r} 0.268 \\ 1.93 \\ + 142.881 \\ \hline \end{array}$$
21. $-25.76 \div (-46)$
22. $-27.09 \div 43$
23.
$$\begin{array}{r} 12.004 \\ \times 2.3 \\ \hline \end{array}$$
24.
$$\begin{array}{r} 28.006 \\ \times 5.2 \\ \hline \end{array}$$

25. Subtract 4.6 from 10. 26. Subtract 18 from 0.26. 27. $-268.19 - 146.25$

25. _____

26. _____

27. _____

28. $-860.18 - 434.85$ 29. $\frac{2.958}{-0.087}$ 30. $\frac{-1.708}{0.061}$

28. _____

29. _____

31. $160 - 43.19$ 32. $120 - 101.21$ 33. 15.62×10

30. _____

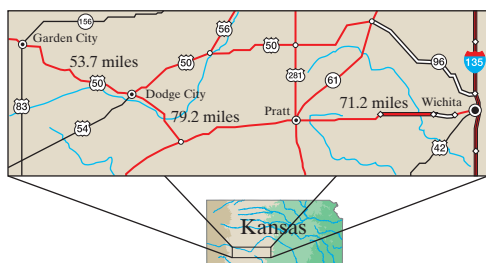
31. _____

34. $15.62 \div 10$ 35. $15.62 + 10$ 36. $15.62 - 10$

32. _____

33. _____

37. Find the distance in miles between Garden City, Kansas, and Wichita, Kansas. Next, estimate the distance by rounding each given distance to the nearest ten.



38. It costs \$7.96 to send a 5-pound package locally via parcel post at a U.S. Post Office. To send the same package Priority Mail costs \$9.85. How much more does it cost to send a package as Priority Mail? (Source: United States Postal Service)

34. _____

35. _____

36. _____

37. _____

38. _____

39. In 2011, sales of Blu-ray Discs were \$2 billion, but DVD sales dropped to \$6.8 billion. Find the total spent to buy Blu-ray Discs or DVDs in 2011. Write the total in billions of dollars and also in standard notation. (Source: USA Today)

39. _____

5.5 Fractions, Decimals, and Order of Operations

Objectives

- A** Write Fractions as Decimals. ▶
- B** Compare Decimals and Fractions. ▶
- C** Simplify Expressions Containing Decimals and Fractions Using Order of Operations. ▶
- D** Solve Area Problems Containing Fractions and Decimals. ▶
- E** Evaluate Expressions Given Decimal Replacement Values. ▶

Practice 1

- a. Write $\frac{2}{5}$ as a decimal.
- b. Write $\frac{9}{40}$ as a decimal.

Practice 2

Write $-\frac{3}{8}$ as a decimal.

Practice 3

- a. Write $\frac{5}{6}$ as a decimal.
- b. Write $\frac{2}{9}$ as a decimal.

Answers

1. a. 0.4 b. 0.225 2. -0.375
3. a. $0.8\bar{3}$ b. $0.\bar{2}$

Objective A Writing Fractions as Decimals ▶

To write a fraction as a decimal, we interpret the fraction bar to mean division and find the quotient.

Writing Fractions as Decimals

To write a fraction as a decimal, divide the numerator by the denominator.

Example 1 Write $\frac{1}{4}$ as a decimal.

Solution: $\frac{1}{4} = 1 \div 4$

$$\begin{array}{r} 0.25 \\ 4 \overline{)1.00} \\ \underline{-8} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

Thus, $\frac{1}{4}$ written as a decimal is 0.25.

Work Practice 1

Example 2 Write $-\frac{5}{8}$ as a decimal.

Solution: $-\frac{5}{8} = -(5 \div 8) = -0.625$

$$\begin{array}{r} 0.625 \\ 8 \overline{)5.000} \\ \underline{-48} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Work Practice 2

Example 3 Write $\frac{2}{3}$ as a decimal.

Solution: $0.666 \dots$

$$\begin{array}{r} 3 \overline{)2.000} \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 2 \end{array}$$

Remainder is 2, then 0 is brought down.

Remainder is 2, then 0 is brought down.

Remainder is 2.

Notice that the digit 2 keeps occurring as the remainder. This will continue so that the digit 6 will keep repeating in the quotient. We place a bar over the digit 6 to indicate that it repeats.

$$\frac{2}{3} = 0.666\ldots = 0.\overline{6}$$

We can also write a decimal approximation for $\frac{2}{3}$. For example, $\frac{2}{3}$ rounded to the nearest hundredth is 0.67. This can be written as $\frac{2}{3} \approx 0.67$.

Work Practice 3

Example 4 Write $\frac{22}{7}$ as a decimal. (Recall that the fraction $\frac{22}{7}$ is an approximation for π .) Round to the nearest hundredth.

Solution: $\frac{3.142}{7} \approx 3.14$ Carry the division out to the thousandths place.

$$\begin{array}{r} 3.142 \\ 7 \overline{)22.000} \\ \underline{-21} \\ 10 \\ \underline{-7} \\ 30 \\ \underline{-28} \\ 20 \\ \underline{-14} \\ 6 \end{array}$$

The fraction $\frac{22}{7}$ in decimal form is approximately 3.14.

Work Practice 4

Example 5 Write $2\frac{3}{16}$ as a decimal.

Solution:

Option 1. Write the fractional part only as a decimal.

$$\begin{array}{r} 0.1875 \\ \frac{3}{16} \rightarrow 16 \overline{)3.0000} \\ \underline{-16} \\ 140 \\ \underline{-128} \\ 120 \\ \underline{-112} \\ 80 \\ \underline{-80} \\ 0 \end{array}$$

Thus $2\frac{3}{16} = 2.1875$

Option 2. Write $2\frac{3}{16}$ as an improper fraction, and divide.

$$\begin{array}{r} 2.1875 \\ 2\frac{3}{16} = \frac{35}{16} \rightarrow 16 \overline{)35.0000} \\ \underline{-32} \\ 30 \\ \underline{-16} \\ 140 \\ \underline{-128} \\ 120 \\ \underline{-112} \\ 80 \\ \underline{-80} \\ 0 \end{array}$$

Work Practice 5

Practice 4

Write $\frac{28}{13}$ as a decimal. Round to the nearest thousandth.

Practice 5

Write $3\frac{5}{16}$ as a decimal.

Answers

4. 2.154 5. 3.3125

Some fractions may be written as decimals using our knowledge of decimals. From Section 5.1, we know that if the denominator of a fraction is 10, 100, 1000, or so on, we can immediately write the fraction as a decimal. For example,

$$\frac{4}{10} = 0.4, \quad \frac{12}{100} = 0.12, \text{ and so on}$$

Practice 6

Write $\frac{3}{5}$ as a decimal.

Practice 7

Write $\frac{3}{50}$ as a decimal.

Practice 8

Insert $<$, $>$, or $=$ to form a true statement.

$$\frac{1}{5} \quad 0.25$$

Answers

6. 0.6 7. 0.06 8. $<$

✓ Concept Check Answer

$\frac{9}{16}$ is less than 1 while 1.735 is greater than 1.

Example 6 Write $\frac{4}{5}$ as a decimal.

Solution: Let's write $\frac{4}{5}$ as an equivalent fraction with a denominator of 10.

$$\frac{4}{5} = \frac{4}{5} \cdot \frac{2}{2} = \frac{8}{10} = 0.8$$

Work Practice 6

Example 7 Write $\frac{1}{25}$ as a decimal.

Solution: $\frac{1}{25} = \frac{1}{25} \cdot \frac{4}{4} = \frac{4}{100} = 0.04$

Work Practice 7

✓ **Concept Check** Suppose you are writing the fraction $\frac{9}{16}$ as a decimal. How do you know you have made a mistake if your answer is 1.735?

Objective B Comparing Decimals and Fractions

Now we can compare decimals and fractions by writing fractions as equivalent decimals.

Example 8 Insert $<$, $>$, or $=$ to form a true statement.

$$\frac{1}{8} \quad 0.12$$

Solution: First we write $\frac{1}{8}$ as an equivalent decimal. Then we compare decimal places.

$$\begin{array}{r} 0.125 \\ 8 \overline{)1.000} \\ \underline{-8} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Original numbers	$\frac{1}{8}$	0.12
Decimals	0.125	0.120
Compare	0.125 $>$ 0.12	

Thus, $\frac{1}{8} > 0.12$

Work Practice 8

Example 9 Insert $<$, $>$, or $=$ to form a true statement.

$$0.\overline{7} \quad \frac{7}{9}$$

Solution: We write $\frac{7}{9}$ as a decimal and then compare.

$$\begin{array}{r} 0.77 \dots = 0.\overline{7} \\ 9 \overline{) 7.00} \\ \underline{-6.3} \\ 70 \\ \underline{-63} \\ 7 \end{array}$$

Original numbers	$0.\overline{7}$	$\frac{7}{9}$
Decimals	$0.\overline{7}$	$0.\overline{7}$
Compare	$0.\overline{7} = 0.\overline{7}$	

Thus, $0.\overline{7} = \frac{7}{9}$

Work Practice 9

Example 10 Write the numbers in order from smallest to largest.

$$\frac{9}{20}, \frac{4}{9}, 0.456$$

Solution:

Original numbers	$\frac{9}{20}$	$\frac{4}{9}$	0.456
Decimals	0.450	0.444...	0.456
Compare in order	2nd	1st	3rd

Written in order, we have

$$\begin{array}{ccc} 1\text{st} & 2\text{nd} & 3\text{rd} \\ \downarrow & \downarrow & \downarrow \\ \frac{4}{9}, & \frac{9}{20}, & 0.456 \end{array}$$

Work Practice 10

Objective C Simplifying Expressions with Decimals and Fractions

In the remaining examples, we will review the order of operations by simplifying expressions that contain decimals.

Order of Operations

1. Perform all operations within parentheses $()$, brackets $[\]$, or other grouping symbols such as fraction bars.
2. Evaluate any expressions with exponents.
3. Multiply or divide in order from left to right.
4. Add or subtract in order from left to right.

Practice 9

Insert $<$, $>$, or $=$ to form a true statement.

a. $\frac{1}{2}$ 0.54 b. $0.\overline{5}$ $\frac{5}{9}$

c. $\frac{5}{7}$ 0.72

Practice 10

Write the numbers in order from smallest to largest.

a. $\frac{1}{3}$, 0.302, $\frac{3}{8}$

b. 1.26, $1\frac{1}{4}$, $1\frac{2}{5}$

c. 0.4, 0.41, $\frac{3}{7}$

Answers

9. a. $<$ b. $=$ c. $<$

10. a. $0.302, \frac{1}{3}, \frac{3}{8}$ b. $1\frac{1}{4}, 1.26, 1\frac{2}{5}$

c. 0.4, 0.41, $\frac{3}{7}$

Practice 11Simplify: $897.8 \div 100 \times 10$ **Practice 12**Simplify: $-8.69(3.2 - 1.8)$ **Practice 13**Simplify: $(-0.7)^2 + 2.1$ **Practice 14**Simplify: $\frac{20.06 - (1.2)^2 \div 10}{0.02}$ **Answers**

11. 89.78 12. -12.166
 13. 2.59 14. 995.8

Example 11 Simplify: $723.6 \div 1000 \times 10$ **Solution:** Multiply or divide in order from left to right.

$$\begin{aligned} 723.6 \div 1000 \times 10 &= 0.7236 \times 10 && \text{Divide.} \\ &= 7.236 && \text{Multiply.} \end{aligned}$$

Work Practice 11**Example 12** Simplify: $-0.5(8.6 - 1.2)$ **Solution:** According to the order of operations, we simplify inside the parentheses first.

$$\begin{aligned} -0.5(8.6 - 1.2) &= -0.5(7.4) && \text{Subtract.} \\ &= -3.7 && \text{Multiply.} \end{aligned}$$

Work Practice 12**Example 13** Simplify: $(-1.3)^2 + 2.4$ **Solution:** Recall the meaning of an exponent.

$$\begin{aligned} (-1.3)^2 + 2.4 &= (-1.3)(-1.3) + 2.4 && \text{Use the definition of an exponent.} \\ &= 1.69 + 2.4 && \text{Multiply. The product of two negative} \\ &&& \text{numbers is a positive number.} \\ &= 4.09 && \text{Add.} \end{aligned}$$

Work Practice 13**Example 14** Simplify: $\frac{5.68 + (0.9)^2 \div 100}{0.2}$ **Solution:** First we simplify the numerator of the fraction. Then we divide.

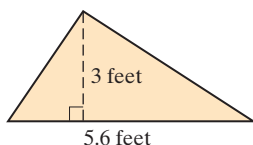
$$\begin{aligned} \frac{5.68 + (0.9)^2 \div 100}{0.2} &= \frac{5.68 + 0.81 \div 100}{0.2} && \text{Simplify } (0.9)^2. \\ &= \frac{5.68 + 0.0081}{0.2} && \text{Divide.} \\ &= \frac{5.6881}{0.2} && \text{Add.} \\ &= 28.4405 && \text{Divide.} \end{aligned}$$

Work Practice 14**Objective D** Solving Area Problems Containing Fractions and Decimals

Sometimes real-life problems contain both fractions and decimals. In the next example, we review the area of a triangle, and when values are substituted, the result may be an expression containing both fractions and decimals.

**Example 15**

The area of a triangle is $\text{Area} = \frac{1}{2} \cdot \text{base} \cdot \text{height}$. Find the area of the triangle shown.

**Solution:**

$$\begin{aligned}
 \text{Area} &= \frac{1}{2} \cdot \text{base} \cdot \text{height} \\
 &= \frac{1}{2} \cdot 5.6 \cdot 3 \\
 &= 0.5 \cdot 5.6 \cdot 3 && \text{Write } \frac{1}{2} \text{ as the decimal } 0.5. \\
 &= 8.4
 \end{aligned}$$

The area of the triangle is 8.4 square feet.

Work Practice 15

Objective E Using Decimals as Replacement Values

Example 16

Evaluate $-2x + 5$ for $x = 3.8$.

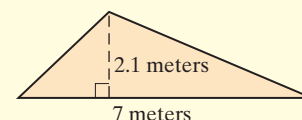
Solution: Replace x with 3.8 in the expression $-2x + 5$ and simplify.

$$\begin{aligned}
 -2x + 5 &= -2(3.8) + 5 && \text{Replace } x \text{ with } 3.8. \\
 &= -7.6 + 5 && \text{Multiply.} \\
 &= -2.6 && \text{Add.}
 \end{aligned}$$

Work Practice 16

Practice 15

Find the area of the triangle.

**Practice 16**

Evaluate $1.7y - 2$ for $y = 2.3$.

Answers

15. 7.35 sq m 16. 1.91

Vocabulary, Readiness & Video Check

Answer each exercise “true” or “false.”

- The number $0.\bar{5}$ means 0.555. _____
- To write $\frac{9}{19}$ as a decimal, perform the division $19\overline{)9}$. _____
- $(-1.2)^2$ means $(-1.2)(-1.2)$ or -1.44 . _____
- To simplify $8.6(4.8 - 9.6)$, we first subtract. _____

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 5.5

- Objective A** 5. In Example 2, why is the bar placed over just the 6?
- Objective B** 6. In Example 3, why do we write the fraction as a decimal rather than the decimal as a fraction?
- Objective C** 7. In Example 4, besides meaning division, what other purpose does the fraction bar serve?
- Objective D** 8. What formula is used to solve Example 5? What is the final answer?
- Objective E** 9. In Example 6, once all replacement values are put into the variable expression, what is the resulting expression to evaluate?

5.5 Exercise Set MyLab Math

Objective A Write each number as a decimal. See Examples 1 through 7.

- | | | | | |
|-----------------------|--------------------|----------------------|---------------------|-----------------------|
| 1. $\frac{1}{5}$ | 2. $\frac{1}{20}$ | 3. $\frac{17}{25}$ | 4. $\frac{13}{25}$ | ▶ 5. $\frac{3}{4}$ |
| 6. $\frac{1}{8}$ | 7. $-\frac{2}{25}$ | 8. $-\frac{3}{25}$ | 9. $\frac{9}{4}$ | 10. $\frac{8}{5}$ |
| ▶ 11. $\frac{11}{12}$ | 12. $\frac{5}{12}$ | 13. $\frac{17}{40}$ | 14. $\frac{19}{25}$ | 15. $\frac{18}{40}$ |
| 16. $\frac{31}{40}$ | 17. $-\frac{1}{3}$ | 18. $-\frac{7}{9}$ | 19. $\frac{7}{16}$ | 20. $\frac{9}{16}$ |
| 21. $\frac{7}{11}$ | 22. $\frac{9}{11}$ | 23. $5\frac{17}{20}$ | 24. $4\frac{7}{8}$ | 25. $\frac{78}{125}$ |
| | | | | 26. $\frac{159}{375}$ |

Round each number as indicated. See Example 4.

- | | |
|--|--|
| 27. Round your decimal answer to Exercise 17 to the nearest hundredth. | 28. Round your decimal answer to Exercise 18 to the nearest hundredth. |
| 29. Round your decimal answer to Exercise 19 to the nearest hundredth. | 30. Round your decimal answer to Exercise 20 to the nearest hundredth. |
| 31. Round your decimal answer to Exercise 21 to the nearest tenth. | 32. Round your decimal answer to Exercise 22 to the nearest tenth. |

Write each fraction as a decimal. If necessary, round to the nearest hundredth. See Examples 1 through 7.

- | | |
|---|---|
| 33. Of the U.S. mountains that are over 14,000 feet in elevation, $\frac{56}{91}$ are located in Colorado. (Source: U.S. Geological Survey) | 34. About $\frac{9}{20}$ of all U.S. citizens have type O blood. (Source: American Red Cross Biomedical Services) |
|---|---|

- 35.** About $\frac{43}{50}$ of Americans are Internet users. (*Source:* Digitalcenter)
- 37.** When first launched, the Hubble Space Telescope's primary mirror was out of shape on the edges by $\frac{1}{50}$ of a human hair. This very small defect made it difficult to focus faint objects being viewed. Because the HST was in low Earth orbit, it was serviced by a shuttle and the defect was corrected.
- 36.** About $\frac{14}{25}$ of Americans use the Internet through a wireless device. (*Source:* Digitalcenter)
- 38.** The two mirrors currently in use in the Hubble Space Telescope were ground so that they do not deviate from a perfect curve by more than $\frac{1}{800,000}$ of an inch. Do not round this number.

Objective B Insert $<$, $>$, or $=$ to form a true statement. See Examples 8 and 9.

- 39.** 0.562 0.569 **40.** 0.983 0.988 **41.** 0.215 $\frac{43}{200}$ **42.** $\frac{29}{40}$ 0.725
- 43.** -0.0932 -0.0923 **44.** -0.00563 -0.00536 **45.** $0.\overline{6}$ $\frac{5}{6}$ **46.** $0.\overline{1}$ $\frac{2}{17}$
- 47.** $\frac{51}{91}$ $0.56\overline{4}$ **48.** $0.58\overline{3}$ $\frac{6}{11}$ **49.** $\frac{4}{7}$ 0.14 **50.** $\frac{5}{9}$ 0.557
- 51.** 1.38 $\frac{18}{13}$ **52.** 0.372 $\frac{22}{59}$ **53.** 7.123 $\frac{456}{64}$ **54.** 12.713 $\frac{89}{7}$

Write the numbers in order from smallest to largest. See Example 10.

- 55.** 0.34, 0.35, 0.32 **56.** 0.47, 0.42, 0.40 **57.** 0.49, 0.491, 0.498 **58.** 0.72, 0.727, 0.728
- 59.** 5.23, $\frac{42}{8}$, 5.34 **60.** 7.56, $\frac{67}{9}$, 7.562 **61.** $\frac{5}{8}$, 0.612, 0.649 **62.** $\frac{5}{6}$, 0.821, 0.849

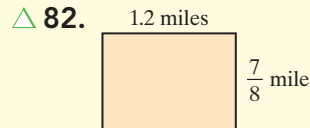
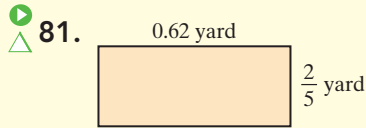
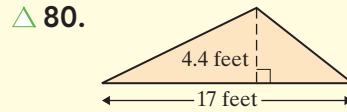
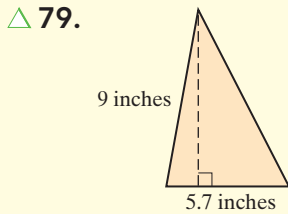
Objective C Simplify each expression. See Examples 11 through 14.

- 63.** $(0.3)^2 + 0.5$ **64.** $(-2.5)(3) - 4.7$ **65.** $\frac{1 + 0.8}{-0.6}$ **66.** $(-0.05)^2 + 3.13$
- 67.** $(-2.3)^2 (0.3 + 0.7)$ **68.** $(8.2)(100) - (8.2)(10)$ **69.** $(5.6 - 2.3)(2.4 + 0.4)$ **70.** $\frac{0.222 - 2.13}{12}$
- 71.** $\frac{(4.5)^2}{100}$ **72.** $0.9(5.6 - 6.5)$ **73.** $\frac{7 + 0.74}{-6}$ **74.** $(1.5)^2 + 0.5$

Find the value of each expression. Give the result as a decimal. See Examples 11 through 14.

- 75.** $\frac{1}{5} - 2(7.8)$ **76.** $\frac{3}{4} - (9.6)(5)$ **77.** $\frac{1}{4} (-9.6 - 5.2)$ **78.** $\frac{3}{8} (4.7 - 5.9)$

Objective D Find the area of each triangle or rectangle. See Example 15.



Objective E Evaluate each expression for $x = 6$, $y = 0.3$, and $z = -2.4$. See Example 16.

83. z^2

84. y^2

85. $x - y$

86. $x - z$

87. $4y - z$

88. $\frac{x}{y} + 2z$

Review

Simplify. See Sections 4.3 and 4.5.

89. $\frac{9}{10} + \frac{16}{25}$

90. $\frac{4}{11} - \frac{19}{22}$

91. $\left(\frac{2}{5}\right)\left(\frac{5}{2}\right)^2$

92. $\left(\frac{2}{3}\right)^2\left(\frac{3}{2}\right)^3$

Concept Extensions

Without calculating, describe each number as < 1 , $= 1$, or > 1 . See the Concept Check in this section.

93. 1.0

94. 1.0000

95. 1.00001

96. $\frac{101}{99}$

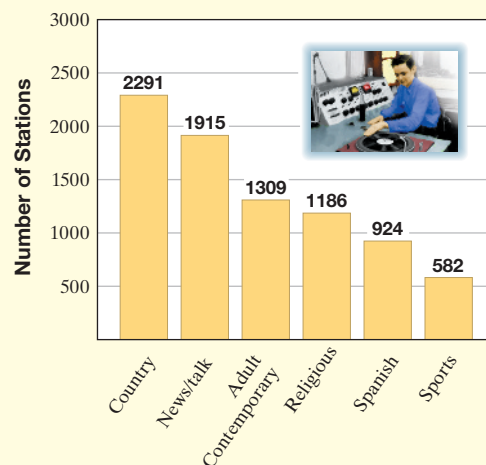
97. $\frac{99}{100}$

98. $\frac{99}{99}$

In 2016, there were 10,181 commercial radio stations in the United States. The most popular formats are listed in the table along with their counts. Use this graph to answer Exercises 99 through 102.

99. Write as a decimal the fraction of radio stations that are sports. Round to the nearest thousandth.
100. Write as a decimal the fraction of radio stations with a Spanish format. Round to the nearest hundredth.
101. Estimate, by rounding each number in the table to the nearest hundred, the total number of stations with the top six formats in 2016.
102. Use your estimate from Exercise 101 to write the fraction of radio stations accounted for by the top six formats as a decimal. Round to the nearest hundredth.

Top Commercial Radio Station Formats in 2016



Format (Total stations: 10,181)

103. Describe two ways to write fractions as decimals.
104. Describe two ways to write mixed numbers as decimals.

5.6 Solving Equations Containing Decimals

Objective A Solving Equations Containing Decimals

In this section, we continue our work with decimals and algebra by solving equations containing decimals. First, we review the steps given earlier for solving an equation.

Steps for Solving an Equation in x

Step 1: If fractions are present, multiply both sides of the equation by the LCD of the fractions.

Step 2: If parentheses are present, use the distributive property.

Step 3: Combine any like terms on each side of the equation.

Step 4: Use the addition property of equality to rewrite the equation so that variable terms are on one side of the equation and constant terms are on the other side.

Step 5: Divide both sides by the numerical coefficient of x to solve.

Step 6: Check your answer in the **original equation**.

Example 1 Solve: $x - 1.5 = 8$

Solution: Steps 1 through 3 are not needed for this equation, so we begin with Step 4. To get x alone on one side of the equation, add 1.5 to both sides.

$$\begin{aligned}x - 1.5 &= 8 && \text{Original equation} \\x - 1.5 + 1.5 &= 8 + 1.5 && \text{Add 1.5 to both sides.} \\x &= 9.5 && \text{Simplify.}\end{aligned}$$

Check: To check, replace x with 9.5 in the *original equation*.

$$\begin{aligned}x - 1.5 &= 8 && \text{Original equation} \\9.5 - 1.5 &\stackrel{?}{=} 8 && \text{Replace } x \text{ with 9.5.} \\8 &= 8 && \text{True}\end{aligned}$$

Since $8 = 8$ is a true statement, 9.5 is the solution of the equation.

Work Practice 1

Example 2 Solve: $-2y = 6.7$

Solution: Steps 1 through 4 are not needed for this equation, so we begin with Step 5. To solve for y , divide both sides by the coefficient of y , which is -2 .

$$\begin{aligned}-2y &= 6.7 && \text{Original equation} \\ \frac{-2y}{-2} &= \frac{6.7}{-2} && \text{Divide both sides by } -2. \\ y &= -3.35 && \text{Simplify.}\end{aligned}$$

Check: To check, replace y with -3.35 in the original equation.

$$\begin{aligned}-2y &= 6.7 && \text{Original equation} \\ -2(-3.35) &\stackrel{?}{=} 6.7 && \text{Replace } y \text{ with } -3.35. \\ 6.7 &= 6.7 && \text{True}\end{aligned}$$

Thus -3.35 is the solution of the equation $-2y = 6.7$.

Work Practice 2

Objective

A Solve Equations Containing Decimals.

Practice 1

Solve: $z + 0.9 = 1.3$

Practice 2

Solve: $0.17x = -0.34$

Answers

1. 0.4 2. -2

Practice 3Solve: $2.9 = 1.7 + 0.3x$ **Practice 4**Solve: $8x + 4.2 = 10x + 11.6$ **Practice 5**Solve: $6.3 - 5x = 3(x + 2.9)$ **Example 3** Solve: $1.2x + 5.8 = 8.2$ **Solution:** We begin with Step 4 and get the variable term alone by subtracting 5.8 from both sides.

$$\begin{aligned}
 1.2x + 5.8 &= 8.2 \\
 1.2x + 5.8 - 5.8 &= 8.2 - 5.8 && \text{Subtract 5.8 from both sides.} \\
 1.2x &= 2.4 && \text{Simplify.} \\
 \frac{1.2x}{1.2} &= \frac{2.4}{1.2} && \text{Divide both sides by 1.2.} \\
 x &= 2 && \text{Simplify.}
 \end{aligned}$$

To check, replace x with 2 in the original equation. The solution is 2.**Work Practice 3****Example 4** Solve: $7x + 3.2 = 4x - 1.6$ **Solution:** We start with Step 4 to get variable terms on one side and constant terms on the other.

$$\begin{aligned}
 7x + 3.2 &= 4x - 1.6 \\
 7x + 3.2 - 3.2 &= 4x - 1.6 - 3.2 && \text{Subtract 3.2 from both sides.} \\
 7x &= 4x - 4.8 && \text{Simplify.} \\
 7x - 4x &= 4x - 4.8 - 4x && \text{Subtract 4x from both sides.} \\
 3x &= -4.8 && \text{Simplify.} \\
 \frac{3x}{3} &= \frac{-4.8}{3} && \text{Divide both sides by 3.} \\
 x &= -1.6 && \text{Simplify.}
 \end{aligned}$$

Check to see that -1.6 is the solution.**Work Practice 4****Example 5** Solve: $5(x - 0.36) = -x + 2.4$ **Solution:** First use the distributive property to distribute the factor 5.

$$\begin{aligned}
 5(x - 0.36) &= -x + 2.4 && \text{Original equation} \\
 5x - 1.8 &= -x + 2.4 && \text{Apply the distributive property.}
 \end{aligned}$$

Next, get the variable terms alone on the left side of the equation by adding 1.8 to both sides of the equation and then adding x to both sides of the equation.

$$\begin{aligned}
 5x - 1.8 + 1.8 &= -x + 2.4 + 1.8 && \text{Add 1.8 to both sides.} \\
 5x &= -x + 4.2 && \text{Simplify.} \\
 5x + x &= -x + 4.2 + x && \text{Add } x \text{ to both sides.} \\
 6x &= 4.2 && \text{Simplify.} \\
 \frac{6x}{6} &= \frac{4.2}{6} && \text{Divide both sides by 6.} \\
 x &= 0.7 && \text{Simplify.}
 \end{aligned}$$

To verify that 0.7 is the solution, replace x with 0.7 in the original equation.**Work Practice 5****Answers**3. 4 4. -3.7 5. -0.3

Instead of solving equations with decimals, sometimes it may be easier to first rewrite the equation so that it contains integers only. Recall that multiplying a decimal by a power of 10 such as 10, 100, or 1000 has the effect of moving the decimal point to the right. We can use the multiplication property of equality to multiply both sides of an equation by an appropriate power of 10. The resulting equivalent equation will contain integers only.

Example 6 Solve: $0.5y + 2.3 = 1.65$

Solution: Multiply both sides of the equation by 100. This will move the decimal point in each term two places to the right.

$$0.5y + 2.3 = 1.65 \quad \text{Original equation}$$

$$100(0.5y + 2.3) = 100(1.65) \quad \text{Multiply both sides by 100.}$$

$$100(0.5y) + 100(2.3) = 100(1.65) \quad \text{Apply the distributive property.}$$

$$50y + 230 = 165 \quad \text{Simplify.}$$

Now the equation contains integers only. Continue solving by subtracting 230 from both sides.

$$50y + 230 = 165$$

$$50y + 230 - 230 = 165 - 230 \quad \text{Subtract 230 from both sides.}$$

$$50y = -65 \quad \text{Simplify.}$$

$$\frac{50y}{50} = \frac{-65}{50} \quad \text{Divide both sides by 50.}$$

$$y = -1.3 \quad \text{Simplify.}$$

Check to see that -1.3 is the solution by replacing y with -1.3 in the original equation.

Work Practice 6

✓Concept Check By what number would you multiply both sides of the following equation to make calculations easier? Explain your choice.

$$1.7x + 3.655 = -14.2$$

Practice 6

$$\text{Solve: } 0.2y + 2.6 = 4$$

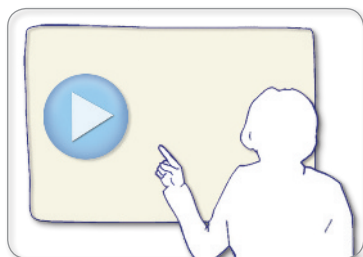
Answer

6. 7

✓Concept Check Answer
Multiply by 1000.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 5.6

- Objective A**
1. In Example 3, why did we multiply both sides of the equation by 10?
 2. In Example 4, we subtracted $3x$ from both sides, but what would have been a potential benefit of subtracting $2x$ instead?

5.6 Exercise Set MyLab Math **Objective A** Solve each equation. See Examples 1 and 2.

1. $x + 1.2 = 7.1$

2. $y - 0.5 = 9$

3. $-5y = 2.15$

4. $-0.4x = 50$

5. $6.2 = y - 4$

6. $9.7 = x + 11.6$

7. $3.1x = -13.95$

8. $3y = -25.8$

Solve each equation. See Examples 3 through 5.

9. $-3.5x + 2.8 = -11.2$

10. $7.1 - 0.2x = 6.1$

 11. $6x + 8.65 = 3x + 10$

12. $7x - 9.64 = 5x + 2.32$

13. $2(x - 1.3) = 5.8$

14. $5(x + 2.3) = 19.5$

Solve each equation by first multiplying both sides by an appropriate power of 10 so that the equation contains integers only. See Example 6.

15. $0.4x + 0.7 = -0.9$

16. $0.7x + 0.1 = 1.5$

17. $7x - 10.8 = x$

18. $3y = 7y + 24.4$

 19. $2.1x + 5 - 1.6x = 10$

20. $1.5x + 2 - 1.2x = 12.2$

Solve. See Examples 1 through 6.

21. $y - 3.6 = 4$

22. $x + 5.7 = 8.4$

23. $-0.02x = -1.2$

24. $-9y = -0.162$

25. $6.5 = 10x + 7.2$

26. $2x - 4.2 = 8.6$

27. $2.7x - 25 = 1.2x + 5$

28. $9y - 6.9 = 6y - 11.1$

29. $200x - 0.67 = 100x + 0.81$

30. $2.3 + 500x = 600x - 0.2$

 31. $3(x + 2.71) = 2x$

32. $7(x + 8.6) = 6x$

33. $8x - 5 = 10x - 8$

34. $24y - 10 = 20y - 17$

 35. $1.2 + 0.3x = 0.9$

36. $1.5 = 0.4x + 0.5$

37. $-0.9x + 2.65 = -0.5x + 5.45$

38. $-50x + 0.81 = -40x - 0.48$

39. $4x + 7.6 = 2(3x - 3.2)$

40. $4(2x - 1.6) = 5x - 6.4$

41. $0.7x + 13.8 = x - 2.16$

42. $y - 5 = 0.3y + 4.1$

Review

Simplify each expression by combining like terms. See Section 3.1.

43. $2x - 7 + x - 9$

44. $x + 14 - 5x - 17$

Perform the indicated operation. See Sections 4.3, 4.5, and 4.7.

45. $\frac{6x}{5} \cdot \frac{1}{2x^2}$

46. $5\frac{1}{3} \div 9\frac{1}{6}$

47. $\frac{x}{3} + \frac{2x}{7}$

48. $50 - 14\frac{9}{13}$

Concept Extensions

Mixed Practice (Sections 5.2 and 5.6) This section of exercises contains equations and expressions. If the exercise contains an equation, solve it for the variable. If the exercise contains an expression, simplify it by combining any like terms.

49. $b + 4.6 = 8.3$

50. $y - 15.9 = -3.8$

51. $2x - 0.6 + 4x - 0.01$

52. $-x - 4.1 - x - 4.02$

53. $5y - 1.2 - 7y + 8$

54. $9a - 5.6 - 3a + 6$

55. $2.8 = z - 6.3$

56. $9.7 = x + 4.3$

57. $4.7x + 8.3 = -5.8$

58. $2.8x + 3.4 = -13.4$

59. $7.76 + 8z - 12z + 8.91$

60. $9.21 + x - 4x + 11.33$

61. $5(x - 3.14) = 4x$

62. $6(x + 1.43) = 5x$

63. $2.6y + 8.3 = 4.6y - 3.4$

64. $8.4z - 2.6 = 5.4z + 10.3$

65. $9.6z - 3.2 - 11.7z - 6.9$

66. $-3.2x + 12.6 - 8.9x - 15.2$

-  67. Explain in your own words the property of equality that allows us to multiply both sides of an equation by a power of 10.
-  68. By what number would you multiply both sides of $8x - 7.6 = 4.23$ to make calculations easier? Explain your choice.

69. Construct an equation whose solution is 1.4.

70. Construct an equation whose solution is -8.6 .

Solve.

 71. $-5.25x = -40.33575$

 72. $7.68y = -114.98496$

 73. $1.95y + 6.834 = 7.65y - 19.8591$

 74. $6.11x + 4.683 = 7.51x + 18.235$

5.7 Decimal Applications: Mean, Median, and Mode

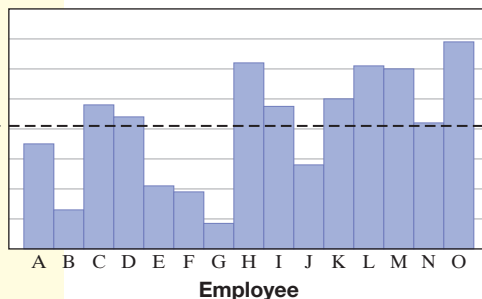
Objectives

- A** Find the Mean (or Average) of a List of Numbers.
- B** Find the Median of a List of Numbers.
- C** Find the Mode of a List of Numbers.

We are certainly familiar with the word “average.” The next four examples show real-life averages. For example,

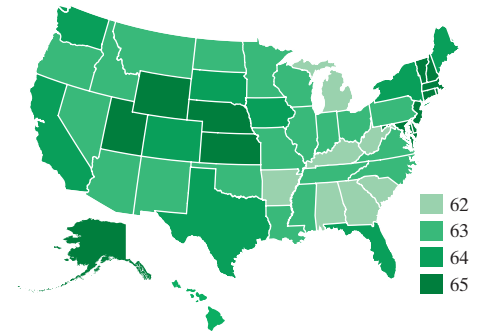
- Adults employed in the United States report working an **average** of 47 hours per week, according to a Gallup poll.
- The U.S. miles per gallon **average** for light vehicles is 25.5, according to autonews.com (*Automotive News*).

Average Annual Sales for Employees A through O



Average Shown on Dashed Line

The Average Retirement Age in the U.S.



Based on U.S. Census Bureau labor force participation data.

As our accumulation of data increases, our ability to gather, store, and present these tremendous amounts of data increases. Sometimes it is desirable to be able to describe a set of data by a single “middle” number or a measure of central tendency. Three of the most common **measures of central tendency** are the **mean** (or average), the **median**, and the **mode**.

Objective A Finding the Mean

The most common measure of central tendency is the mean (sometimes called the “arithmetic mean” or the “average”). Recall that we first introduced finding the average of a list of numbers in Section 1.6.

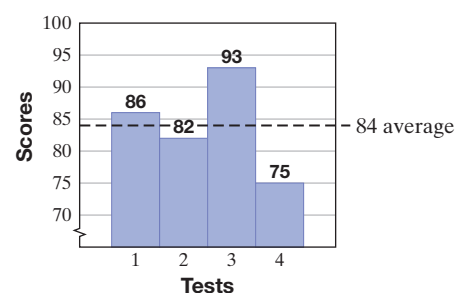
The **mean (average)** of a set of number items is the sum of the items divided by the number of items.

$$\text{mean} = \frac{\text{sum of items}}{\text{number of items}}$$

For example: To find the mean of four test scores—86, 82, 93, and 75—we find the sum of the scores and then divide by the number of scores, 4.

$$\text{mean} = \frac{86 + 82 + 93 + 75}{4} = \frac{336}{4} = 84$$

Notice that by looking at a bar graph of the scores with a dashed line at the mean, it is reasonable that 84 is one *measure of central tendency*.



Example 1 Finding the Mean Time in an Experiment

Seven students in a psychology class conducted an experiment on mazes. Each student was given a pencil and asked to successfully complete the same maze. The timed results are below:

Student	Ann	Thanh	Carlos	Jesse	Melinda	Ramzi	Dayni
Time (Seconds)	13.2	11.8	10.7	16.2	15.9	13.8	18.5

- Who completed the maze in the shortest time? Who completed the maze in the longest time?
- Find the mean time.
- How many students took longer than the mean time? How many students took shorter than the mean time?

Solution:

- Carlos completed the maze in 10.7 seconds, the shortest time. Dayni completed the maze in 18.5 seconds, the longest time.
- To find the mean (or average), we find the sum of the items and divide by 7, the number of items.

$$\begin{aligned}\text{mean} &= \frac{\text{sum of items}}{\text{number of items}} = \frac{13.2 + 11.8 + 10.7 + 16.2 + 15.9 + 13.8 + 18.5}{7} \\ &= \frac{100.1}{7} = 14.3\end{aligned}$$

- Three students, Jesse, Melinda, and Dayni, had times longer than the mean time. Four students, Ann, Thanh, Carlos, and Ramzi, had times shorter than the mean time.

Work Practice 1

 **Concept Check** Estimate the mean of the following set of data:

5, 10, 10, 10, 10, 15

The mean has one main disadvantage. This measure of central tendency can be greatly affected by *outliers*. (**Outliers** are values that are especially large or small when compared with the rest of the data set.) Let's see an example of this next.

Example 2 The table lists the rounded salary of 10 staff numbers.

Staff	A	B	C	D	E	F	G	H	I	J
Salary (in thousands)	\$32	\$34	\$46	\$38	\$42	\$95	\$102	\$50	\$42	\$41

- Find the mean of all 10 staff members.
- Find the mean of all staff members except F and G.

Solution:

$$\begin{aligned}\text{a. mean} &= \frac{32 + 34 + 46 + 38 + 42 + 95 + 102 + 50 + 42 + 41}{10} = \frac{522}{10} \\ &= 52.2\end{aligned}$$

The mean salary is \$52.2 thousand or \$52,200.

(Continued on next page)

Practice 1

Find the mean of the following test scores: 87, 75, 96, 91, and 78.

Practice 2

Use the table in Example 2 and find the mean salary of all staff members except G. Round thousands of dollars to 2 decimal places.

Answers

1. 85.4 2. \$46.67 thousand or \$46,670

 **Concept Check Answer**
10

b.
$$\text{mean} = \frac{32 + 34 + 46 + 38 + 42 + 50 + 42 + 41}{8} = \frac{325}{8}$$
$$= 40.625$$

Now, the mean salary is \$40.625 thousand or \$40,625.

Work Practice 2

The mean in part **a.** does not appear to be a measure of central tendency because this mean, \$52.2 thousand, is greater than all salaries except 2 of the 10. Also, notice the difference in the means for parts **a.** and **b.** By removing the 2 outliers, the mean was greatly reduced.

Although the mean was calculated correctly each time, parts **a.** and **b.** of Example 2 show one disadvantage of the mean. That is, a few numerical outliers can greatly affect the mean.

In Section 8.1 we will discuss the range of a data set as well as calculate measures of central tendency from frequency distribution tables and graphs.

Helpful Hint

Remember an important disadvantage of the mean.
If our data set has a few outliers, the mean may not be the best measure of central tendency.

Often in college, the calculation of a **grade point average (GPA)** is a **weighted mean** and is calculated as shown in Example 3.

Practice 3

Find the grade point average if the following grades were earned in one semester. Round to 2 decimal places.

Grade	Credit Hours
A	2
B	4
C	5
D	2
A	2

Answer
3. 2.67

Example 3 Calculating Grade Point Average (GPA)

The following grades were earned by a student during one semester. Find the student's grade point average.

Course	Grade	Credit Hours
College mathematics	A	3
Biology	B	3
English	A	3
PE	C	1
Social studies	D	2

Solution: To calculate the grade point average, we need to know the point values for the different possible grades. The point values of grades commonly used in colleges and universities are given below:

A: 4, B: 3, C: 2, D: 1, F: 0

Now, to find the grade point average, we multiply the number of credit hours for each course by the point value of each grade. The grade point average is the sum of these products divided by the sum of the credit hours.

Course	Grade	Point Value of Grade	Credit Hours	Point Value of Credit Hours
College mathematics	A	4	3	12
Biology	B	3	3	9
English	A	4	3	12
PE	C	2	1	2
Social studies	D	1	2	2
		Totals:	12	37

grade point average = $\frac{37}{12} \approx 3.08$ rounded to two decimal places

The student earned a grade point average of 3.08.

Work Practice 3

Objective B Finding the Median

You may have noticed that a very low number or a very high number can affect the mean of a list of numbers. Because of this, you may sometimes want to use another measure of central tendency. A second measure of central tendency is called the **median**. The median of a list of numbers is not affected by a low or high number in the list.

The **median** of a set of numbers in numerical order is the middle number. If the number of items is odd, the median is the middle number. If the number of items is even, the median is the mean of the two middle numbers.

Example 4 Find the median of the following list of numbers:

25, 54, 56, 57, 60, 71, 98

Solution: Because this list is in numerical order, the median is the middle number, 57.

Work Practice 4

Example 5 Find the median of the following list of scores: 67, 91, 75, 86, 55, 91

Solution: First we list the scores in numerical order and then we find the middle number.

55, 67, 75, 86, 91, 91

Since there is an even number of scores, there are two middle numbers, 75 and 86. The median is the mean of the two middle numbers.

$$\text{median} = \frac{75 + 86}{2} = 80.5$$

The median is 80.5.

Work Practice 5



Don't forget to write the numbers in order from smallest to largest before finding the median.

Objective C Finding the Mode

The last common measure of central tendency is called the **mode**.

The **mode** of a set of numbers is the number that occurs most often. (It is possible for a set of numbers to have more than one mode or to have no mode.)

Example 6 Find the mode of the list of numbers:

11, 14, 14, 16, 31, 56, 65, 77, 77, 78, 79

Solution: There are two numbers that occur the most often. They are 14 and 77. This list of numbers has two modes, 14 and 77.

Work Practice 6

Practice 4

Find the median of the list of numbers: 5, 11, 14, 23, 24, 35, 38, 41, 43

Practice 5

Find the median of the list of scores: 36, 91, 78, 65, 95, 95, 88, 71

Practice 6

Find the mode of the list of numbers: 14, 10, 10, 13, 15, 15, 15, 17, 18, 18, 20

Answers

4. 24 5. 83 6. 15

Practice 7

Find the median and the mode of the list of numbers:

26, 31, 15, 15, 26, 30, 16, 18, 15, 35

Answer

7. median: 22; mode: 15

✓ **Concept Check Answer**
false; a set of numbers may have no mode

Example 7

Find the median and the mode of the following set of numbers. These numbers were high temperatures for 14 consecutive days in a city in Montana.

76, 80, 85, 86, 89, 87, 82, 77, 76, 79, 82, 89, 89, 92

Solution: First we write the numbers in numerical order.

76, 76, 77, 79, 80, 82, 82, 85, 86, 87, 89, 89, 89, 92

Since there is an even number of items, the median is the mean of the two middle numbers, 82 and 85.

$$\text{median} = \frac{82 + 85}{2} = 83.5$$

The mode is 89, since 89 occurs most often.

Work Practice 7

✓ **Concept Check** True or false? Every set of numbers *must* have a mean, median, and mode. Explain your answer.

Helpful Hint

Don't forget that it is possible for a list of numbers to have no mode. For example, the list

2, 4, 5, 6, 8, 9

has no mode. There is no number or numbers that occur more often than the others.

Remember, in Section 8.1, we will calculate mean, median and mode from frequency distribution tables and graphs.

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

mean mode grade point average
median average

- Another word for "mean" is _____.
- The number that occurs most often in a set of numbers is called the _____.
- The _____ of a set of number items is $\frac{\text{sum of items}}{\text{number of items}}$.
- The _____ of a set of numbers is the middle number. If the number of numbers is even, it is the _____ of the two middle numbers.
- An example of weighted mean is a calculation of _____.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.

See Video 5.7 **Objective A** 6. Why is the $\approx$ symbol used in  Example 1? **Objective B** 7. From  Example 3, what is always the first step when finding the median of a set of data numbers? **Objective C** 8. From  Example 4, why do you think it is helpful to have data numbers in numerical order when finding the mode? **5.7** Exercise Set MyLab Math **Objectives A B C Mixed Practice** For each set of numbers, find the mean, median, and mode. If necessary, round the mean to one decimal place. See Examples 1, 2, and 4 through 6.

1. 15, 23, 24, 18, 25
2. 45, 36, 28, 46, 52
-  3. 7.6, 8.2, 8.2, 9.6, 5.7, 9.1
4. 4.9, 7.1, 6.8, 6.8, 5.3, 4.9
5. 0.5, 0.2, 0.2, 0.6, 0.3, 1.3, 0.8, 0.1, 0.5
6. 0.6, 0.6, 0.8, 0.4, 0.5, 0.3, 0.7, 0.8, 0.1
7. 231, 543, 601, 293, 588, 109, 334, 268
8. 451, 356, 478, 776, 892, 500, 467, 780

The 10 tallest buildings in the world, completed as of the end of 2016, are listed in the following table. Use this table to answer Exercises 9 through 14. If necessary, round results to one decimal place. See Examples 1, 2, and 4 through 6.

9. Find the mean height of the five tallest buildings.
10. Find the median height of the five tallest buildings.
11. Find the median height of the six tallest buildings.
12. Find the mean height of the six tallest buildings.

Building	Height in Feet
Burj Khalifa, Dubai	2717
Shanghai Tower, Shanghai	2073
Makkah Royal Clock Tower, Mecca	1972
Ping An Finance Center	1965
Lotte World Tower	1819
One World Trade Center, New York City	1776
Guangzhou CTF Finance Center, Guangzhou	1739
Taipei 101, Taipei	1667
Shanghai World Financial Center, Shanghai	1614
International Commerce Center, Hong Kong	1588

(Source: Council on Tall Buildings and Urban Habitat)

-  13. Given the building heights, explain how you know, without calculating, that the answer to Exercise 10 is greater than the answer to Exercise 11.
-  14. Given the building heights, explain how you know, without calculating, that the answer to Exercise 12 is less than the answer to Exercise 9.

For Exercises 15 through 18, the grades are given for a student for a particular semester. Find the grade point average. If necessary, round the grade point average to the nearest hundredth. See Example 3.

15.

Grade	Credit Hours
B	3
C	3
A	4
C	4

16.

Grade	Credit Hours
D	1
F	1
C	4
B	5

17.

Grade	Credit Hours
A	3
A	3
A	4
B	3
C	1

18.

Grade	Credit Hours
B	2
B	2
C	3
A	3
B	3

During an experiment, the following times (in seconds) were recorded:
7.8, 6.9, 7.5, 4.7, 6.9, 7.0.

19. Find the mean.
20. Find the median.
21. Find the mode.

In a mathematics class, the following test scores were recorded for a student: 93, 85, 89, 79, 88, 91.

22. Find the mean.
23. Find the median.
24. Find the mode.

The following pulse rates were recorded for a group of 15 students:
78, 80, 66, 68, 71, 64, 82, 71, 70, 65, 70, 75, 77, 86, 72.

25. Find the mean.
26. Find the median.
27. Find the mode.
28. How many pulse rates were higher than the mean?
29. How many pulse rates were lower than the mean?
30. Explain how to find the position of the median.

Below are lengths for the six longest rivers in the world.

Name	Length (miles)
Nile	4160
Amazon	4000
Yangtze	3915
Mississippi-Missouri	3709
Ob-Irtysh	3459
Huang Ho	3395

Find the mean and the median for each of the following.

31. the six longest rivers

32. the three longest rivers

Review

Write each fraction in simplest form. See Section 4.2.

33. $\frac{18}{30y}$

34. $\frac{4x}{36}$

35. $\frac{55y^2}{75y^2}$

36. $\frac{35a^3}{100a^2}$

Concept Extensions

Find the missing numbers in each set of numbers.

37. _____, _____, _____, 40, _____. The mode is 35. The median is 37. The mean is 38.

38. 16, 18, _____, _____, _____. The mode is 21. The median is 20.

 39. Without making any computations, decide whether the median of the following list of numbers will be a whole number. Explain your reasoning.
36, 77, 29, 58, 43

 40. Write a list of numbers for which you feel the median would be a better measure of central tendency than the mean.

Chapter 5 Group Activity

Maintaining a Checking Account

(Sections 5.1, 5.2, 5.3)

This activity may be completed by working in groups or individually.

A checking account is a convenient way of handling money and paying bills. To open a checking account, the bank or savings and loan association requires a customer to make a deposit. Then the customer receives a checkbook that contains checks, deposit slips, and a register for recording checks written and deposits made. It is important to record all payments and deposits that affect the account. It is also important to keep the checkbook balance current by subtracting checks written and adding deposits made.

About once a month, checking customers receive a statement from the bank listing all activity that the account has had in the last month. The statement lists a beginning balance, all checks and deposits, any service charges made against the account, and an ending balance. Because it may take several days for checks that a customer has written to clear the banking system, the check register may list checks that do not appear on the monthly bank statement. These checks are called **outstanding checks**. Deposits that are recorded in the check register but do not appear on the statement are called **deposits in transit**. Because of these differences,

For the checkbook register and monthly bank statement given:

- update the checkbook register
- list the outstanding checks and their total, and deposits in transit
- balance the checkbook—be sure to update the register with any interest or service fees

it is important to balance, or reconcile, the checkbook against the monthly statement. The steps for doing so are listed below.

Balancing or Reconciling a Checkbook

- Step 1:** Place a check mark in the checkbook register next to each check and deposit listed on the monthly bank statement. Any entries in the register without a check mark are outstanding checks or deposits in transit.
- Step 2:** Find the ending checkbook register balance and add to it any outstanding checks and any interest paid on the account.
- Step 3:** From the total in Step 2, subtract any deposits in transit and any service charges.
- Step 4:** Compare the amount found in Step 3 with the ending balance listed on the bank statement. If they are the same, the checkbook balances with the bank statement. Be sure to update the check register with service charges and interest.
- Step 5:** If the checkbook does not balance, recheck the balancing process. Next, make sure that the running checkbook register balance was calculated correctly. Finally, compare the checkbook register with the statement to make sure that each check was recorded for the correct amount.

Checkbook Register						
#	Date	Description	Payment	✓	Deposit	Balance 425.86
114	4/1	Market Basket	30.27			
115	4/3	May's Texaco	8.50			
	4/4	Cash at ATM	50.00			
116	4/6	UNO Bookstore	121.38			
	4/7	Deposit			100.00	
117	4/9	MasterCard	84.16			
118	4/10	Redbox	6.12			
119	4/12	Kroger	18.72			
120	4/14	Parking sticker	18.50			
	4/15	Direct deposit			294.36	
121	4/20	Rent	395.00			
122	4/25	Student fees	20.00			
	4/28	Deposit			75.00	

First National Bank Monthly Statement 4/30		
BEGINNING BALANCE:		425.86
Date	Number	Amount
CHECKS AND ATM WITHDRAWALS		
4/3	114	30.27
4/4	ATM	50.00
4/11	117	84.16
4/13	115	8.50
4/15	119	18.72
4/22	121	395.00
DEPOSITS		
4/7		100.00
4/15	Direct deposit	294.36
SERVICE CHARGES		
Low balance fee		7.50
INTEREST		
Credited 4/30		1.15
ENDING BALANCE:		227.22

Chapter 5 Vocabulary Check

Fill in each blank with one of the choices listed below. Some choices may be used more than once and some not at all.

vertically	decimal	and	right triangle
standard form	mean	median	circumference
sum	denominator	numerator	mode

- Like fractional notation, _____ notation is used to denote a part of a whole.
- To write fractions as decimals, divide the _____ by the _____.
- To add or subtract decimals, write the decimals so that the decimal points line up _____.
- When writing decimals in words, write “_____” for the decimal point.
- When multiplying decimals, the decimal point in the product is placed so that the number of decimal places in the product is equal to the _____ of the number of decimal places in the factors.
- The _____ of a set of numbers is the number that occurs most often.
- The distance around a circle is called the _____.
- The _____ of a set of numbers in numerical order is the middle number. If there is an even number of numbers, the median is the _____ of the two middle numbers.
- The _____ of a list of items with number values is $\frac{\text{sum of items}}{\text{number of items}}$.
- When 2 million is written as 2,000,000, we say it is written in _____.

Helpful Hint

▶ Are you preparing for your test?
To help, don't forget to take these:

- Chapter 5 Getting Ready for the Test on page 410
- Chapter 5 Test on page 411

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

5 Chapter Highlights

Definitions and Concepts

Examples

Section 5.1 Introduction to Decimals

Place-Value Chart

hundreds	tens	ones		tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths
100	10	1	$\uparrow$ decimal point	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10,000}$	$\frac{1}{100,000}$

4.265 means

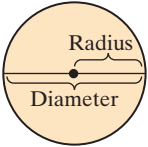
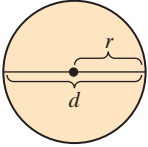
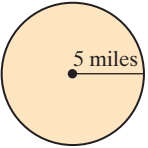
$$4 \cdot 1 + 2 \cdot \frac{1}{10} + 6 \cdot \frac{1}{100} + 5 \cdot \frac{1}{1000}$$

or

$$4 + \frac{2}{10} + \frac{6}{100} + \frac{5}{1000}$$

(continued)

Definitions and Concepts	Examples
Section 5.1 Introduction to Decimals (continued)	
Writing (or Reading) a Decimal in Words Step 1: Write the whole number part in words. Step 2: Write “and” for the decimal point. Step 3: Write the decimal part in words as though it were a whole number, followed by the place value of the last digit. A decimal written in words can be written in standard form by reversing the above procedure. To Round a Decimal to a Place Value to the Right of the Decimal Point Step 1: Locate the digit to the right of the given place value. Step 2: If this digit is 5 or greater, add 1 to the digit in the given place value and drop all digits to its right. If this digit is less than 5, drop all digits to the right of the given place value.	Write 3.08 in words. Three and eight hundredths Write “negative four and twenty-one thousandths” in standard form. −4.021 Round 86.1256 to the nearest hundredth. Step 1: 86.12 5 6 ← hundredths place ↑ digit to the right Step 2: Since the digit to the right is 5 or greater, we add 1 to the digit in the hundredths place and drop all digits to its right. 86.1256 rounded to the nearest hundredth is 86.13.
Section 5.2 Adding and Subtracting Decimals	
To Add or Subtract Decimals Step 1: Write the decimals so that the decimal points line up vertically. Step 2: Add or subtract as with whole numbers. Step 3: Place the decimal point in the sum or difference so that it lines up vertically with the decimal points in the problem.	Add: $4.6 + 0.28$ Subtract: $2.8 - 1.04$ $\begin{array}{r} 4.60 \\ + 0.28 \\ \hline 4.88 \end{array}$ $\begin{array}{r} 2.80 \\ - 1.04 \\ \hline 1.76 \end{array}$
Section 5.3 Multiplying Decimals and Circumference of a Circle	
To Multiply Decimals Step 1: Multiply the decimals as though they are whole numbers. Step 2: The decimal point in the product is placed so that the number of decimal places in the product is equal to the <i>sum</i> of the number of decimal places in the factors.	Multiply: 1.48×5.9 $\begin{array}{r} 1.48 \leftarrow 2 \text{ decimal places} \\ \times 5.9 \leftarrow 1 \text{ decimal place} \\ \hline 1332 \\ 7400 \\ \hline 8.732 \leftarrow 3 \text{ decimal places} \end{array}$

Definitions and Concepts	Examples
Section 5.3 Multiplying Decimals and Circumference of a Circle (continued)	
The circumference of a circle is the distance around the circle.  $C = \pi d$ or $C = 2\pi r$ where $\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$. or 	Find the exact circumference of a circle with radius 5 miles and an approximation by using 3.14 for π.  $C = 2\pi r$ $= 2\pi(5)$ $= 10\pi$ $\approx 10(3.14)$ ≈ 31.4 The circumference is exactly 10π miles and approximately 31.4 miles.
Section 5.4 Dividing Decimals	
To Divide Decimals Step 1: If the divisor is not a whole number, move the decimal point in the divisor to the right until the divisor is a whole number. Step 2: Move the decimal point in the dividend to the right the <i>same number of places</i> as the decimal point was moved in Step 1. Step 3: Divide. The decimal point in the quotient is directly over the moved decimal point in the dividend.	Divide: $1.118 \div 2.6$ $\begin{array}{r} 0.43 \\ 2.6 \overline{) 1.118} \\ \underline{-104} \\ 78 \\ \underline{-78} \\ 0 \end{array}$
Section 5.5 Fractions, Decimals, and Order of Operations	
To write fractions as decimals, divide the numerator by the denominator. Order of Operations 1. Perform all operations within parentheses (), brackets [], or other grouping symbols such as fraction bars. 2. Evaluate any expressions with exponents. 3. Multiply or divide in order from left to right. 4. Add or subtract in order from left to right.	Write $\frac{3}{8}$ as a decimal. $\begin{array}{r} 0.375 \\ 8 \overline{) 3.000} \\ \underline{-24} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$ Simplify. $ \begin{aligned} -1.9(12.8 - 4.1) &= -1.9(8.7) && \text{Subtract.} \\ &= -16.53 && \text{Multiply.} \end{aligned} $

Definitions and Concepts	Examples
Section 5.6 Solving Equations Containing Decimals	
Steps for Solving an Equation in x Step 1: If fractions are present, multiply both sides of the equation by the LCD of the fractions. Step 2: If parentheses are present, use the distributive property. Step 3: Combine any like terms on each side of the equation. Step 4: Use the addition property of equality to rewrite the equation so that variable terms are on one side of the equation and constant terms are on the other side. Step 5: Divide both sides by the numerical coefficient of x to solve. Step 6: Check your answer in the <i>original equation</i>.	Solve: $3(x + 2.6) = 10.92$ $3x + 7.8 = 10.92$ Apply the distributive property. $3x + 7.8 - 7.8 = 10.92 - 7.8$ Subtract 7.8 from both sides. $3x = 3.12$ Simplify. $\frac{3x}{3} = \frac{3.12}{3}$ Divide both sides by 3. $x = 1.04$ Simplify. Check 1.04 in the original equation.
Section 5.7 Decimal Applications: Mean, Median, and Mode	
The mean (or average) of a set of number items is $\text{mean} = \frac{\text{sum of items}}{\text{number of items}}$ The median of a set of numbers in numerical order is the middle number. If the number of items is even, the median is the mean of the two middle numbers. The mode of a set of numbers is the number that occurs most often. (A set of numbers may have no mode or more than one mode.)	Find the mean, median, and mode of the following set of numbers: 33, 35, 35, 43, 68, 68 $\text{mean} = \frac{33 + 35 + 35 + 43 + 68 + 68}{6} = 47$ The median is the mean of the two middle numbers, 35 and 43 $\text{median} = \frac{35 + 43}{2} = 39$ There are two modes because there are two numbers that occur twice: 35 and 68

Chapter 5

Review

(5.1) Determine the place value of the number 4 in each decimal.

1. 23.45
2. 0.000345

Write each decimal in words.

3. −23.45
4. 0.00345
5. 109.23
6. 200.000032

Write each decimal in standard form.

7. Eight and six hundredths

8. Negative five hundred three and one hundred two thousandths

9. Sixteen thousand twenty-five and fourteen ten-thousandths

10. Fourteen and eleven thousandths

Write each decimal as a fraction or a mixed number.

11. 0.16

12. -12.023

Write each fraction or mixed number as a decimal.

13. $\frac{231}{100,000}$

14. $25\frac{1}{4}$

Insert $<$, $>$, or $=$ between each pair of numbers to make a true statement.

15. 0.49 0.43

16. 0.973 0.9730

17. -38.0027 -38.00056

18. -0.230505 -0.23505

Round each decimal to the given place value.

19. 0.623, nearest tenth

20. 0.9384, nearest hundredth

21. -42.895, nearest hundredth

22. 16.34925, nearest thousandth

Write each number in standard notation.

23. Saturn is a distance of about 890.5 million miles from the Sun.

24. The tail of a comet can be over 600 thousand miles long.

(5.2) Add.

25. $8.6 + 9.5$

26. $3.9 + 1.2$

27. $-6.4 + (-0.88)$

28. $-19.02 + 6.98$

29. $200.49 + 16.82 + 103.002$

30. $0.00236 + 100.45 + 48.29$

Subtract.

31. $4.9 - 3.2$

32. $5.23 - 2.74$

33. $-892.1 - 432.4$

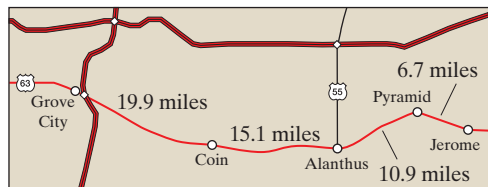
34. $0.064 - 10.2$

35. $100 - 34.98$

36. $200 - 0.00198$

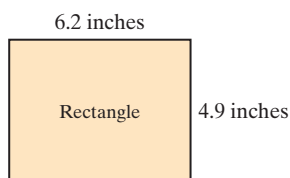
Solve.

37. Find the total distance between Grove City and Jerome.

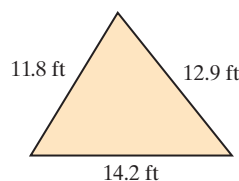


38. Evaluate $x - y$ for $x = 1.2$ and $y = 6.9$.

- △ 39. Find the perimeter.



- △ 40. Find the perimeter.



(5.3) Multiply.

41. 7.2×10

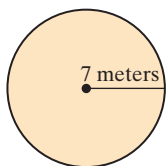
42. 9.345×1000

43. -34.02×2.3

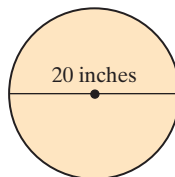
44. $-839.02 \times (-87.3)$

Find the exact circumference of each circle. Then use the approximation 3.14 for π and approximate the circumference.

- △ 45.



- △ 46.



(5.4) Divide. Round the quotient to the nearest thousandth if necessary.

47. $3 \overline{)0.2631}$

48. $20 \overline{)316.5}$

49. $-21 \div (-0.3)$

50. $-0.0063 \div 0.03$

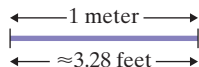
51. $0.34 \overline{)2.74}$

52. $19.8 \overline{)601.92}$

53. $\frac{23.65}{1000}$

54. $\frac{93}{-10}$

55. There are approximately 3.28 feet in 1 meter. Find how many meters are in 24 feet to the nearest tenth of a meter.



56. George Strait pays \$69.71 per month to pay back a loan of \$3136.95. In how many months will the loan be paid off?

(5.5) Write each fraction or mixed number as a decimal. Round to the nearest thousandth if necessary.

57. $\frac{4}{5}$

58. $-\frac{12}{13}$

59. $2\frac{1}{3}$

60. $\frac{13}{60}$

Insert $<$, $>$, or $=$ to make a true statement.

61. 0.392 0.39200 62. -0.0231 -0.0221 63. $\frac{4}{7}$ 0.625 64. 0.293 $\frac{5}{17}$

Write the numbers in order from smallest to largest.

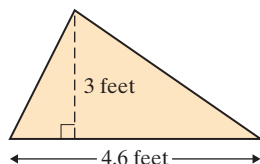
65. 0.837, 0.839, 0.832 66. 0.685, 0.626, $\frac{5}{8}$ 67. $\frac{3}{7}$, 0.42, 0.43 68. $\frac{18}{11}$, 1.63, $\frac{19}{12}$

Simplify each expression.

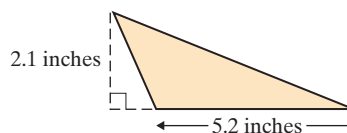
69. $-7.6 \times 1.9 + 2.5$ 70. $(-2.3)^2 - 1.4$ 71. $0.0726 \div 10 \times 1000$
72. $0.9(6.5 - 5.6)$ 73. $\frac{(1.5)^2 + 0.5}{0.05}$ 74. $\frac{7 + 0.74}{-0.06}$

Find each area.

△ 75.



△ 76.



(5.6) Solve.

77. $x + 3.9 = 4.2$ 78. $70 = y - 22.81$ 79. $2x = 17.2$
80. $-1.1y = 88$ 81. $3x - 0.78 = 1.2 + 2x$ 82. $-x + 0.6 - 2x = -4x - 0.9$
83. $-1.3x - 9.4 = -0.4x + 8.6$ 84. $3(x - 1.1) = 5x - 5.3$

(5.7) Find the mean, median, and any mode(s) for each list of numbers. If necessary, round to the nearest tenth.

85. 13, 23, 33, 14, 6 86. 45, 86, 21, 60, 86, 64, 45
87. 14,000, 20,000, 12,000, 20,000, 36,000, 45,000 88. 560, 620, 123, 400, 410, 300, 400, 780, 430, 450

For Exercises 89 and 90, the grades are given for a student for a particular semester. Find each grade point average. If necessary, round the grade point average to the nearest hundredth.

89.

Grade	Credit Hours
A	3
A	3
C	2
B	3
C	1

90.

Grade	Credit Hours
B	3
B	4
C	2
D	2
B	3

Mixed Review

91. Write 200.0032 in words.

92. Write negative sixteen and nine hundredths in standard form.

93. Write 0.0847 as a fraction or a mixed number.

94. Write the numbers $\frac{6}{7}$, $\frac{8}{9}$, 0.75 in order from smallest to largest.

Write each fraction as a decimal. Round to the nearest thousandth, if necessary.

95. $-\frac{7}{100}$

96. $\frac{9}{80}$ (Do not round.)

97. $\frac{8935}{175}$

Insert $<$, $>$, or $=$ to make a true statement.

98. -402.000032 -402.00032

99. $\frac{6}{11}$ 0.55

Round each decimal to the given place value.

100. 86.905, nearest hundredth

101. 3.11526, nearest thousandth

Round each money amount to the nearest dollar.

102. \$123.46

103. \$3645.52

Add or subtract as indicated.

104. $3.2 - 4.9$

105. $9.12 - 3.86$

106. $-102.06 + 89.3$

107. $-4.021 + (-10.83) + (-0.056)$

Multiply or divide as indicated. Round to the nearest thousandth, if necessary.

108.
$$\begin{array}{r} 2.54 \\ \times 3.2 \\ \hline \end{array}$$

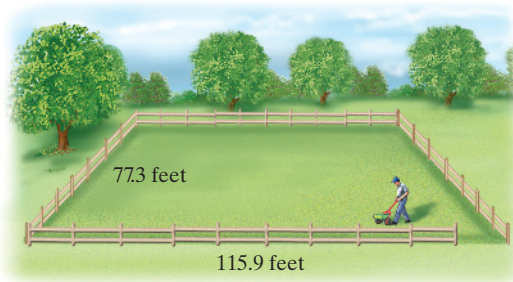
109. $(-3.45)(2.1)$

110. $0.005 \overline{)24.5}$

111. $2.3 \overline{)54.98}$

Solve.

- △ 112. Tomaso is going to fertilize his lawn, a rectangle that measures 77.3 feet by 115.9 feet. Approximate the area of the lawn by rounding each measurement to the nearest ten feet.



113. Estimate the cost of the items to see whether the groceries can be purchased with a \$10 bill.



Simplify each expression.

114.
$$\frac{(3.2)^2}{100}$$

115. $(2.6 + 1.4)(4.5 - 3.6)$

Find the mean, median, and any mode(s) for each list of numbers. If needed, round answers to the nearest hundredth.

116. 73, 82, 95, 68, 54

117. 952, 327, 566, 814, 327, 729

MULTIPLE CHOICE All the exercises below are **Multiple Choice**. Choose the correct letter.

For Exercises 1 through 4, name the place value for the given digit. The number is 1026.89704 and choices are:

- A. tens B. hundreds C. tenths D. thousandths E. ten-thousandths

- ▶ 1. the digit 8 ▶ 2. the digit 2
▶ 3. the digit 0 to the left of decimal point ▶ 4. the digit 0 to the right of the decimal point

For Exercises 5 through 8, let $x = 3$ and $y = 0.2$. Choose the expression that gives each answer.

- A. $x + y$ B. $x - y$ C. xy D. $x \div y$

- ▶ 5. answer: 0.6 ▶ 6. answer: 3.2
▶ 7. answer: 2.8 ▶ 8. answer: 15

For Exercises 9 through 11, choose the correct letter.

- ▶ 9. The expression $2(0.5)^2 + 0.3$ simplifies to
A. 1.3 B. 0.8 C. 2.3 D. 1.28
▶ 10. One way to solve $5(x - 3.2) = 9.5$ is to first use the distributive property to multiply on the left side of the equation. Once this is done, the equivalent equation is
A. $5x - 3.2 = 9.5$ B. $5x - 16 = 9.5$ C. $5x - 16 = 95$
▶ 11. One way to solve $0.3x + 1.8 = 2.16$ is to first multiply both sides of the equation by 100. Once this is done, the equivalent equation is
A. $3x + 18 = 216$ B. $3x + 1.8 = 216$ C. $30x + 180 = 216$

For Exercises 12 through 14, choose the correct directions that lead to the given correct answer for the data set: 7, 9, 10, 13, 13

- A. Find the mean. B. Find the median. C. Find the mode.

- ▶ 12. answer: 10 ▶ 13. answer: 13 ▶ 14. answer: 10.4

Write each decimal as indicated.

► 1. 45.092, in words

► 2. Three thousand and fifty-nine thousandths, in standard form

Perform each indicated operation. Round the result to the nearest thousandth if necessary.

► 3. $2.893 + 4.21 + 10.492$

► 4. $-47.92 - 3.28$

► 5. $9.83 - 30.25$

► 6. 10.2×4.01

► 7. $-0.00843 \div (-0.23)$

Round each decimal to the indicated place value.

► 8. 34.8923, nearest tenth

► 9. 0.8623, nearest thousandth

Insert $<$, $>$, or $=$ between each pair of numbers to form a true statement.

► 10. 25.0909 25.9090

► 11. $\frac{4}{9}$ 0.445

Write each decimal as a fraction or a mixed number.

► 12. 0.345

► 13. -24.73

Write each fraction as a decimal. If necessary, round to the nearest thousandth.

► 14. $-\frac{13}{26}$

► 15. $\frac{16}{17}$

Simplify.

► 16. $(-0.6)^2 + 1.57$

► 17. $\frac{0.23 + 1.63}{-0.3}$

► 18. $2.4x - 3.6 - 1.9x - 9.8$

Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____

19. _____

Solve.

▶ 19. $0.2x + 1.3 = 0.7$

▶ 20. $2(x + 5.7) = 6x - 3.4$

20. _____

Find the mean, median, and mode of each list of numbers.

▶ 21. 26, 32, 42, 43, 49

▶ 22. 8, 10, 16, 16, 14, 12, 12, 13

21. _____

Find the grade point average. If necessary, round to the nearest hundredth.

▶ 23.

Grade	Credit Hours
A	3
B	3
C	3
B	4
A	1

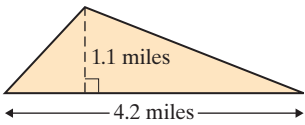
22. _____

23. _____

Solve.

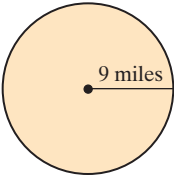
▶ 24. At its farthest, Pluto is 4583 million miles from the Sun. Write this number using standard notation.

▶ 25. Find the area.



25. _____

▶ 26. Find the exact circumference of the circle. Then use the approximation 3.14 for π and approximate the circumference.



▶ 27. Vivian Thomas is going to put insecticide on her lawn to control grubs. The lawn is a rectangle that measures 123.8 feet by 80 feet. The amount of insecticide required is 0.02 ounce per square foot.

a. Find the area of her lawn.

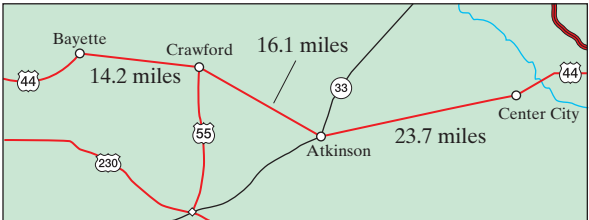
b. Find how much insecticide Vivian needs to purchase.

26. _____

27. a. _____

b. _____

▶ 28. Find the total distance from Bayette to Center City.



28. _____

Write each number in words.

1. 72

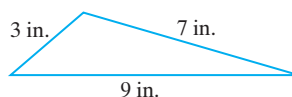
2. 107

3. 546

4. 5026

5. Add: $46 + 713$

6. Find the perimeter.



7. Subtract: $543 - 29$. Then check by adding.

8. Divide: $3268 \div 27$

9. Round 278,362 to the nearest thousand.

10. Write the prime factorization of 30.

11. Multiply: 236×86

12. Multiply: $236 \times 86 \times 0$

13. Find each quotient and then check the answer by multiplying.

a. $1\overline{)7}$

b. $12 \div 1$

c. $\frac{6}{6}$

d. $9 \div 9$

e. $\frac{20}{1}$

f. $18\overline{)18}$

14. Find the average of 25, 17, 19, and 39.

15. Simplify: $2 \cdot 4 - 3 \div 3$

16. Simplify: $77 \div 11 \cdot 7$

Evaluate.

17. 9^2

18. 5^3

Answers

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

13. a. _____

b. _____

c. _____

d. _____

e. _____

f. _____

14. _____

15. _____

16. _____

17. _____

18. _____

19.

20.

21.

22.

23. a.

b.

c.

24. a.

b.

c.

25.

26.

27.

28.

29.

30.

31.

32.

33.

34.

35.

36.

37.

38.

19. 3^4

20. 10^3

21. Evaluate $\frac{x - 5y}{y}$ for $x = 35$ and $y = 5$.

22. Evaluate $\frac{2a + 4}{c}$ for $a = 7$ and $c = 3$.

23. Find the opposite of each number.

a. 13

b. -2

c. 0

24. Find the opposite of each number.

a. -7

b. 4

c. -1

25. Add: $-2 + (-21)$

26. Add: $-7 + (-15)$

Find the value of each expression.

27. $5 \cdot 6^2$

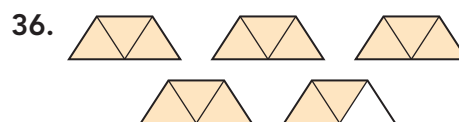
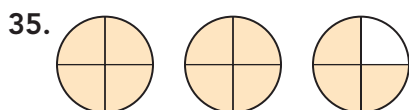
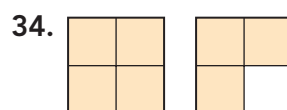
28. $4 \cdot 2^3$

29. -7^2

30. $(-2)^5$

31. $(-5)^2$

32. -3^2

Represent the shaded part as an improper fraction and a mixed number.

37. Write the prime factorization of 252.

38. Find the difference of 87 and 25.

39. Write $-\frac{72}{26}$ in simplest form.

40. Write $9\frac{7}{8}$ as an improper fraction.

41. Determine whether $\frac{16}{40}$ and $\frac{10}{25}$ are equivalent.

42. Insert $<$ or $>$ to form a true statement. $\frac{4}{7}$ $\frac{5}{9}$

Multiply.

43. $\frac{2}{3} \cdot \frac{5}{11}$

44. $2\frac{5}{8} \cdot \frac{4}{7}$

45. $\frac{1}{4} \cdot \frac{1}{2}$

46. $7 \cdot 5\frac{2}{7}$

Solve.

47. $\frac{z}{-4} = 11 - 5$

48. $6x - 12 - 5x = -20$

49. Add: $763.7651 + 22.001 + 43.89$

50. Add: $89.27 + 14.361 + 127.2318$

51. Multiply: 23.6×0.78

52. Multiply: 43.8×0.645

39. _____

40. _____

41. _____

42. _____

43. _____

44. _____

45. _____

46. _____

47. _____

48. _____

49. _____

50. _____

51. _____

52. _____

6

Having studied fractions in Chapter 4, we are ready to explore the useful notations of ratio and proportion.

Ratio is another name for *quotient* and is usually written in fraction form. A **proportion** is an equation with two equal ratios.

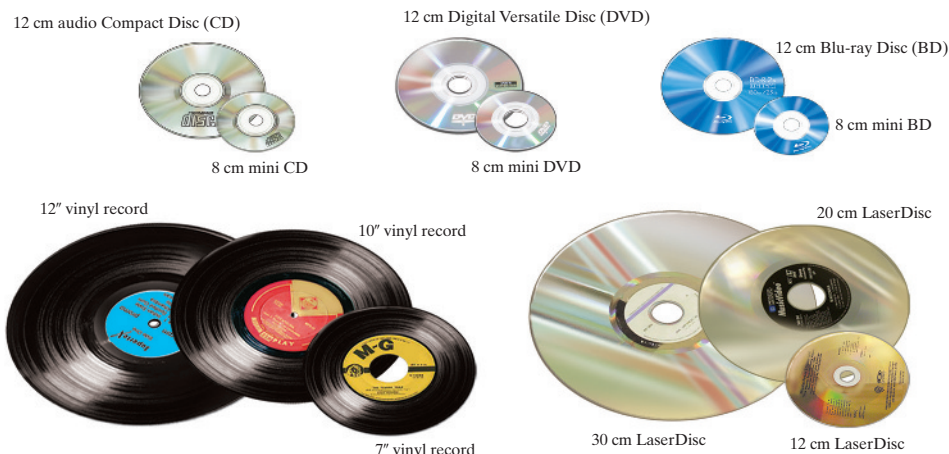
Sections

- 6.1 Ratios and Rates
- 6.2 Proportions
 - Integrated Review**—Ratio, Rate, and Proportion
- 6.3 Proportions and Problem Solving
- 6.4 Square Roots and the Pythagorean Theorem
- 6.5 Congruent and Similar Triangles

Check Your Progress

Vocabulary Check
Chapter Highlights
Chapter Review
Getting Ready for the Test
Chapter Test
Cumulative Review

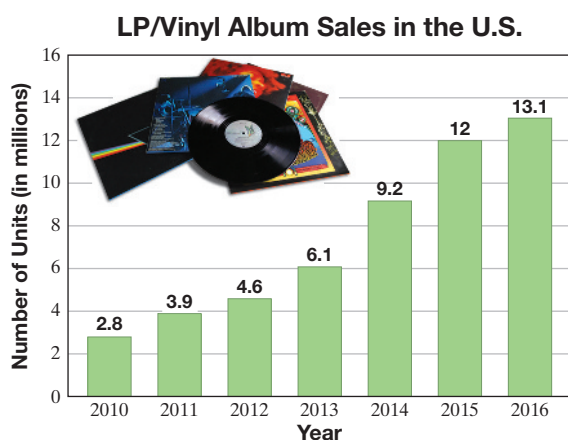
Ratio, Proportion, and Triangle Applications



How Do You Listen to Music? Downloading? Streaming? A Physical CD or LP/Vinyl?

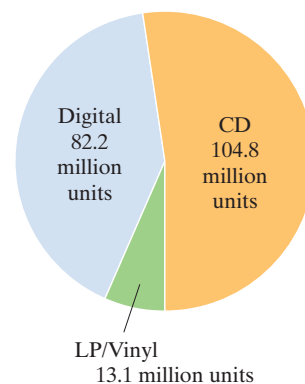
No matter how you listen to music, this industry is booming. In the United States, the music industry has grown to an estimated \$15 billion. Subscription streaming is increasing in the number of paid subscribers, and the number of digital music downloads is expected to decline. Interestingly enough, LP/vinyl albums are making a comeback. Below the bar graph shows this recent increase, and the circle graph shows the small piece of the U.S. album sales pie belonging to LP/vinyl album sales.

Above are physical drawings of various music and storage disks (or discs), including the sizes of these disks. In Section 6.1, Exercises 25 through 28, we find the ratios of diameters of certain disk storing mechanisms. (Source: IFPI.org)



Source: IFPI.org, Nieslon.com

U.S. Album Sales (LP/Vinyl, Digital, and CD)



6.1 Ratios and Rates

Objective A Writing Ratios as Fractions

A **ratio** is the quotient of two quantities. A ratio, in fact, is no different from a fraction, except that a ratio is sometimes written using notation other than fractional notation. For example, the ratio of 1 to 2 can be written as

$$\begin{array}{ccccc} 1 \text{ to } 2 & \text{or} & \frac{1}{2} & \text{or} & 1 : 2 \\ & & \nearrow & & \uparrow \\ & & \text{fractional notation} & & \text{colon notation} \end{array}$$

These ratios are all read as, “the ratio of 1 to 2.”

✓ Concept Check How should each ratio be read aloud?

a. $\frac{8}{5}$ b. $\frac{5}{8}$

In this section, we write ratios using fractional notation. If the fraction happens to be an improper fraction, do not write the fraction as a mixed number. Why? The mixed number form is not a ratio or quotient of two quantities.

Writing a Ratio as a Fraction

The order of the quantities is important when writing ratios. To write a ratio as a fraction, write the *first number* of the ratio as the *numerator* of the fraction and the *second number* as the *denominator*.

Helpful Hint

The ratio of 6 to 11 is $\frac{6}{11}$, *not* $\frac{11}{6}$.

Example 1 Write the ratio of 21 to 29 using fractional notation.

Solution: The ratio is $\frac{21}{29}$.

Work Practice 1

To simplify a ratio, we just write the fraction in simplest form. Common factors as well as common units can be divided out.

Example 2 Write the ratio of \$15 to \$10 as a fraction in simplest form.

Solution:

$$\frac{\$15}{\$10} = \frac{15}{10} = \frac{3 \cdot \cancel{5}}{2 \cdot \cancel{5}} = \frac{3}{2}$$

Work Practice 2

Objectives

A Write Ratios as Fractions.

B Write Rates as Fractions.

C Find Unit Rates.

D Find Unit Prices.

Practice 1

Write the ratio of 19 to 30 using fractional notation.

Practice 2

Write the ratio of \$16 to \$12 as a fraction in simplest form.

Answers

1. $\frac{19}{30}$ 2. $\frac{4}{3}$

✓ Concept Check Answers

a. “the ratio of eight to five”

b. “the ratio of five to eight”

Helpful Hint!

In the previous example, although $\frac{3}{2} = 1\frac{1}{2}$, a ratio is a quotient of *two* quantities. For that reason, ratios are not written as mixed numbers.

If a ratio contains decimal numbers or mixed numbers, we simplify by writing the ratio as a ratio of whole numbers.

Practice 3

Write the ratio of 1.68 to 4.8 as a fraction in simplest form.

Practice 4

Write the ratio of $2\frac{2}{3}$ to $1\frac{13}{15}$ as a fraction in simplest form.

Practice 5

Use the circle graph for Example 5 to write the ratio of work miles to total miles as a fraction in simplest form.

Answers

3. $\frac{7}{20}$ 4. $\frac{10}{7}$ 5. $\frac{8}{25}$

Example 3

Write the ratio of 2.5 to 3.15 as a fraction in simplest form.

Solution: The ratio in fraction form is

$$\frac{2.5}{3.15}$$

Now let's clear the ratio of decimals.

$$\frac{2.5}{3.15} = \frac{2.5}{3.15} \cdot 1 = \frac{2.5}{3.15} \cdot \frac{100}{100} = \frac{2.5 \cdot 100}{3.15 \cdot 100} = \frac{250}{315} = \frac{50}{63} \quad \text{Simplest form}$$

Work Practice 3

Example 4

Write the ratio of $2\frac{5}{8}$ to $8\frac{3}{4}$ as a fraction in simplest form.

Solution: The ratio in fraction form is $\frac{2\frac{5}{8}}{8\frac{3}{4}}$.

To simplify, remember that the fraction bar means division.

$$\frac{2\frac{5}{8}}{8\frac{3}{4}} = 2\frac{5}{8} \div 8\frac{3}{4} = \frac{21}{8} \div \frac{35}{4} = \frac{21}{8} \cdot \frac{4}{35} = \frac{3 \cdot \cancel{7} \cdot 1}{2 \cdot \cancel{4} \cdot 5 \cdot \cancel{7}} = \frac{3}{10} \quad \text{Simplest form}$$

Work Practice 4

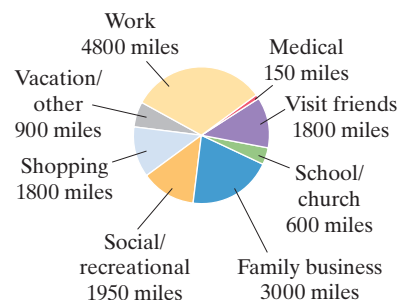
Example 5

Writing a Ratio from a Circle Graph

The circle graph at the right shows the part of a car's total mileage that falls into a particular category. Write the ratio of family business miles to total miles as a fraction in simplest form.

Solution:

$$\begin{aligned} \frac{\text{family business miles}}{\text{total miles}} &= \frac{3000 \text{ miles}}{15,000 \text{ miles}} \\ &= \frac{3000}{15,000} \\ &= \frac{\cancel{3000}^1}{5 \cdot \cancel{3000}_1} \\ &= \frac{1}{5} \end{aligned}$$



Total yearly mileage: 15,000

Sources: The American Automobile Manufacturers Association and the National Automobile Dealers Association.

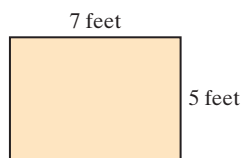
Work Practice 5



Example 6

Given the rectangle shown:

- Find the ratio of its width (shorter side) to its length (longer side).
- Find the ratio of its length to its perimeter.



Solution:

- The ratio of its width to its length is

$$\frac{\text{width}}{\text{length}} = \frac{5 \text{ feet}}{7 \text{ feet}} = \frac{5}{7}$$

- Recall that the perimeter of a rectangle is the distance around the rectangle: $7 + 5 + 7 + 5 = 24$ feet. The ratio of its length to its perimeter is

$$\frac{\text{length}}{\text{perimeter}} = \frac{7 \text{ feet}}{24 \text{ feet}} = \frac{7}{24}$$

Work Practice 6

Objective B Writing Rates as Fractions

A special type of ratio is a rate. **Rates** are used to compare *different* kinds of quantities. For example, suppose that a recreational runner can run 3 miles in 33 minutes. If we write this rate as a fraction, we have

$$\frac{3 \text{ miles}}{33 \text{ minutes}} = \frac{1 \text{ mile}}{11 \text{ minutes}} \quad \text{In simplest form}$$

Helpful Hint

When comparing quantities with different units, write the units as part of the comparison. Units do not divide out unless they are the same.

Same Units: $\frac{3 \text{ inches}}{12 \text{ inches}} = \frac{1}{4}$ Units are the same and divide out.

Different Units: $\frac{2 \text{ miles}}{20 \text{ minutes}} = \frac{1 \text{ mile}}{10 \text{ minutes}}$ Units are still written.

Examples

Write each rate as a fraction in simplest form.

7. \$2160 for 12 weeks is $\frac{2160 \text{ dollars}}{12 \text{ weeks}} = \frac{180 \text{ dollars}}{1 \text{ week}}$

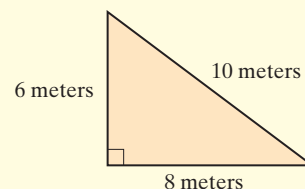
8. 360 miles on 16 gallons of gasoline is $\frac{360 \text{ miles}}{16 \text{ gallons}} = \frac{45 \text{ miles}}{2 \text{ gallons}}$

Work Practice 7-8

✓ **Concept Check** True or false? $\frac{16 \text{ gallons}}{4 \text{ gallons}}$ is a rate. Explain.

Practice 6

Given the triangle shown:



- Find the ratio of the length of the shortest side to the length of the longest side.
- Find the ratio of the length of the longest side to the perimeter of the triangle.

Practice 7-8

Write each rate as a fraction in simplest form.

7. \$1350 for 6 weeks

8. 295 miles on 15 gallons of gasoline

Answers

6. a. $\frac{3}{5}$ b. $\frac{5}{12}$ 7. $\frac{\$225}{1 \text{ wk}}$ 8. $\frac{59 \text{ mi}}{3 \text{ gal}}$

✓ Concept Check Answer

false; a rate compares different kinds of quantities

Objective C Finding Unit Rates

A **unit rate** is a rate with a denominator of 1. A familiar example of a unit rate is 55 mph, read as “55 **miles per hour**.” This means 55 miles per 1 hour or

$$\frac{55 \text{ miles}}{1 \text{ hour}} \quad \text{Denominator of 1}$$

Helpful Hint

In this context, the word *per* translates to “division.”

Writing a Rate as a Unit Rate

To write a rate as a unit rate, divide the numerator of the rate by the denominator.

Practice 9

Write as a unit rate: 3200 feet every 8 seconds

Example 9

Write as a unit rate: \$31,500 every 7 months

Solution:

$$\frac{31,500 \text{ dollars}}{7 \text{ months}} \quad \begin{array}{r} 4,500 \\ 7 \overline{)31,500} \end{array}$$

The unit rate is

$$\frac{4500 \text{ dollars}}{1 \text{ month}} \text{ or } 4500 \text{ dollars/month} \quad \text{Read as, “4500 dollars per month.”}$$

Work Practice 9**Practice 10**

Write as a unit rate: 78 bushels of fruit from 12 trees

Example 10

Write as a unit rate: 337.5 miles every 15 gallons of gas

Solution:

$$\frac{337.5 \text{ miles}}{15 \text{ gallons}} \quad \begin{array}{r} 22.5 \\ 15 \overline{)337.5} \end{array}$$

The unit rate is

$$\frac{22.5 \text{ miles}}{1 \text{ gallon}} \text{ or } 22.5 \text{ miles/gallon} \quad \text{Read as, “22.5 miles per gallon.”}$$

Work Practice 10**Objective D** Finding Unit Prices

Rates are used extensively in sports, business, medicine, and science. One of the most common uses of rates is in consumer economics. When a unit rate is “money per item,” it is also called a **unit price**.

$$\text{unit price} = \frac{\text{price}}{\text{number of units}}$$

Answers

9. $\frac{400 \text{ ft}}{1 \text{ sec}}$ or 400 ft/sec

10. $\frac{6.5 \text{ bushels}}{1 \text{ tree}}$ or 6.5 bushels/tree

Example 11 Finding Unit Price

A store charges \$3.36 for a 16-ounce jar of picante sauce. What is the unit price in dollars per ounce?

Solution:

$$\text{unit price} = \frac{\$3.36}{16 \text{ ounces}} = \frac{\$0.21}{1 \text{ ounce}} \quad \text{or} \quad \$0.21 \text{ per ounce}$$

Work Practice 11

Example 12 Finding the Best Buy

Approximate each unit price to decide which is the better buy: 4 bars of soap for \$3.99 or 5 bars of soap for \$4.59.



Solution:

$$\begin{aligned} \frac{\text{unit}}{\text{price}} &= \frac{\text{price}}{\text{no. of units}} = \frac{\$3.99}{4 \text{ bars}} \approx \$1.00 \text{ per bar of soap} & \frac{0.997}{4} \approx 1.00 & \text{ ("is approximately")} \\ \frac{\text{unit}}{\text{price}} &= \frac{\text{price}}{\text{no. of units}} = \frac{\$4.59}{5 \text{ bars}} \approx \$0.92 \text{ per bar of soap} & \frac{0.918}{5} \approx 0.92 & \end{aligned}$$

Since the 5-bar package has a cheaper price per bar, it is the better buy.

Work Practice 12

Practice 11

An automobile rental agency charges \$170 for 5 days for a certain model car. What is the unit price in dollars per day?

Practice 12

Approximate each unit price to decide which is the better buy for a bag of nacho chips: 11 ounces for \$3.99 or 16 ounces for \$5.99.

**Answers**

11. \$34 per day 12. 11-oz bag

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

rate	different	denominator	unit
numerator	unit price	true	
division	ratio	false	

1. A rate with a denominator of 1 is called a(n) _____ rate.
2. When a rate is written as money per item, a unit rate is called a(n) _____.
3. The word *per* translates to “_____.”
4. Rates are used to compare _____ types of quantities.
5. To write a rate as a unit rate, divide the _____ of the rate by the _____.
6. The quotient of two quantities is called a(n) _____.
7. Answer true or false: The ratio $\frac{7}{5}$ means the same as the ratio $\frac{5}{7}$. _____

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 6.1

- Objective A** 8. How is the ratio in Example 2 rewritten as an equivalent ratio containing no decimals?
- Objective B** 9. Why can't we divide out the units in Example 5 as we did in Example 4?
- Objective C** 10. Why did we divide the first quantity of the rate in Example 8 by the second quantity?
- Objective D** 11. From Example 9, unit prices can be especially helpful when?

6.1 Exercise Set MyLab Math

Objective A Write each ratio as a ratio of whole numbers using fractional notation. Write the fraction in simplest form. See Examples 1 through 6.

1. 16 to 24 2. 25 to 150 3. 7.7 to 10 4. 8.1 to 10
5. 4.63 to 8.21 6. 9.61 to 7.62 7. 6 ounces to 16 ounces 8. 35 meters to 100 meters
9. \$32 to \$100 10. \$46 to \$102 11. 24 days to 14 days 12. 120 miles to 80 miles
13. $3\frac{1}{2}$ to $12\frac{1}{4}$ 14. $3\frac{1}{3}$ to $4\frac{1}{6}$ 15. $7\frac{3}{5}$ hours to $1\frac{9}{10}$ hours 16. $25\frac{1}{2}$ days to $2\frac{5}{6}$ days

Find the ratio described in each exercise as a fraction in simplest form. See Examples 5 and 6.

17.

Average Weight of Mature Whales	
Blue Whale	Fin Whale
145 tons	50 tons

Use the table to find the ratio of the weight of an average mature fin whale to the weight of an average mature blue whale.

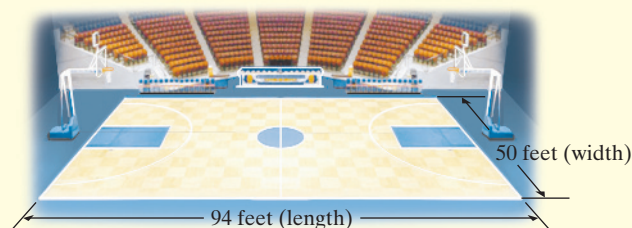
18.

Countries with Small Land Areas	
Tuvalu	San Marino
10 sq mi	24 sq mi

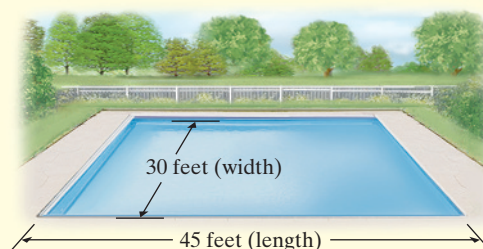
(Source: World Almanac)

Use the table to find the ratio of the land area of Tuvalu to the land area of San Marino.

19. Find the ratio of the width of a regulation size basketball court to its perimeter.



20. Find the ratio of the width to the perimeter shown of the swimming pool.



At the Hidalgo County School Board meeting one night, there were 125 women and 100 men present.

21. Find the ratio of women to men.
22. Find the ratio of men to the total number of people present.

Blood contains three types of cells: red blood cells, white blood cells, and platelets. For approximately every 600 red blood cells in healthy humans, there are 40 platelets and 1 white blood cell. Use this information for Exercises 23 and 24. (Source: American Red Cross Biomedical Services)

23. Write the ratio of red blood cells to platelet cells.
24. Write the ratio of white blood cells to red blood cells.

Exercises 25–28 have to do with disk storage. See the Chapter Opener for drawings of various disk storage devices.

25. A standard DVD has a diameter of 12 centimeters while a miniDVD has a diameter of 8 centimeters. Write the ratio of the miniDVD diameter to the standard DVD diameter.
26. LaserDiscs have a diameter of 12 centimeters and 20 centimeters. Write the ratio of the smaller diameter to the larger diameter.
27. Hard drives come in many diameters. Find the ratio of a $2\frac{1}{2}$ " diameter to a $5\frac{1}{4}$ " diameter hard drive.
28. Floppy disks come in many diameters. Find the ratio of a $3\frac{1}{2}$ " diameter to a $5\frac{1}{4}$ " diameter floppy disk.

- △ 29. Find the ratio of the longest side to the perimeter of the right-triangular-shaped billboard.



- △ 30. Find the ratio of the base to the perimeter of the triangular mainsail.



31. Citizens of Mexico consume the most Coca-Cola beverages per capita of any country in the world. Mexicans drink an average of the equivalent of about 744 8-oz beverages per year each. In contrast, the average American consumes about the equivalent of about 402 8-oz beverages per year. Find the ratio of the average amount of Coca-Cola beverages drunk by Mexicans to the average amount of Coca-Cola beverages drunk by Americans. (Source: Coca-Cola Company)
32. A large order of McDonald's french fries has 510 calories. Of this total, 216 calories are from fat. Find the ratio of the calories from fat to total calories in a large order of McDonald's french fries. (Source: McDonald's Corporation)

Objective B Write each rate as a fraction in simplest form. See Examples 7 and 8.

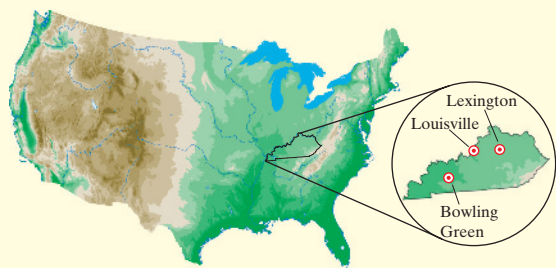
- ▶ 33. 5 shrubs every 15 feet
- 35. 15 returns for 100 sales
- ▶ 37. 6 laser printers for 28 computers
- 39. 18 gallons of pesticide for 4 acres of crops
- 34. 14 lab tables for 28 students
- 36. 8 phone lines for 36 employees
- 38. 4 inches of rain in 18 hours
- 40. 150 graduate students for 8 advisors

Objective C Write each rate as a unit rate. See Examples 9 and 10.

- 41. 330 calories in a 3-ounce serving
- ▶ 43. 375 riders in 5 subway cars
- 45. A hummingbird moves its wings at a rate of 5400 wingbeats a minute. Write this rate in wingbeats per second.
- 42. 275 miles in 11 hours
- 44. 18 signs in 6 blocks
- 46. A bat moves its wings at a rate of 1200 wingbeats a minute. Write this rate in wingbeats per second.



- ▶ 47. A \$1,000,000 lottery winning paid over 20 years
- 49. The state of Kentucky has 3,311,950 registered voters for two senators. (Source: Kentucky.gov)
- 48. 400,000 library books for 8000 students
- 50. The 2020 projected population of Louisiana is approximately 4,588,800 residents for 64 parishes. (Note: Louisiana is the only U.S. state with parishes instead of counties.) (Source: Louisiana.gov)



- 51. 12,000 good assembly line products to 40 defective products
- 53. The combined salary for the top 15 players on the 2016 World Series Champion Chicago Cubs was approximately \$67,763,381. Round to the nearest dollar. (Source: ESPN)
- 52. 5,000,000 lottery tickets for 4 lottery winners
- 54. The top-grossing concert tour was the 2009–2011 U2 360° tour, which grossed over \$735,900 thousand for 110 shows worldwide. (Source: Pollstar)

- 55.** Charlie Catlett can assemble 250 computer boards in an 8-hour shift while Suellen Catlett can assemble 402 computer boards in a 12-hour shift.
- Find the unit rate of Charlie.
 - Find the unit rate of Suellen.
 - Who can assemble computer boards faster, Charlie or Suellen?
- 56.** Jerry Stein laid 713 bricks in 46 minutes while his associate, Bobby Burns, laid 396 bricks in 30 minutes.
- Find the unit rate of Jerry.
 - Find the unit rate of Bobby.
 - Who is the faster bricklayer?

For Exercises 57 and 58, round the rates to the nearest tenth.

- 57.** One student drove 400 miles in his car on 14.5 gallons of gasoline. His sister drove 270 miles in her truck on 9.25 gallons of gasoline.
- Find the unit rate of the car.
 - Find the unit rate of the truck.
 - Which vehicle gets better gas mileage?
- 58.** Charlotte Leal is a grocery scanner who can scan an average of 100 items in 3.5 minutes while her cousin Leo can scan 148 items in 5.5 minutes.
- Find the unit rate of Charlotte.
 - Find the unit rate of Leo.
 - Who is the faster scanner?

Objective D Find each unit price. See Example 11.

- 59.** \$57.50 for 5 compact discs
- 60.** \$0.87 for 3 apples
- 61.** \$1.19 for 7 bananas
- 62.** \$73.50 for 6 lawn chairs

Find each unit price and decide which is the better buy. Round to three decimal places. Assume that we are comparing different sizes of the same brand. See Example 12.

- 63.** Crackers:
\$3.29 for 8 ounces
\$4.79 for 12 ounces
- 64.** Pickles:
\$2.79 for 32 ounces
\$1.49 for 18 ounces
- 65.** Frozen orange juice:
\$1.89 for 16 ounces
\$0.69 for 6 ounces
- 66.** Eggs:
\$1.56 for a dozen
\$3.69 for a flat $\left(2\frac{1}{2}\text{ dozen}\right)$
- 67.** Soy sauce:
12 ounces for \$2.29
8 ounces for \$1.49
- 68.** Shampoo:
20 ounces for \$1.89
32 ounces for \$3.19
- 69.** Napkins:
100 for \$0.59
180 for \$0.93
- 70.** Crackers:
20 ounces for \$2.39
8 ounces for \$0.99

Review

See Section 5.4.

- 71.** $9\overline{)20.7}$
- 72.** $7\overline{)60.2}$
- 73.** $3.7\overline{)0.555}$
- 74.** $4.6\overline{)1.15}$

Concept Extensions

- 75.** Is the ratio $\frac{11}{15}$ the same as the ratio $\frac{15}{11}$? Explain your answer.
- 76.** Explain why the ratio $\frac{5}{1}$ is incorrect for Exercise 20.

Decide whether each value is a ratio written as a fraction in simplest form. If not, write it as a fraction in simplest form.

77. $\frac{6 \text{ inches}}{15 \text{ inches}}$

78. $4\frac{1}{2}$

79. A panty hose manufacturing machine will be repaired if the ratio of defective panty hose to good panty hose is at least 1 to 20. A quality control engineer found 10 defective panty hose in a batch of 200. Determine whether the machine should be repaired.

80. A grocer will refuse a shipment of tomatoes if the ratio of bruised tomatoes to the total batch is at least 1 to 10. A sample is found to contain 3 bruised tomatoes and 33 good tomatoes. Determine whether the shipment should be refused.

Fill in the table to calculate miles per gallon.

	Beginning Odometer Reading	Ending Odometer Reading	Miles Driven	Gallons of Gas Used	Miles per Gallon (round to the nearest tenth)
81.	29,286	29,543		13.4	
82.	16,543	16,895		15.8	
83.	79,895	80,242		16.1	
84.	31,623	32,056		11.9	

Find each unit rate.

85. The longest stairway is the service stairway for the Niesenbahn Cable railway near Spiez, Switzerland. It has 11,674 steps and rises to a height of 7759 feet. Find the unit rate of steps per foot rounded to the nearest tenth of a step. (Source: Guinness World Records)



86. In the United States in 2016, the total number of students enrolled in public schools was 50,400,000. There are 98,300 public schools. Write a unit rate in students per school. Round to the nearest whole. (Source: National Center for Education Statistics)



-  87. In 2016, 19 states had mandatory helmet laws. (Source: Insurance Institute for Highway Safety)
- a. Find the ratio of states with mandatory helmet laws to total U.S. states.
 - b. Find the ratio of states with mandatory helmet laws to states without mandatory helmet laws.
 -  c. Are your ratios for parts **a** and **b** the same? Explain why or why not.

 89. In your own words, define the phrase “unit rate.”

 91. Should the rate $\frac{3 \text{ lights}}{2 \text{ feet}}$ be written as $\frac{3}{2}$? Explain why or why not.

 88. Suppose that the amount of a product decreases, say from an 80-ounce container to a 70-ounce container, but the price of the container remains the same. Does the unit price increase or decrease? Explain why.

 90. In your own words, define the phrase “unit price.”

92. Find an item in the grocery store and calculate its unit price.

6.2 Proportions

Objective A Writing Proportions

A **proportion** is a statement that two ratios or rates are equal.

Proportion

A proportion states that two ratios are equal. If $\frac{a}{b}$ and $\frac{c}{d}$ are two ratios, then

$$\frac{a}{b} = \frac{c}{d}$$

is a proportion.

For example,

$$\frac{5}{6} = \frac{10}{12}$$

is a proportion. We can read this as, “5 is to 6 as 10 is to 12.”

Example 1 Write each sentence as a proportion.

- 12 diamonds is to 15 rubies as 4 diamonds is to 5 rubies.
- 5 hits is to 9 at bats as 20 hits is to 36 at bats.

Solution:

- | | | | | | | |
|----------|---|-----------------|---|---------------|---|----------|
| diamonds | → | $\frac{12}{15}$ | = | $\frac{4}{5}$ | ← | diamonds |
| rubies | → | | | | ← | rubies |
- | | | | | | | |
|---------|---|---------------|---|-----------------|---|---------|
| hits | → | $\frac{5}{9}$ | = | $\frac{20}{36}$ | ← | hits |
| at bats | → | | | | ← | at bats |

Work Practice 1

Helpful Hint

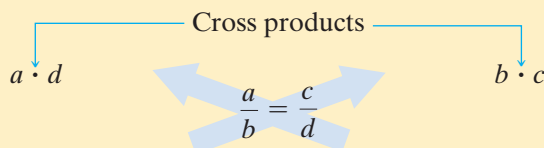
Notice in the above examples of proportions that the numerators contain the same units and the denominators contain the same units. In this text, proportions will be written so that this is the case.

Objective B Determining Whether Proportions Are True

Like other mathematical statements, a proportion may be either true or false. A proportion is true if its ratios are equal. Since ratios are fractions, one way to determine whether a proportion is true is to write both fractions in simplest form and compare them.

Another way is to compare cross products as we did in Section 4.2.

Using Cross Products to Determine Whether Proportions Are True or False



If cross products are **equal**, the proportion is **true**. If cross products are **not equal**, the proportion is **false**.

Objectives

- Write Sentences as Proportions.
- Determine Whether Proportions Are True.
- Find an Unknown Number in a Proportion.

Practice 1

Write each sentence as a proportion.

- 24 cups is to 6 cups as 4 cups is to 1 cup.
- 560 students is to 25 instructors as 112 students is to 5 instructors.

Answers

- $\frac{24}{6} = \frac{4}{1}$
 - $\frac{560}{25} = \frac{112}{5}$

Practice 2

Is $\frac{4}{8} = \frac{10}{20}$ a true proportion?

Practice 3

Is $\frac{4.2}{6} = \frac{4.8}{8}$ a true proportion?

Practice 4

Is $\frac{3\frac{3}{10}}{1\frac{5}{6}} = \frac{4\frac{1}{5}}{2\frac{1}{3}}$ a true proportion?

Example 2 Is $\frac{2}{3} = \frac{4}{6}$ a true proportion?

Solution:

Cross products

$$2 \cdot 6 \quad \quad \quad 3 \cdot 4$$

$$\frac{2}{3} = \frac{4}{6}$$

$$2 \cdot 6 \stackrel{?}{=} 3 \cdot 4 \quad \text{Are cross products equal?}$$

$$12 = 12 \quad \text{Equal, so proportion is true.}$$

Since the cross products are equal, the proportion is true.

Work Practice 2

Example 3 Is $\frac{4.1}{7} = \frac{2.9}{5}$ a true proportion?

Solution:

Cross products

$$4.1 \cdot 5 \quad \quad \quad 7 \cdot 2.9$$

$$\frac{4.1}{7} = \frac{2.9}{5}$$

$$4.1 \cdot 5 \stackrel{?}{=} 7 \cdot 2.9 \quad \text{Are cross products equal?}$$

$$20.5 \neq 20.3 \quad \text{Not equal, so proportion is false.}$$

Since the cross products are not equal, $\frac{4.1}{7} \neq \frac{2.9}{5}$. The proportion is false.

Work Practice 3

Example 4 Is $\frac{1\frac{1}{6}}{10\frac{1}{2}} = \frac{\frac{1}{2}}{4\frac{1}{2}}$ a true proportion?

Solution:

$$\frac{1\frac{1}{6}}{10\frac{1}{2}} = \frac{\frac{1}{2}}{4\frac{1}{2}}$$

$$1\frac{1}{6} \cdot 4\frac{1}{2} \stackrel{?}{=} 10\frac{1}{2} \cdot \frac{1}{2} \quad \text{Are cross products equal?}$$

$$\frac{7}{6} \cdot \frac{9}{2} \stackrel{?}{=} \frac{21}{2} \cdot \frac{1}{2} \quad \text{Write mixed numbers as improper fractions.}$$

$$\frac{21}{4} = \frac{21}{4} \quad \text{Equal, so proportion is true.}$$

Since the cross products are equal, the proportion is true.

Work Practice 4

✓ **Concept Check** Using the numbers in the proportion $\frac{21}{27} = \frac{7}{9}$, write two other true proportions.

Objective C Finding Unknown Numbers in Proportions

When one number of a proportion is unknown, we can use cross products to find the unknown number. For example, to find the unknown number x in the proportion

$\frac{2}{3} = \frac{x}{30}$, we use cross products.

Answers

2. yes 3. no 4. yes

✓ **Concept Check Answer**

possible answers: $\frac{27}{21} = \frac{9}{7}$ and $\frac{9}{27} = \frac{7}{21}$

Example 5 Solve $\frac{2}{3} = \frac{x}{30}$ for x .

Solution: If the cross products are equal, then the proportion is true. We begin, then, by setting cross products equal to each other.

$$\frac{2}{3} = \frac{x}{30}$$

$$2 \cdot 30 = 3 \cdot x \quad \text{Set cross products equal.}$$

$$60 = 3x \quad \text{Multiply.}$$

Recall that to find x , we divide both sides of the equation by 3.

$$\frac{60}{3} = \frac{3x}{3} \quad \text{Divide both sides by 3.}$$

$$20 = x \quad \text{Simplify.}$$

Check: To check, we replace x with 20 in the original proportion to see if the result is a true statement.

$$\frac{2}{3} = \frac{x}{30} \quad \text{Original proportion}$$

$$\frac{2}{3} \stackrel{?}{=} \frac{20}{30} \quad \text{Replace } x \text{ with 20.}$$

$$\frac{2}{3} = \frac{2}{3} \quad \text{True}$$

Since $\frac{2}{3} = \frac{2}{3}$ is a true statement, 20 is the solution.

Work Practice 5

Example 6 Solve $\frac{51}{34} = \frac{-3}{x}$ for x .

Solution:

$$\frac{51}{34} = \frac{-3}{x}$$

$$51 \cdot x = 34 \cdot -3 \quad \text{Set cross products equal.}$$

$$51x = -102 \quad \text{Multiply.}$$

$$\frac{51x}{51} = \frac{-102}{51} \quad \text{Divide both sides by 51.}$$

$$x = -2 \quad \text{Simplify.}$$

Check: $\frac{51}{34} = \frac{-3}{x} \quad \text{Original proportion}$

$$\frac{51}{34} \stackrel{?}{=} \frac{-3}{-2} \quad \text{Replace } x \text{ with } -2.$$

$$\frac{51}{34} \stackrel{?}{=} \frac{-3 \cdot -17}{-2 \cdot -17}$$

$$\frac{51}{34} = \frac{51}{34} \quad \text{True}$$

Work Practice 6

Practice 5

Solve $\frac{2}{5} = \frac{x}{25}$ for x .

Practice 6

Solve $\frac{-15}{2} = \frac{60}{x}$ for x .

Answers

5. 10 6. -8

Practice 7

Solve for z:

$$\frac{7}{8} = \frac{2}{z}$$

Practice 8Solve for y: $\frac{y}{9} = \frac{0.6}{1.2}$ **Answers**7. $\frac{3}{4}$ 8. 4.5**Example 7**Solve for y: $\frac{1}{2} = \frac{3}{\frac{4}{5}y}$ **Solution:**

$$\frac{1}{2} = \frac{3}{\frac{4}{5}y}$$

$$\frac{1}{2} \cdot y = \frac{4}{5} \cdot \frac{3}{4} \quad \text{Set cross products equal.}$$

$$\frac{1}{2}y = \frac{3}{5} \quad \text{Multiply.}$$

$$2 \cdot \frac{1}{2}y = 2 \cdot \frac{3}{5} \quad \text{Multiply both sides by 2.}$$

$$y = \frac{6}{5} \quad \text{Simplify.}$$

Verify that $\frac{6}{5}$ is the solution.**Check:** We will check by cross products.

$$\frac{1}{2} = \frac{3}{\frac{4}{5}y}$$

$$\frac{1}{2} = \frac{3}{\frac{4}{5} \cdot \frac{6}{5}}$$

$$\frac{1}{2} \cdot \frac{6}{5} = \frac{4}{5} \cdot \frac{3}{4} \quad \text{Set cross products equal.}$$

$$\frac{3}{5} = \frac{3}{5} \quad \text{True}$$

True, so the solution is $\frac{6}{5}$.**Work Practice 7****Example 8**Solve for x: $\frac{x}{3} = \frac{0.8}{1.5}$ **Solution:**

$$\frac{x}{3} = \frac{0.8}{1.5}$$

$$x \cdot 1.5 = 3(0.8) \quad \text{Set cross products equal.}$$

$$1.5x = 2.4 \quad \text{Multiply.}$$

$$\frac{1.5x}{1.5} = \frac{2.4}{1.5} \quad \text{Divide both sides by 1.5.}$$

$$x = 1.6 \quad \text{Simplify.}$$

Check:

$$\frac{x}{3} = \frac{0.8}{1.5}$$

$$\frac{1.6}{3} = \frac{0.8}{1.5}$$

$$(1.6)(1.5) = 3(0.8) \quad \text{Set cross products equal.}$$

$$2.4 = 2.4 \quad \text{True}$$

True, so the solution is 1.6.

Work Practice 8

Example 9 Solve for y : $\frac{14}{y} = \frac{12}{16}$

Solution:

$$\frac{14}{y} = \frac{12}{16}$$

$$14 \cdot 16 = y \cdot 12 \quad \text{Set cross products equal.}$$

$$224 = 12y \quad \text{Multiply.}$$

$$\frac{224}{12} = \frac{12y}{12} \quad \text{Divide both sides by 12.}$$

$$\frac{56}{3} = y \quad \text{Simplify.}$$

Check to see that the solution is $\frac{56}{3}$.

Work Practice 9

Helpful Hint

In Example 9, the fraction $\frac{12}{16}$ may be simplified to $\frac{3}{4}$ before solving the equation. The solution will remain the same.

✓ Concept Check True or false: The first step in solving the proportion $\frac{4}{z} = \frac{12}{15}$ yields the equation $4z = 180$. If false, give the correct cross product equation.

Example 10 Solve for x : $\frac{1.6}{1.1} = \frac{x}{0.3}$. Round the solution to the nearest hundredth.

Solution:

$$\frac{1.6}{1.1} = \frac{x}{0.3}$$

$$(1.6)(0.3) = 1.1 \cdot x \quad \text{Set cross products equal.}$$

$$0.48 = 1.1x \quad \text{Multiply.}$$

$$\frac{0.48}{1.1} = \frac{1.1x}{1.1} \quad \text{Divide both sides by 1.1.}$$

$$0.44 \approx x \quad \text{Round to the nearest hundredth.}$$

Work Practice 10

Practice 9

Solve for z : $\frac{17}{z} = \frac{8}{10}$

Practice 10

Solve for y : $\frac{4.5}{1.8} = \frac{y}{3}$

Answers

9. $\frac{85}{4}$ or 21.25 10. 7.5

✓ Concept Check Answer
false; $60 = 12z$

Vocabulary, Readiness & Video Check

Use the words and phrases below to fill in each blank.

ratio
false

cross products
proportion

true

- $\frac{4.2}{8.4} = \frac{1}{2}$ is called a _____ while $\frac{7}{8}$ is called a _____.
- In $\frac{a}{b} = \frac{c}{d}$, $a \cdot d$ and $b \cdot c$ are called _____.
- In a proportion, if cross products are equal, the proportion is _____.
- In a proportion, if cross products are not equal, the proportion is _____.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 6.2

- Objective A** 5. From Example 1, what does “as” translate to in a proportion statement?
- Objective B** 6. What types of fractions are in the equation in Example 3?
- Objective C** 7. How do we know the problem in Example 6 is a proportion?
8. In checking our answer to Example 6, why won't the cross products be exactly equal?

6.2 Exercise Set MyLab Math

Objective A Translating Write each sentence as a proportion. See Example 1.

1. 10 diamonds is to 6 opals as 5 diamonds is to 3 opals.
2. 1 raisin is to 5 cornflakes as 8 raisins is to 40 cornflakes.
3. 20 students is to 5 microscopes as 4 students is to 1 microscope.
4. 4 hit songs is to 16 releases as 1 hit song is to 4 releases.
5. 6 eagles is to 58 sparrows as 3 eagles is to 29 sparrows.
6. 12 errors is to 8 pages as 1.5 errors is to 1 page.
7. $2\frac{1}{4}$ cups of flour is to 24 cookies as $6\frac{3}{4}$ cups of flour is to 72 cookies.
8. $1\frac{1}{2}$ cups milk is to 10 bagels as $\frac{3}{4}$ cup milk is to 5 bagels.
9. 22 vanilla wafers is to 1 cup of cookie crumbs as 55 vanilla wafers is to 2.5 cups of cookie crumbs.
(Source: Based on data from *Family Circle* magazine)
10. 1 cup of instant rice is to 1.5 cups cooked rice as 1.5 cups of instant rice is to 2.25 cups of cooked rice.
(Source: Based on data from *Family Circle* magazine)

Objective B Determine whether each proportion is true or false. See Examples 2 through 4.

11. $\frac{15}{9} = \frac{5}{3}$

12. $\frac{8}{6} = \frac{20}{15}$

13. $\frac{5}{8} = \frac{4}{7}$

14. $\frac{7}{3} = \frac{9}{5}$

15. $\frac{9}{36} = \frac{2}{8}$

16. $\frac{8}{24} = \frac{3}{9}$

17. $\frac{5}{8} = \frac{625}{1000}$

18. $\frac{30}{50} = \frac{600}{1000}$

19. $\frac{0.8}{0.3} = \frac{0.2}{0.6}$

20. $\frac{0.7}{0.4} = \frac{0.3}{0.1}$

21. $\frac{4.2}{8.4} = \frac{5}{10}$

22. $\frac{8}{10} = \frac{5.6}{0.7}$

▶ 23. $\frac{\frac{3}{4}}{\frac{4}{3}} = \frac{\frac{1}{2}}{\frac{8}{9}}$

24. $\frac{\frac{2}{3}}{\frac{8}{5}} = \frac{\frac{1}{7}}{\frac{3}{10}}$

25. $\frac{\frac{2\frac{2}{5}}{2}}{\frac{5}{3}} = \frac{\frac{10}{9}}{\frac{1}{4}}$

26. $\frac{5\frac{5}{8}}{\frac{5}{3}} = \frac{4\frac{1}{2}}{1\frac{1}{5}}$

27. $\frac{\frac{4}{5}}{\frac{5}{6}} = \frac{\frac{6}{5}}{\frac{5}{9}}$

28. $\frac{\frac{6}{7}}{\frac{3}{5}} = \frac{\frac{10}{7}}{\frac{7}{5}}$

Objectives A B Mixed Practice—Translating Write each sentence as a proportion. Then determine whether the proportion is a true proportion. See Examples 1 through 4.

29. Ten is to fifteen as four is to six.

30. Six is to eight as nine is to twelve.

31. Eleven is to four as five is to two.

32. Five is to three as seven is to five.

33. Fifteen hundredths is to three as thirty-five hundredths is to seven.

34. One and eight tenths is to two as four and five tenths is to five.

35. Two thirds is to one fifth as two fifths is to one ninth.

36. Ten elevenths is to three fourths as one fourth is to one half.

Objective C Solve each proportion for the given variable. Round the solution where indicated. See Examples 5 through 10.

37. $\frac{x}{5} = \frac{6}{10}$

38. $\frac{x}{3} = \frac{12}{9}$

▶ 39. $\frac{-18}{54} = \frac{3}{n}$

40. $\frac{25}{100} = \frac{-7}{n}$

41. $\frac{30}{10} = \frac{15}{y}$

42. $\frac{16}{20} = \frac{z}{35}$

43. $\frac{8}{15} = \frac{z}{6}$

44. $\frac{12}{10} = \frac{z}{16}$

45. $\frac{24}{x} = \frac{60}{96}$

46. $\frac{26}{x} = \frac{28}{49}$

47. $\frac{-3.5}{12.5} = \frac{-7}{n}$

48. $\frac{-0.2}{0.7} = \frac{-8}{n}$

▶ 49. $\frac{n}{0.6} = \frac{0.05}{12}$

50. $\frac{7.8}{13} = \frac{n}{2.6}$

51. $\frac{\frac{8}{1}}{\frac{1}{3}} = \frac{24}{n}$

52. $\frac{12}{\frac{3}{4}} = \frac{48}{n}$

53. $\frac{\frac{1}{3}}{\frac{3}{8}} = \frac{\frac{2}{5}}{n}$

54. $\frac{\frac{7}{9}}{\frac{8}{27}} = \frac{\frac{1}{4}}{n}$

55. $\frac{\frac{12}{n}}{\frac{3}{9}} = \frac{\frac{2}{3}}{\frac{6}{9}}$

56. $\frac{24}{n} = \frac{\frac{8}{15}}{\frac{5}{9}}$

$$57. \frac{n}{1\frac{1}{5}} = \frac{4\frac{1}{6}}{6\frac{2}{3}}$$

$$58. \frac{n}{3\frac{1}{8}} = \frac{7\frac{3}{5}}{2\frac{3}{8}}$$

$$59. \frac{25}{n} = \frac{3}{\frac{7}{30}}$$

$$60. \frac{9}{n} = \frac{5}{\frac{11}{15}}$$

$$61. \frac{3.2}{0.3} = \frac{x}{1.4}$$

Round to the nearest tenth.

$$62. \frac{1.8}{z} = \frac{2.5}{8.4}$$

Round to the nearest tenth.

$$63. \frac{z}{5.2} = \frac{0.08}{6}$$

Round to the nearest hundredth.

$$64. \frac{4.25}{6.03} = \frac{5}{y}$$

Round to the nearest hundredth.

 65. $\frac{7}{18} = \frac{x}{5}$

Round to the nearest tenth.

$$66. \frac{17}{x} = \frac{9}{4}$$

Round to the nearest thousandth.

$$67. \frac{43}{17} = \frac{8}{z}$$

Round to the nearest thousandth.

$$68. \frac{x}{12} = \frac{18}{7}$$

Round to the nearest hundredth.

Review

Insert $<$ or $>$ to form a true statement. See Sections 4.7 and 5.1.

$$69. 8.01 \quad 8.1$$

$$70. 7.26 \quad 7.026$$

$$71. 2\frac{1}{2} \quad 2\frac{1}{3}$$

$$72. 9\frac{1}{5} \quad 9\frac{1}{4}$$

Simplify each fraction. See Section 4.2.

$$73. \frac{75}{125}$$

$$74. \frac{11y}{99y}$$

$$75. \frac{12x}{42}$$

$$76. \frac{28y^2}{42y^3}$$

Concept Extensions

Use the numbers in each proportion to write two other true proportions. See the first Concept Check in this section.

$$77. \frac{9}{15} = \frac{3}{5}$$

$$78. \frac{1}{4} = \frac{5}{20}$$

$$79. \frac{6}{18} = \frac{1}{3}$$

$$80. \frac{2}{7} = \frac{4}{14}$$

81. If the proportion $\frac{a}{b} = \frac{c}{d}$ is a true proportion, write two other true proportions using the same letters.

82. Write a true proportion.

 83. Explain the difference between a ratio and a proportion.

 84. Explain how to find the unknown number in a proportion such as $\frac{n}{18} = \frac{12}{8}$.

For each proportion, solve for the variable.

$$85. \frac{x}{7} = \frac{0}{8}$$

$$86. \frac{0}{2} = \frac{y}{3.5}$$

$$\text{87. } \frac{z}{1150} = \frac{588}{483}$$

$$\text{88. } \frac{585}{x} = \frac{117}{474}$$

$$\text{89. } \frac{222}{1515} = \frac{37}{y}$$

$$\text{90. } \frac{1425}{1062} = \frac{z}{177}$$

Ratio, Rate, and Proportion

Write each ratio as a ratio of whole numbers using fractional notation. Write the fraction in simplest form.

1. 27 to 30
2. 18 to 50
3. 9.4 to 10
4. 3.2 to 9.2
5. 8.65 to 6.95
6. 3.6 to 4.2
7. $\frac{7}{2}$ to 13
8. $1\frac{2}{3}$ to $2\frac{3}{4}$
9. 16 inches to 24 inches
10. 5 hours to 40 hours

Find the ratio described in each problem.

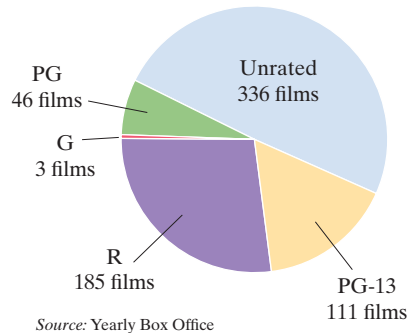
11. Find the ratio of the width (shorter side) to the length (longer side) of the sign below.



12. The circle graph below shows the ratings of films released in 2016. Use this graph to answer the questions.

- a. How many films were rated R?
- b. Find the ratio of PG-13 films to total films.

Rating Distribution of Films Released in 2016



Write each rate as a fraction in simplest form.

13. 4 professors for every 20 graduate assistants
14. 6 lights every 20 feet
15. 100 U.S. Senators for 50 states
16. 5 teachers for every 140 students
17. 21 inches every 7 seconds
18. \$40 every 5 hours

Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. a. _____
b. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____

19. _____

20. _____

21. _____

22. _____

23. _____

24. _____

25. _____

26. _____

27. _____

28. _____

29. _____

30. _____

31. _____

32. _____

33. _____

34. _____

35. _____

36. _____

37. _____

38. _____

19. 76 households with computers for every 100 households

20. 538 electoral votes for 50 states

Write each rate as a unit rate.

21. 560 feet in 4 seconds

22. 195 miles in 3 hours

23. 63 employees per 3 fax lines

24. 85 phone calls for 5 teenagers

25. 156 miles per 6 gallons

26. 112 teachers for 7 computers

27. 8125 books for 1250 college students

28. 2310 pounds for 14 adults

*Write each unit price and decide which is the better buy. Round to 3 decimal places.*29. Cat food:
8 pounds for \$2.16
18 pounds for \$4.9930. Paper plates:
100 for \$1.98
500 for \$8.9931. Microwave popcorn:
3 packs for \$2.39
8 packs for \$5.9932. AA batteries:
4 for \$4.69
10 for \$14.89*Determine whether each proportion is true.*

33. $\frac{7}{4} = \frac{5}{3}$

34. $\frac{8.2}{2} = \frac{16.4}{4}$

Solve each proportion for the given variable.

35. $\frac{5}{3} = \frac{40}{x}$

36. $\frac{y}{10} = \frac{13}{4}$

37. $\frac{6}{11} = \frac{z}{5}$

38. $\frac{21}{x} = \frac{\frac{7}{2}}{3}$

6.3 Proportions and Problem Solving

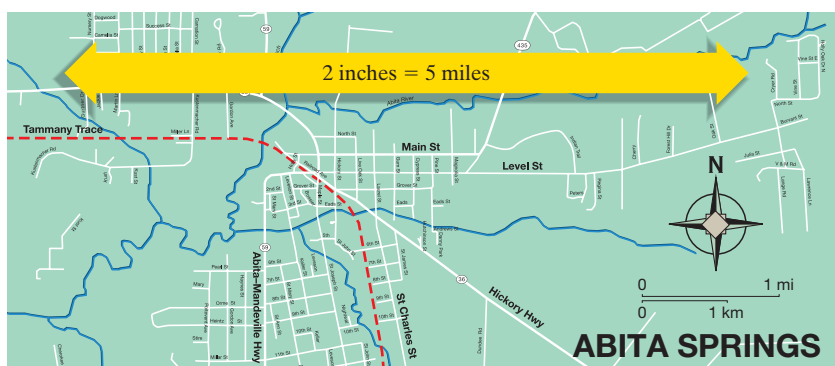
Objective A Solving Problems by Writing Proportions

Writing proportions is a powerful tool for solving problems in almost every field, including business, chemistry, biology, health sciences, and engineering, as well as in daily life. Given a specified ratio (or rate) of two quantities, a proportion can be used to determine an unknown quantity.

In this section, we use the same problem-solving steps that we have used earlier in this text.

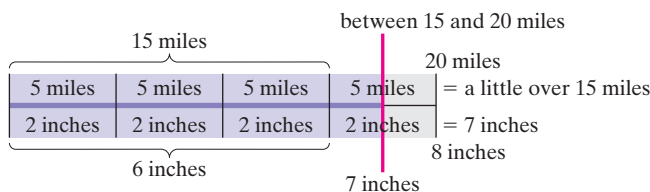
Example 1 Determining Distances from a Map

On a chamber of commerce map of Abita Springs, 5 miles corresponds to 2 inches. How many miles correspond to 7 inches?



Solution:

1. UNDERSTAND. Read and reread the problem. You may want to draw a diagram.



From the diagram we can see that a reasonable solution should be between 15 and 20 miles.

2. TRANSLATE. We will let x be our unknown number. Since 5 miles corresponds to 2 inches as x miles corresponds to 7 inches, we have the proportion

$$\begin{array}{lcl} \text{miles} & \rightarrow & \frac{5}{2} = \frac{x}{7} \leftarrow \text{miles} \\ \text{inches} & \rightarrow & \end{array}$$

3. SOLVE: In earlier sections, we estimated to obtain a reasonable answer. Notice we did this in Step 1 above.

$$\frac{5}{2} = \frac{x}{7}$$

$$5 \cdot 7 = 2 \cdot x$$

$$35 = 2x$$

$$\frac{35}{2} = \frac{2x}{2}$$

$$17 \frac{1}{2} = x \text{ or } x = 17.5$$

Set the cross products equal to each other.

Multiply.

Divide both sides by 2.

Simplify.

(Continued on next page)

Objective

- A Solve Problems by Writing Proportions.

Practice 1

On an architect's blueprint, 1 inch corresponds to 4 feet. How long is a wall represented by a $4 \frac{1}{4}$ -inch line on the blueprint?

Answer

1. 17 ft

4. **INTERPRET.** Check your work. This result is reasonable since it is between 15 and 20 miles. State your conclusion: 7 inches corresponds to 17.5 miles.

Work Practice 1

Helpful Hint

We can also solve Example 1 by writing the proportion

$$\frac{2 \text{ inches}}{5 \text{ miles}} = \frac{7 \text{ inches}}{x \text{ miles}}$$

Although other proportions may be used to solve Example 1, we will solve by writing proportions so that the numerators have the same unit measures and the denominators have the same unit measures.

Practice 2

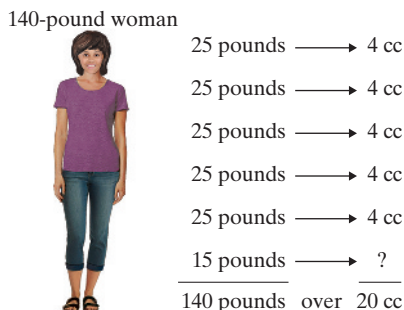
An auto mechanic recommends that 5 ounces of isopropyl alcohol be mixed with a tankful of gas (16 gallons) to increase the octane of the gasoline for better engine performance. At this rate, how many gallons of gas can be treated with an 8-ounce bottle of alcohol?

Example 2 Finding Medicine Dosage

The standard dose of an antibiotic is 4 cc (cubic centimeters) for every 25 pounds (lb) of body weight. At this rate, find the standard dose for a 140-lb woman.

Solution:

1. **UNDERSTAND.** Read and reread the problem. You may want to draw a diagram to estimate a reasonable solution.



From the diagram, we can see that a reasonable solution is a little over 20 cc.

2. **TRANSLATE.** We will let x be the unknown number. From the problem, we know that 4 cc is to 25 pounds as x cc is to 140 pounds, or

$$\begin{array}{ccccc} \text{cubic centimeters} & \rightarrow & 4 & = & \frac{x}{140} & \leftarrow & \text{cubic centimeters} \\ \text{pounds} & \rightarrow & 25 & = & 140 & \leftarrow & \text{pounds} \end{array}$$

3. **SOLVE:**

$$\frac{4}{25} = \frac{x}{140}$$

$$4 \cdot 140 = 25 \cdot x$$

$$560 = 25x$$

$$\frac{560}{25} = \frac{25x}{25}$$

$$22\frac{2}{5} = x \text{ or } x = 22.4$$

Set the cross products equal to each other.

Multiply.

Divide both sides by 25.

Simplify.

4. **INTERPRET.** Check your work. This result is reasonable since it is a little over 20 cc. State your conclusion: The standard dose for a 140-lb woman is 22.4 cc.

Work Practice 2

Answer

2. $25\frac{3}{5}$ or 25.6 gal

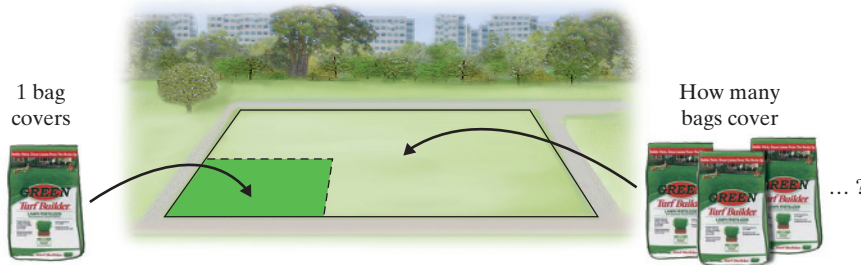


Example 3 Calculating Supplies Needed to Fertilize a Lawn

A 50-pound bag of fertilizer covers 2400 square feet of lawn. How many bags of fertilizer are needed to cover a town square containing 15,360 square feet of lawn? Round the answer up to the nearest whole bag.

Solution:

1. UNDERSTAND. Read and reread the problem. Draw a picture.



Since one bag covers 2400 square feet, let's see how many 2400s there are in 15,360. We will estimate. The number 15,360 rounded to the nearest thousand is 15,000 and 2400 rounded to the nearest thousand is 2000. Then $15,000 \div 2000 = 7\frac{1}{2}$ or 7.5.

2. TRANSLATE. We'll let x be the unknown number. From the problem, we know that 1 bag is to 2400 square feet as x bags is to 15,360 square feet.

$$\begin{array}{ccccc} \text{bags} & \rightarrow & 1 & = & x & \leftarrow & \text{bags} \\ \text{square feet} & \rightarrow & 2400 & = & 15,360 & \leftarrow & \text{square feet} \end{array}$$

3. SOLVE:

$$\frac{1}{2400} = \frac{x}{15,360}$$

$$1 \cdot 15,360 = 2400 \cdot x \quad \text{Set the cross products equal to each other.}$$

$$15,360 = 2400 \cdot x \quad \text{Multiply.}$$

$$\frac{15,360}{2400} = \frac{2400x}{2400} \quad \text{Divide both sides by 2400.}$$

$$6.4 = x \quad \text{Simplify.}$$

4. INTERPRET. Check that replacing x with 6.4 makes the proportion true. Is the answer reasonable? Yes, since it's close to $7\frac{1}{2}$ or 7.5. Since we must buy whole bags of fertilizer, 7 bags are needed. State your conclusion: To cover 15,360 square feet of lawn, 7 bags are needed.

Work Practice 3

✓ Concept Check You are told that 12 ounces of ground coffee will brew enough coffee to serve 20 people. How could you estimate how much ground coffee will be needed to serve 95 people?

Practice 3

If a gallon of paint covers 450 square feet, how many gallons are needed to paint a retaining wall that is 270 feet long and 11 feet high? Round the answer up to the nearest whole gallon.

Answer

3. 7 gal

✓ Concept Check Answer

Find how much will be needed for 100 people (20×5) by multiplying 12 ounces by 5, which is 60 ounces.

Martin-Gay Interactive Videos Watch the section lecture video and answer the following questions.



See Video 6.3

- Objective A**
1. In Example 1, rounding to the nearest week or whole number means rounding to what place value?
 2. What is the another way to set up a proportion to solve Example 2?

6.3 Exercise Set MyLab Math

Objective A Solve. For Exercises 1 and 2, the solutions have been started for you. See Examples 1 through 3. An NBA basketball player averages 45 baskets for every 100 attempts.

1. If he attempted 800 field goals, how many field goals did he make?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed. Let's let

x = how many field goals he made

2. TRANSLATE into an equation.

baskets (field goals) $\rightarrow \frac{45}{100} = \frac{x}{800}$ ← baskets (field goals)
attempts $\rightarrow$ ← attempts

3. SOLVE the equation. Set cross products equal to each other and solve.

$$\frac{45}{100} \times \frac{x}{800}$$

Finish by SOLVING and **4. INTERPRET.**

2. If he made 225 baskets, how many did he attempt?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed. Let's let

x = how many baskets attempted

2. TRANSLATE into an equation.

baskets $\rightarrow \frac{45}{100} = \frac{225}{x}$ ← baskets
attempts $\rightarrow$ ← attempts

3. SOLVE the equation. Set cross products equal to each other and solve.

$$\frac{45}{100} \times \frac{225}{x}$$

Finish by SOLVING and **4. INTERPRET.**

It takes a word processor 30 minutes to word process and spell check 4 pages.

3. Find how long it takes her to word process and spell check 22 pages.

4. Find how many pages she can word process and spell check in 4.5 hours.

University Law School accepts 2 out of every 7 applicants.

5. If the school accepted 180 students, find how many applications they received.

6. If the school accepted 150 students, find how many applications they received.

On an architect's blueprint, 1 inch corresponds to 8 feet.

7. Find the length of a wall represented by a line $2\frac{7}{8}$ inches long on the blueprint.

8. Find the length of a wall represented by a line $5\frac{1}{4}$ inches long on the blueprint.

A human-factors expert recommends that there be at least 9 square feet of floor space in a college classroom for every student in the class.

- △ 9. Find the minimum floor space that 30 students require.
- △ 10. Due to a lack of space, a university converts a 21-by-15-foot conference room into a classroom. Find the maximum number of students the room can accommodate.

A Honda Civic Hybrid averages 627 miles on a 12.3-gallon tank of gas.

11. Manuel Lopez is planning a 1250-mile vacation trip in his Honda Civic Hybrid. Find how many gallons of gas he can expect to burn. Round to the nearest gallon.
12. Ramona Hatch has enough money to put 6.9 gallons of gas in her Honda Civic Hybrid. She is planning on driving home from college for the weekend. If her home is 290 miles away, should she make it home before she runs out of gas?

The scale on an Italian map states that 1 centimeter corresponds to 30 kilometers.

13. Find how far apart Milan and Rome are if their corresponding points on the map are 15 centimeters apart.
14. On the map, a small Italian village is located 0.4 centimeter from the Mediterranean Sea. Find the actual distance.



A bag of Scott fertilizer covers 3000 square feet of lawn.

- △ 15. Find how many bags of fertilizer should be purchased to cover a rectangular lawn 260 feet by 180 feet.
- △ 16. Find how many bags of fertilizer should be purchased to cover a square lawn measuring 160 feet on each side.

A Cubs baseball player gets 3 hits every 8 times at bat.

17. If this Cubs player comes up to bat 40 times in the World Series, find how many hits he would be expected to get.
18. At this rate, if he got 12 hits, find how many times he batted.

A survey reveals that 2 out of 3 people prefer Coke to Pepsi.

19. In a room of 40 people, how many people are likely to prefer Coke? Round the answer to the nearest person.
20. In a college class of 36 students, find how many students are likely to prefer Pepsi.

A self-tanning lotion advertises that a 3-oz bottle will provide four applications.

21. Jen Haddad found a great deal on a 14-oz bottle of the self-tanning lotion she had been using. Based on the advertising claims, how many applications of the self-tanner should Jen expect? Round down to the nearest whole number.
22. The Community College thespians need fake tans for a play they are doing. If the play has a cast of 35, how many ounces of self-tanning lotion should the cast purchase? Round up to the next whole number of ounces.

The school's computer lab goes through 5 reams of printer paper every 3 weeks.

- ▶ 23. Find out how long a case of printer paper is likely to last (a case of paper holds 8 reams of paper). Round to the nearest week.
24. How many cases of printer paper should be purchased to last the entire semester of 15 weeks? Round up to the next case.

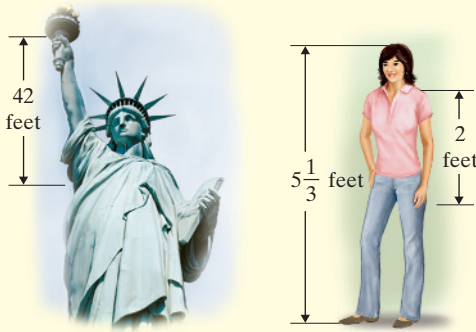
A recipe for pancakes calls for 2 cups flour and $1\frac{1}{2}$ cups milk to make a serving for four people.

25. Ming has plenty of flour, but only 4 cups milk. How many servings can he make?
26. The swim team has a weekly breakfast after early practice. How much flour will it take to make pancakes for 18 swimmers?
27. In the Seattle Space Needle, the elevators whisk you to the revolving restaurant at a speed of 800 feet in 60 seconds. If the revolving restaurant is 500 feet up, how long does it take you to reach the restaurant by elevator? (*Source: Seattle Space Needle*)
28. A 16-oz grande Tazo Black Iced Tea at Starbucks has 80 calories. How many calories are there in a 24-oz venti Tazo Black Iced Tea? (*Source: Starbucks Coffee Company*)
29. Mosquitos are annoying insects. To eliminate mosquito larvae, a certain granular substance can be applied to standing water in a ratio of 1 tsp per 25 sq ft of standing water.
- At this rate, find how many teaspoons of granules must be used for 450 square feet.
 - If 3 tsp = 1 tbsp, how many tablespoons of granules must be used?
30. Another type of mosquito control is liquid, where 3 oz of pesticide is mixed with 100 oz of water. This mixture is sprayed on roadsides to control mosquito breeding grounds hidden by tall grass.
- If one mixture of water with this pesticide can treat 150 feet of roadway, how many ounces of pesticide are needed to treat one mile? (*Hint: 1 mile = 5280 feet*)
 - If 8 liquid ounces equals one cup, write your answer to part **a** in cups. Round to the nearest cup.



31. The daily supply of oxygen for one person is provided by 625 square feet of lawn. A total of 3750 square feet of lawn would provide the daily supply of oxygen for how many people? (*Source: Professional Lawn Care Association of America*)
32. In 2015, approximately \$20 billion of the \$60 billion Americans spent on their pets was spent on pet food. Petsmart had about \$6,962,700,000 in net sales that year. How much of Petsmart's net sales would you expect to have been spent on pet food? (*Source: American Pet Products Manufacturers Association and Petsmart*)

33. A student would like to estimate the height of the Statue of Liberty in New York City's harbor. The length of the Statue of Liberty's right arm is 42 feet. The student's right arm is 2 feet long and her height is $5\frac{1}{3}$ feet. Use this information to estimate the height of the Statue of Liberty. How close is your estimate to the statue's actual height of 111 feet, 1 inch from heel to top of head? (Source: National Park Service)



34. The length of the Statue of Liberty's index finger is 8 feet while the height to the top of the head is about 111 feet. Suppose your measurements are proportionally the same as this statue's and your height is 5 feet.
- Use this information to find the proposed length of your index finger. Give the exact measurement and then a decimal rounded to the nearest hundredth.
 -  Measure your index finger and write it as a decimal in feet rounded to the nearest hundredth. How close is the length of your index finger to the answer to part a? Explain why.

- ▶ 35. There are 72 milligrams of cholesterol in a 3.5-ounce serving of lobster. How much cholesterol is in 5 ounces of lobster? Round to the nearest tenth of a milligram. (Source: The National Institutes of Health)
36. There are 76 milligrams of cholesterol in a 3-ounce serving of skinless chicken. How much cholesterol is in 8 ounces of chicken? (Source: USDA)
37. The Comcast Building in New York City is 850 feet tall and contains 69 stories (above ground). The Empire State Building contains 102 stories. If the Empire State Building has the same number of feet per floor as the Comcast Building, approximate its height rounded to the nearest foot. (Source: Skyscrapers.com)
38. Two out of every 5 men blame their poor eating habits on too much fast food. In a room of 40 men, how many would you expect to blame their not eating well on fast food? (Source: Healthy Choice Mixed Grills survey)
- ▶ 39. Medication is prescribed in 7 out of every 10 hospital emergency room visits that involve an injury. If a large urban hospital had 620 emergency room visits involving an injury in the past month, how many of these visits would you expect to have included a prescription for medication? (Source: National Center for Health Statistics)
40. Currently in the American population of people aged 65 years old and older, there are approximately 130 women for every 100 men. In a nursing home with 280 male residents over the age of 65, how many female residents over the age of 65 would be expected? (Source: U.S. Bureau of the Census)
41. One out of three American adults got his or her first job in the restaurant industry. In an office of 84 workers, how many of these people would you expect to have gotten their first job in the restaurant industry? (Source: National Restaurant Association)
42. One pound of firmly packed brown sugar yields $2\frac{1}{4}$ cups. How many pounds of brown sugar will be required in a recipe that calls for 6 cups of firmly packed brown sugar? (Source: Based on data from *Family Circle* magazine)

When making homemade ice cream in a hand-cranked freezer, the tub containing the ice cream mix is surrounded by a brine (water/salt) solution. To freeze the ice cream mix rapidly so that smooth and creamy ice cream results, the brine solution should combine crushed ice and rock salt in a ratio of 5 to 1. Use this for Exercises 43 and 44. (Source: White Mountain Freezers, The Rival Company)

- 43.** A small ice cream freezer requires 12 cups of crushed ice. How much rock salt should be mixed with the ice to create the necessary brine solution?
- 44.** A large ice cream freezer requires $18\frac{3}{4}$ cups of crushed ice. How much rock salt will be needed to create the necessary brine solution?
- 45.** The gas/oil ratio for a certain chainsaw is 50 to 1.
- How much oil (in gallons) should be mixed with 5 gallons of gasoline?
 - If 1 gallon equals 128 fluid ounces, write the answer to part **a** in fluid ounces. Round to the nearest whole ounce.
- 46.** The gas/oil ratio for a certain tractor mower is 20 to 1.
- How much oil (in gallons) should be mixed with 10 gallons of gas?
 - If 1 gallon equals 4 quarts, write the answer to part **a** in quarts.
- 47.** The adult daily dosage for a certain medicine is 150 mg (milligrams) of medicine for every 20 pounds of body weight.
- At this rate, find the daily dose for a man who weighs 275 pounds.
 - If the man is to receive 500 mg of this medicine every 8 hours, is he receiving the proper dosage?
- 48.** The adult daily dosage for a certain medicine is 80 mg (milligrams) for every 25 pounds of body weight.
- At this rate, find the daily dose for a woman who weighs 190 pounds.
 - If she is to receive this medicine every 6 hours, find the amount to be given every 6 hours.

Review

Find the prime factorization of each number. See Section 4.2.

49. 200

50. 300

51. 32

52. 81

Concept Extensions

As we have seen earlier, proportions are often used in medicine dosage calculations. The exercises below have to do with liquid drug preparations, where the weight of the drug is contained in a volume of solution. The description of mg and ml below will help. We will study metric units further in Chapter 9.

mg means milligrams (A paper clip weighs about a gram. A milligram is about the weight of $\frac{1}{1000}$ of a paper clip.)

ml means milliliter (A liter is about a quart. A milliliter is about the amount of liquid in $\frac{1}{1000}$ of a quart.)

One way to solve the applications below is to set up the proportion $\frac{\text{mg}}{\text{ml}} = \frac{\text{mg}}{\text{ml}}$.

A solution strength of 15 mg of medicine in 1 ml of solution is available.

- 53.** If a patient needs 12 mg of medicine, how many ml do you administer?
- 54.** If a patient needs 33 mg of medicine, how many ml do you administer?

A solution strength of 8 mg of medicine in 1 ml of solution is available.

- 55.** If a patient needs 10 mg of medicine, how many ml do you administer?
- 56.** If a patient needs 6 mg of medicine, how many ml do you administer?

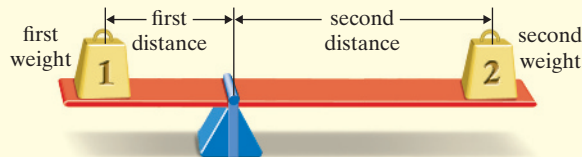
Estimate the following. See the Concept Check in this section.

57. It takes 1.5 cups of milk to make 11 muffins. Estimate the amount of milk needed to make 8 dozen muffins. Explain your calculation.

58. A favorite chocolate chip cookie recipe calls for $2\frac{1}{2}$ cups of flour to make 2 dozen cookies. Estimate the amount of flour needed to make 50 cookies. Explain your calculation.

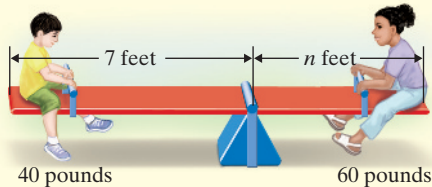
A board such as the one pictured below will balance if the following proportion is true:

$$\frac{\text{first weight}}{\text{second distance}} = \frac{\text{second weight}}{\text{first distance}}$$

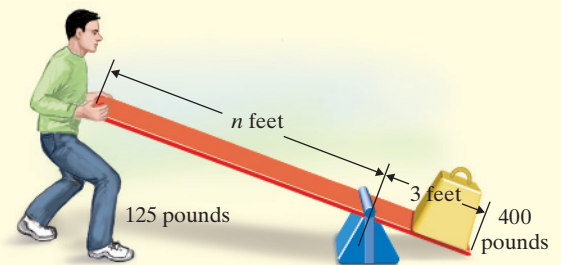


Use this proportion to solve Exercises 59 and 60.

59. Find the distance n that will allow the board to balance.



60. Find the length n needed to lift the weight below.



61. Describe a situation in which writing a proportion might solve a problem related to driving a car.

6.4 Square Roots and the Pythagorean Theorem

Now that we know how to write ratios and solve proportions, in Section 6.5 we use proportions to help us find unknown sides of similar triangles. In this section, we prepare for work on triangles by studying right triangles and their applications.

First, let's practice finding square roots.

Objective A Finding Square Roots

The **square** of a number is the number times itself. For example,

The square of 5 is 25 because 5^2 or $5 \cdot 5 = 25$.

The square of -5 is also 25 because $(-5)^2$ or $(-5)(-5) = 25$.

The reverse process of squaring is finding a **square root**. For example,

A square root of 25 is 5 because $5 \cdot 5$ or $5^2 = 25$.

A square root of 25 is also -5 because $(-5)(-5)$ or $(-5)^2 = 25$.

Objectives

- A** Find the Square Root of a Number.
- B** Approximate Square Roots.
- C** Use the Pythagorean Theorem.

Every positive number has two square roots. We see on the previous page that the square roots of 25 are 5 and -5 .

We use the symbol $\sqrt{\quad}$, called a **radical sign**, to indicate the positive square root of a nonnegative number. For example,

$$\sqrt{25} = 5 \text{ because } 5^2 = 25 \text{ and } 5 \text{ is positive.}$$

$$\sqrt{9} = 3 \text{ because } 3^2 = 9 \text{ and } 3 \text{ is positive.}$$

Square Root of a Number

The square root, $\sqrt{\quad}$, of a positive number a is the positive number b whose square is a . In symbols,

$$\sqrt{a} = b, \quad \text{if } b^2 = a$$

$$\text{Also, } \sqrt{0} = 0.$$

Helpful Hint

Remember that the radical sign $\sqrt{\quad}$ is used to indicate the **positive** (or principal) **square root** of a nonnegative number.

Practice 1–6

Find each square root.

1. $\sqrt{100}$
2. $\sqrt{64}$
3. $\sqrt{169}$
4. $\sqrt{0}$
5. $\sqrt{\frac{1}{4}}$
6. $\sqrt{\frac{9}{16}}$

Examples

Find each square root.

1. $\sqrt{49} = 7$ because $7^2 = 49$.
2. $\sqrt{36} = 6$ because $6^2 = 36$.
3. $\sqrt{1} = 1$ because $1^2 = 1$.
4. $\sqrt{81} = 9$ because $9^2 = 81$.
5. $\sqrt{\frac{1}{36}} = \frac{1}{6}$ because $\left(\frac{1}{6}\right)^2$ or $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$.
6. $\sqrt{\frac{4}{25}} = \frac{2}{5}$ because $\left(\frac{2}{5}\right)^2$ or $\frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25}$.

Work Practice 1–6

Objective B Approximating Square Roots

Thus far, we have found square roots of perfect squares. Numbers like $\frac{1}{4}$, 36 , $\frac{4}{25}$, and 1 are called **perfect squares** because their square root is a whole number or a fraction. A square root such as $\sqrt{5}$ cannot be written as a whole number or a fraction since 5 is not a perfect square.

Although $\sqrt{5}$ cannot be written as a whole number or a fraction, it can be approximated by estimating, by using a table (as in the appendix), or by using a calculator.

Example 7

Use Appendix A.4 or a calculator to approximate each square root to the nearest thousandth.

- a. $\sqrt{43} \approx 6.557$ is approximately
- b. $\sqrt{80} \approx 8.944$

Work Practice 7

Practice 7

Use Appendix A.4 or a calculator to approximate each square root to the nearest thousandth.

- a. $\sqrt{10}$
- b. $\sqrt{62}$

Answers

1. 10
2. 8
3. 13
4. 0
5. $\frac{1}{2}$
6. $\frac{3}{4}$
7. a. 3.162 b. 7.874

Helpful Hint

$\sqrt{80}$, on the previous page, is *approximately* 8.944. This means that if we multiply 8.944 by 8.944, the product is *close* to 80.

$$8.944 \times 8.944 \approx 79.995$$

It is possible to approximate a square root to the nearest whole number without the use of a calculator or table. To do so, study the number line below and look for patterns.



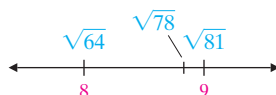
Above the number line, notice that as the numbers under the radical signs increase, their value, and thus their placement on the number line, increase also.

Example 8 Without a calculator or table:

- Determine which two whole numbers $\sqrt{78}$ is between.
- Use part **a** to approximate $\sqrt{78}$ to the nearest whole.

Solution:

- Review perfect squares and recall that $\sqrt{64} = 8$ and $\sqrt{81} = 9$. Since 78 is between 64 and 81, $\sqrt{78}$ is between $\sqrt{64}$ (or 8) and $\sqrt{81}$ (or 9).



Thus, $\sqrt{78}$ is between 8 and 9.

- Since $\sqrt{78}$ is closer to $\sqrt{81}$ (or 9) than $\sqrt{64}$ (or 8), then (as our number line shows), $\sqrt{78}$ approximated to the nearest whole is 9.

Work Practice 8**Practice 8**

Without a calculator or table, approximate $\sqrt{62}$ to the nearest whole.

Objective C Using the Pythagorean Theorem

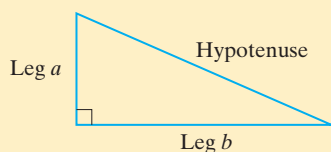
One important application of square roots has to do with right triangles. Recall that a **right triangle** is a triangle in which one of the angles is a right angle, or measures 90° (degrees). The **hypotenuse** of a right triangle is the side opposite the right angle. The **legs** of a right triangle are the other two sides. These are shown in the following figure. The right angle in the triangle is indicated by the small square drawn in that angle.

The following theorem is true for all right triangles.

Pythagorean Theorem

If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then

$$a^2 + b^2 = c^2$$

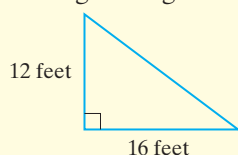


In other words, $(\text{leg})^2 + (\text{other leg})^2 = (\text{hypotenuse})^2$.

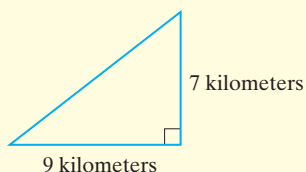
Answer
8. 8

Practice 9

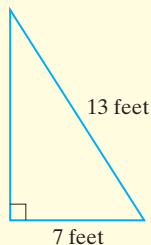
Find the length of the hypotenuse of the given right triangle.

**Practice 10**

Approximate the length of the hypotenuse of the given right triangle. Round to the nearest whole unit.

**Practice 11**

Find the length of the leg in the given right triangle. Give the exact length and a two-decimal-place approximation.

**Answers**

9. 20 feet 10. 11 kilometers
11. $\sqrt{120}$ feet $\approx$ 10.95 feet

✓ **Concept Check Answer**

a

Example 9

Find the length of the hypotenuse of the given right triangle.

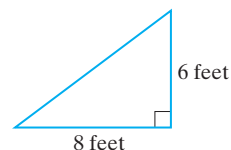
Solution: Let $a = 6$ and $b = 8$. According to the Pythagorean theorem,

$$a^2 + b^2 = c^2$$

$$6^2 + 8^2 = c^2 \quad \text{Let } a = 6 \text{ and } b = 8.$$

$$36 + 64 = c^2 \quad \text{Evaluate } 6^2 \text{ and } 8^2.$$

$$100 = c^2 \quad \text{Add.}$$



In the equation $c^2 = 100$, the solutions of c are the square roots of 100. Since $10 \cdot 10 = 100$ and $(-10)(-10) = 100$, both 10 and -10 are square roots of 100. Since c represents a length, we are only interested in the positive square root of c^2 .

$$\begin{aligned} c &= \sqrt{100} \\ &= 10 \end{aligned}$$

The hypotenuse is 10 feet long.

Work Practice 9

Example 10

Approximate the length of the hypotenuse of the given right triangle. Round the length to the nearest whole unit.

Solution: Let $a = 17$ and $b = 10$.

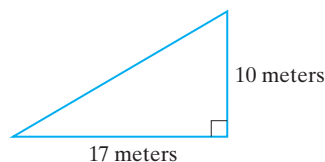
$$a^2 + b^2 = c^2$$

$$17^2 + 10^2 = c^2$$

$$289 + 100 = c^2$$

$$389 = c^2$$

$$\sqrt{389} = c \text{ or } c \approx 20 \quad \text{From Appendix A.4 or a calculator}$$

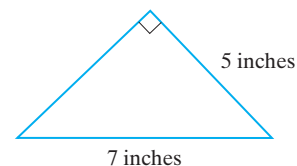


The hypotenuse is exactly $\sqrt{389}$ meters, which is approximately 20 meters.

Work Practice 10

Example 11

Find the length of the leg in the given right triangle. Give the exact length and a two-decimal-place approximation.



Solution: Notice that the hypotenuse measures 7 inches and that the length of one leg measures 5 inches. Thus, let $c = 7$ and a or b be 5. We will let $a = 5$.

$$a^2 + b^2 = c^2$$

$$5^2 + b^2 = 7^2$$

Let $a = 5$ and $c = 7$.

$$25 + b^2 = 49$$

Evaluate 5^2 and 7^2 .

$$b^2 = 24$$

Subtract 25 from both sides.

$$b = \sqrt{24} \approx 4.90$$

The length of the leg is exactly $\sqrt{24}$ inches and approximately 4.90 inches.

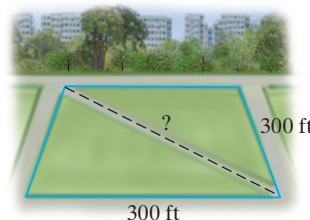
Work Practice 11

✓ **Concept Check** The following lists are the lengths of the sides of two triangles. Which set forms a right triangle?

- a. 8, 15, 17 b. 24, 30, 40

Example 12 Finding the Dimensions of a Park

An inner-city park is in the shape of a square that measures 300 feet on a side. A sidewalk is to be constructed along the diagonal of the park. Find the length of the sidewalk rounded to the nearest whole foot.



Solution: The diagonal is the hypotenuse of a right triangle, which we label c .

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 300^2 + 300^2 &= c^2 && \text{Let } a = 300 \text{ and } b = 300. \\ 90,000 + 90,000 &= c^2 && \text{Evaluate } (300)^2. \\ 180,000 &= c^2 && \text{Add.} \\ \sqrt{180,000} &= c \text{ or } c \approx 424 \end{aligned}$$

The length of the sidewalk is approximately 424 feet.

Work Practice 12

Practice 12

A football field is a rectangle measuring 100 yards by 53 yards. Draw a diagram and find the length of the diagonal of a football field to the nearest yard.

Answer

12. 113 yards



Calculator Explorations Finding and Approximating Square Roots

To simplify or approximate square roots using a calculator, locate the key marked $\sqrt{}$.

To simplify $\sqrt{64}$, for example, press the keys

$\boxed{64} \sqrt{}$ or $\sqrt{} \boxed{64}$

The display should read $\boxed{8}$. Then

$$\sqrt{64} = 8$$

To approximate $\sqrt{10}$, press the keys

$\boxed{10} \sqrt{}$ or $\sqrt{} \boxed{10}$

The display should read $\boxed{3.16227766}$. This is an approximation for $\sqrt{10}$. A three-decimal-place approximation is

$$\sqrt{10} \approx 3.162$$

Is this answer reasonable? Since 10 is between the perfect squares 9 and 16, $\sqrt{10}$ is between $\sqrt{9} = 3$ and $\sqrt{16} = 4$. Our answer is reasonable since 3.162 is between 3 and 4.

Simplify.

1. $\sqrt{1024}$
2. $\sqrt{676}$

Approximate each square root. Round each answer to the nearest thousandth.

3. $\sqrt{15}$
4. $\sqrt{19}$
5. $\sqrt{97}$
6. $\sqrt{56}$

Vocabulary, Readiness & Video Check

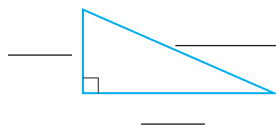
Use the choices below to fill in each blank. Some choices will be used more than once.

squaring	Pythagorean theorem	radical	-10	leg
hypotenuse	perfect squares	10	c^2	b^2

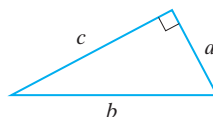
1. The square roots of 100 are _____ and _____ because $10 \cdot 10 = 100$ and $(-10)(-10) = 100$.
2. $\sqrt{100} = \underline{\hspace{2cm}}$ because $10 \cdot 10 = 100$ and 10 is positive.
3. The _____ sign is used to denote the positive square root of a nonnegative number.
4. The reverse process of _____ a number is finding a square root of a number.

5. The numbers 9, 1, and $\frac{1}{25}$ are called _____.

6. Label the parts of the right triangle.



7. In the given triangle, $a^2 + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$.



8. The _____ can be used for right triangles.

Martin-Gay Interactive Videos Watch the section lecture video and answer the following questions.



See Video 6.4

Objective A 9. From the lecture before [Example 1](#), list the square root(s) of 49. How is this different from finding $\sqrt{49}$?

Objective B 10. In [Example 5](#), how do we know $\sqrt{15}$ is closer to 4 than to 3?

Objective C 11. From [Example 6](#), how do we know which side of a right triangle is the hypotenuse?

6.4 Exercise Set MyLab Math

Objective A Find each square root. See Examples 1 through 6.

1. $\sqrt{4}$

2. $\sqrt{9}$

3. $\sqrt{121}$

4. $\sqrt{144}$

5. $\sqrt{\frac{1}{81}}$

6. $\sqrt{\frac{1}{64}}$

7. $\sqrt{\frac{16}{64}}$

8. $\sqrt{\frac{36}{81}}$

Objective B Use Appendix A.4 or a calculator to approximate each square root. Round the square root to the nearest thousandth. See Example 7.

9. $\sqrt{3}$

10. $\sqrt{5}$

11. $\sqrt{15}$

12. $\sqrt{17}$

13. $\sqrt{31}$

14. $\sqrt{85}$

15. $\sqrt{26}$

16. $\sqrt{35}$

Determine what two whole numbers each square root is between without using a calculator or table. Then use a calculator or Appendix A.4 to check. See Example 8.

17. $\sqrt{38}$

18. $\sqrt{27}$

19. $\sqrt{101}$

20. $\sqrt{85}$

Objectives A B Mixed Practice Find each square root. If necessary, round the square root to the nearest thousandth. See Examples 1 through 8.

21. $\sqrt{256}$

22. $\sqrt{625}$

23. $\sqrt{92}$

24. $\sqrt{18}$

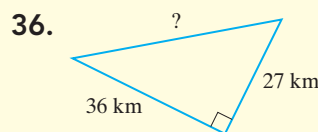
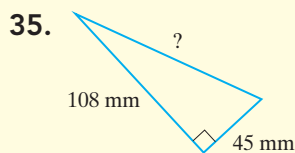
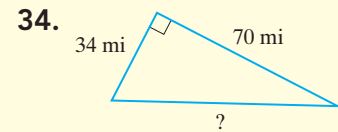
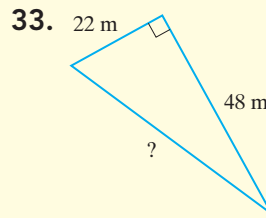
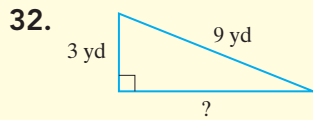
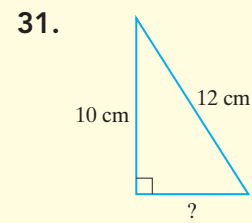
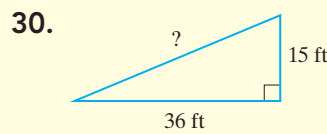
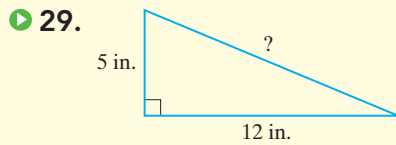
25. $\sqrt{\frac{49}{144}}$

26. $\sqrt{\frac{121}{169}}$

27. $\sqrt{71}$

28. $\sqrt{62}$

Objective C Find the unknown length in each right triangle. If necessary, approximate the length to the nearest thousandth. See Examples 9 through 12.



Sketch each right triangle and find the length of the side not given. If necessary, approximate the length to the nearest thousandth. (Each length is in units.) See Examples 9 through 12.

37. leg = 3, leg = 4

38. leg = 9, leg = 12

39. leg = 5, hypotenuse = 13

40. leg = 6, hypotenuse = 10

41. leg = 10, leg = 14

42. leg = 2, leg = 16

43. leg = 35, leg = 28

44. leg = 30, leg = 15

45. leg = 30, leg = 30

46. leg = 21, leg = 21

► 47. hypotenuse = 2, leg = 1

48. hypotenuse = 9, leg = 8

49. leg = 7.5, leg = 4

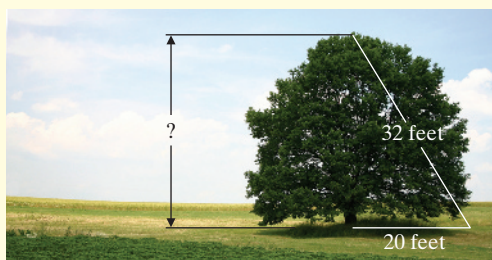
50. leg = 12, leg = 22.5

Solve. See Example 12.

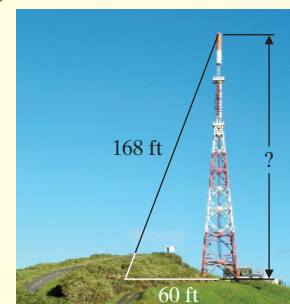
51. A standard city block is a square with each side measuring 100 yards. Find the length of the diagonal of a city block to the nearest hundredth yard.

52. A section of land is a square with each side measuring 1 mile. Find the length of the diagonal of the section of land to the nearest thousandth mile.

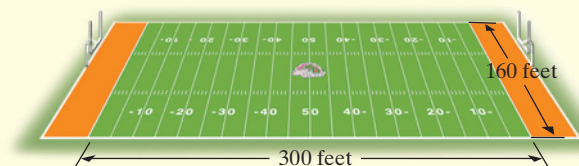
53. Find the height of the tree. Round the height to one decimal place.



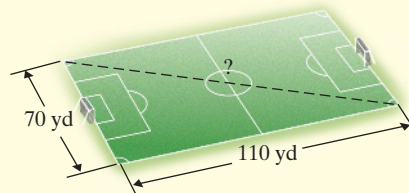
54. Find the height of the antenna. Round the height to one decimal place.



55. The playing field for football is a rectangle that is 300 feet long by 160 feet wide. Find the length of a straight-line run that started at one corner and went diagonally to end at the opposite corner. Round to the nearest foot, if necessary.



56. A soccer field is in the shape of a rectangle and its dimensions depend on the age of the players. The dimensions of the soccer field below are the minimum dimensions for international play. Find the length of the diagonal of this rectangle. Round the answer to the nearest tenth of a yard.



Review

Write each fraction in simplest form. See Section 4.2.

57. $\frac{10}{12}$

58. $\frac{10}{15}$

59. $\frac{2x}{60}$

60. $\frac{35}{75y}$

Perform the indicated operations. See Sections 4.3 and 4.4.

61. $\frac{9}{13y} + \frac{12}{13y}$

62. $\frac{3x}{9} - \frac{5}{9}$

63. $\frac{9}{8} \cdot \frac{x}{8}$

64. $\frac{7x}{11} \div \frac{8x}{11}$

Concept Extensions

Use the results of Exercises 17–20 and approximate each square root to the nearest whole without using a calculator or table. Then use a calculator or Appendix A.4 to check. See Example 8.

65. $\sqrt{38}$

66. $\sqrt{27}$

67. $\sqrt{101}$

68. $\sqrt{85}$

69. Without using a calculator, explain how you know that $\sqrt{105}$ is not approximately 9.875.

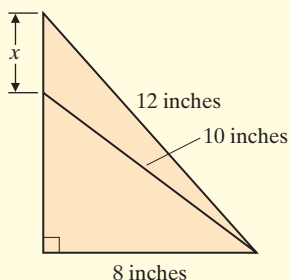
70. Without using a calculator, explain how you know that $\sqrt{27}$ is not approximately 3.296.

Does the set form the lengths of the sides of a right triangle? See the Concept Check in this section.

71. 25, 60, 65

72. 20, 45, 50

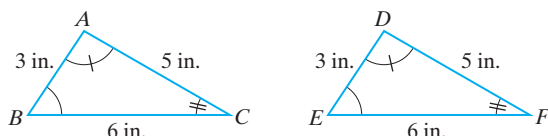
- △ 73. Find the exact length of x . Then give a two-decimal-place approximation.



6.5 Congruent and Similar Triangles

Objective A Deciding Whether Two Triangles Are Congruent

Two triangles are **congruent** when they have the same shape and the same size. In congruent triangles, the measures of corresponding angles are equal and the lengths of corresponding sides are equal. The following triangles are congruent:



Since these triangles are congruent, the measures of corresponding angles are equal.

Angles with equal measure: $\angle A$ and $\angle D$, $\angle B$ and $\angle E$, $\angle C$ and $\angle F$. Also, the lengths of corresponding sides are equal.

Equal corresponding sides: $\overline{AB}$ and $\overline{DE}$, $\overline{BC}$ and $\overline{EF}$, $\overline{CA}$ and $\overline{FD}$

Any one of the following may be used to determine whether two triangles are congruent:

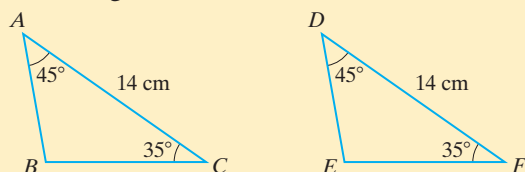
Objectives

- A** Decide Whether Two Triangles Are Congruent.
- B** Find the Ratio of Corresponding Sides in Similar Triangles.
- C** Find Unknown Lengths of Sides in Similar Triangles.

Congruent Triangles

Angle-Side-Angle (ASA)

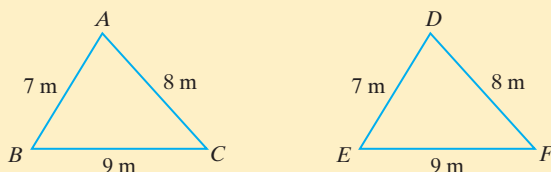
If the measures of two angles of a triangle equal the measures of two angles of another triangle, and the lengths of the sides between each pair of angles are equal, the triangles are congruent.



For example, these two triangles are congruent by Angle-Side-Angle.

Side-Side-Side (SSS)

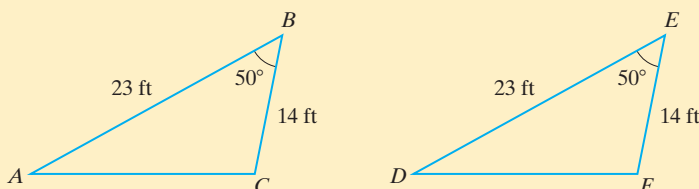
If the lengths of the three sides of a triangle equal the lengths of the corresponding sides of another triangle, the triangles are congruent.



For example, these two triangles are congruent by Side-Side-Side.

Side-Angle-Side (SAS)

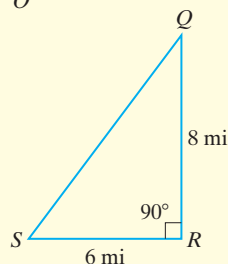
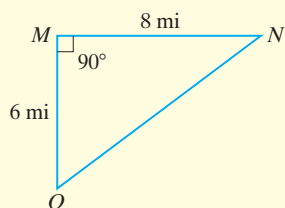
If the lengths of two sides of a triangle equal the lengths of corresponding sides of another triangle, and the measures of the angles between each pair of sides are equal, the triangles are congruent.



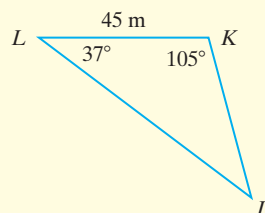
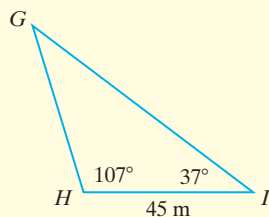
For example, these two triangles are congruent by Side-Angle-Side.

Practice 1

- a. Determine whether triangle MNO is congruent to triangle RQS .

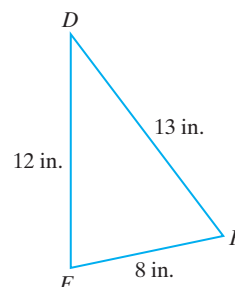
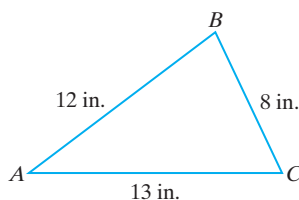


- b. Determine whether triangle GHI is congruent to triangle JKL .



Example 1

Determine whether triangle ABC is congruent to triangle DEF .



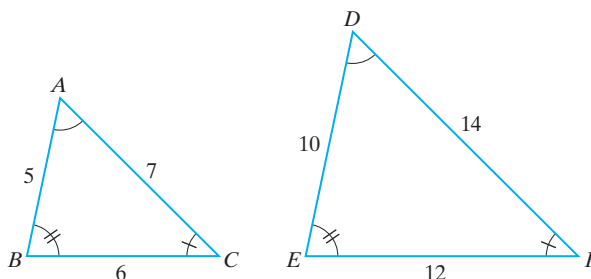
Solution: Since the lengths of all three sides of triangle ABC equal the lengths of all three sides of triangle DEF , the triangles are congruent.

Work Practice 1

In Example 1, notice that as soon as we know that the two triangles are congruent, we know that all three corresponding angles are congruent.

Objective B Finding the Ratios of Corresponding Sides in Similar Triangles

Two triangles are **similar** when they have the same shape but not necessarily the same size. In similar triangles, the measures of corresponding angles are equal and corresponding sides are in proportion. The following triangles are similar:



Since these triangles are similar, the measures of corresponding angles are equal. (Note: The triangles above are not drawn to scale.)

Angles with equal measure: $\angle A$ and $\angle D$, $\angle B$ and $\angle E$, $\angle C$ and $\angle F$. Also, the lengths of corresponding sides are in proportion.

Sides in proportion: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ or, in this particular case,

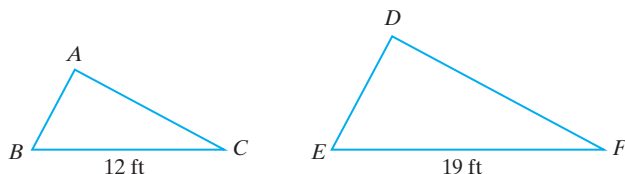
$$\frac{AB}{DE} = \frac{5}{10} = \frac{1}{2}, \frac{BC}{EF} = \frac{6}{12} = \frac{1}{2}, \frac{CA}{FD} = \frac{7}{14} = \frac{1}{2}$$

The ratio of corresponding sides is $\frac{1}{2}$.

Answers

1. a. congruent b. not congruent

Example 2 Find the ratio of corresponding sides for the similar triangles ABC and DEF .



Solution: We are given the lengths of two corresponding sides. Their ratio is

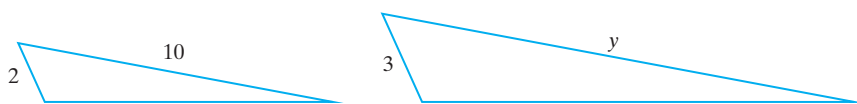
$$\frac{12 \text{ feet}}{19 \text{ feet}} = \frac{12}{19}$$

Work Practice 2

Objective C Finding Unknown Lengths of Sides in Similar Triangles

Because the ratios of lengths of corresponding sides are equal, we can use proportions to find unknown lengths in similar triangles.

Example 3 Given that the triangles are similar, find the missing length y .



Solution: Since the triangles are similar, corresponding sides are in proportion. Thus, the ratio of 2 to 3 is the same as the ratio of 10 to y , or

$$\frac{2}{3} = \frac{10}{y}$$

To find the unknown length y , we set cross products equal.

$$\frac{2}{3} = \frac{10}{y}$$

$$2 \cdot y = 3 \cdot 10 \quad \text{Set cross products equal.}$$

$$2y = 30 \quad \text{Multiply.}$$

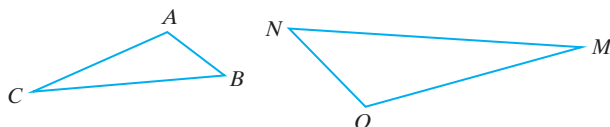
$$\frac{2y}{2} = \frac{30}{2} \quad \text{Divide both sides by 2.}$$

$$y = 15 \quad \text{Simplify.}$$

The missing length is 15 units.

Work Practice 3

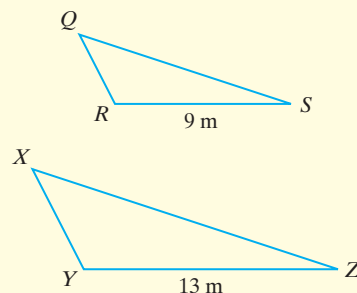
✓Concept Check The following two triangles are similar. Which vertices of the first triangle appear to correspond to which vertices of the second triangle?



Many applications involve a diagram containing similar triangles. Surveyors, astronomers, and many other professionals continually use similar triangles in their work.

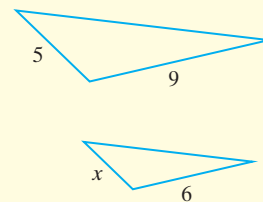
Practice 2

Find the ratio of corresponding sides for the similar triangles QRS and XYZ .



Practice 3

Given that the triangles are similar, find the missing length x .



Answers

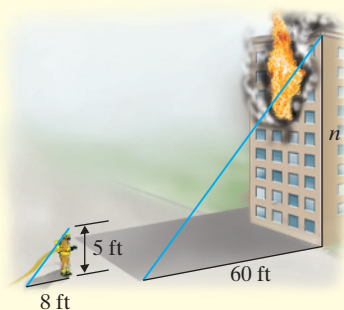
2. $\frac{9}{13}$ 3. $x = \frac{10}{3}$ or $3\frac{1}{3}$ units

✓Concept Check Answer

A corresponds to O ; B corresponds to N ; C corresponds to M

Practice 4

Tammy Shultz, a firefighter, needs to estimate the height of a burning building. She estimates the length of her shadow to be 8 feet long and the length of the building's shadow to be 60 feet long. Find the approximate height of the building if she is 5 feet tall.

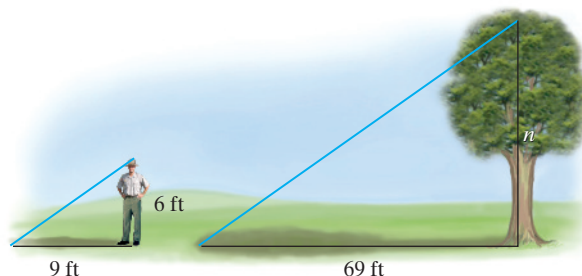


Answer

4. approximately 37.5 ft

Example 4 Finding the Height of a Tree

Mel Wagstaff is a 6-foot-tall park ranger who needs to know the height of a particular tree. He measures the shadow of the tree to be 69 feet long when his own shadow is 9 feet long. Find the height of the tree.



Solution:

1. **UNDERSTAND.** Read and reread the problem. Notice that the triangle formed by the sun's rays, Mel, and his shadow is similar to the triangle formed by the sun's rays, the tree, and its shadow.
2. **TRANSLATE.** Write a proportion from the similar triangles formed.

$$\begin{array}{lcl} \text{Mel's height} & \rightarrow & \frac{6}{n} = \frac{9}{69} \leftarrow \text{length of Mel's shadow} \\ \text{height of tree} & \rightarrow & \text{length of tree's shadow} \\ & \text{or } \frac{6}{n} = \frac{3}{23} & \text{Simplify } \frac{9}{69} \text{ (ratio in lowest terms).} \end{array}$$

3. **SOLVE** for n :

$$\frac{6}{n} = \frac{3}{23}$$

$$6 \cdot 23 = n \cdot 3 \quad \text{Set cross products equal.}$$

$$138 = 3n \quad \text{Multiply.}$$

$$\frac{138}{3} = \frac{3n}{3} \quad \text{Divide both sides by 3.}$$

$$46 = n$$

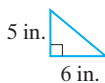
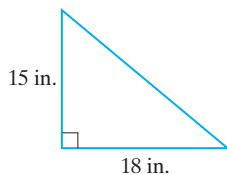
4. **INTERPRET.** Check to see that replacing n with 46 in the proportion makes the proportion true. State your conclusion: The height of the tree is 46 feet.

Work Practice 4

Vocabulary, Readiness & Video Check

Answer each question true or false.

1. Two triangles that have the same shape but not necessarily the same size are congruent. _____
2. Two triangles are congruent if they have the same shape and size. _____
3. Congruent triangles are also similar. _____
4. Similar triangles are also congruent. _____
5. For the two similar triangles, the ratio of corresponding sides is $\frac{5}{6}$. _____



Martin-Gay Interactive Videos



See Video 6.5

Watch the section lecture video and answer the following questions.

Objective A 6. How did we decide which congruency rule to use to determine if the two triangles in [Example 1](#) are congruent?

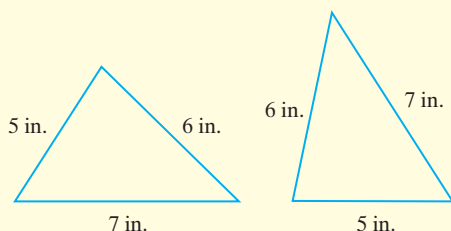
Objective B 7. From [Example 2](#), what does “corresponding sides are in proportion” mean?

Objective C 8. In [Example 4](#), what is another proportion we could have used to solve the application problem?

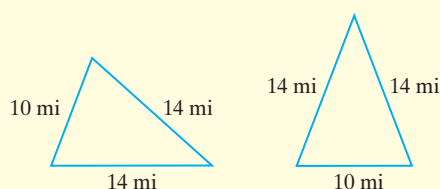
6.5 Exercise Set MyLab Math

Objective A Determine whether each pair of triangles is congruent. If congruent, state the reason why, such as SSS, SAS, or ASA. See Example 1.

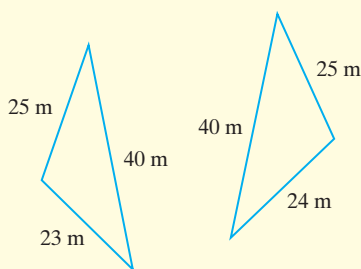
1.



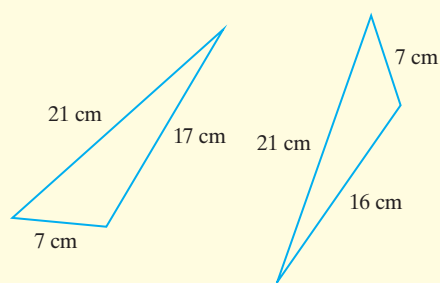
2.



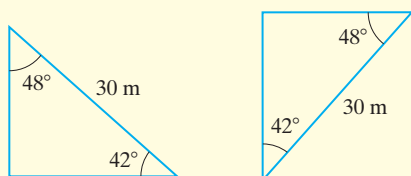
3.



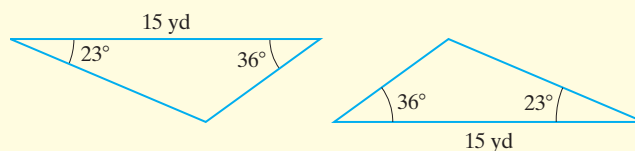
4.



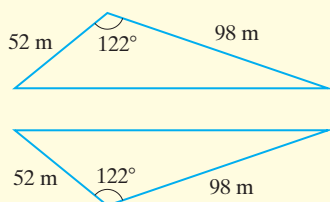
5.



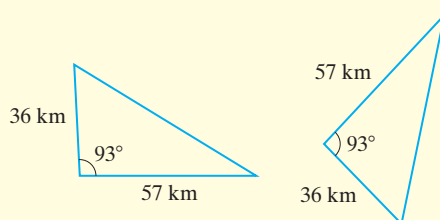
6.



7.

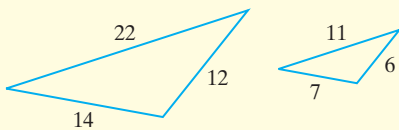


8.

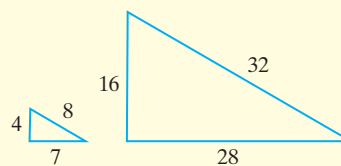


Objective B Find each ratio of the corresponding sides of the given similar triangles. See Example 2.

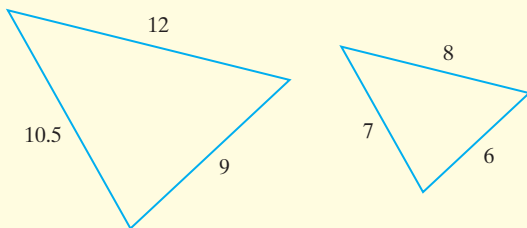
9.



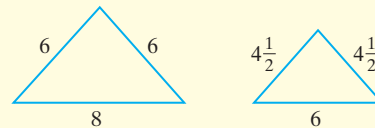
10.



11.

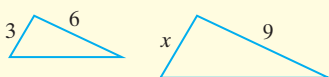


12.

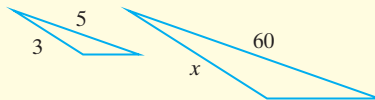


Objective C Given that the pairs of triangles are similar, find the unknown length of the side labeled with a variable. See Example 3.

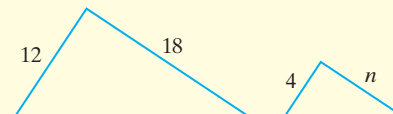
13.



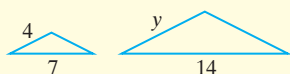
14.



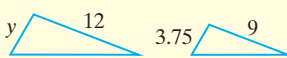
15.



16.



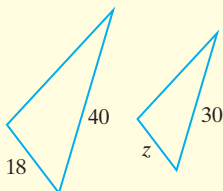
17.



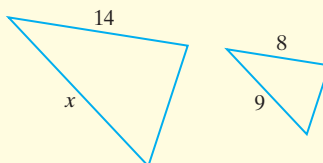
18.



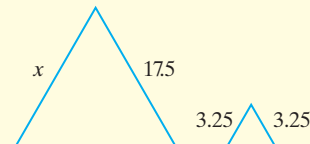
19.



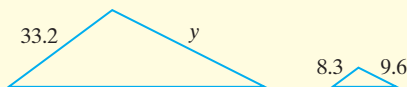
20.



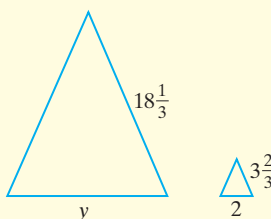
21.



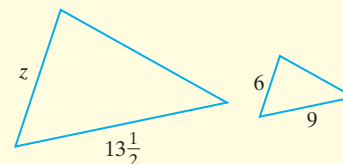
22.



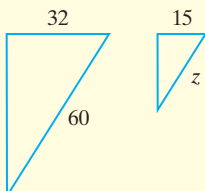
23.



24.



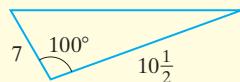
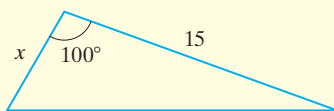
25.



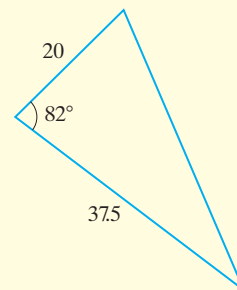
26.



27.

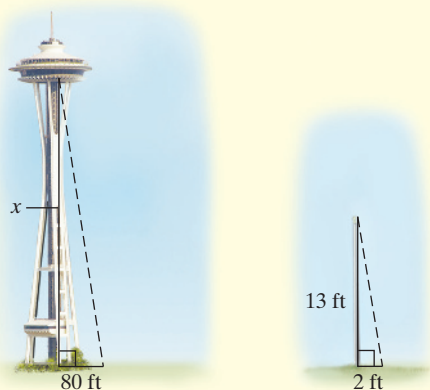


28.



Solve. For Exercises 29 and 30, the solutions have been started for you. See Example 4.

29. Given the following diagram, approximate the height of the observation deck in the Seattle Space Needle in Seattle, Washington. (Source: Seattle Space Needle)



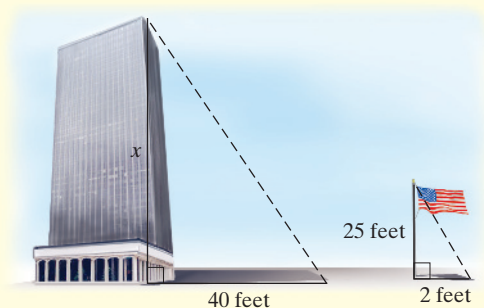
Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into a proportion using the similar triangles formed. (Fill in the blanks.)

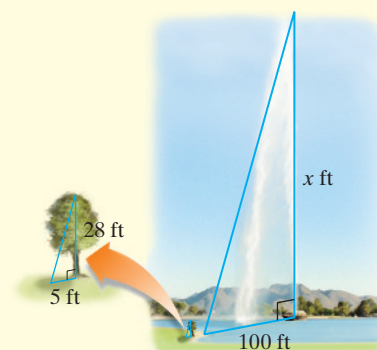
$$\begin{array}{lcl} \text{height of} & & \text{length of Space} \\ \text{observation deck} & \rightarrow \frac{x}{13} = & \leftarrow \text{Needle shadow} \\ \text{height of pole} & \rightarrow \frac{13}{2} = & \leftarrow \text{length of pole} \\ & & \text{shadow} \end{array}$$

3. SOLVE by setting cross products equal.
4. INTERPRET.

31. Given the following diagram, approximate the height of the Bank One Tower in Oklahoma City, Oklahoma. (Source: *The World Almanac*)



30. Fountain Hills, Arizona, boasts the tallest fountain in the United States. The fountain sits in a 28-acre lake and shoots up a column of water every hour. Based on the diagram below, what is the approximate height of the fountain?



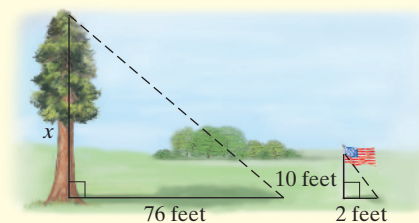
Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into a proportion using the similar triangles formed. (Fill in the blanks.)

$$\begin{array}{lcl} \text{height of tree} & \rightarrow \frac{28}{x} = & \leftarrow \text{length of tree} \\ \text{height of fountain} & \rightarrow \frac{x}{100} = & \leftarrow \text{shadow} \\ & & \text{length of fountain} \\ & & \text{shadow} \end{array}$$

3. SOLVE by setting cross products equal.
4. INTERPRET.

32. The tallest tree currently growing is Hyperion, a redwood located in the Redwood National Park in California. Given the following diagram, approximate its height. (Source: *Guinness World Records*) (Note: The tree's current recorded height is 379.1 ft.)



33. If a 30-foot tree casts an 18-foot shadow, find the length of the shadow cast by a 24-foot tree.

34. If a 24-foot flagpole casts a 32-foot shadow, find the length of the shadow cast by a 44-foot antenna. Round to the nearest tenth.

Review

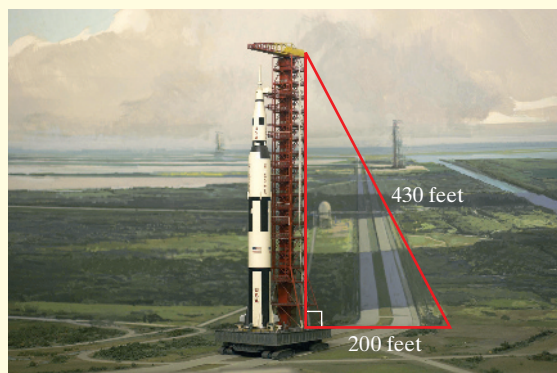
Solve. See Section 6.3.

35. For the health of his fish, Pete's Sea World uses the standard that a 20-gallon tank should house only 19 neon tetras. Find the number of neon tetras that Pete would place into a 55-gallon tank.

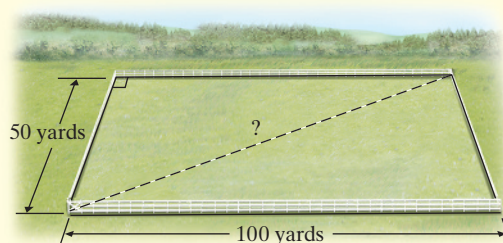
36. A local package express deliveryman is traveling the city expressway at 45 mph when he is forced to slow down due to traffic ahead. His truck slows at the rate of 3 mph every 5 seconds. Find his speed 8 seconds after braking.

Solve. See Section 6.4.

37. Launch Umbilical Tower 1 is the name of the gantry used for the *Apollo* launch that took Neil Armstrong and Buzz Aldrin to the moon. Find the height of the gantry to the nearest whole foot.



38. Arena polo, popular in the United States and England, is played on a field that is 100 yards long and usually 50 yards wide. Find the length, to the nearest yard, of the diagonal of this field.



Perform the indicated operation. See Sections 5.2 through 5.4.

39. $3.6 + 0.41$

40. $0.41 - 3.6$

41. $(0.41)(-3)$

42. $-0.48 \div 3$

Concept Extensions

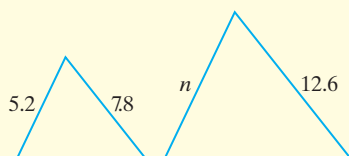
43. The print area on a particular page measures 7 inches by 9 inches. A printing shop is to copy the page and reduce the print area so that its length is 5 inches. What will its width be? Will the print now fit on a 3-by-5-inch index card?

44. The art sample for a banner measures $\frac{1}{3}$ foot in width by $1\frac{1}{2}$ feet in length. If the completed banner is to have a length of 9 feet, find its width.

Given that the pairs of triangles are similar, find the length of the side labeled n . Round your results to 1 decimal place.



45.



46.



47. In your own words, describe any differences in similar triangles and congruent triangles.



48. Describe a situation where similar triangles would be useful for a contractor building a house.

49. A triangular park is planned and waiting to be approved by the city zoning commission. A drawing of the park shows sides of length 5 inches, $7\frac{1}{2}$ inches, and $10\frac{5}{8}$ inches. If the scale on the drawing is $\frac{1}{4}$ in. = 10 ft, find the actual proposed dimensions of the park.
50. John and Robyn Costello draw a triangular deck on their house plans. Robyn measures sides of the deck drawing on the plans to be 3 inches, $4\frac{1}{2}$ inches, and 6 inches. If the scale on the drawing is $\frac{1}{4}$ in. = 1 foot, find the lengths of the sides of the deck they want built.

Chapter 6 Group Activity

Investigating Scale Drawings

Sections 6.1, 6.2, and 6.3

Materials:

- ruler
- tape measure
- grid paper (optional)

This activity may be completed by working in groups or individually.

Scale drawings are used by architects, engineers, interior designers, ship builders, and others. In a scale drawing, each unit measurement on the drawing represents a fixed length on the object being drawn. For instance, in an architect's scale drawing, 1 inch on the drawing may represent 10 feet on a building. The scale describes the relationship between the measurements. If the measurements have the same units, the scale can be expressed as a ratio. In this case, the ratio would be 1:120, representing 1 inch to 120 inches (or 10 feet).

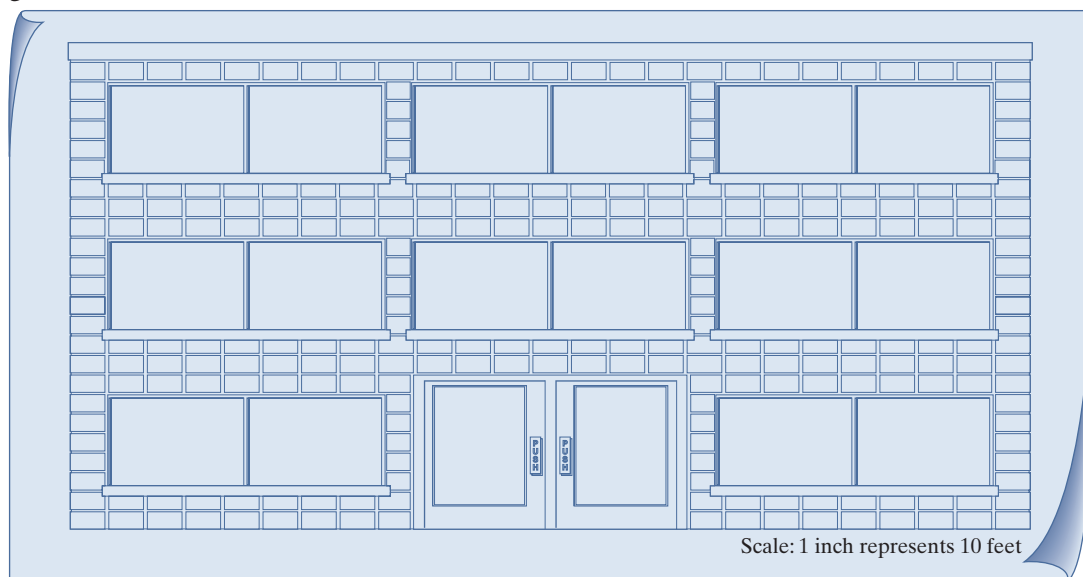
Use a ruler and the scale drawing of a college building below to answer the following questions.

1. How wide is each of the front doors of the college building?

2. How long is the front of the college building?
3. How tall is the front of the college building?

Now you will draw your own scale floor plan. First choose a room to draw—it can be your math classroom, your living room, your dormitory room, or any room that can be easily measured. Start by using a tape measure to measure the distances around the base of the walls in the room you are drawing.

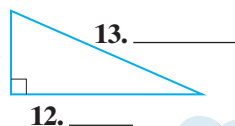
4. Choose a scale for your floor plan.
5. Convert each measurement in the room you are drawing to the corresponding lengths needed for the scale drawing.
6. Complete your floor plan (you may find it helpful to use grid paper). Mark the locations of doors and windows on your floor plan. Be sure to indicate on the drawing the scale used in your floor plan.



Fill in each blank with one of the words or phrases listed below. Some choices may be used more than once.

1. A(n) _____ is the quotient of two numbers. It can be written as a fraction, using a colon, or using the word *to*.
2. $\frac{x}{2} = \frac{7}{16}$ is an example of a(n) _____.
3. A(n) _____ is a rate with a denominator of 1.
4. A(n) _____ is a “money per item” unit rate.
5. A(n) _____ is used to compare different kinds of quantities.
6. In the proportion $\frac{x}{2} = \frac{7}{16}$, $x \cdot 16$ and $2 \cdot 7$ are called _____.
7. If cross products are _____, the proportion is true.
8. If cross products are _____, the proportion is false.
9. _____ triangles have the same shape and the same size.
10. _____ triangles have exactly the same shape but not necessarily the same size.

11. _____



15. In the right triangle , $a^2 + b^2 = c^2$ is called the _____ theorem.

▶ Are you preparing for your test?
Don't forget to take these:

- Chapter 6 Getting Ready for the Test on page 469
- Chapter 6 Test on page 470

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

Chapter Highlights

Copyright 2019 Pearson Education, Inc.

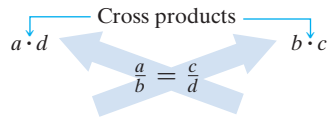
Definitions and Concepts

Examples

Section 6.2 Proportions

A **proportion** is a statement that two ratios or rates are equal.

Using **Cross Products** to Determine Whether Proportions Are True or False



If cross products are equal, the proportion is true.

If $ad = bc$, then the proportion is true.

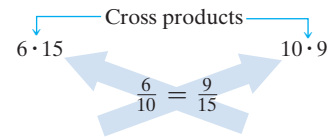
If cross products are not equal, the proportion is false.

If $ad \neq bc$, then the proportion is false.

To find an unknown value x in a proportion, we set the cross products equal to each other and solve the resulting equation.

$\frac{1}{2} = \frac{4}{8}$ is a proportion.

Is $\frac{6}{10} = \frac{9}{15}$ a true proportion?



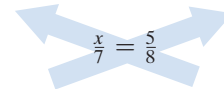
$$6 \cdot 15 \stackrel{?}{=} 10 \cdot 9$$

Are cross products equal?

$$90 = 90$$

Since cross products are equal, the proportion is a true proportion.

Find x : $\frac{x}{7} = \frac{5}{8}$



$$x \cdot 8 = 7 \cdot 5$$

Set the cross products equal to each other.

$$8x = 35$$

Multiply.

$$\frac{8x}{8} = \frac{35}{8}$$

Divide both sides by 8.

$$x = 4\frac{3}{8}$$

Section 6.3 Proportions and Problem Solving

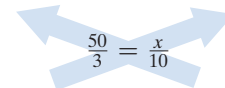
Given a specified ratio (or rate) of two quantities, a proportion can be used to determine an unknown quantity.

On a map, 50 miles corresponds to 3 inches. How many miles correspond to 10 inches?

1. UNDERSTAND. Read and reread the problem.
2. TRANSLATE. We let x represent the unknown number. We are given that 50 miles is to 3 inches as x miles is to 10 inches.

$$\begin{array}{lcl} \text{miles} \rightarrow & \frac{50}{3} = \frac{x}{10} & \leftarrow \text{miles} \\ \text{inches} \rightarrow & & \leftarrow \text{inches} \end{array}$$

3. SOLVE:



$$50 \cdot 10 = 3 \cdot x$$

Set the cross products equal to each other.

$$500 = 3x$$

Multiply.

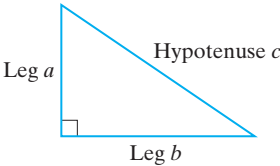
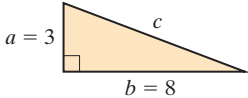
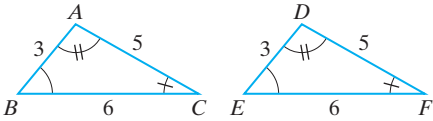
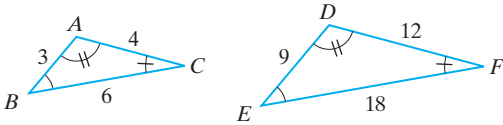
$$\frac{500}{3} = \frac{3x}{3}$$

Divide both sides by 3.

$$x = 166\frac{2}{3}$$

4. INTERPRET. Check your work. State your conclusion:

On the map, $166\frac{2}{3}$ miles corresponds to 10 inches.

Definitions and Concepts	Examples
Section 6.4 Square Roots and the Pythagorean Theorem	
Square Root of a Number The square root of a positive number a is the positive number b whose square is a. In symbols, $\sqrt{a} = b, \quad \text{if} \quad b^2 = a$ Also, $\sqrt{0} = 0$. Pythagorean Theorem If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$ 	$\sqrt{9} = 3, \quad \sqrt{100} = 10, \quad \sqrt{1} = 1$ Find c.  $\begin{aligned} a^2 + b^2 &= c^2 \\ 3^2 + 8^2 &= c^2 \\ 9 + 64 &= c^2 \\ 73 &= c^2 \\ \sqrt{73} &= c \text{ or } c \approx 8.5 \end{aligned}$ Let $a = 3$ and $b = 8$. Multiply. Simplify.
Section 6.5 Congruent and Similar Triangles	
Congruent triangles have the same shape and the same size. Corresponding angles are equal, and corresponding sides are equal. Similar triangles have exactly the same shape but not necessarily the same size. Corresponding angles are equal, and the ratios of the lengths of corresponding sides are equal.	 Congruent triangles  Similar triangles $\frac{AB}{DE} = \frac{3}{9} = \frac{1}{3}, \quad \frac{BC}{EF} = \frac{6}{18} = \frac{1}{3},$ $\frac{CA}{FD} = \frac{4}{12} = \frac{1}{3}$

(6.1) Write each ratio as a fraction in simplest form.

1. 23 to 37

2. \$121 to \$143

3. 4.25 yards to
8.75 yards

4. $2\frac{1}{4}$ to $4\frac{3}{8}$

Use the garden shown for Exercises 5 and 6. Write each ratio as a fraction in simplest form.



5. Find the ratio of the garden's length (longer side) to the garden's width (shorter side).

△ 6. Find the ratio of the width to the perimeter of the rectangular garden.

Write each rate as a fraction in simplest form.

7. 6000 people to 2400 pets

8. 15 pages printed in 6 minutes

Write each rate as a unit rate.

9. 468 miles in 9 hours

10. 180 feet in 12 seconds

11. \$6.96 for 4 diskettes

12. 104 bushels of fruit from 8 trees

Find each unit price and decide which is the better buy. Round to 3 decimal places. Assume that we are comparing different sizes of the same brand.

13. Taco sauce: 8 ounces for \$0.99 or 12 ounces for \$1.69



14. Peanut butter: 18 ounces for \$1.49 or 28 ounces for \$2.39



(6.2) **Translating** Write each sentence as a proportion.

15. 24 uniforms is to 8 players as 3 uniforms is to 1 player.

16. 12 tires is to 3 cars as 4 tires is to 1 car.

Determine whether each proportion is true.

17. $\frac{19}{8} = \frac{14}{6}$

18. $\frac{3.75}{3} = \frac{7.5}{6}$

Find the unknown number x in each proportion.

19. $\frac{x}{3} = \frac{30}{18}$

20. $\frac{x}{9} = \frac{7}{3}$

21. $\frac{-8}{5} = \frac{9}{x}$

22. $\frac{-27}{\frac{9}{4}} = \frac{x}{-5}$

23. $\frac{0.4}{x} = \frac{2}{4.7}$

24. $\frac{x}{4\frac{1}{2}} = \frac{2\frac{1}{10}}{8\frac{2}{5}}$

25. $\frac{x}{0.4} = \frac{4.7}{3}$

Round to the nearest hundredth.

26. $\frac{0.07}{0.3} = \frac{7.2}{n}$

Round to the nearest tenth.

(6.3) Solve.

The ratio of a quarterback's completed passes to attempted passes is 3 to 7.

27. If he attempted 32 passes, find how many passes he completed. Round to the nearest whole pass.

28. If he completed 15 passes, find how many passes he attempted.

One bag of pesticide covers 4000 square feet of garden.

- △ 29. Find how many bags of pesticide should be purchased to cover a rectangular garden that is 180 feet by 175 feet.

- △ 30. Find how many bags of pesticide should be purchased to cover a square garden that is 250 feet on each side.

On a road map of Texas, 0.75 inch represents 80 miles.

31. Find the distance from Houston to Corpus Christi if the distance on the map is about 2 inches.

32. The distance from El Paso to Dallas is 1025 miles. Find the distance between these cities on the map. Round to the nearest tenth of an inch.

(6.4) Find each square root. If necessary, round the square root to the nearest thousandth.

33. $\sqrt{64}$

34. $\sqrt{144}$

35. $\sqrt{12}$

36. $\sqrt{15}$

37. $\sqrt{0}$

38. $\sqrt{1}$

39. $\sqrt{50}$

40. $\sqrt{65}$

41. $\sqrt{\frac{4}{25}}$

42. $\sqrt{\frac{1}{100}}$

Find the unknown length in each given right triangle. If necessary, round to the nearest tenth.

△ 43. leg = 12, leg = 5

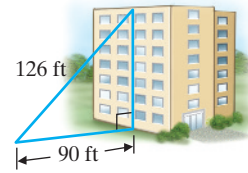
△ 44. leg = 20, leg = 21

△ 45. leg = 9, hypotenuse = 14

△ 46. leg = 66, hypotenuse = 86

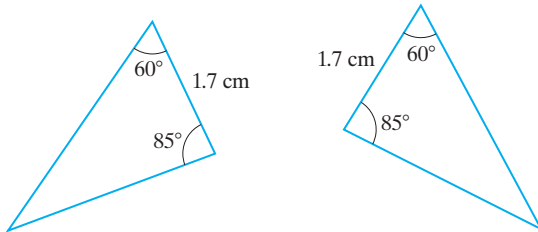
- △ 47. Find the length, to the nearest hundredth, of the diagonal of a square that has a side of length 20 centimeters.

- △ 48. Find the height of the building rounded to the nearest tenth.



(6.5) Determine whether each pair of triangles is congruent. If congruent, state the reason why, such as SSS, SAS, or ASA.

49.

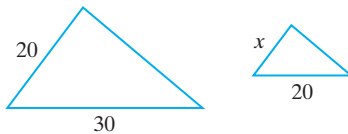


50.

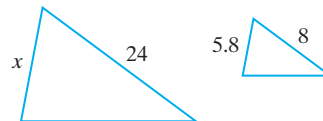


Given that the pairs of triangles are similar, find the unknown length x .

51.



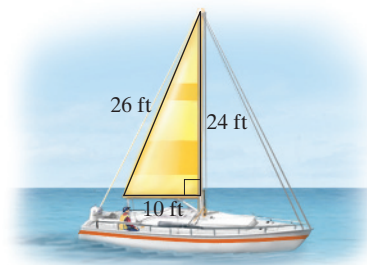
52.



Solve.

53. A housepainter needs to estimate the height of a condominium. He estimates the length of his shadow to be 7 feet long and the length of the building's shadow to be 42 feet long. Find the approximate height of the building if the housepainter is $5\frac{1}{2}$ feet tall.

54. A design company is making a triangular sail for a model sailboat. The model sail is to be the same shape as a life-size sailboat's sail. Use the following diagram to find the unknown lengths x and y .



Mixed Review

Write each ratio as a fraction in simplest form.

55. 15 to 25

56. 3 pints to 81 pints

Write each rate as a fraction in simplest form.

57. 2 teachers for 18 students

58. 6 nurses for 24 patients

Write each rate as a unit rate.

59. 136 miles in 4 hours

60. 12 gallons of milk from 6 cows

For Exercises 61 and 62, write the ratio or rate in simplest form.

61. During the 2016 Summer Olympics in Rio, a total of 2102 medals were awarded. Athletes from Italy won a total of 28 medals. Find the ratio of medals won by Italy to the total medals awarded. (Source: Rio2016)

62. Paul Crake of Canberra, Australia, holds the record for the Empire State Building Run Up in New York. He ran up 1576 steps in about 9.5 minutes. Find the number of steps per minute. Round to the nearest step. (Source: Guinness Book of World Records)

Find each unit price and decide which is the better buy. Round to 3 decimal places. Assume that we are comparing different sizes of the same brand.

63. Cold medicine:
\$4.94 for 4 oz.
\$9.98 for 8 oz.

64. Juice:
12 oz for \$0.65
64 oz for \$2.98

Translating Write each sentence as a proportion.

65. 2 cups of cookie dough is to 30 cookies as 4 cups of cookie dough is to 60 cookies.
66. 5 nickels is to 3 dollars as 20 nickels is to 12 dollars.

Find the unknown number x in each proportion.

67. $\frac{3}{x} = \frac{15}{8}$

68. $\frac{5}{4} = \frac{x}{20}$

69. $\frac{x}{3} = \frac{7.5}{6}$

70. $\frac{\frac{1}{3}}{25} = \frac{x}{30}$

Find each square root. If necessary, approximate and round to the nearest thousandth.

71. $\sqrt{36}$

72. $\sqrt{\frac{16}{81}}$

73. $\sqrt{105}$

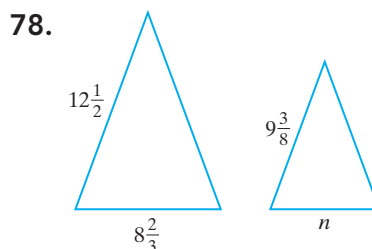
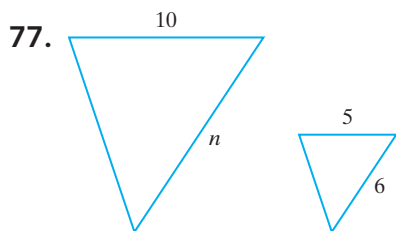
74. $\sqrt{32}$

Find the unknown length in each given right triangle. If necessary, round to the nearest tenth.

75. leg = 66, leg = 56

76. leg = 12, hypotenuse = 24

Given that the pairs of triangles are similar, find the unknown length n .



MULTIPLE CHOICE Exercises 1 through 4 are Multiple Choice. Choose the correct letter for each exercise.

- 1. The phrase “6 is to 7 as 18 is to 21” translates to

A. $\frac{6}{7} = \frac{18}{21}$
B. $\frac{6}{18} = \frac{21}{7}$
C. $\frac{6}{21} = \frac{7}{18}$

- 2. Use cross products to determine which proportion is *not* equivalent to the proportion $\frac{3}{x} = \frac{5}{11}$

A. $\frac{3}{11} = \frac{x}{5}$
B. $\frac{x}{3} = \frac{11}{5}$
C. $\frac{3}{5} = \frac{x}{11}$
D. $\frac{11}{x} = \frac{5}{3}$

- 3. Which square root is between the numbers 8 and 10?

A. $\sqrt{36}$
B. $\sqrt{25}$
C. $\sqrt{49}$
D. $\sqrt{81}$

- 4. Which square root is between the numbers 6 and 7?

A. $\sqrt{10}$
B. $\sqrt{13}$
C. $\sqrt{50}$
D. $\sqrt{40}$

TRUE OR FALSE Choose the correct letter for Exercises 5 through 8.

A. True or B. False

- 5. The Pythagorean theorem applies to right triangles only.
► 6. A right triangle can have two 90° angles.
► 7. A right triangle has 3 sides, 1 side is called a leg and the other 2 sides are each called a hypotenuse.
► 8. The hypotenuse of a right triangle is the longest side.

Answers

Write each ratio or rate as a fraction in simplest form.

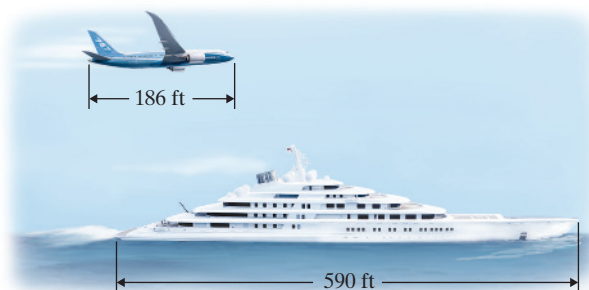
▶ 1. 4500 trees to 6500 trees

▶ 2. 9 inches of rain in 30 days

▶ 3. 8.6 to 10

▶ 4. $5\frac{7}{8}$ to $9\frac{3}{4}$

- ▶ 5. The world's largest yacht, the Azzam, measures 590 feet. A Boeing 787-8 Dreamliner measures 186 feet long. Find the ratio of the length of the Azzam to the length of a 787-8. (Source: CNN)



Find each unit rate.

▶ 6. 650 kilometers in 8 hours

▶ 7. 140 students for 5 teachers

- ▶ 8. The Sojourner is a 6-wheeled vehicle that was used in the exploration of Mars, but was remotely controlled from Earth. Each wheel was designed to move independently, allowing the capability of traversing various obstacles and the ability to be turned around in place. This little vehicle was capable of traveling about 960 inches each 60 minutes. (Source: NASA)



Find each unit price and decide which is the better buy. Round to three decimal places.

- ▶ 9. Steak sauce:
8 ounces for \$1.19
12 ounces for \$1.89

- ▶ 10. Jelly:
16 ounces for \$1.49
24 ounces for \$2.39

Determine whether the proportion is true.

▶ 11. $\frac{28}{16} = \frac{14}{8}$

▶ 12. $\frac{3.6}{2.2} = \frac{1.9}{1.2}$

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

Solve each proportion for the given variable.

▶ 13. $\frac{n}{3} = \frac{15}{9}$

▶ 14. $\frac{8}{x} = \frac{11}{6}$

▶ 15. $\frac{4}{\frac{3}{7}} = \frac{y}{\frac{1}{4}}$

▶ 16. $\frac{1.5}{5} = \frac{2.4}{n}$

Solve.

- ▶ 17. On an architect's drawing, 2 inches corresponds to 9 feet. Find the length of a home represented by a line that is 11 inches long.

- ▶ 18. If a car can be driven 80 miles in 3 hours, how long will it take to travel 100 miles?

- ▶ 19. The standard dose of medicine for a dog is 10 grams for every 15 pounds of body weight. What is the standard dose for a dog that weighs 80 pounds?

Find each square root and simplify. Round to the nearest thousandth if necessary.

▶ 20. $\sqrt{49}$

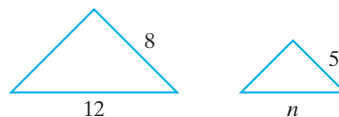
▶ 21. $\sqrt{157}$

▶ 22. $\sqrt{\frac{64}{100}}$

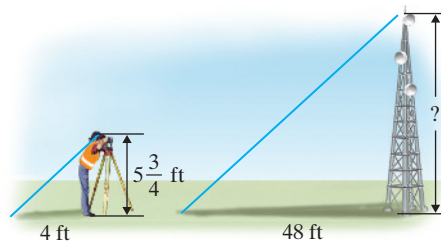
Solve.

- ▶ 23. Approximate, to the nearest hundredth of a centimeter, the unknown length of the side of a right triangle with legs of 4 centimeters each.

- ▶ 24. Given that the following triangles are similar, find the unknown length n .



- ▶ 25. A surveyor needs to estimate the height of a tower. She estimates the length of her shadow to be 4 feet long and the length of the tower's shadow to be 48 feet long. Find the height of the tower if she is $5\frac{3}{4}$ feet tall.



13. _____

14. _____

15. _____

16. _____

17. _____

18. _____

19. _____

20. _____

21. _____

22. _____

23. _____

24. _____

25. _____

Answers

1. a.
b.
c.
d.
2. a.
b.
c.
d.
- 3.
- 4.
5. a.
b.
- 6.
- 7.
- 8.
- 9.
- 10.
- 11.
- 12.
- 13.
- 14.
- 15.
- 16.
- 17.
- 18.
- 19.
- 20.

1. Subtract. Check each answer by adding.
 - a. $12 - 9$
 - b. $22 - 7$
 - c. $35 - 35$
 - d. $70 - 0$
2. Multiply
 - a. $20 \cdot 0$
 - b. $20 \cdot 1$
 - c. $0 \cdot 20$
 - d. $1 \cdot 20$

3. Round 248,982 to the nearest hundred.
4. Round 248,982 to the nearest thousand.

Perform the indicated operations.

5. a.
$$\begin{array}{r} 25 \\ \times 8 \\ \hline \end{array}$$
 b.
$$\begin{array}{r} 246 \\ \times 5 \\ \hline \end{array}$$
6. $10,468 \div 28$
7. $1 + (-10) + (-8) + 9$
8. $-12(7)$

9. Write the prime factorization of 80.
10. Find $3^2 - 1^2$.

11. Write $\frac{12}{20}$ in simplest form.
12. Find $9^2 \cdot 3$.

Multiply.

13. $\left(-\frac{6}{13}\right)\left(-\frac{26}{30}\right)$
14. $3\frac{3}{8} \cdot 4\frac{5}{9}$

Perform the indicated operation and simplify.

15. $\frac{7}{8} + \frac{6}{8} + \frac{3}{8}$
16. $\frac{7}{10} - \frac{3}{10} + \frac{4}{10}$
17. Find the LCD of $\frac{3}{7}$ and $\frac{5}{14}$.
18. Add: $\frac{17}{25} + \frac{3}{10}$

19. Write an equivalent fraction with the indicated denominator. $\frac{3}{4} = \frac{\quad}{20}$
20. Determine whether these fractions are equivalent.
 $\frac{10}{55}$ $\frac{6}{33}$

21. Subtract: $\frac{2}{3} - \frac{10}{11}$

23. A flight from Tucson to Phoenix, Arizona, requires $\frac{5}{12}$ of an hour. If the plane has been flying $\frac{1}{4}$ of an hour, find how much time remains before landing.

22. Subtract: $17\frac{5}{24} - 9\frac{5}{9}$



24. Simplify: $80 \div 8 \cdot 2 + 7$

26. Find the average of $\frac{3}{5}$, $\frac{4}{9}$, and $\frac{11}{15}$.

25. Add: $2\frac{1}{3} + 5\frac{3}{8}$

27. Insert $<$ or $>$ to form a true statement.
 $\frac{3}{4}$ $\frac{9}{11}$

Solve.

28. $5y - 8y = 24$

30. $3y - 6 = 7y - 6$

32. $4(y + 1) - 3 = 21$

34. Write “seventy-five thousandths” in standard form.

36. Round 736.2359 to the nearest thousandth.

38. Subtract: $700 - 18.76$

40. Write $\frac{3}{8}$ as a decimal.

42. Write 7.9 as an improper fraction.

44. Find the unknown number n .
 $\frac{n}{4} = \frac{12}{16}$

46. Write the rate as a unit rate.
 700 meters in 5 seconds

Write each ratio as a fraction in simplest form.

47. The ratio of \$15 to \$10

49. The ratio of 2.5 to 3.15

29. $y - 5 = -2 - 6$

31. $3a - 6 = a + 4$

33. $3(2x - 6) + 6 = 0$

35. Round 736.2359 to the nearest tenth.

37. Add: $23.85 + 1.604$

39. Multiply: 0.0531×16

41. Divide: $-5.98 \div 115$

43. Simplify: $-0.5(8.6 - 1.2)$

45. Write the numbers in order from smallest to largest.
 $\frac{9}{20}$, $\frac{4}{9}$, 0.456

48. The ratio of 7 to 21

50. The ratio of 900 to 9000

21. _____
22. _____
23. _____
24. _____
25. _____
26. _____
27. _____
28. _____
29. _____
30. _____
31. _____
32. _____
33. _____
34. _____
35. _____
36. _____
37. _____
38. _____
39. _____
40. _____
41. _____
42. _____
43. _____
44. _____
45. _____
46. _____
47. _____
48. _____
49. _____
50. _____

7

Percent

This chapter is devoted to percent, a concept used virtually every day in ordinary and business life. Understanding percent and using it efficiently depend on understanding ratios, because a percent is a ratio whose denominator is 100. We present techniques to write percents as fractions and as decimals. We then solve problems relating to interest rates, sales tax, discounts, and other real-life situations by writing percent equations.

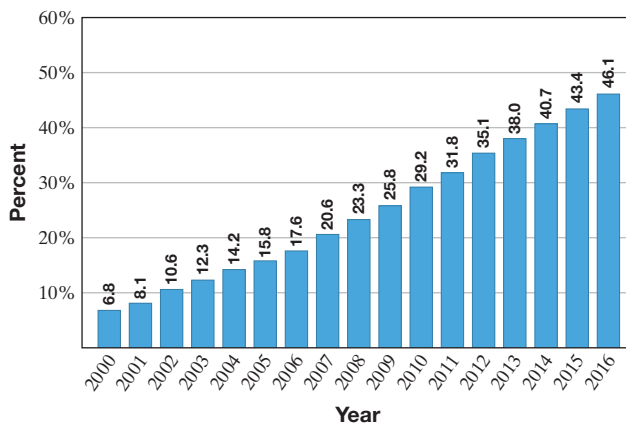


How Many Internet Users Are There in the World?

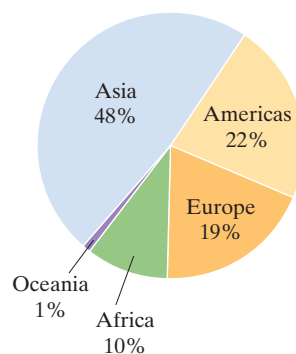
We have a world population that is approaching 7.5 billion. As our population grows, so does access to the Internet. Below is an attempt to show the increase in the world use of the Internet via what we will call an “Internet user.” For the use of our graph below, an Internet user is defined to be someone who can access the Internet, either through a computer or a mobile device, in the location where he or she calls home. The graph on the left illustrates the percent of world Internet users increasing, and the circle graph on the right shows these Internet users by region.

In Section 7.4, Exercises 71 through 76, we study the number of Internet users in selected countries.

Percent of World Population Identified as Internet Users



Internet Users by Region



Source: Internet Live Stats (www.InternetLiveStats.com)

Sections

- 7.1** Percents, Decimals, and Fractions
- 7.2** Solving Percent Problems with Equations
- 7.3** Solving Percent Problems with Proportions
- Integrated Review**—Percent and Percent Problems
- 7.4** Applications of Percent
- 7.5** Percent and Problem Solving: Sales Tax, Commission, and Discount
- 7.6** Percent and Problem Solving: Interest

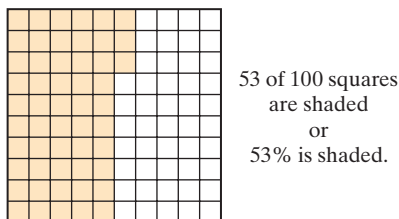
Check Your Progress

- Vocabulary Check
- Chapter Highlights
- Chapter Review
- Getting Ready for the Test
- Chapter Test
- Cumulative Review

7.1 Percents, Decimals, and Fractions

Objective A Understanding Percent

The word **percent** comes from the Latin phrase *per centum*, which means “**per 100**.” For example, 53% (53 percent) means 53 per 100. In the square below, 53 of the 100 squares are shaded. Thus, 53% of the figure is shaded.



Since 53% means 53 per 100, 53% is the ratio of 53 to 100, or $\frac{53}{100}$.

$$53\% = \frac{53}{100}$$

Also,

$$7\% = \frac{7}{100} \quad 7 \text{ parts per 100 parts}$$

$$73\% = \frac{73}{100} \quad 73 \text{ parts per 100 parts}$$

$$109\% = \frac{109}{100} \quad 109 \text{ parts per 100 parts}$$

Percent

Percent means **per one hundred**. The “%” symbol is used to denote percent.

Percent is used in a variety of everyday situations. For example,

- 88.5% of the U.S. population are Internet users.
- The store is having a 20%-off sale.
- For the past two years, the enrollment in community colleges increased 15%.
- The South is the home of 49% of all frequent paintball participants.
- 50% of the cinema screens in the United States are digital non-3-D.



Example 1

In a survey of 100 people, 17 people drive blue cars. What percent of the people drive blue cars?

Solution: Since 17 people out of 100 drive blue cars, the fraction is $\frac{17}{100}$. Then

$$\frac{17}{100} = 17\%$$

Work Practice 1

Objectives

- A** Understand Percent.
- B** Write Percents as Decimals or Fractions.
- C** Write Decimals or Fractions as Percents.
- D** Solve Applications with Percents, Decimals, and Fractions.

Practice 1

Of 100 students in a club, 27 are freshmen. What percent of the students are freshmen?

Answer

1. 27%

Practice 2

31 out of 100 college instructors are in their forties. What percent of these instructors are in their forties?

Example 2

45 out of every 100 young college graduates return home to live (at least temporarily). What percent of college graduates is this? (Source: Independent Insurance Agents of America)

Solution:

$$\frac{45}{100} = 45\%$$

Work Practice 2**Objective B Writing Percents as Decimals or Fractions**

Since percent means “per hundred,” we have that

$$1\% = \frac{1}{100} = 0.01$$

In other words, the percent symbol means “per hundred” or, equivalently, “ $\frac{1}{100}$ ” or “0.01.” Thus

$$\text{Write } 87\% \text{ as a fraction: } 87\% = 87 \times \frac{1}{100} = \frac{87}{100}$$

or

$$\text{Write } 87\% \text{ as a decimal: } 87\% = 87 \times (0.01) = 0.87$$

Results are the same.

Of course, we know that the end results are the same; that is,

$$\frac{87}{100} = 0.87$$

The above gives us two options for converting percents. We can replace the percent symbol, %, by $\frac{1}{100}$ or by 0.01 and then multiply.

For consistency, when we

- convert from a percent to a *decimal*, we will drop the % symbol and multiply by 0.01.
- convert from a percent to a *fraction*, we will drop the % symbol and multiply by $\frac{1}{100}$.

Let's practice writing percents as decimals, then writing percents as fractions.

Writing a Percent as a Decimal

Replace the percent symbol with its decimal equivalent, 0.01; then multiply.

$$43\% = 43(0.01) = 0.43$$

Helpful Hint!

If it helps, think of writing a percent as a decimal by

Percent →

Remove the % symbol and move the decimal point 2 places to the left.

→ Decimal

Answer

2. 31%

Examples Write each percent as a decimal.

3. $23\% = 23(0.01) = 0.23$ Replace the percent symbol with 0.01. Then multiply.

4. $4.6\% = 4.6(0.01) = 0.046$ Replace the percent symbol with 0.01. Then multiply.

5. $190\% = 190(0.01) = 1.90$ or 1.9

6. $0.74\% = 0.74(0.01) = 0.0074$

7. $100\% = 100(0.01) = 1.00$ or 1

Helpful HintWe just learned that $100\% = 1$.**Work Practice 3–7****✓ Concept Check** Why is it incorrect to write the percent 0.033% as 3.3 in decimal form?

Now let's write percents as fractions.

Writing a Percent as a FractionReplace the percent symbol with its fraction equivalent, $\frac{1}{100}$; then multiply. Don't forget to simplify the fraction if possible.

$$43\% = 43 \cdot \frac{1}{100} = \frac{43}{100}$$

Examples Write each percent as a fraction or mixed number in simplest form.

8. $40\% = 40 \cdot \frac{1}{100} = \frac{40}{100} = \frac{2 \cdot \cancel{20}}{5 \cdot \cancel{20}} = \frac{2}{5}$

9. $1.9\% = 1.9 \cdot \frac{1}{100} = \frac{1.9}{100}$. We don't want the numerator of the fraction to contain a decimal, so we multiply by 1 in the form of $\frac{10}{10}$.

$$= \frac{1.9}{100} \cdot \frac{10}{10} = \frac{1.9 \cdot 10}{100 \cdot 10} = \frac{19}{1000}$$

10. $125\% = 125 \cdot \frac{1}{100} = \frac{125}{100} = \frac{5 \cdot \cancel{25}}{4 \cdot \cancel{25}} = \frac{5}{4}$ or $1 \frac{1}{4}$

11. $33\frac{1}{3}\% = 33\frac{1}{3} \cdot \frac{1}{100} = \frac{100}{3} \cdot \frac{1}{100} = \frac{\cancel{100} \cdot 1}{3 \cdot \cancel{100}} = \frac{1}{3}$
Write as an improper fraction.

12. $100\% = 100 \cdot \frac{1}{100} = \frac{100}{100} = 1$

Helpful HintJust as in Example 7, we confirm that $100\% = 1$.**Work Practice 8–12****Objective C** Writing Decimals or Fractions as PercentsTo write a decimal or fraction as a percent, we use the result of Examples 7 and 12. In these examples, we found that $1 = 100\%$.

Write 0.38 as a percent: $0.38 = 0.38(1) = 0.38(100\%) = 38\%$

Write $\frac{1}{5}$ as a percent: $\frac{1}{5} = \frac{1}{5}(1) = \frac{1}{5} \cdot 100\% = \frac{100}{5}\% = 20\%$

First, let's practice writing decimals as percents.

Practice 3–7

Write each percent as a decimal.

3. 49%

4. 3.1%

5. 175%

6. 0.46%

7. 600%

Practice 8–12

Write each percent as a fraction or mixed number in simplest form.

8. 50%

9. 2.3%

10. 150%

11. $66\frac{2}{3}\%$

12. 12%

Answers

3. 0.49 4. 0.031 5. 1.75 6. 0.0046

7. 6 8. $\frac{1}{2}$ 9. $\frac{23}{1000}$ 10. $\frac{3}{2}$ or $1\frac{1}{2}$

11. $\frac{2}{3}$ 12. $\frac{3}{25}$

✓ Concept Check

To write a percent as a decimal, the decimal point should be moved two places to the left, not to the right. So the correct answer is 0.00033.

Writing a Decimal as a Percent

Multiply by 1 in the form of 100%.

$$0.27 = 0.27(100\%) = 27\%$$

Helpful Hint

If it helps, think of writing a decimal as a percent by reversing the steps in the Helpful Hint on page 476.

Percent ←

Move the decimal point
2 places to the right and
attach a % symbol.

← Decimal

Practice 13–16

Write each decimal as a percent.

13. 0.14 **14.** 1.75

15. 0.057 **16.** 0.5

Examples

Write each decimal as a percent.

13. $0.65 = 0.65(100\%) = 65\%$ or 65% *Multiply by 100%.*

14. $1.25 = 1.25(100\%) = 125\%$ or 125%

15. $0.012 = 0.012(100\%) = 001.2\%$ or 1.2%

16. $0.6 = 0.6(100\%) = 060\%$ or 60%

Helpful Hint

A zero was inserted as a placeholder.

Work Practice 13–16

✓ **Concept Check** Why is it incorrect to write the decimal 0.0345 as 34.5% in percent form?

Now let's write fractions as percents.

Writing a Fraction as a Percent

Multiply by 1 in the form of 100%.

$$\frac{1}{8} = \frac{1}{8} \cdot 100\% = \frac{1}{8} \cdot \frac{100}{1}\% = \frac{100}{8}\% = 12\frac{1}{2}\% \text{ or } 12.5\%$$

Helpful Hint

From Examples 7 and 12, we know that

$$100\% = 1$$

Recall that when we multiply a number by 1, we are not changing the value of that number. This means that when we multiply a number by 100%, we are not changing its value but rather writing the number as an equivalent percent.

Answers

13. 14% **14.** 175% **15.** 5.7%

16. 50%

✓ **Concept Check Answer**

To change a decimal to a percent, multiply by 100%, or move the decimal point *only* two places to the right. So the correct answer is 3.45%.

Examples

Write each fraction or mixed number as a percent.

$$17. \frac{7}{20} = \frac{7}{20} \cdot 100\% = \frac{7}{20} \cdot \frac{100}{1}\% = \frac{700}{20}\% = 35\%$$

$$18. \frac{2}{3} = \frac{2}{3} \cdot 100\% = \frac{2}{3} \cdot \frac{100}{1}\% = \frac{200}{3}\% = 66\frac{2}{3}\%$$

$$19. 2\frac{1}{4} = \frac{9}{4} \cdot 100\% = \frac{9}{4} \cdot \frac{100}{1}\% = \frac{900}{4}\% = 225\%$$

Helpful Hint

$$\frac{200}{3} = 66.\bar{6}$$

Thus, another way to write $\frac{200}{3}\%$ is $66.\bar{6}\%$.**Work Practice 17–19****✓ Concept Check** Which digit in the percent 76.4582% represents

- a. A tenth percent? b. A thousandth percent?
c. A hundredth percent? d. A ten percent?

Example 20Write $\frac{1}{12}$ as a percent. Round to the nearest hundredth percent.**Solution:**

$$\frac{1}{12} = \frac{1}{12} \cdot 100\% = \frac{1}{12} \cdot \frac{100}{1}\% = \frac{100}{12}\% \approx 8.33\%$$

“approximately”

$$\begin{array}{r} 8.333 \approx 8.33 \\ 12 \overline{)100.000} \\ \underline{-96} \\ 40 \\ \underline{-36} \\ 40 \\ \underline{-36} \\ 4 \end{array}$$

Thus, $\frac{1}{12}$ is approximately 8.33%.**Work Practice 20****Objective D** Solving Applications with Percents, Decimals, and Fractions

Let's summarize what we have learned so far about percents, decimals, and fractions:

Summary of Converting Percents, Decimals, and Fractions

- To write a percent as a decimal, replace the % symbol with its decimal equivalent, 0.01; then multiply.
- To write a percent as a fraction, replace the % symbol with its fraction equivalent, $\frac{1}{100}$; then multiply.
- To write a decimal or fraction as a percent, multiply by 100%.

Practice 17–19

Write each fraction or mixed number as a percent.

$$17. \frac{3}{25} \quad 18. \frac{9}{40} \quad 19. 5\frac{1}{2}$$

Practice 20Write $\frac{3}{17}$ as a percent. Round to the nearest hundredth percent.**Answers**

$$17. 12\% \quad 18. 22\frac{1}{2}\% \quad 19. 550\% \\ 20. 17.65\%$$

✓ Concept Check Answers

a. 4 b. 8 c. 5 d. 7

If we let x represent a number, below we summarize using symbols.

Write a percent
as a decimal:

$$x\% = x(0.01)$$

Write a percent
as a fraction:

$$x\% = x \cdot \frac{1}{100}$$

Write a number
as a percent:

$$x = x \cdot 100\%$$

Practice 21

A family decides to spend no more than 27.5% of its monthly income on rent. Write 27.5% as a decimal and as a fraction.

Practice 22

Provincetown's budget for waste disposal increased by $1\frac{3}{4}$ times over the budget from last year. What percent of increase is this?

Answers

21. $0.275, \frac{11}{40}$ 22. 175%

Example 21

In the last ten years, automobile thefts in the continental United States have decreased 40.8%. Write this percent as a decimal and as a fraction. (*Source:* Insurance Information Institute)

Solution:



As a decimal: $40.8\% = 40.8(0.01) = 0.408$

As a fraction: $40.8\% = 40.8 \cdot \frac{1}{100} = \frac{40.8}{100} = \frac{40.8}{100} \cdot \frac{10}{10} = \frac{408}{1000} = \frac{8 \cdot 51}{8 \cdot 125} = \frac{51}{125}$

Thus, 40.8% written as a decimal is 0.408, and written as a fraction is $\frac{51}{125}$.

Work Practice 21

Example 22

An advertisement for a stereo system reads " $\frac{1}{4}$ off." What percent off is this?

Solution: Write $\frac{1}{4}$ as a percent.

$$\frac{1}{4} = \frac{1}{4} \cdot 100\% = \frac{1}{4} \cdot \frac{100\%}{1} = \frac{100}{4}\% = 25\%$$

Thus, " $\frac{1}{4}$ off" is the same as "25% off."

Work Practice 22

Note: It is helpful to know a few basic percent conversions. Appendix A.2 contains a handy reference of percent, decimal, and fraction equivalencies.

Also, Appendix A.3 shows how to find common percents of a number.

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

$\frac{1}{100}$ 0.01 100% percent

- _____ means “per hundred.”
- _____ = 1.
- The % symbol is read as _____.
- To write a decimal or a fraction as a percent, multiply by 1 in the form of _____.
- To write a percent as a *decimal*, drop the % symbol and multiply by _____.
- To write a percent as a *fraction*, drop the % symbol and multiply by _____.

Martin-Gay Interactive Videos



See Video 7.1

Watch the section lecture video and answer the following questions.

- Objective A** 7. From the lecture before Example 1, what is the most important thing to remember about percent? ▶
- Objective B** 8. From Example 4, what is the percent equivalent of 1? ▶
- Objective C** 9. Complete this statement based on Example 9: Multiplying by 100% is the same as multiplying by _____. ▶
- Objective D** 10. From Example 15, what is the main difference between writing a percent as a decimal and writing a percent as a fraction? ▶

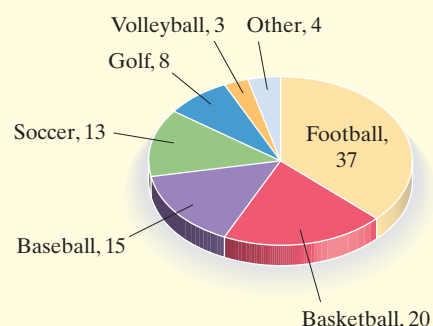
7.1 Exercise Set MyLab Math

Objective A Solve. See Examples 1 and 2.

- In a survey of 100 college students, 96 use the Internet. What percent use the Internet?
- A basketball player makes 81 out of 100 attempted free throws. What percent of free throws are made?

One hundred adults were asked to name their favorite sport, and the results are shown in the circle graph.

- What sport was preferred by most adults? What percent preferred this sport?
- What sport was preferred by the least number of adults? What percent preferred this sport?
- What percent of adults preferred football or soccer?
- What percent of adults preferred basketball or baseball?



Objective B Write each percent as a decimal. See Examples 3 through 7.

- | | | | |
|------------|----------|------------|------------|
| ▶ 7. 41% | 8. 62% | ▶ 9. 6% | 10. 3% |
| ▶ 11. 100% | 12. 136% | 13. 73.6% | 14. 45.7% |
| ▶ 15. 2.8% | 16. 1.4% | 17. 0.6% | 18. 0.9% |
| 19. 300% | 20. 500% | 21. 32.58% | 22. 72.18% |

Write each percent as a fraction or mixed number in simplest form. See Examples 8 through 12.

- | | | | |
|-----------------------|-----------------------|-------------------------|----------------------|
| ▶ 23. 8% | 24. 22% | 25. 4% | 26. 2% |
| 27. 4.5% | 28. 7.5% | ▶ 29. 175% | 30. 275% |
| 31. 6.25% | 32. 8.75% | ▶ 33. $10\frac{1}{3}\%$ | 34. $7\frac{3}{4}\%$ |
| 35. $22\frac{3}{8}\%$ | 36. $21\frac{7}{8}\%$ | | |

Objective C Write each decimal as a percent. See Examples 13 through 16.

- | | | | |
|------------|------------|-------------|-----------|
| ▶ 37. 0.22 | 38. 0.44 | ▶ 39. 0.006 | 40. 0.008 |
| 41. 5.3 | 42. 2.7 | 43. 0.056 | 44. 0.019 |
| 45. 0.2228 | 46. 0.1115 | ▶ 47. 3.00 | 48. 9.00 |
| 49. 0.7 | 50. 0.8 | | |

Write each fraction or mixed number as a percent. See Examples 17 through 19.

- | | | | |
|---------------------|---------------------|---------------------|--------------------|
| 51. $\frac{7}{10}$ | 52. $\frac{9}{10}$ | ▶ 53. $\frac{4}{5}$ | 54. $\frac{2}{5}$ |
| 55. $\frac{34}{50}$ | 56. $\frac{41}{50}$ | 57. $\frac{3}{8}$ | 58. $\frac{5}{16}$ |
| ▶ 59. $\frac{1}{3}$ | 60. $\frac{5}{6}$ | 61. $4\frac{1}{2}$ | 62. $6\frac{1}{5}$ |
| 63. $1\frac{9}{10}$ | 64. $2\frac{7}{10}$ | | |

Write each fraction as a percent. Round to the nearest hundredth percent. See Example 20.

- | | | | |
|--------------------|---------------------|----------------------|---------------------|
| 65. $\frac{9}{11}$ | 66. $\frac{11}{12}$ | ▶ 67. $\frac{4}{15}$ | 68. $\frac{10}{11}$ |
|--------------------|---------------------|----------------------|---------------------|

Objectives A B C Mixed Practice Complete each table. See Examples 1 through 20.

69.

Percent	Decimal	Fraction
60%		
	0.235	
		$\frac{4}{5}$
$33\frac{1}{3}\%$		
		$\frac{7}{8}$
75%		

70.

Percent	Decimal	Fraction
	0.525	
		$\frac{3}{4}$
$66\frac{1}{2}\%$		
		$\frac{5}{6}$
100%		
		$\frac{7}{50}$

71.

Percent	Decimal	Fraction
200%		
	2.8	
705%		
		$4\frac{27}{50}$

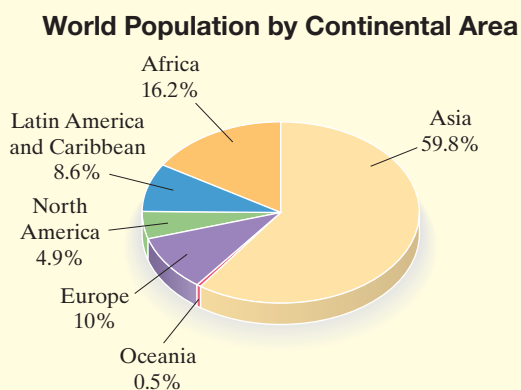
72.

Percent	Decimal	Fraction
800%		
	3.2	
608%		
		$9\frac{13}{50}$

Objective D Write each percent as a decimal and a fraction. See Examples 21 and 22.

73. People take aspirin for a variety of reasons. The most common use of aspirin is to prevent heart disease, accounting for 38% of all aspirin use. (Source: Bayer Market Research)
74. Spending for advertisements on the Internet in 2016 increased by 14.2% over spending in 2015. (Source: USA Today)
75. Together, the Greenland and Antarctic ice sheets contains more than 99% of the freshwater ice on earth. (Source: National Snow and Ice Data Center)
76. In 2016, geothermal sources accounted for 66% of Iceland's energy use. (Source: National Energy Authority of Iceland)
77. In the United States recently, 49.2% of households had no landlines, just cell phones. (Source: CTIA—The Wireless Association)
78. From 2005 to 2015, use of smoked tobacco products in the United States decreased by 28.6%. (Source: Centers for Disease Control and Prevention)

In Exercises 79 through 82, write the percent from the circle graph as a decimal and a fraction. See Examples 21 and 22.



79. Oceania: 0.5% 80. Europe: 10%
81. Africa: 16.2% 82. Asia: 59.8%

Solve. See Examples 21 and 22.

- 83.** In a particular year, 0.781 of all electricity produced in France was nuclear generated. Write this decimal as a percent.
- 85.** The mirrors on the Hubble Space Telescope are able to lock onto a target without deviating more than $\frac{7}{1000}$ of an arc-second. Write this fraction as a percent. (Source: NASA)
- 87.** In 2016, the U.S. Postal Service processed and delivered approximately 47% of the world's card and letter mail volume. Write this percent as a decimal. (Source: U.S. Postal Service)
- 84.** In 2015, the United States' share of global passenger vehicle production was 0.1496. Write this decimal as a percent. (Source: OICA)
- 86.** In a particular year, $\frac{1}{4}$ of light trucks sold in the United States were white. Write this fraction as a percent.
- 88.** In 2015, 81% of U.S. households had at least one high-definition television. Write this percent as a fraction. (Source: Leichtman Research Group)

Review

Perform the indicated operations. See Sections 4.6 and 4.7.

- 89.** $\frac{3}{4} - \frac{1}{2} \cdot \frac{8}{9}$ **90.** $\left(\frac{2}{11} + \frac{5}{11}\right)\left(\frac{2}{11} - \frac{5}{11}\right)$ **91.** $6\frac{2}{3} - 4\frac{5}{6}$ **92.** $6\frac{2}{3} \div 4\frac{5}{6}$

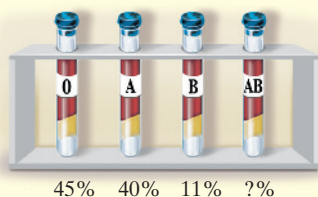
Concept Extensions

Solve. See the Concept Checks in this section.

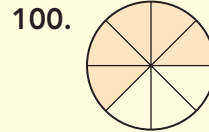
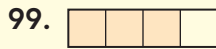
- 93.** Given the percent 52.8647%, round as indicated.
- Round to the nearest tenth percent.
 - Round to the nearest hundredth percent.
- 95.** Which of the following are correct?
- $6.5\% = 0.65$
 - $7.8\% = 0.078$
 - $120\% = 0.12$
 - $0.35\% = 0.0035$
- 94.** Given the percent 0.5269%, round as indicated.
- Round to the nearest tenth percent.
 - Round to the nearest hundredth percent.
- 96.** Which of the following are correct?
- $0.231 = 23.1\%$
 - $5.12 = 0.0512\%$
 - $3.2 = 320\%$
 - $0.0175 = 0.175\%$

Recall that $1 = 100\%$. This means that 1 whole is 100%. Use this for Exercises 97 and 98. (Source: Some Body by Dr. Pete Rowen)

- 97.** The four blood types are A, B, O, and AB. (Each blood type can also be further classified as Rh-positive or Rh-negative depending upon whether your blood contains protein or not.) Given the percent blood types for people in the United States below, calculate the percent of the U.S. population with AB blood type.
- 98.** The top four components of bone are below. Find the missing percent.
- Minerals—45%
 - Living tissue—30%
 - Water—20%
 - Other—?



What percent of the figure is shaded?



Fill in the blanks.

101. A fraction written as a percent is greater than 100% when the numerator is _____ than the denominator. (greater/less)

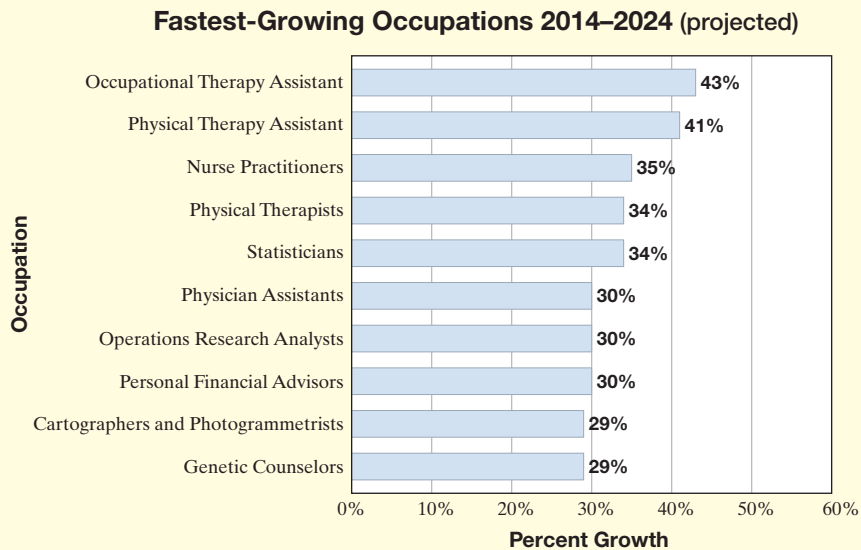
102. A decimal written as a percent is less than 100% when the decimal is _____ than 1. (greater/less)

Write each fraction as a decimal and then write each decimal as a percent. Round the decimal to three decimal places (nearest thousandth) and the percent to the nearest tenth percent.

 103. $\frac{21}{79}$

 104. $\frac{56}{102}$

The bar graph shows the predicted fastest-growing occupations by percent that require an associate degree or more education. Use this graph for Exercises 105 through 108.



Source: Bureau of Labor Statistics

105. What occupation is predicted to be the fastest growing?

106. What occupation is predicted to be the second fastest growing?

107. Write the percent change for physician assistants as a decimal.

108. Write the percent change for statisticians as a decimal.

 109. In your own words, explain how to write a percent as a decimal.

 110. In your own words, explain how to write a decimal as a percent.

7.2 Solving Percent Problems with Equations

Objectives

- A** Write Percent Problems as Equations.
- B** Solve Percent Problems.

Practice 1

Translate: 8 is what percent of 48?

Practice 2

Translate: 2.6 is 40% of what number?

Practice 3

Translate: What number is 90% of 0.045?

Answers

- 1. $8 = x \cdot 48$
- 2. $2.6 = 40\% \cdot x$
- 3. $x = 90\% \cdot 0.045$

Sections 7.2 and 7.3 introduce two methods for solving percent problems. It may not be necessary for you to study both sections. You may want to check with your instructor for further advice.

To solve percent problems in this section, we will translate the problems into mathematical statements, or equations.

Objective A Writing Percent Problems as Equations

Recognizing key words in a percent problem is helpful in writing the problem as an equation. Three key words in the statement of a percent problem and their meanings are as follows:

of means **multiplication** ($\cdot$)

is means **equal** ($=$)

what (or some equivalent) means **the unknown number**

In our examples, we will let the letter x stand for the unknown number.

Example 1 Translate to an equation:

5 is what percent of 20?

Solution:

5	is	what percent	of	20?
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
5	=	x	$\cdot$	20

Work Practice 1

Helpful Hint

Remember that an equation is simply a mathematical statement that contains an equal sign ($=$).

$$5 = 20x$$

$\uparrow$
equal sign

Example 2 Translate to an equation:

1.2 is 30% of what number?

Solution:

1.2	is	30%	of	what number?
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
1.2	=	30%	$\cdot$	x

Work Practice 2

Example 3 Translate to an equation:

What number is 25% of 0.008?

Solution:

What number	is	25%	of	0.008?
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
x	=	25%	$\cdot$	0.008

Work Practice 3

Examples

Translate each question to an equation.

4. 38% of 200 is what number?

$$\begin{array}{ccccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 38\% & \cdot & 200 & = & x \end{array}$$

5. 40% of what number is 80?

$$\begin{array}{ccccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 40\% & \cdot & x & = & 80 \end{array}$$

6. What percent of 85 is 34?

$$\begin{array}{ccccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ x & \cdot & 85 & = & 34 \end{array}$$

Work Practice 4–6

 **Concept Check** In the equation $2x = 10$, what step is taken to solve the equation?

Objective B Solving Percent Problems

You may have noticed by now that each percent problem has contained three numbers—in our examples, two are known and one is unknown. Each of these numbers is given a special name.

$$\begin{array}{ccccccc} 15\% & \text{of} & 60 & \text{is} & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 15\% & \cdot & 60 & = & 9 \\ \text{percent} & & \text{base} & & \text{amount} \end{array}$$

We call this equation the **percent equation**.

Percent Equation

$$\text{percent} \cdot \text{base} = \text{amount}$$

Once a percent problem has been written as a percent equation, we can use the equation to find the unknown number, whether it is the percent, the base, or the amount.

Example 7 Solving Percent Equations for the Amount**Solution:**

$$\begin{array}{ccccccc} \text{What number} & \text{is} & 35\% & \text{of} & 60? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ x & = & 35\% & \cdot & 60 & \text{Translate to an equation.} \\ x & = & 0.35 & \cdot & 60 & \text{Write 35\% as 0.35.} \\ x & = & 21 & & & \text{Multiply: } \begin{array}{r} 60 \\ \times 0.35 \\ \hline 300 \\ 1800 \\ \hline 21.00 \end{array} \end{array}$$

Then 21 is 35% of 60. Is this reasonable? To see, round 35% to 40%. Then 40% of 60 or $0.40(60)$ is 24. Our result is reasonable since 21 is close to 24.

Work Practice 7**Practice 4–6**

Translate each question to an equation.

4. 56% of 180 is what number?

5. 12% of what number is 21?

6. What percent of 95 is 76?

Practice 7

What number is 25% of 90?

Answers4. $56\% \cdot 180 = x$ 5. $12\% \cdot x = 21$ 6. $x \cdot 95 = 76$ 7. 22.5

 **Concept Check Answer**

Divide both sides of the equation by 2.

Helpful Hint

When solving a percent equation, write the percent as a decimal or fraction.

Practice 8

95% of 400 is what number?

Example 8

85% of 300 is what number?

Solution:

$$\begin{array}{rclcl}
 85\% & \cdot & 300 & = & x & \text{Translate to an equation.} \\
 0.85 & \cdot & 300 & = & x & \text{Write 85\% as 0.85.} \\
 255 & = & x & & & \text{Multiply: } 0.85 \cdot 300 = 255.
 \end{array}$$

Then 85% of 300 is 255. Is this result reasonable? To see, round 85% to 90%. Then 90% of 300 or $0.90(300) = 270$, which is close to 255.

Work Practice 8

Practice 9

15% of what number is 2.4?

Example 9

Solving Percent Equations for the Base

12% of what number is 0.6?

Solution:

$$\begin{array}{rclcl}
 12\% & \cdot & x & = & 0.6 & \text{Translate to an equation.} \\
 0.12 & \cdot & x & = & 0.6 & \text{Write 12\% as 0.12.} \\
 \frac{0.12 \cdot x}{0.12} & = & \frac{0.6}{0.12} & & & \text{Divide both sides by 0.12.} \\
 x & = & 5 & & & \begin{array}{r} 5. \\ 0.12 \overline{)0.60} \\ \underline{-60} \\ 0 \end{array}
 \end{array}$$

Then 12% of 5 is 0.6. Is this reasonable? To see, round 12% to 10%. Then 10% of 5 or $0.10(5) = 0.5$, which is close to 0.6.

Work Practice 9

Practice 10

18 is $4\frac{1}{2}\%$ of what number?

Example 10

13 is $6\frac{1}{2}\%$ of what number?

Solution:

$$\begin{array}{rclcl}
 13 & = & 6\frac{1}{2}\% \cdot x & & & \text{Translate to an equation.} \\
 13 & = & 0.065 \cdot x & & & 6\frac{1}{2}\% = 6.5\% = 0.065 \\
 \frac{13}{0.065} & = & \frac{0.065 \cdot x}{0.065} & & & \text{Divide both sides by 0.065.} \\
 200 & = & x & & & \begin{array}{r} 200. \\ 0.065 \overline{)13.000} \\ \underline{-130} \\ 0 \end{array}
 \end{array}$$

Then 13 is $6\frac{1}{2}\%$ of 200. Check to see if this result is reasonable.

Work Practice 10

Answers

8. 380 9. 16 10. 400

Example 11 Solving Percent Equations for the Percent

What percent of 12 is 9?

Solution:

$$\begin{array}{ccccccc}
 \text{What percent} & \text{of} & 12 & \text{is} & 9? \\
 \downarrow & & \downarrow & \downarrow & \downarrow \\
 x & \cdot & 12 & = & 9 & \text{Translate to an equation.} \\
 \frac{x \cdot 12}{12} & = & \frac{9}{12} & \text{Divide both sides by 12.} \\
 \text{or } x & = & 0.75
 \end{array}$$

Next, since we are looking for percent, we can write $\frac{9}{12}$ or 0.75 as a percent.

$$x = 75\%$$

Then 75% of 12 is 9. To check, see that $75\% \cdot 12 = 9$.

Work Practice 11**Helpful Hint**

If your unknown in the percent equation is a percent, don't forget to convert your answer to a percent.

Example 12 78 is what percent of 65?**Solution:**

$$\begin{array}{ccccccc}
 78 & \text{is} & \text{what percent} & \text{of} & 65? \\
 \downarrow & & \downarrow & \downarrow & \downarrow \\
 78 & = & x & \cdot & 65 & \text{Translate to an equation.} \\
 \frac{78}{65} & = & \frac{x \cdot 65}{65} & & & \text{Divide both sides by 65.} \\
 1.2 & = & x & & & \\
 120\% & = & x & & & \text{Write 1.2 as a percent.}
 \end{array}$$

Then 78 is 120% of 65. Check this result.

Work Practice 12

✓Concept Check Consider these problems.

- 75% of 50 =
 a. 50 b. a number greater than 50 c. a number less than 50
- 40% of a number is 10. Is the number
 a. 10? b. less than 10? c. greater than 10?
- 800 is 120% of what number? Is the number
 a. 800? b. less than 800? c. greater than 800?

Helpful Hint

Use the following to see if your answers are reasonable.

$$\begin{array}{l}
 (100\%) \text{ of a number} = \text{the number} \\
 \left(\begin{array}{c} \text{a percent} \\ \text{greater than} \\ 100\% \end{array} \right) \text{ of a number} = \text{a number greater} \\
 \hspace{10em} \text{than the original number} \\
 \left(\begin{array}{c} \text{a percent} \\ \text{less than 100\%} \end{array} \right) \text{ of a number} = \text{a number less} \\
 \hspace{10em} \text{than the original number}
 \end{array}$$

Practice 11

What percent of 90 is 27?

Practice 12

63 is what percent of 45?

Answers

11. 30% 12. 140%

✓Concept Check Answers

1. c 2. c 3. b

Vocabulary, Readiness & Video Check

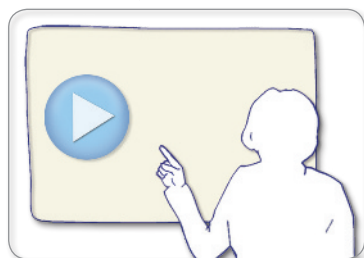
Use the choices below to fill in each blank.

percent	amount	of	less
base	the number	is	greater

- The word _____ translates to “=”.
- The word _____ usually translates to “multiplication.”
- In the statement “10% of 90 is 9,” the number 9 is called the _____, 90 is called the _____, and 10 is called the _____.
- 100% of a number = _____.
- Any “percent greater than 100%” of “a number” = “a number _____ than the original number.”
- Any “percent less than 100%” of “a number” = “a number _____ than the original number.”

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 7.2

Objective A 7. What are the three translations we need to remember from the lecture before Example 1?

Objective B 8. What is different about the translated equation in Example 5?

7.2 Exercise Set MyLab Math

Objective A Translating Translate each to an equation. Do not solve. See Examples 1 through 6.

- 18% of 81 is what number?
- 36% of 72 is what number?
- 20% of what number is 105?
- 40% of what number is 6?
- 0.6 is 40% of what number?
- 0.7 is 20% of what number?
- What percent of 80 is 3.8?
- 9.2 is what percent of 92?
- What number is 9% of 43?
- What number is 25% of 55?
- What percent of 250 is 150?
- What percent of 375 is 300?

Objective B Solve. See Examples 7 and 8.

▶ 13. 10% of 35 is what number?

14. 25% of 68 is what number?

15. What number is 14% of 205?

16. What number is 18% of 425?

Solve. See Examples 9 and 10.

▶ 17. 1.2 is 12% of what number?

18. 0.22 is 44% of what number?

19. $8\frac{1}{2}\%$ of what number is 51?

20. $4\frac{1}{2}\%$ of what number is 45?

Solve. See Examples 11 and 12.

21. What percent of 80 is 88?

22. What percent of 40 is 60?

23. 17 is what percent of 50?

24. 48 is what percent of 50?

Objectives A B Mixed Practice Solve. See Examples 1 through 12.

25. 0.1 is 10% of what number?

26. 0.5 is 5% of what number?

27. 150% of 430 is what number?

28. 300% of 56 is what number?

29. 82.5 is $16\frac{1}{2}\%$ of what number?

30. 7.2 is $6\frac{1}{4}\%$ of what number?

▶ 31. 2.58 is what percent of 50?

32. 2.64 is what percent of 25?

33. What number is 42% of 60?

34. What number is 36% of 80?

35. What percent of 184 is 64.4?

36. What percent of 120 is 76.8?

37. 120% of what number is 42?

38. 160% of what number is 40?

39. 2.4% of 26 is what number?

40. 4.8% of 32 is what number?

41. What percent of 600 is 3?

42. What percent of 500 is 2?

43. 6.67 is 4.6% of what number?

44. 9.75 is 7.5% of what number?

45. 1575 is what percent of 2500?

46. 2520 is what percent of 3500?

47. 2 is what percent of 50?

48. 2 is what percent of 40?

Review

Find the value of x in each proportion. See Section 6.2.

49. $\frac{27}{x} = \frac{9}{10}$

50. $\frac{35}{x} = \frac{7}{5}$

51. $\frac{x}{5} = \frac{8}{11}$

52. $\frac{x}{3} = \frac{6}{13}$

Write each sentence as a proportion. See Section 6.2.

53. 17 is to 12 as x is to 20. 54. 20 is to 25 as x is to 10. 55. 8 is to 9 as 14 is to x . 56. 5 is to 6 as 15 is to x .

Concept Extensions

For each equation, determine the next step taken to find the value of n . See the first Concept Check in this section.

57. $5 \cdot n = 32$

- a. $n = 5 \cdot 32$ b. $n = \frac{5}{32}$ c. $n = \frac{32}{5}$ d. none of these

58. $68 = 8 \cdot n$

- a. $n = 8 \cdot 68$ b. $n = \frac{68}{8}$ c. $n = \frac{8}{68}$ d. none of these

59. $0.06 = n \cdot 7$

- a. $n = 0.06 \cdot 7$ b. $n = \frac{0.06}{7}$ c. $n = \frac{7}{0.06}$ d. none of these

60. $n = 0.7 \cdot 12$

- a. $n = 8.4$ b. $n = \frac{12}{0.7}$ c. $n = \frac{0.7}{12}$ d. none of these

-  61. Write a word statement for the equation $20\% \cdot x = 18.6$. Use the phrase “what number” for “ x .”

-  62. Write a word statement for the equation $x = 33\frac{1}{3}\% \cdot 24$. Use the phrase “what number” for “ x .”

For each exercise, determine whether the percent, x , is (a) 100%, (b) greater than 100%, or (c) less than 100%. See the second Concept Check in this section.

63. $x\%$ of 20 is 30 64. $x\%$ of 98 is 98 65. $x\%$ of 120 is 85 66. $x\%$ of 35 is 50

For each exercise, determine whether the number, y , is (a) equal to 45, (b) greater than 45, or (c) less than 45.

67. 55% of 45 is y 68. 230% of 45 is y 69. 100% of 45 is y
70. 30% of y is 45 71. 100% of y is 45 72. 180% of y is 45

-  73. In your own words, explain how to solve a percent equation.  74. Write a percent problem that uses the percent 50%.

Solve.

 75. 1.5% of 45,775 is what number?

 76. What percent of 75,528 is 27,945.36?

 77. 22,113 is 180% of what number?

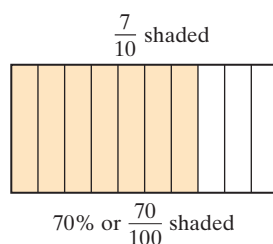
7.3 Solving Percent Problems with Proportions

There is more than one method that can be used to solve percent problems. (See the note at the beginning of Section 7.2.) In the last section, we used the percent equation. In this section, we will use proportions.

Objective A Writing Percent Problems as Proportions

To understand the proportion method, recall that 70% means the ratio of 70 to 100, or $\frac{70}{100}$.

$$70\% = \frac{70}{100} = \frac{7}{10}$$



Since the ratio $\frac{70}{100}$ is equal to the ratio $\frac{7}{10}$, we have the proportion

$$\frac{7}{10} = \frac{70}{100}$$

We call this proportion the **percent proportion**. In general, we can name the parts of this proportion as follows:

Percent Proportion

$$\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100} \quad \leftarrow \text{always } 100$$

or

$$\begin{array}{lcl} \text{amount} \rightarrow & a & \\ \text{base} \rightarrow & b & \end{array} \quad \frac{p}{100} \quad \leftarrow \text{percent}$$

When we translate percent problems to proportions, the **percent**, p , can be identified by looking for the symbol % or the word *percent*. The **base**, b , usually follows the word *of*. The **amount**, a , is the part compared to the whole.

Helpful Hint

This table may be useful when identifying the parts of a proportion.

Part of Proportion	How It's Identified
Percent	% or <i>percent</i>
Base	Appears after <i>of</i>
Amount	Part compared to whole

Objectives

- A** Write Percent Problems as Proportions.
- B** Solve Percent Problems.

Practice 1

Translate to a proportion.
27% of what number is 54?

Practice 2

Translate to a proportion.
30 is what percent of 90?

Practice 3

Translate to a proportion.
What number is 25% of 116?

Practice 4

Translate to a proportion.
680 is 65% of what number?

Answers

1. $\frac{54}{b} = \frac{27}{100}$ 2. $\frac{30}{90} = \frac{p}{100}$
3. $\frac{a}{116} = \frac{25}{100}$ 4. $\frac{680}{b} = \frac{65}{100}$

Example 1 Translate to a proportion.

12% of what number is 47?

Solution:

percent	base It appears after the word <i>of</i> .	amount It is the part compared to the whole.
---------	---	---

$$\begin{array}{l} \text{amount} \rightarrow \frac{47}{b} = \frac{12}{100} \leftarrow \text{percent} \\ \text{base} \rightarrow \end{array}$$

Work Practice 1**Example 2** Translate to a proportion.

101 is what percent of 200?

Solution:

amount It is the part compared to the whole.	percent	base It appears after the word <i>of</i> .
---	---------	---

$$\begin{array}{l} \text{amount} \rightarrow \frac{101}{200} = \frac{p}{100} \leftarrow \text{percent} \\ \text{base} \rightarrow \end{array}$$

Work Practice 2**Example 3** Translate to a proportion.

What number is 90% of 45?

Solution:

amount It is the part compared to the whole.	percent	base It appears after the word <i>of</i> .
---	---------	---

$$\begin{array}{l} \text{amount} \rightarrow \frac{a}{45} = \frac{90}{100} \leftarrow \text{percent} \\ \text{base} \rightarrow \end{array}$$

Work Practice 3**Example 4** Translate to a proportion.

238 is 40% of what number?

Solution:

amount	percent	base
--------	---------	------

$$\frac{238}{b} = \frac{40}{100}$$

Work Practice 4

Example 5 Translate to a proportion.

What percent of 30 is 75?

Solution:

percent

base

amount

$$\frac{75}{30} = \frac{p}{100}$$

Work Practice 5**Example 6** Translate to a proportion.

45% of 105 is what number?

Solution:

percent

base

amount

$$\frac{a}{105} = \frac{45}{100}$$

Work Practice 6

✓ **Concept Check** Consider the statement “78 is what percent of 350?” Which part of the percent proportion is unknown?

- a. the amount
- b. the base
- c. the percent

Consider another statement: “14 is 10% of some number.” Which part of the percent proportion is unknown?

- a. the amount
- b. the base
- c. the percent

Objective B Solving Percent Problems

The proportions that we have written in this section contain three values that can change: the percent, the base, and the amount. If any two of these values are known, we can find the third (the unknown) value. To do this, we write a percent proportion and find the unknown value as we did in Section 6.2.

Example 7 Solving Percent Proportions for the Amount

What number is 30% of 9?

Solution:

amount

percent

base

$$\frac{a}{9} = \frac{30}{100}$$

(Continued on next page)

Practice 5

Translate to a proportion.
What percent of 40 is 75?

Practice 6

Translate to a proportion.
46% of 80 is what number?

Practice 7

What number is 8% of 120?

Answers

5. $\frac{75}{40} = \frac{p}{100}$ 6. $\frac{a}{80} = \frac{46}{100}$ 7. 9.6

✓ **Concept Check Answers**
c, b

Helpful Hint

The proportion in Example 7 contains the ratio $\frac{30}{100}$. A ratio in a proportion may be simplified before solving the proportion. The unknown number in both

$$\frac{a}{9} = \frac{30}{100} \text{ and } \frac{a}{9} = \frac{3}{10} \text{ is } 2.7.$$

Practice 8

65% of what number is 52?

Practice 9

15.4 is 5% of what number?

Answers

8. 80 9. 308

✓ **Concept Check Answer**
by putting the result into the proportion and checking that the proportion is true

To solve, we set cross products equal to each other.

$$\frac{a}{9} = \frac{30}{100}$$

$$a \cdot 100 = 9 \cdot 30 \quad \text{Set cross products equal.}$$

$$100a = 270 \quad \text{Multiply.}$$

$$\frac{100a}{100} = \frac{270}{100} \quad \text{Divide both sides by 100, the coefficient of } a.$$

$$a = 2.7 \quad \text{Simplify.}$$

Thus, 2.7 is 30% of 9.

Work Practice 7**Example 8** Solving Percent Problems for the Base

150% of what number is 30?

Solution: percent base amount

$$\frac{30}{b} = \frac{150}{100} \quad \text{Write the proportion.}$$

$$\frac{30}{b} = \frac{3}{2} \quad \text{Simplify } \frac{150}{100} \text{ and write as } \frac{3}{2}.$$

$$30 \cdot 2 = b \cdot 3 \quad \text{Set cross products equal.}$$

$$60 = 3b \quad \text{Multiply.}$$

$$\frac{60}{3} = \frac{3b}{3} \quad \text{Divide both sides by 3.}$$

$$20 = b \quad \text{Simplify.}$$

Thus, 150% of 20 is 30.

Work Practice 8

✓ **Concept Check** When solving a percent problem by using a proportion, describe how you can check the result.

Example 9

20.8 is 40% of what number?

Solution: amount percent base

$$\frac{20.8}{b} = \frac{40}{100} \text{ or } \frac{20.8}{b} = \frac{2}{5} \quad \text{Write the proportion and simplify } \frac{40}{100}.$$

$$20.8 \cdot 5 = b \cdot 2 \quad \text{Set cross products equal.}$$

$$104 = 2b \quad \text{Multiply.}$$

$$\frac{104}{2} = \frac{2b}{2} \quad \text{Divide both sides by 2.}$$

$$52 = b \quad \text{Simplify.}$$

So, 20.8 is 40% of 52.

Work Practice 9

Example 10 Solving Percent Problems for the Percent

What percent of 50 is 8?

Solution:

percent base amount

$$\frac{8}{50} = \frac{p}{100} \quad \text{or} \quad \frac{4}{25} = \frac{p}{100} \quad \text{Write the proportion and simplify } \frac{8}{50}.$$

$$4 \cdot 100 = 25 \cdot p$$

Set cross products equal.

$$400 = 25p$$

Multiply.

$$\frac{400}{25} = \frac{25p}{25}$$

Divide both sides by 25.

$$16 = p$$

Simplify.

So, 16% of 50 is 8.

Helpful Hint

Recall from our percent proportion that this number already is a percent. Just keep the number as is and attach a % symbol.

Example 11

504 is what percent of 360?

Solution:

amount percent base

$$\frac{504}{360} = \frac{p}{100}$$

Let's choose not to simplify the ratio $\frac{504}{360}$.

$$504 \cdot 100 = 360 \cdot p \quad \text{Set cross products equal.}$$

$$50,400 = 360p \quad \text{Multiply.}$$

$$\frac{50,400}{360} = \frac{360p}{360} \quad \text{Divide both sides by 360.}$$

$$140 = p \quad \text{Simplify.}$$

Notice that by choosing not to simplify $\frac{504}{360}$, we had larger numbers in our equation. Either way, we find that 504 is 140% of 360.

Work Practice 11**Practice 10**

What percent of 40 is 8?

Practice 11

414 is what percent of 180?

Helpful Hint

Use the following to see whether your answers to the above examples and practice problems are reasonable.

100% of a number = the number

$\left(\begin{array}{c} \text{a percent} \\ \text{greater than} \\ 100\% \end{array} \right)$ of a number = a number larger than the original number

$\left(\begin{array}{c} \text{a percent} \\ \text{less than 100\%} \end{array} \right)$ of a number = a number less than the original number

Answers

10. 20% 11. 230%

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. These choices will be used more than once.

amount base percent

1. When translating the statement “20% of 15 is 3” to a proportion, the number 3 is called the _____, 15 is the _____, and 20 is the _____.
2. In the question “50% of what number is 28?,” which part of the percent proportion is unknown? _____
3. In the question “What number is 25% of 200?,” which part of the percent proportion is unknown? _____
4. In the question “38 is what percent of 380?,” which part of the percent proportion is unknown? _____

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 7.3

Objective A 5. In Example 1, how did we identify what part of the percent proportion 45 is?

Objective B 6. From Examples 4–6, what number is *always* part of the cross product equation of a percent proportion?

7.3 Exercise Set MyLab Math

Objective A Translating Translate each to a proportion. Do not solve. See Examples 1 through 6.

- ▶ 1. 98% of 45 is what number?
2. 92% of 30 is what number?
3. What number is 4% of 150?
4. What number is 7% of 175?
5. 14.3 is 26% of what number?
6. 1.2 is 47% of what number?
7. 35% of what number is 84?
8. 85% of what number is 520?
- ▶ 9. What percent of 400 is 70?
10. What percent of 900 is 216?
11. 8.2 is what percent of 82?
12. 9.6 is what percent of 96?

Objective B Solve. See Example 7.

13. 40% of 65 is what number?
14. 25% of 84 is what number?
15. What number is 18% of 105?
16. What number is 60% of 29?

Solve. See Examples 8 and 9.

17. 15% of what number is 90?

18. 55% of what number is 55?

▶ 19. 78 is 78% of what number?

20. 1.1 is 44% of what number?

Solve. See Examples 10 and 11.

21. What percent of 35 is 42?

22. What percent of 98 is 147?

23. 14 is what percent of 50?

24. 24 is what percent of 50?

Objectives A B Mixed Practice *Solve. See Examples 1 through 11.*

25. 3.7 is 10% of what number?

26. 7.4 is 5% of what number?

27. 2.4% of 70 is what number?

28. 2.5% of 90 is what number?

29. 160 is 16% of what number?

30. 30 is 6% of what number?

31. 394.8 is what percent of 188?

32. 550.4 is what percent of 172?

33. What number is 89% of 62?

34. What number is 53% of 130?

▶ 35. What percent of 6 is 2.7?

36. What percent of 5 is 1.6?

37. 140% of what number is 105?

38. 170% of what number is 221?

39. 1.8% of 48 is what number?

40. 7.8% of 24 is what number?

41. What percent of 800 is 4?

42. What percent of 500 is 3?

43. 3.5 is 2.5% of what number?

44. 9.18 is 6.8% of what number?

▶ 45. 20% of 48 is what number?

46. 75% of 14 is what number?

47. 2486 is what percent of 2200?

48. 9310 is what percent of 3800?

Review

Add or subtract the fractions. See Sections 4.4, 4.5, and 4.7.

$$49. -\frac{11}{16} + \left(-\frac{3}{16}\right)$$

$$50. \frac{7}{12} - \frac{5}{8}$$

$$51. 3\frac{1}{2} - \frac{11}{30}$$

$$52. 2\frac{2}{3} + 4\frac{1}{2}$$

Add or subtract the decimals. See Section 5.2.

$$53. \begin{array}{r} 0.41 \\ + 0.29 \\ \hline \end{array}$$

$$54. \begin{array}{r} 10.78 \\ 4.3 \\ + 0.21 \\ \hline \end{array}$$

$$55. \begin{array}{r} 2.38 \\ - 0.19 \\ \hline \end{array}$$

$$56. \begin{array}{r} 16.37 \\ - 2.61 \\ \hline \end{array}$$

Concept Extensions

-  **57.** Write a word statement for the proportion $\frac{x}{28} = \frac{25}{100}$. Use the phrase “what number” for “ x .”

-  **58.** Write a percent statement that translates to $\frac{16}{80} = \frac{20}{100}$.

Suppose you have finished solving four percent problems using proportions that you set up correctly. Check each answer to see if each makes the proportion a true proportion. If any proportion is not true, solve it to find the correct solution. See the Concept Checks in this section.

$$59. \frac{a}{64} = \frac{25}{100}$$

Is the amount equal to 17?

$$60. \frac{520}{b} = \frac{65}{100}$$

Is the base equal to 800?

$$61. \frac{p}{100} = \frac{13}{52}$$

Is the percent equal to 25 (25%)?

$$62. \frac{36}{12} = \frac{p}{100}$$

Is the percent equal to 50 (50%)?

-  **63.** In your own words, describe how to identify the percent, the base, and the amount in a percent problem.

-  **64.** In your own words, explain how to use a proportion to solve a percent problem.

Solve. Round to the nearest tenth, if necessary.

 **65.** What number is 22.3% of 53,862?

 **66.** What percent of 110,736 is 88,542?

 **67.** 8652 is 119% of what number?

Percent and Percent Problems

Write each number as a percent.

1. 0.94

2. 0.17

3. $\frac{3}{8}$

4. $\frac{7}{2}$

5. 4.7

6. 8

7. $\frac{9}{20}$

8. $\frac{53}{50}$

9. $6\frac{3}{4}$

10. $3\frac{1}{4}$

11. 0.02

12. 0.06

Write each percent as a decimal.

13. 71%

14. 31%

15. 3%

16. 4%

17. 224%

18. 700%

19. 2.9%

20. 6.6%

Write each percent as a decimal and as a fraction or mixed number in simplest form. (If necessary when writing as a decimal, round to the nearest thousandth.)

21. 7%

22. 5%

23. 6.8%

24. 11.25%

Answers

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

13. _____

14. _____

15. _____

16. _____

17. _____

18. _____

19. _____

20. _____

21. _____

22. _____

23. _____

24. _____

25. _____

25. 74%

26. 45%

27. $16\frac{1}{3}\%$ 28. $12\frac{2}{3}\%$

26. _____

27. _____

28. _____

Solve each percent problem.

29. 15% of 90 is what number?

30. 78 is 78% of what number?

29. _____

30. _____

31. 297.5 is 85% of what number?

32. 78 is what percent of 65?

31. _____

32. _____

33. _____

33. 23.8 is what percent of 85?

34. 38% of 200 is what number?

34. _____

35. _____

35. What number is 40% of 85?

36. What percent of 99 is 128.7?

36. _____

37. _____

37. What percent of 250 is 115?

38. What number is 45% of 84?

38. _____

39. _____

39. 42% of what number is 63?

40. 95% of what number is 58.9?

40. _____

7.4 Applications of Percent

Objective A Solving Applications Involving Percent

Percent is used in a variety of everyday situations. The next examples show just a few ways that percent occurs in real-life settings. (Each of these examples shows two ways of solving these problems. If you studied Section 7.2 only, see *Method 1*. If you studied Section 7.3 only, see *Method 2*.)

The next example has to do with the Appalachian Trail, a hiking trail conceived by a forester in 1921 and diagrammed to the right. (*Note:* The trail mileage changes from year to year as maintenance groups reroute the trail as needed.)



Example 1

The circle graph in the margin shows the Appalachian Trail mileage by state. If the total mileage of the trail is 2174, use the circle graph to determine the number of miles in the state of New York. Round to the nearest whole mile.

Solution: *Method 1.* First, we state the problem in words.

In words: What number is 4% of 2174?

Translate: $x = 4\% \cdot 2174$

To solve for x , we find $4\% \cdot 2174$.

$$x = 0.04 \cdot 2174 \quad \text{Write 4\% as a decimal.}$$

$$x = 86.96 \quad \text{Multiply.}$$

$$x \approx 87 \quad \text{Round to the nearest whole.}$$

Rounded to the nearest whole mile, we have that approximately 87 miles of the Appalachian Trail are in New York state.

Method 2. State the problem in words; then translate.

In words: What number is 4% of 2174?

Translate: $\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100}$

$$\frac{a}{2174} = \frac{4}{100}$$

Next, we solve for a .

$$a \cdot 100 = 2174 \cdot 4 \quad \text{Set cross products equal.}$$

$$100a = 8696 \quad \text{Multiply.}$$

$$\frac{100a}{100} = \frac{8696}{100} \quad \text{Divide both sides by 100.}$$

$$a = 86.96 \quad \text{Simplify.}$$

$$a \approx 87 \quad \text{Round to the nearest whole.}$$

Rounded to the nearest whole mile, we have that approximately 87 miles of the Appalachian Trail are in New York state.

Work Practice 1

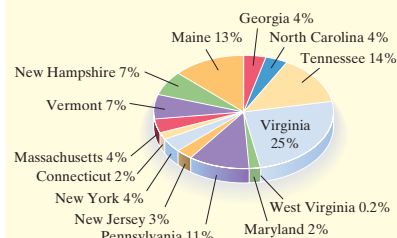
Objectives

- A** Solve Applications Involving Percent.
- B** Find Percent of Increase and Percent of Decrease.

Practice 1

If the total mileage of the Appalachian Trail is 2174, use the circle graph to determine the number of miles in the state of Virginia.

Appalachian Trail Mileage by State Percent



Total miles: 2174

(*Due to rounding, these percents have a sum greater than 100%.)
Source: purebound.com

Answer

1. 543.5 mi

Practice 2

From 2014 to 2024, it is projected that the number of employed nurses will grow by 439,300. If the number of nurses employed in 2014 was 2,751,000, find the percent of increase in nurses employed from 2014 to 2024. Round to the nearest percent. (Source: Bureau of Labor Statistics)



Practice 3

The freshmen class of 864 students is 32% of all students at Euclid University. How many students go to Euclid University?

Answers

2. 16% 3. 2700

Example 2 Finding Percent of Nursing School Applications Accepted

There continues to be a shortage of nursing school facilities. Recently, of the 266 thousand applications to bachelor degree nursing schools, 120 thousand of these were accepted. What percent of these applications were accepted? Round to the nearest percent. (Source: Bureau of Labor Statistics)

Solution: *Method 1.* First, we state the problem in words.

In words: 120 is what percent of 266?

Translate: $\begin{array}{ccccccc} 120 & = & & x & \cdot & 266 \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \text{or } 120 & = & & 266x \end{array}$

Next, solve for x .

$$\frac{120}{266} = \frac{266x}{266} \quad \text{Divide both sides by 266.}$$

$$0.45 \approx x \quad \text{Divide and round to the nearest hundredth.}$$

$$45\% \approx x \quad \text{Write as a percent.}$$

About 45% of nursing school applications were accepted.

Method 2.

In words: 120 is what percent of 266?

$\begin{array}{ccc} \downarrow & & \downarrow \\ \text{amount} & & \text{percent} \\ \downarrow & & \downarrow \\ \text{base} & & \end{array}$

Translate: $\begin{array}{l} \text{amount} \rightarrow 120 \\ \text{base} \rightarrow 266 \end{array} \rightarrow \frac{120}{266} = \frac{p}{100} \quad \leftarrow \text{percent}$

Next, solve for p .

$$120 \cdot 100 = 266 \cdot p \quad \text{Set cross products equal.}$$

$$12,000 = 266p \quad \text{Multiply.}$$

$$\frac{12,000}{266} = \frac{266p}{266} \quad \text{Divide both sides by 266.}$$

$$45 \approx p$$

About 45% of nursing school applications were accepted.

Work Practice 2

Example 3 Finding the Base Number of Absences

Mr. Percy, the principal at Slidell High School, counted 31 freshmen absent during a particular day. If this is 4% of the total number of freshmen, how many freshmen are there at Slidell High School?

Solution: *Method 1.* First we state the problem in words; then we translate.

In words: 31 is 4% of what number?

Translate: $\begin{array}{ccccccc} 31 & = & 4\% & \cdot & x \\ \downarrow & & \downarrow & & \downarrow \\ 31 & = & 4\% & \cdot & x \end{array}$

Next, we solve for x .

$$31 = 0.04 \cdot x \quad \text{Write 4\% as a decimal.}$$

$$\frac{31}{0.04} = \frac{0.04x}{0.04} \quad \text{Divide both sides by 0.04.}$$

$$775 = x \quad \text{Simplify.}$$

There are 775 freshmen at Slidell High School.

Method 2. First we state the problem in words; then we translate.

In words: 31 is 4% of what number?

$\downarrow$ $\downarrow$ $\downarrow$
 amount percent base

Translate: $\frac{\text{amount} \rightarrow 31}{\text{base} \rightarrow b} = \frac{4}{100}$ ← percent

Next, we solve for b .

$$31 \cdot 100 = b \cdot 4 \quad \text{Set cross products equal.}$$

$$3100 = 4b \quad \text{Multiply.}$$

$$\frac{3100}{4} = \frac{4b}{4} \quad \text{Divide both sides by 4.}$$

$$775 = b \quad \text{Simplify.}$$

There are 775 freshmen at Slidell High School.

Work Practice 3

Example 4 Finding the Base Increase in Licensed Drivers

From 2015 to 2016, the number of licensed drivers on the road in the United States increased by 5.8%. In 2015, there were about 210 million licensed drivers on the road.

- Find the increase in licensed drivers from 2015 to 2016.
- Find the number of licensed drivers on the road in 2016. (Source: Federal Highway Administration)



Solution: **Method 1.** First we find the increase in licensed drivers.

In words: What number is 5.8% of 210?

Translate: $x = 5.8\% \cdot 210$

Next, we solve for x .

$$x = 0.058 \cdot 210 \quad \text{Write 5.8\% as a decimal.}$$

$$x = 12.18 \quad \text{Multiply.}$$

- The increase in licensed drivers was 12.18 million.
- This means that the number of licensed drivers in 2016 was

$$\begin{aligned}
 \text{Number of licensed drivers in 2016} &= \text{Number of licensed drivers in 2015} + \text{Increase in number of licensed drivers} \\
 &= 210 \text{ million} + 12.18 \text{ million} \\
 &= 222.18 \text{ million}
 \end{aligned}$$

Practice 4

From 2015 to 2016, the number of registered cars and light trucks on the road in the United States increased by 1.5%. In 2015, the number of registered cars and light trucks on the road was 260 million.

- Find the increase in the number of registered cars and light trucks on the road in 2016.
- Find the total number of registered cars and light trucks on the road in 2016.

(Source: Hedges Company and Federal Highway Administration)

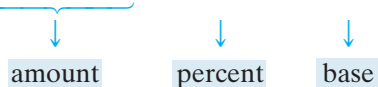
Answers

4. a. 3.9 million b. 263.9 million

(Continued on next page)

Method 2. First we find the increase in licensed drivers.

In words: What number is 5.8% of 210?



Translate: $\frac{\text{amount} \rightarrow a}{\text{base} \rightarrow 210} = \frac{5.8 \leftarrow \text{percent}}{100}$

Next, we solve for a .

$$a \cdot 100 = 210 \cdot 5.8 \quad \text{Set cross products equal.}$$

$$100a = 1218 \quad \text{Multiply.}$$

$$\frac{100a}{100} = \frac{1218}{100} \quad \text{Divide both sides by 100.}$$

$$a = 12.18 \quad \text{Simplify.}$$

- a. The increase in licensed drivers was 12.18 million.
- b. This means that the number of licensed drivers in 2016 was

$$\begin{aligned}
 \begin{array}{c} \text{Number of} \\ \text{licensed drivers} \\ \text{in 2016} \end{array} &= \begin{array}{c} \text{Number of} \\ \text{licensed drivers} \\ \text{in 2015} \end{array} + \begin{array}{c} \text{Increase} \\ \text{in number of} \\ \text{licensed drivers} \end{array} \\
 &= 210 \text{ million} + 12.18 \text{ million} \\
 &= 222.18 \text{ million}
 \end{aligned}$$

Work Practice 4

Objective B Finding Percent of Increase and Percent of Decrease

We often use percents to show how much an amount has increased or decreased.

Suppose that the population of a town is 10,000 people and then it increases by 2000 people. The **percent of increase** is

$$\frac{\text{amount of increase} \rightarrow 2000}{\text{original amount} \rightarrow 10,000} = 0.2 = 20\%$$

In general, we have the following.

Percent of Increase

$$\text{percent of increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

Then write the quotient as a percent.

Practice 5

The number of people attending the local play, *Peter Pan*, increased from 285 on Friday to 333 on Saturday. Find the percent of increase in attendance. Round to the nearest tenth percent.

Answer

5. 16.8%

Example 5 Finding Percent of Increase

The number of applications for a mathematics scholarship at one university increased from 34 to 45 in one year. What is the percent of increase? Round to the nearest whole percent.

Solution: First we find the amount of increase by subtracting the original number of applicants from the new number of applicants.

$$\text{amount of increase} = 45 - 34 = 11$$

The amount of increase is 11 applicants. To find the percent of increase,

$$\text{percent of increase} = \frac{\text{amount of increase}}{\text{original amount}} = \frac{11}{34} \approx 0.32 = 32\%$$

The number of applications increased by about 32%.



Helpful Hint Make sure that this number is the original number and not the new number.

Work Practice 5

✓ **Concept Check** A student is calculating the percent of increase in enrollment from 180 students one year to 200 students the next year. Explain what is wrong with the following calculations:

~~$$\text{Amount of increase} = 200 - 180 = 20$$~~

~~$$\text{Percent of increase} = \frac{20}{200} = 0.1 = 10\%$$~~

Suppose that your income was \$300 a week and then it decreased by \$30. The **percent of decrease** is

$$\begin{aligned} \text{amount of decrease} &\rightarrow \$30 \\ \text{original amount} &\rightarrow \$300 \end{aligned} \quad \frac{30}{300} = 0.1 = 10\%$$

Percent of Decrease

$$\text{percent of decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$

Then write the quotient as a percent.

Example 6 Finding Percent of Decrease

In response to a decrease in sales, a company with 1500 employees reduces the number of employees to 1230. What is the percent of decrease?

Solution: First we find the amount of decrease by subtracting 1230 from 1500.

$$\text{amount of decrease} = 1500 - 1230 = 270$$

The amount of decrease is 270. To find the percent of decrease,

$$\text{percent of decrease} = \frac{\text{amount of decrease}}{\text{original amount}} = \frac{270}{1500} = 0.18 = 18\%$$

The number of employees decreased by 18%.

Work Practice 6

✓ **Concept Check** An ice cream stand sold 6000 ice cream cones last summer. This year the same stand sold 5400 cones. Was there a 10% increase, a 10% decrease, or neither? Explain.

Practice 6

A town with a population of 20,200 in 2003 decreased to 18,483 in 2013. What was the percent of decrease?

Answer

6. 8.5%

✓ Concept Check Answers

To find the percent of increase, you have to divide the amount of increase (20) by the original amount (180); 10% decrease.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 7.4

Objective A 1. How do we interpret the answer 175,000 in Example 1? ▶

Objective B 2. In Example 3, what does the improper fraction tell us? ▶

7.4 Exercise Set MyLab Math

Objective A Solve. For Exercises 1 and 2, the solutions have been started for you. See Examples 1 through 4. If necessary, round percents to the nearest tenth and all other answers to the nearest whole.

1. An inspector found 24 defective bolts during an inspection. If this is 1.5% of the total number of bolts inspected, how many bolts were inspected?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.

Go to *Method 1* or *Method 2*.

Method 1.

2. TRANSLATE into an equation. (Fill in the boxes.)

$$\begin{array}{ccccccc} 24 & \text{is} & 1.5\% & \text{of} & \text{what number} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 24 & \square & 1.5\% & \square & x \end{array}$$

3. SOLVE for x . (See Example 3, Method 1, for help.)
4. INTERPRET. The total number of bolts inspected was _____.

Method 2.

2. TRANSLATE into a proportion. (Fill in the first two blanks with “amount” or “base.”)

$$\begin{array}{ccccccc} 24 & \text{is} & 1.5\% & \text{of} & \text{what number} \\ \downarrow & & \searrow & & \downarrow \\ \underline{\hspace{1cm}} & & \text{percent} & & \underline{\hspace{1cm}} \\ \text{amount} \rightarrow \underline{\hspace{1cm}} & = & \frac{1.5}{100} & \leftarrow \text{percent} \\ \text{base} \rightarrow \underline{\hspace{1cm}} & & & & \end{array}$$

3. SOLVE the proportion. (See Example 3, Method 2, for help.)
4. INTERPRET. The total number of bolts inspected was _____.
3. One model of Total Gym provides weight resistance through adjustments of incline. The minimum weight resistance is 4% of the weight of the person using the Total Gym. Find the minimum weight resistance possible for a 220-pound man. (*Source:* Total Gym)
5. A student’s cost for last semester at her community college was \$2700. She spent \$378 of that on books. What percent of last semester’s college costs was spent on books?

2. A day care worker found 28 children absent one day during an epidemic of chicken pox. If this was 35% of the total number of children attending the day care center, how many children attend this day care center?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.

Go to *Method 1* or *Method 2*.

Method 1.

2. TRANSLATE into an equation. (Fill in the boxes.)

$$\begin{array}{ccccccc} 28 & \text{is} & 35\% & \text{of} & \text{what number} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 28 & \square & 35\% & \square & x \end{array}$$

3. SOLVE for x . (See Example 3, Method 1, for help.)
4. INTERPRET. The total number of children attending the day care center is _____.

Method 2.

2. TRANSLATE into a proportion. (Fill in the first two blanks with “amount” or “base.”)

$$\begin{array}{ccccccc} 28 & \text{is} & 35\% & \text{of} & \text{what number} \\ \downarrow & & \searrow & & \downarrow \\ \underline{\hspace{1cm}} & & \text{percent} & & \underline{\hspace{1cm}} \\ \text{amount} \rightarrow \underline{\hspace{1cm}} & = & \frac{35}{100} & \leftarrow \text{percent} \\ \text{base} \rightarrow \underline{\hspace{1cm}} & & & & \end{array}$$

3. SOLVE the proportion. (See Example 3, Method 2, for help.)
4. INTERPRET. The total number of children attending the day care center is _____.
4. The maximum weight resistance for one model of Total Gym is 60% of the weight of the person using it. Find the maximum weight resistance possible for a 220-pound man. (See Exercise 3 if needed.)
6. Pierre Sampeau belongs to his local food cooperative, where he receives a percentage of what he spends each year as a dividend. He spent \$3850 last year at the food cooperative store and received a dividend of \$154. What percent of his total spending at the food cooperative did he receive as a dividend?

In 2016, there were about 43,500 cinemas screens in the United States and Canada. Use this information for Exercises 7 and 8. (Source: MPAA)

7. If about 38.5% of the total screens in the United States and Canada were digital 3-D screens, find the approximate number of digital 3-D screens. Round to the nearest whole.
8. If about 2% of the total screens in the United States and Canada were analog screens, find the approximate number of analog screens. Round to the nearest whole.

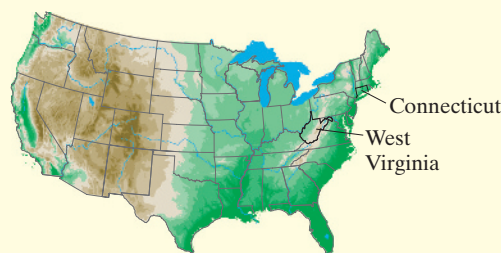
Solve.

9. In 2016, there were 36,899 McDonald's restaurants worldwide, with 14,155 of them located in the United States. Determine the percent of McDonald's restaurants in the United States in 2016. Round to the nearest percent. (Source: McDonald's Corporation)
10. Of the 66,800 veterinarians in private practice in the United States in 2015, approximately 36,900 are women. Determine the percent of female veterinarians in private practice in the United States in 2015. Round to the nearest percent. (Source: American Veterinary Medical Association)
11. A furniture company currently produces 6200 chairs per month. If production decreases by 8%, find the decrease and the new number of chairs produced each month.
12. The enrollment at a local college decreased by 5% over last year's enrollment of 7640. Find the decrease in enrollment and the current enrollment.
13. From 2014 to 2024, the number of people employed as occupational therapy assistants in the United States is expected to increase by 43%. The number of people employed as occupational therapy assistants in 2014 was 41,900. Find the predicted number of occupational therapy assistants in 2024. (Source: Bureau of Labor Statistics)
14. From 2012 to 2016, the number of households owning reptiles increased by 63%. The number of households owning reptiles in 2012 was 3 million. Find the number of households owning reptiles in 2016. (Source: American Veterinary Medical Association)



Two States, West Virginia and Connecticut, decreased in population from 2014 to 2016. Their locations are shown on the U.S. map below. (Source: United States Census Bureau)

15. In 2014, the population of West Virginia was approximately 1,853,000. If the population decrease was about 0.49%, find the population of West Virginia in 2016. Round to the nearest whole number.
16. In 2014, the population of Connecticut was approximately 3,595,000. If the population decrease was about 0.001%, find the population of Connecticut in 2016. Round to the nearest whole number.



A popular extreme sport is snowboarding. Ski trails are marked with difficulty levels of easy ●, intermediate ■, difficult ◆, expert ◆◆, and other variations. Use this information for Exercises 17 and 18. Round each percent to the nearest whole.

17. At Keystone ski area in Colorado, approximately 41 of the 135 total ski runs are rated intermediate. What percent of the runs are intermediate?
18. At Telluride ski area in Colorado, about 28 of the 115 total ski runs are rated easy. What percent of the runs are easy?

For each food described, find the percent of total calories from fat. If necessary, round to the nearest tenth percent. See Example 2.

19. Ranch dressing serving size of 2 tablespoons

	Calories
Total	40
From fat	20

20. Unsweetened cocoa powder serving size of 1 tablespoon

	Calories
Total	20
From fat	5

21. **Nutrition Facts**

Serving Size 1 pouch (20g)	
Servings Per Container 6	
Amount Per Serving	
Calories	80
Calories from fat	10
% Daily Value*	
Total Fat 1g	2%
Sodium 45mg	2%
Total Carbohydrate 17g	6%
Sugars 9g	
Protein 0g	
Vitamin C	25%
Not a significant source of saturated fat, cholesterol, dietary fiber, vitamin A, calcium and iron.	
*Percent Daily Values are based on a 2,000 calorie diet.	

Artificial Fruit Snacks

22. **Nutrition Facts**

Serving Size $\frac{1}{4}$ cup (33g)	
Servings Per Container About 9	
Amount Per Serving	
Calories 190	Calories from Fat 130
% Daily Value	
Total Fat 16g	24%
Saturated Fat 3g	16%
Cholesterol 0mg	0%
Sodium 135mg	6%
Total Carbohydrate 9g	3%
Dietary Fiber 1g	5%
Sugars 2g	
Protein 5g	
Vitamin A 0% • Vitamin C 0%	
Calcium 0% • Iron 8%	

Peanut Mixture

23. **Nutrition Facts**

Serving Size 18 crackers (29g)	
Servings Per Container About 9	
Amount Per Serving	
Calories 120	Calories from Fat 35
% Daily Value*	
Total Fat 4g	6%
Saturated Fat 0.5g	3%
Polyunsaturated Fat 0g	
Monounsaturated Fat 1.5g	
Cholesterol 0mg	0%
Sodium 220mg	9%
Total Carbohydrate 21g	7%
Dietary Fiber 2g	7%
Sugars 3g	
Protein 2g	
Vitamin A 0% • Vitamin C 0%	
Calcium 2% • Iron 4%	
Phosphorus 10%	

Snack Crackers

24. **Nutrition Facts**

Serving Size 28 crackers (31g)	
Servings Per Container About 6	
Amount Per Serving	
Calories 130	Calories from Fat 35
% Daily Value*	
Total Fat 4g	6%
Saturated Fat 2g	10%
Polyunsaturated Fat 1g	
Monounsaturated Fat 1g	
Cholesterol 0mg	0%
Sodium 470mg	20%
Total Carbohydrate 23g	8%
Dietary Fiber 1g	4%
Sugars 4g	
Protein 2g	
Vitamin A 0% • Vitamin C 0%	
Calcium 0% • Iron 2%	

Snack Crackers

Solve. If necessary, round money amounts to the nearest cent and all other amounts to the nearest tenth. See Examples 1 through 4.

25. A family paid \$26,250 as a down payment for a home. If this represents 15% of the price of the home, find the price of the home.
26. A banker learned that \$842.40 is withheld from his monthly check for taxes and insurance. If this represents 18% of his total pay, find the total pay.

27. An owner of a repair service company estimates that for every 40 hours a repairperson is on the job, he can bill for only 78% of the hours. The remaining hours, the repairperson is idle or driving to or from a job. Determine the number of hours per 40-hour week the owner can bill for a repairperson.
28. A manufacturer of electronic components expects 1.04% of its products to be defective. Determine the number of defective components expected in a batch of 28,350 components. Round to the nearest whole component.
29. A car manufacturer announced that next year, the price of a certain model of car will increase by 4.5%. This year the price is \$19,286. Find the increase in price and the new price.
30. A union contract calls for a 6.5% salary increase for all employees. Determine the increase and the new salary that a worker currently making \$58,500 under this contract can expect.

A popular extreme sport is artificial wall climbing. The photo shown is an artificial climbing wall. Exercises 31–32 are about the Footsloggers Climbing Tower in Boone, North Carolina.

31. A climber is resting at a height of 21 feet while on the Footsloggers Climbing Tower. If this is 60% of the tower's total height, find the height of the tower.
32. A group plans to climb the Footsloggers Climbing Tower at the group rate, once they save enough money. Thus far, \$126 has been saved. If this is 70% of the total amount needed for the group, find the total price.



33. Tuition for an Ohio resident at the Columbus campus of Ohio State University was \$8994 in 2010. The tuition increased by 11.6% during the period from 2010 to 2016. Find the increase and the tuition for the 2016–2017 school year. Round the increase to the nearest whole dollar. (Source: Ohio State University)
34. The population of Americans aged 65 and older was 40 million in 2010. That population is projected to increase by 80.5% by 2030. Find the increase and the projected 2030 population. (Source: Bureau of the Census)
35. From 2014–2015 to 2024–2025, the number of students enrolled in associate degree programs is projected to increase by 22.3%. If the enrollment in associate degree programs in 2014–2015 was 6,700,000, find the increase and the projected number of students enrolled in an associate degree program in 2024–2025. (Source: National Center for Educational Statistics)
36. From 2010–2011 to 2021–2022, the number of bachelor degrees awarded is projected to increase by 174%. If the number of bachelor degrees awarded in 2010–2011 was 1,703,000, find the increase and the projected number of bachelor degrees awarded in the 2021–2022 school year. (Source: National Center for Educational Statistics)

Objective B Find the amount of increase and the percent of increase. See Example 5.

	Original Amount	New Amount	Amount of Increase	Percent of Increase
37.	50	80		
38.	8	12		
39.	65	117		
40.	68	170		

Find the amount of decrease and the percent of decrease. See Example 6.

	Original Amount	New Amount	Amount of Decrease	Percent of Decrease
41.	8	6		
42.	25	20		
43.	160	40		
44.	200	162		

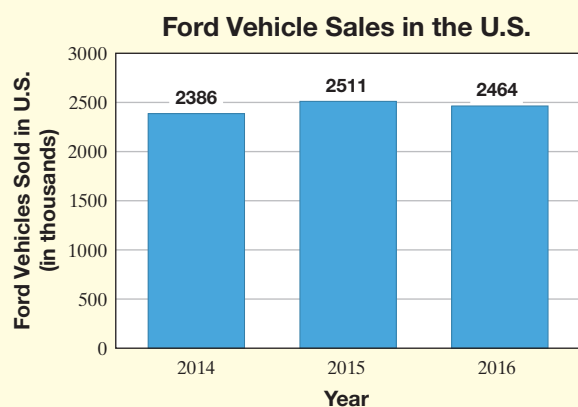
Solve. Round percents to the nearest tenth, if necessary. See Examples 5 and 6.

45. There are 150 calories in a cup of whole milk and only 84 in a cup of skim milk. In switching to skim milk, find the percent of decrease in number of calories per cup.
46. In reaction to a slow economy, the number of employees at a soup company decreased from 530 to 477. What was the percent of decrease in the number of employees?
47. Before taking a typing course, Geoffry Landers could type 32 words per minute. By the end of the course, he was able to type 76 words per minute. Find the percent of increase.
48. The number of cable TV systems recently decreased from 10,845 to 10,700. Find the percent of decrease.
49. In 2016, there were approximately 71 million virtual reality devices in use worldwide. This is expected to grow to 337 million in 2020. What is the projected percent of increase? (Source: CTIA—The Wireless Association)
50. The population of Japan is expected to decrease from 127,799 thousand in 2011 to 97,076 thousand in 2050. Find the percent of decrease. (Source: International Programs Center, Bureau of the Census, U.S. Dept. of Commerce)



51. In 2014, there were 3782 thousand elementary and secondary teachers employed in the United States. This number is expected to increase to 4151 thousand teachers in 2021. What is the percent of increase? (Source: National Center for Educational Statistics)
52. In 2014, approximately 475 thousand correctional officers were employed in the United States. By 2024, this number is expected to increase to 493 thousand correctional officers. What is the percent of increase? (Source: Bureau of Labor Statistics)
53. Between 2014 and 2015, permanent digital downloads of singles decreased from approximately 1199 million to approximately 1021 million. What is the percent of decrease? (Source: Recording Industry Association of America)
54. As the largest health care occupation, registered nurses held about 3.1 million jobs in 2015. The number of registered nurses is expected to be 3.9 million by 2025. What is the percent of increase? (Source: American Association of Colleges of Nursing)

Ford is U.S. automobile brand, named after its founder, Henry Ford. Use the graph below to answer Exercises 55 and 56.



Source: Ford Motor Company

55. Find the percent of decrease in sales of Ford vehicles in the United States from 2015 to 2016.

56. Find the percent of increase in sales of Ford vehicles in the United States from 2014 to 2015.

- ▶ 57. In 1940, the average size of a farm in the United States was 174 acres. In a recent year, the average size of a farm in the United States had increased to 420 acres. What is this percent of increase? (Source: National Agricultural Statistics Service)



58. In 2012, there were 651 deaths from boating accidents in the United States. By 2015, the number of deaths from boating accidents had decreased to 626. What was the percent of decrease? (Source: U.S. Coast Guard)



In 1999, Napster, a free online file-sharing service, debuted. iTunes, which debuted in 2003, is given credit for getting people to start paying for digital music. In 2015, for the first time, streaming music was the largest revenue producer in the music industry.

59. In 1999, total revenue from U.S. music sales and licensing was \$14.6 billion. By 2015, this revenue had dropped to \$7.1 billion. Find this percent of decrease in music revenue. (Source: Recording Industry Association of America)
60. By comparing prices, a particular music album downloads from a low of \$2.99 to a high of \$7.99. Find the percent of increase from \$2.99 to \$7.99.

Review

Perform each indicated operation. See Sections 4.3 through 4.5, 4.7, and 5.2 through 5.4.

61. 0.12×38

62. $29.4 \div 0.7$

63. $9.2 + 1.98$

64. $78 - 19.46$

65. $-\frac{3}{8} + \frac{5}{12}$

66. $\left(-\frac{3}{8}\right)\left(-\frac{5}{12}\right)$

67. $2\frac{4}{5} \div 3\frac{9}{10}$

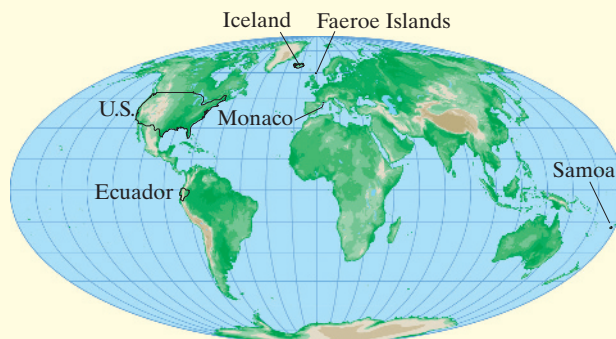
68. $2\frac{4}{5} - 3\frac{9}{10}$

Concept Extensions

69. If a number is increased by 100%, how does the increased number compare with the original number? Explain your answer.
70. In your own words, explain what is wrong with the following statement: “Last year we had 80 students attend. This year we have a 50% increase or a total of 160 students attend.”

Check the Chapter Opener bar graph. If the number of Internet users in 2016 was 46.1% of the world population, it does not necessarily mean that each country of the world has 46.1% of its population identified as Internet users. In fact, there is a country that claims to have 100% of its population as Internet users. In the table below, we have listed a few selected countries along with Internet user data. Use your knowledge of percent and fill in the table. If needed, round percents to the nearest tenth and populations to the nearest hundred. (Source: Internet Live Stats)

	Country	Population	Internet Users % of Population	Number of Internet Users
71.	Iceland	331,778	100%	
72.	Faeroe Islands	48,239	98.5%	
73.	Monaco	37,863		35,196
74.	Samoa	194,523		56,373
75.	United States		88.5%	286,942,400
76.	Ecuador		43.1%	7,055,575



77. Check the bar graph for Exercises 55 and 56. Are your answers for Exercises 55 and 56 reasonable? Explain why or why not.
78. Check the circle graph in Section 7.1, Exercises 79 through 82. If the total world population of approximately 7417 million people represents 100% of this circle graph, do the percents calculated seem reasonable? Explain why or why not.

Explain what errors were made by each student when solving percent of increase or decrease problems and then correct the errors. See the Concept Checks in this section.

“The population of a certain rural town was 150 in 1990, 180 in 2000, and 150 in 2010.”

79. Find the percent of increase in population from 1990 to 2000.

Miranda's solution: Percent of increase = $\frac{30}{180} = 0.1\bar{6} \approx 16.7\%$

80. Find the percent of decrease in population from 2000 to 2010.

Jeremy's solution: Percent of decrease = $\frac{30}{150} = 0.20 = 20\%$

81. The percent of increase from 1990 to 2000 is the same as the percent decrease from 2000 to 2010. True or false?

Chris's answer: True because they had the same amount of increase as the amount of decrease.

7.5 Percent and Problem Solving: Sales Tax, Commission, and Discount

Objective A Calculating Sales Tax and Total Price

Percents are frequently used in the retail trade. For example, most states charge a tax on certain items when purchased. This tax is called a **sales tax**, and retail stores collect it for the state. Sales tax is almost always stated as a percent of the purchase price.

A 9% sales tax rate on a purchase of a \$10 calculator gives a sales tax of

$$\text{sales tax} = 9\% \text{ of } \$10 = 0.09 \cdot \$10.00 = \$0.90$$

The total price to the customer would be

$$\begin{array}{rcccl} \text{purchase price} & \text{plus} & \text{sales tax} & & \\ \downarrow & \downarrow & \downarrow & & \\ \$10.00 & + & \$0.90 & = & \$10.90 \end{array}$$

This example suggests the following equations:

Sales Tax and Total Price

$$\text{sales tax} = \text{tax rate} \cdot \text{purchase price}$$

$$\text{total price} = \text{purchase price} + \text{sales tax}$$

In this section we round dollar amounts to the nearest cent.

Example 1 Finding Sales Tax and Purchase Price

Find the sales tax and the total price on the purchase of an \$85.50 atlas in a city where the sales tax rate is 7.5%.



Solution: The purchase price is \$85.50 and the tax rate is 7.5%.

$$\begin{array}{rcll} \text{sales tax} & = & \text{tax rate} \cdot \text{purchase price} & \\ \downarrow & & \downarrow & \swarrow \\ \text{sales tax} & = & 7.5\% \cdot \$85.50 & \\ & = & 0.075 \cdot \$85.5 & \text{Write 7.5\% as a decimal.} \\ & \approx & \$6.41 & \text{Rounded to the nearest cent} \end{array}$$

Thus, the sales tax is \$6.41. Next find the total price.

$$\begin{array}{rcll} \text{total price} & = & \text{purchase price} + \text{sales tax} & \\ \downarrow & & \downarrow & \swarrow \\ \text{total price} & = & \$85.50 + \$6.41 & \\ & = & \$91.91 & \end{array}$$

The sales tax on \$85.50 is \$6.41, and the total price is \$91.91.

Work Practice 1

Objectives

- A** Calculate Sales Tax and Total Price.
- B** Calculate Commissions.
- C** Calculate Discount and Sale Price.

Practice 1

If the sales tax rate is 8.5%, what is the sales tax and the total amount due on a \$59.90 Goodgrip tire? (Round the sales tax to the nearest cent.)

Answer

1. tax: \$5.09; total: \$64.99

Practice 2

The sales tax on an \$18,500 automobile is \$1665. Find the sales tax rate.

Practice 3

A sales representative for Office Product Copiers sold \$47,632 worth of copy equipment and supplies last month. What is his commission for the month if he is paid a commission of 6.6% of his total sales for the month?

Answers

2. 9% 3. \$3143.71

✓ Concept Check Answer

Since $10\% = \frac{1}{10}$, the sales tax is

$\frac{\$50}{10} = \$5$. The total price should have been \$55.

✓ **Concept Check** The purchase price of a textbook is \$50 and sales tax is 10%. If you are told by the cashier that the total price is \$75, how can you tell that a mistake has been made?

Example 2 Finding a Sales Tax Rate

The sales tax on a \$310 Sony flat-screen digital 32-inch television is \$26.35. Find the sales tax rate.

Solution: Let r represent the unknown sales tax rate. Then

$$\begin{array}{rcl} \text{sales tax} & = & \text{tax rate} \cdot \text{purchase price} \\ \downarrow & & \downarrow \quad \downarrow \\ \$26.35 & = & r \cdot \$310 \\ \frac{26.35}{310} & = & \frac{r \cdot 310}{310} \quad \text{Divide both sides by 310.} \\ 0.085 & = & r \quad \text{Simplify.} \\ 8.5\% & = & r \quad \text{Write 0.085 as a percent.} \end{array}$$

The sales tax rate is 8.5%.

Work Practice 2

Objective B Calculating Commissions

A **wage** is payment for performing work. Hourly wage, commissions, and salary are some of the ways wages can be paid. Many people who work in sales are paid a commission. An employee who is paid a **commission** is paid a percent of his or her total sales.

Commission

$$\text{commission} = \text{commission rate} \cdot \text{sales}$$

Example 3 Finding the Amount of Commission

Sherry and Cindy Souter, real estate broker partners for Wealth Investments, sold a house for \$214,000 last week. If their commission is 1.5% of the selling price of the home, find the amount of their commission.

Solution:

$$\begin{array}{rcl} \text{commission} & = & \text{commission rate} \cdot \text{sales} \\ \downarrow & & \downarrow \quad \downarrow \\ \text{commission} & = & 1.5\% \cdot \$214,000 \\ & = & 0.015 \cdot \$214,000 \quad \text{Write 1.5\% as 0.015.} \\ & = & \$3210 \quad \text{Multiply.} \end{array}$$



Their commission on the house is \$3210.

Work Practice 3

Example 4 Finding a Commission Rate

A salesperson earned \$1560 for selling \$13,000 worth of electronics equipment. Find the commission rate.

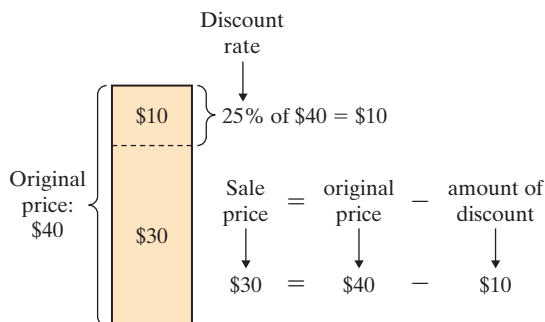
Solution: Let r stand for the unknown commission rate. Then

$$\begin{array}{rclcl}
 \text{commission} & = & \text{commission rate} & \cdot & \text{sales} \\
 \downarrow & & \downarrow & & \downarrow \\
 \$1560 & = & r & \cdot & \$13,000 \\
 \frac{1560}{13,000} & = & r & \text{Divide 1560 by 13,000, the number multiplied by } r. \\
 0.12 & = & r & \text{Simplify.} \\
 12\% & = & r & \text{Write 0.12 as a percent.}
 \end{array}$$

The commission rate is 12%.

Work Practice 4**Objective C** Calculating Discount and Sale Price

Suppose that an item that normally sells for \$40 is on sale for 25% off. This means that the **original price** of \$40 is reduced, or **discounted**, by 25% of \$40, or \$10. The **discount rate** is 25%, the **amount of discount** is \$10, and the **sale price** is \$40 - \$10, or \$30. Study the diagram below to visualize these terms.



To calculate discounts and sale prices, we can use the following equations:

Discount and Sale Price

$$\begin{aligned}
 \text{amount of discount} &= \text{discount rate} \cdot \text{original price} \\
 \text{sale price} &= \text{original price} - \text{amount of discount}
 \end{aligned}$$

Example 5 Finding a Discount and a Sale Price

An electric rice cooker that normally sells for \$65 is on sale for 25% off. What is the amount of discount and what is the sale price?

Solution: First we find the amount of discount, or simply the discount.

$$\begin{array}{rclcl}
 \text{amount of discount} & = & \text{discount rate} & \cdot & \text{original price} \\
 \downarrow & & \downarrow & & \downarrow \\
 \text{amount of discount} & = & 25\% & \cdot & \$65 \\
 & = & 0.25 \cdot \$65 & \text{Write 25\% as 0.25.} \\
 & = & \$16.25 & \text{Multiply.}
 \end{array}$$

(Continued on next page)

Practice 4

A salesperson earns \$645 for selling \$4300 worth of appliances. Find the commission rate.

Practice 5

A discontinued washer and dryer combo is advertised on sale for 35% off the regular price of \$700. Find the amount of discount and the sale price.

Answers

4. 15% 5. \$245; \$455

The discount is \$16.25. Next, find the sale price.

$$\begin{array}{rcl}
 \text{sale price} & = & \text{original price} - \text{discount} \\
 \downarrow & & \downarrow \quad \quad \downarrow \\
 \text{sale price} & = & \$65 - \$16.25 \\
 & = & \$48.75 \quad \text{Subtract.}
 \end{array}$$

The sale price is \$48.75.

Work Practice 5

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

amount of discount	sale price	sales tax
commission	total price	

- _____ = tax rate $\cdot$ purchase price
- _____ = purchase price + sales tax
- _____ = commission rate $\cdot$ sales
- _____ = discount rate $\cdot$ original price
- _____ = original price - amount of discount

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 7.5

- Objective A** 6. In Example 1, what is our first step after translating the problem into an equation?
- Objective B** 7. What is our final step in solving Example 2?
- Objective C** 8. In the lecture before Example 3, since both equations shown involve the “amount of discount,” how can the two equations be combined into one equation?

7.5 Exercise Set MyLab Math

Objective A Solve. See Examples 1 and 2.

- What is the sales tax on a jacket priced at \$150 if the sales tax rate is 5%?
- If the sales tax rate is 6%, find the sales tax on a microwave oven priced at \$188.
- The purchase price of a camcorder is \$799. What is the total price if the sales tax rate is 7.5%? Round to the nearest hundredth.
- A stereo system has a purchase price of \$426. What is the total price if the sales tax rate is 8%?
- A new large-screen television has a purchase price of \$4790. If the sales tax on this purchase is \$335.30, find the sales tax rate.
- The sales tax on the purchase of a \$6800 used car is \$374. Find the sales tax rate.

7. The sales tax on a table saw is \$10.20.
- What is the purchase price of the table saw (before tax) if the sales tax rate is 8.5%? (*Hint: Use the sales tax equation and insert the replacement values.*)
 - Find the total price of the table saw.



8. The sales tax on a one-half-carat diamond ring is \$76.
- Find the purchase price of the ring (before tax) if the sales tax rate is 9.5%. (See the hint for Exercise 7.a.)
 - Find the total price of the ring.



9. A gold and diamond bracelet sells for \$1800. Find the sales tax and the total price if the sales tax rate is 6.5%.
- ▶ 11. The sales tax on the purchase of a futon is \$24.25. If the tax rate is 5%, find the purchase price of the futon.
13. The sales tax is \$98.70 on a stereo sound system purchase of \$1645. Find the sales tax rate.
15. A cell phone costs \$210, a battery recharger costs \$15, and batteries cost \$5. What is the sales tax and total price for purchasing these items if the sales tax rate is 7%?
10. The purchase price of a personal computer is \$1890. If the sales tax rate is 8%, what is the sales tax and the total price?
12. The sales tax on the purchase of a TV-DVD combination is \$32.85. If the tax rate is 9%, find the purchase price of the TV-DVD.
14. The sales tax is \$103.50 on a necklace purchase of \$1150. Find the sales tax rate.
16. Ms. Warner bought a blouse for \$35, a skirt for \$55, and a blazer for \$95. Find the sales tax and the total price she paid, given a sales tax rate of 6.5%.



Objective B Solve. See Examples 3 and 4.

17. A sales representative for a large furniture warehouse is paid a commission rate of 4%. Find her commission if she sold \$1,329,401 worth of furniture last year.
- ▶ 19. A salesperson earned a commission of \$1380.40 for selling \$9860 worth of paper products. Find the commission rate.
18. Rosie Davis-Smith is a beauty consultant for a home cosmetic business. She is paid a commission rate of 12.8%. Find her commission if she sold \$1638 in cosmetics last month.
20. A salesperson earned a commission of \$3575 for selling \$32,500 worth of books to various bookstores. Find the commission rate.

21. How much commission will Jack Pruet make on the sale of a \$325,900 house if he receives 1.5% of the selling price?
22. Frankie Lopez sold \$9638 of jewelry this week. Find her commission for the week if she receives a commission rate of 5.6%.
23. A real estate agent earned a commission of \$5565 for selling a house. If his rate is 3%, find the selling price of the house. (*Hint: Use the commission equation and insert the replacement values.*)
24. A salesperson earned \$1750 for selling fertilizer. If her commission rate is 7%, find the selling price of the fertilizer. (See the hint for Exercise 23.)

Objective C Find the amount of discount and the sale price. See Example 5.

	Original Price	Discount Rate	Amount of Discount	Sale Price
25.	\$89	10%		
26.	\$74	20%		
27.	\$196.50	50%		
28.	\$110.60	40%		
29.	\$410	35%		
30.	\$370	25%		
31.	\$21,700	15%		
32.	\$17,800	12%		

33. A \$300 fax machine is on sale for 15% off. Find the amount of discount and the sale price.
34. A \$4295 designer dress is on sale for 30% off. Find the amount of discount and the sale price.

Objectives A B Mixed Practice Complete each table.

	Purchase Price	Tax Rate	Sales Tax	Total Price
35.	\$305	9%		
36.	\$243	8%		
37.	\$56	5.5%		
38.	\$65	8.4%		

	Sale	Commission Rate	Commission
39.	\$235,800	3%	
40.	\$195,450	5%	
41.	\$17,900		\$1432
42.	\$25,600		\$2304

Review

Multiply. See Sections 4.3, 5.3, and 5.5.

43. $2000 \cdot \frac{3}{10} \cdot 2$

44. $500 \cdot \frac{2}{25} \cdot 3$

45. $400 \cdot \frac{3}{100} \cdot 11$

46. $1000 \cdot \frac{1}{20} \cdot 5$

47. $600 \cdot 0.04 \cdot \frac{2}{3}$

48. $6000 \cdot 0.06 \cdot \frac{3}{4}$

Concept Extensions

Solve. See the Concept Check in this section.

49. Your purchase price is \$68 and the sales tax rate is 9.5%. Round each amount and use the rounded amounts to estimate the total price. Choose the best estimate.
a. \$105 b. \$58 c. \$93 d. \$77
50. Your purchase price is \$200 and the tax rate is 10%. Choose the best estimate of the total price.
a. \$190 b. \$210 c. \$220 d. \$300

Tipping One very useful application of percent is mentally calculating a tip. Recall that to find 10% of a number, simply move the decimal point one place to the left. To find 20% of a number, just double 10% of the number. To find 15% of a number, find 10% and then add to that number half of the 10% amount. Mentally fill in the chart below. To do so, start by rounding the bill amount to the nearest dollar.

Tipping Chart				
	Bill Amount	10%	15%	20%
51.	\$40.21			
52.	\$15.89			
53.	\$72.17			
54.	\$9.33			

55. Suppose that the original price of a shirt is \$50. Which is better, a 60% discount or a discount of 30% followed by a discount of 35% of the reduced price? Explain your answer.
56. Which is better, a 30% discount followed by an additional 25% off or a 20% discount followed by an additional 40% off? To see, suppose an item costs \$100 and calculate each discounted price. Explain your answer.
57. A diamond necklace sells for \$24,966. If the tax rate is 7.5%, find the total price.
58. A house recently sold for \$562,560. The commission rate on the sale is 5.5%. If the real estate agent is to receive 60% of the commission, find the amount received by the agent.

7.6 Percent and Problem Solving: Interest

Objective A Calculating Simple Interest

Interest is money charged for using other people's money. When you borrow money, you pay interest. When you loan or invest money, you earn interest. The money borrowed, loaned, or invested is called the **principal amount**, or simply **principal**. Interest is normally stated in terms of a percent of the principal for a given period of time. The **interest rate** is the percent used in computing the interest. Unless stated otherwise, *the rate is understood to be per year*. When the interest is computed on the original principal, it is called **simple interest**. Simple interest is calculated using the following equation:

Simple Interest

$$\text{Simple Interest} = \text{Principal} \cdot \text{Rate} \cdot \text{Time}$$

$$I = P \cdot R \cdot T$$

where the rate is understood to be per year and time is in years.

Objectives

- A** Calculate Simple Interest.
- B** Calculate Compound Interest.

Practice 1

Find the simple interest after 5 years on \$875 at an interest rate of 7%.

Practice 2

A student borrowed \$1500 for 9 months on her credit card at a simple interest rate of 20%. How much interest did she pay?

Answers

1. \$306.25 2. \$225

✓ **Concept Check Answer**
\$16,000 is too much interest.

Example 1 Finding Simple Interest

Find the simple interest after 2 years on \$500 at an interest rate of 12%.

Solution: In this example, $P = \$500$, $R = 12\%$, and $T = 2$ years. Replace the variables with values in the formula $I = PRT$.

$$I = P \cdot R \cdot T$$

$$I = \$500 \cdot 12\% \cdot 2 \quad \text{Let } P = \$500, R = 12\%, \text{ and } T = 2.$$

$$= \$500 \cdot (0.12) \cdot 2 \quad \text{Write 12\% as a decimal.}$$

$$= \$120 \quad \text{Multiply.}$$

The simple interest is \$120.

Work Practice 1

If time is not given in years, we need to convert the given time to years.

Example 2 Finding Simple Interest

A recent college graduate borrowed \$2400 at 10% simple interest for 8 months to buy a used Toyota Corolla. Find the simple interest he paid.

Solution: Since there are 12 months in a year, we first find what part of a year 8 months is.

$$8 \text{ months} = \frac{8}{12} \text{ year} = \frac{2}{3} \text{ year}$$

Now we find the simple interest.

$$I = P \cdot R \cdot T$$

$$= \$2400 \cdot (0.10) \cdot \frac{2}{3} \quad \text{Let } P = \$2400, R = 10\% \text{ or } 0.10, \text{ and } T = \frac{2}{3}.$$

$$= \$160$$

The interest on his loan is \$160.

Work Practice 2

✓ **Concept Check** Suppose in Example 2 you had obtained an answer of \$16,000. How would you know that you had made a mistake in this problem?

When money is borrowed, the borrower pays the original amount borrowed, or the principal, as well as the interest. When money is invested, the investor receives the original amount invested, or the principal, as well as the interest. In either case, the **total amount** is the sum of the principal and the interest.

Finding the Total Amount of a Loan or Investment

$$\text{total amount (paid or received)} = \text{principal} + \text{interest}$$

Example 3 Finding the Total Amount of an Investment

An accountant invested \$2000 at a simple interest rate of 10% for 2 years. What total amount of money will she have from her investment in 2 years?

Solution: First we find her interest.

$$\begin{aligned} I &= P \cdot R \cdot T \\ &= \$2000 \cdot (0.10) \cdot 2 \quad \text{Let } P = \$2000, R = 10\% \text{ or } 0.10, \text{ and } T = 2. \\ &= \$400 \end{aligned}$$

The interest is \$400.

Next, we add the interest to the principal.

$$\begin{array}{rcl} \text{total amount} & = & \text{principal} + \text{interest} \\ \downarrow & & \downarrow \quad \downarrow \\ \text{total amount} & = & \$2000 + \$400 \\ & = & \$2400 \end{array}$$

After 2 years, she will have a total amount of \$2400.

Work Practice 3

✓ **Concept Check** Which investment would earn more interest: an amount of money invested at 8% interest for 2 years, or the same amount of money invested at 8% for 3 years? Explain.

Objective B Calculating Compound Interest

Recall that simple interest depends on the original principal only. Another type of interest is compound interest. **Compound interest** is computed not only on the principal, but also on the interest already earned in previous compounding periods. Compound interest is used more often than simple interest.

Let's see how compound interest differs from simple interest. Suppose that \$2000 is invested at 7% interest **compounded annually** for 3 years. This means that interest is added to the principal at the end of each year and that next year's interest is computed on this new amount. In this section, we round dollar amounts to the nearest cent.

	Amount at Beginning of Year	Principal	.	Rate	.	Time	= Interest	Amount at End of Year
1st year	\$2000	\$2000	.	0.07	.	1	= \$140	\$2000 + 140 = \$2140
2nd year	\$2140	\$2140	.	0.07	.	1	= \$149.80	\$2140 + 149.80 = \$2289.80
3rd year	\$2289.80	\$2289.80	.	0.07	.	1	≈ \$160.29	\$2289.80 + 160.29 = \$2450.09

The compound interest earned can be found by

$$\begin{array}{rcl} \text{total amount} & - & \text{original principal} = \text{compound interest} \\ \downarrow & & \downarrow \quad \downarrow \\ \$2450.09 & - & \$2000 = \$450.09 \end{array}$$

The simple interest earned would have been

$$\begin{array}{rcl} \text{principal} & \cdot & \text{rate} \cdot \text{time} = \text{interest} \\ \downarrow & & \downarrow \quad \downarrow \quad \downarrow \\ \$2000 & \cdot & 0.07 \cdot 3 = \$420 \end{array}$$

Since compound interest earns “interest on interest,” compound interest earns more than simple interest.

Computing compound interest using the method above can be tedious. We can use a calculator and the compound interest formula on the next page to compute compound interest more quickly.

Practice 3

If \$2100 is borrowed at a simple interest rate of 13% for 6 months, find the total amount paid.

Answer

3. \$2236.50

✓ **Concept Check Answers**

8% for 3 years. Since the interest rate is the same, the longer you keep the money invested, the more interest you earn.

Practice 4

\$3000 is invested at 4% interest compounded annually. Find the total amount after 6 years.

Practice 5

\$5500 is invested at $6\frac{1}{4}\%$ compounded *daily* for 5 years. Find the total amount at the end of 5 years. (Use 1 year = 365 days.)

Answers

4. \$3795.96 5. \$7517.41

Compound Interest Formula

The total amount A in an account is given by

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

where P is the principal, r is the interest rate written as a decimal, t is the length of time in years, and n is the number of times compounded per year.

Example 4

\$1800 is invested at 2% interest compounded annually. Find the total amount after 3 years.

Solution: “Compounded annually” means 1 time a year, so

$n = 1$. Also, $P = \$1800$, $r = 2\% = 0.02$, and $t = 3$ years.

$$\begin{aligned} A &= P\left(1 + \frac{r}{n}\right)^{n \cdot t} \\ &= 1800\left(1 + \frac{0.02}{1}\right)^{1 \cdot 3} \\ &= 1800(1.02)^3 \\ &\approx 1910.17 \end{aligned}$$

Helpful Hint

Remember order of operations. **First** evaluate $(1.02)^3$, then multiply by 1800.

Round to 2 decimal places.

The total amount at the end of 3 years is \$1910.17.

Work Practice 4**Example 5**

Finding Total Amount Received from an Investment

\$4000 is invested at 5.3% compounded quarterly for 10 years. Find the total amount at the end of 10 years.

Solution: “Compounded quarterly” means 4 times a year, so

$n = 4$. Also, $P = \$4000$, $r = 5.3\% = 0.053$, and $t = 10$ years.

$$\begin{aligned} A &= P\left(1 + \frac{r}{n}\right)^{n \cdot t} \\ &= 4000\left(1 + \frac{0.053}{4}\right)^{4 \cdot 10} \\ &= 4000(1.01325)^{40} \\ &\approx 6772.12 \end{aligned}$$

The total amount after 10 years is \$6772.12.

Work Practice 5**Calculator Explorations Compound Interest Formula**

For a review of using your calculator to evaluate compound interest, see this box.

Let's review the calculator keys pressed to evaluate the expression in Example 5,

$$4000\left(1 + \frac{0.053}{4}\right)^{40}$$

To evaluate, press the keys

4000 $\times$ $($ 1 $+$ 0.053 $\div$ 4 $)$ y^x or $\wedge$ 40 then $=$ or ENTER . The display will read 6772.117549 .

Rounded to 2 decimal places, this is 6772.12.

Find the compound interest.

1. \$600, 5 years, 9%, compounded quarterly
2. \$10,000, 15 years, 4%, compounded daily
3. \$1200, 20 years, 11%, compounded annually
4. \$5800, 1 year, 7%, compounded semiannually
5. \$500, 4 years, 6%, compounded quarterly
6. \$2500, 19 years, 5%, compounded daily

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Choices may be used more than once.

total amount simple principal amount compound

- To calculate _____ interest, use $I = P \cdot R \cdot T$.
- To calculate _____ interest, use $A = P \left(1 + \frac{r}{n} \right)^{n \cdot t}$.
- _____ interest is computed not only on the original principal, but also on interest already earned in previous compounding periods.
- When interest is computed on the original principal only, it is called _____ interest.
- _____ (paid or received) = principal + interest
- The _____ is the money borrowed, loaned, or invested.

Martin-Gay Interactive Videos



See Video 7.6

Watch the section lecture video and answer the following questions.

- Objective A** 7. Complete this statement based on the lecture before
 Example 1: Simple interest is charged on the _____ only.
- Objective B** 8. In Example 2, how often is the interest compounded and what number does this translate to in the formula?

7.6 Exercise Set MyLab Math

Objective A Find the simple interest. See Examples 1 and 2.

	Principal	Rate	Time
1.	\$200	8%	2 years
2.	\$800	9%	3 years
3.	\$160	11.5%	4 years
4.	\$950	12.5%	5 years
5.	\$5000	10%	$1\frac{1}{2}$ years

	Principal	Rate	Time
6.	\$1500	14%	$2\frac{1}{4}$ years
7.	\$375	18%	6 months
8.	\$775	15%	8 months
9.	\$2500	16%	21 months
10.	\$1000	10%	18 months

Solve. See Examples 1 through 3.

- A company borrows \$162,500 for 5 years at a simple interest rate of 12.5%. Find the interest paid on the loan and the total amount paid back.
- A money market fund advertises a simple interest rate of 9%. Find the total amount received on an investment of \$5000 for 15 months.
- \$265,000 is borrowed to buy a house. If the simple interest rate on the 30-year loan is 8.25%, find the interest paid on the loan and the total amount paid back.
- The Real Service Company takes out a 270-day (9-month) short-term, simple interest loan of \$4500 to finance the purchase of some new equipment. If the interest rate is 14%, find the total amount that the company pays back.

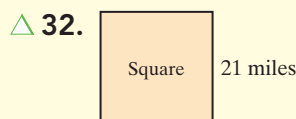
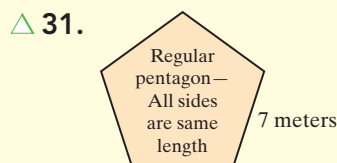
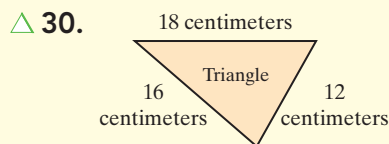
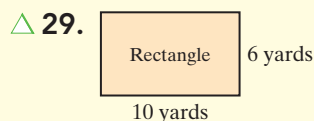
15. Marsha borrows \$8500 and agrees to pay it back in 4 years. If the simple interest rate is 17%, find the total amount she pays back.
16. An 18-year-old is given a high school graduation gift of \$2000. If this money is invested at 8% simple interest for 5 years, find the total amount.

Objective B Find the total amount in each compound interest account. See Examples 4 and 5.

17. \$6150 is compounded semiannually at a rate of 14% for 15 years.
18. \$2060 is compounded annually at a rate of 15% for 10 years.
19. \$1560 is compounded daily at a rate of 8% for 5 years.
20. \$1450 is compounded quarterly at a rate of 10% for 15 years.
21. \$10,000 is compounded semiannually at a rate of 9% for 20 years.
22. \$3500 is compounded daily at a rate of 8% for 10 years.
23. \$2675 is compounded annually at a rate of 9% for 1 year.
24. \$6375 is compounded semiannually at a rate of 10% for 1 year.
25. \$2000 is compounded annually at a rate of 8% for 5 years.
26. \$2000 is compounded semiannually at a rate of 8% for 5 years.
27. \$2000 is compounded quarterly at a rate of 8% for 5 years.
28. \$2000 is compounded daily at a rate of 8% for 5 years.

Review

Find the perimeter of each figure. See Section 1.3.



Perform the indicated operations. See Sections 4.3 through 4.6.

33. $\frac{x}{4} + \frac{x}{5}$

34. $-\frac{x}{4} \div \left(-\frac{x}{5}\right)$

35. $\left(\frac{2}{3}\right)\left(-\frac{1}{3}\right) - \left(\frac{9}{10}\right)\left(\frac{2}{5}\right)$

36. $\frac{3}{11} \div \frac{9}{22} \cdot \frac{1}{3}$

Concept Extensions

37. Review Exercises 25 through 28. As the number of compoundings increases per year, how is the total amount affected?
38. Explain how to find the amount of interest in a compounded account.
39. Compare the following accounts: Account 1: \$1000 is invested for 10 years at a simple interest rate of 6%. Account 2: \$1000 is compounded semiannually at a rate of 6% for 10 years. Discuss how the interest is computed for each account. Determine which account earns more interest. Why?

Chapter 7 Group Activity

Fastest-Growing Occupations

According to U.S. Bureau of Labor Statistics projections, the careers listed below are the top ten fastest-growing jobs ranked by expected percent increase through the year 2024. (*Source:* Bureau of Labor Statistics)

Occupation	Employment in 2014	Percent Change	Expected employment in 2024
Wind turbine technicians	4400	108%	
Occupational therapy assistants and aides	41,900	40%	
Physical therapist assistants and aides	128,700	40%	
Home health aides	913,500	38%	
Commercial divers	3450	37%	
Nurse practitioners	170,400	35%	
Physical therapists	210,900	34%	
Statisticians	30,000	34%	
Ambulance drivers and attendants but not EMTs	19,950	33%	
Physician assistants	94,400	30%	
Operations research analysts	91,300	30%	

What do most of these fast-growing occupations have in common? They require knowledge of math! For some careers, such as nurse practitioners, statisticians, and operations research analysts, the ways math is used on the job may be obvious. For other occupations, the use of math may not be quite as apparent. However, tasks common to many jobs—filling in a time sheet, writing up an expense or mileage report, planning a budget, figuring a bill, ordering supplies, and even making a work schedule—all require math.

This activity may be completed by working in groups or individually.

1. List the top five occupations by order of employment figures for 2014.
2. Using the 2014 employment figures and the percent increase from 2014 to 2024, find the expected 2024 employment figures for each occupation listed in the table. Round to the nearest hundred.
3. List the top five occupations by order of employment figures for 2024. Did the order change at all from 2014? Explain.

Chapter 7 Vocabulary Check

Fill in each blank with one of the words or phrases listed below. Some words may be used more than once.

percent sales tax is 0.01 $\frac{1}{100}$ amount of discount percent of decrease total price
base of amount 100% compound interest percent of increase sale price commission

- 1. In a mathematical statement, _____ usually means “multiplication.”
- 2. In a mathematical statement, _____ means “equal.”
- 3. _____ means “per hundred.”
- 4. _____ is computed not only on the principal, but also on interest already earned in previous compounding periods.
- 5. In the percent proportion, _____ = $\frac{\text{percent}}{100}$.
- 6. To write a decimal or fraction as a percent, multiply by _____.
- 7. The decimal equivalent of the % symbol is _____.
- 8. The fraction equivalent of the % symbol is _____.
- 9. The percent equation is _____ • percent = _____.
- 10. _____ = $\frac{\text{amount of decrease}}{\text{original amount}}$
- 11. _____ = $\frac{\text{amount of increase}}{\text{original amount}}$
- 12. _____ = tax rate • purchase price
- 13. _____ = purchase price + sales tax
- 14. _____ = commission rate • sales
- 15. _____ = discount rate • original price
- 16. _____ = original price – amount of discount



Are you preparing for your test? To help, don't forget to take these:

- Chapter 7 Getting Ready for the Test on page 534
- Chapter 7 Test on page 535

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

7 Chapter Highlights

Definitions and Concepts	Examples
Section 7.1 Percents, Decimals, and Fractions	
Percent means “per hundred.” The % symbol denotes percent.	$51\% = \frac{51}{100}$ 51 per 100
To write a percent as a decimal , replace the % symbol with its decimal equivalent, 0.01, and multiply.	$7\% = \frac{7}{100}$ 7 per 100
To write a decimal as a percent , multiply by 100%.	$32\% = 32(0.01) = 0.32$
To write a percent as a fraction , replace the % symbol with its fraction equivalent, $\frac{1}{100}$ and multiply.	$0.08 = 0.08(100\%) = 8\%$
To write a fraction as a percent , multiply by 100%.	$25\% = 25 \cdot \frac{1}{100} = \frac{25}{100} = \frac{25}{4 \cdot 25} = \frac{1}{4}$
	$\frac{1}{6} = \frac{1}{6} \cdot 100\% = \frac{1}{6} \cdot \frac{100}{1}\% = \frac{100}{6}\% = 16\frac{2}{3}\%$

Definitions and Concepts	Examples
Section 7.2 Solving Percent Problems with Equations	
Three key words in the statement of a percent problem are of, which means multiplication ($\cdot$) is, which means equal ($=$) what (or some equivalent word or phrase), which stands for the unknown (x)	Solve: 6 is 12% of what number? $\begin{array}{ccccccc} \downarrow & \downarrow & & \downarrow & \downarrow & & \downarrow \\ 6 & = & 12\% & \cdot & & & x \\ 6 & = & 0.12 \cdot x & \text{Write 12\% as a decimal.} \\ \frac{6}{0.12} & = & \frac{0.12 \cdot x}{0.12} & \text{Divide both sides by 0.12.} \\ 50 & = & x \end{array}$ Thus, 6 is 12% of 50.
Section 7.3 Solving Percent Problems with Proportions	
Percent Proportion $\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100} \quad \leftarrow \text{always 100}$ or $\begin{array}{l} \text{amount} \rightarrow \frac{a}{b} = \frac{p}{100} \\ \text{base} \rightarrow \end{array} \quad \leftarrow \text{percent}$	Solve: 20.4 is what percent of 85? $\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \text{amount} & \text{percent} & \text{base} \end{array}$ $\begin{array}{l} \text{amount} \rightarrow 20.4 \\ \text{base} \rightarrow 85 \end{array} \quad \frac{p}{100} \quad \leftarrow \text{percent}$ $20.4 \cdot 100 = 85 \cdot p \quad \text{Set cross products equal.}$ $2040 = 85 \cdot p \quad \text{Multiply.}$ $\frac{2040}{85} = \frac{85 \cdot p}{85} \quad \text{Divide both sides by 85.}$ $24 = p \quad \text{Simplify.}$ Thus, 20.4 is 24% of 85.
Section 7.4 Applications of Percent	
Percent of Increase $\text{percent of increase} = \frac{\text{amount of increase}}{\text{original amount}}$ Percent of Decrease $\text{percent of decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$	A town with a population of 16,480 decreased to 13,870 over a 12-year period. Find the percent of decrease. Round to the nearest whole percent. $\begin{aligned} \text{amount of decrease} &= 16,480 - 13,870 \\ &= 2610 \\ \text{percent of decrease} &= \frac{\text{amount of decrease}}{\text{original amount}} \\ &= \frac{2610}{16,480} \\ &\approx 0.16 \\ &= 16\% \end{aligned}$ The town's population decreased by about 16%.

Definitions and Concepts	Examples
Section 7.5 Percent and Problem Solving: Sales Tax, Commission, and Discount	
Sales Tax $\text{sales tax} = \text{sales tax rate} \cdot \text{purchase price}$ $\text{total price} = \text{purchase price} + \text{sales tax}$	Find the sales tax and the total price of a purchase of \$42.00 if the sales tax rate is 9%. $\begin{array}{rcl} \text{sales tax} & = & \text{sales tax rate} \cdot \text{purchase price} \\ \downarrow & & \downarrow \qquad \qquad \downarrow \\ \text{sales tax} & = & 9\% \cdot \$42 \\ & = & 0.09 \cdot \$42 \\ & = & \$3.78 \end{array}$ The total price is $\begin{array}{rcl} \text{total price} & = & \text{purchase price} + \text{sales tax} \\ \downarrow & & \downarrow \qquad \qquad \downarrow \\ \text{total price} & = & \$42.00 + \$3.78 \\ & = & \$45.78 \end{array}$ The total price is \$45.78.
Commission $\text{commission} = \text{commission rate} \cdot \text{sales}$	A salesperson earns a commission of 3%. Find the commission from sales of \$12,500 worth of appliances. $\begin{array}{rcl} \text{commission} & = & \text{commission rate} \cdot \text{sales} \\ \downarrow & & \downarrow \qquad \qquad \downarrow \\ \text{commission} & = & 3\% \cdot \$12,500 \\ & = & 0.03 \cdot 12,500 \\ & = & \$375 \end{array}$ The commission is \$375.
Discount and Sale Price $\text{amount of discount} = \text{discount rate} \cdot \text{original price}$ $\text{sale price} = \text{original price} - \text{amount of discount}$	A suit is priced at \$320 and is on sale today for 25% off. What is the sale price? $\begin{array}{rcl} \text{amount of discount} & = & \text{discount rate} \cdot \text{original price} \\ \downarrow & & \downarrow \qquad \qquad \downarrow \\ \text{amount of discount} & = & 25\% \cdot \$320 \\ & = & 0.25 \cdot 320 \\ & = & \$80 \end{array}$ $\begin{array}{rcl} \text{sale price} & = & \text{original price} - \text{amount of discount} \\ \downarrow & & \downarrow \qquad \qquad \downarrow \\ \text{sale price} & = & \$320 - \$80 \\ & = & \$240 \end{array}$ The sale price is \$240.

Definitions and Concepts	Examples
Section 7.6 Percent and Problem Solving: Interest	
Simple Interest interest = principal · rate · time $I = P \cdot R \cdot T$ where the rate is understood to be per year, unless told otherwise. Compound Interest Formula The total amount A in an account is $A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$ where P is the principal, r is the interest rate written as a decimal, t is time in years, and n is the number of times compounded per year.	Find the simple interest after 3 years on \$800 at an interest rate of 5%. $\begin{array}{rccccccc} \text{interest} & = & \text{principal} & \cdot & \text{rate} & \cdot & \text{time} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \text{interest} & = & \$800 & \cdot & 5\% & \cdot & 3 \\ & = & \$800 \cdot 0.05 \cdot 3 & & \text{Write 5\% as 0.05.} & & \\ & = & \$120 & & \text{Multiply.} & & \end{array}$ The interest is \$120. \$800 is invested at 5% compounded quarterly for 10 years. Find the total amount at the end of 10 years. “Compounded quarterly” means $n = 4$. Also, $P = \\$800$, $r = 5\% = 0.05$, and $t = 10$ years. $\begin{aligned} A &= P\left(1 + \frac{r}{n}\right)^{n \cdot t} \\ &= 800\left(1 + \frac{0.05}{4}\right)^{4 \cdot 10} \\ &= 800(1.0125)^{40} \\ &\approx 1314.90 \end{aligned}$ The total amount is \$1314.90.

Chapter 7

Review

(7.1) Solve.

1. In a survey of 100 adults, 37 preferred pepperoni on their pizzas. What percent preferred pepperoni?

2. A basketball player made 77 out of 100 attempted free throws. What percent of free throws was made?

Write each percent as a decimal.

3. 26%

4. 75%

5. 3.5%

6. 1.5%

7. 275%

8. 400%

9. 47.85%

10. 85.34%

Write each decimal as a percent.

11. 1.6

12. 0.055

13. 0.076

14. 0.085

15. 0.71

16. 0.65

17. 6

18. 9

Write each percent as a fraction or mixed number in simplest form.

19. 7%

20. 15%

21. 25%

22. 8.5%

23. 10.2%

24. $16\frac{2}{3}\%$

25. $33\frac{1}{3}\%$

26. 110%

Write each fraction or mixed number as a percent.

27. $\frac{2}{5}$

28. $\frac{7}{10}$

29. $\frac{7}{12}$

30. $1\frac{2}{3}$

31. $1\frac{1}{4}$

32. $\frac{3}{5}$

33. $\frac{1}{16}$

34. $\frac{5}{8}$

(7.2) Translating Translate each to an equation and solve.

35. 1250 is 1.25% of what number?

36. What number is $33\frac{1}{3}\%$ of 24,000?

37. 124.2 is what percent of 540?

38. 22.9 is 20% of what number?

39. What number is 17% of 640?

40. 693 is what percent of 462?

(7.3) Translating Translate each to a proportion and solve.

41. 104.5 is 25% of what number?

42. 16.5 is 5.5% of what number?

43. What number is 30% of 532?

44. 63 is what percent of 35?

45. 93.5 is what percent of 85?

46. What number is 33% of 500?

(7.4) Solve.

47. In a survey of 2000 people, it was found that 1320 have a microwave oven. Find the percent of people who own microwaves.

48. Of the 12,360 freshmen entering County College, 2000 are enrolled in prealgebra. Find the percent of entering freshmen who are enrolled in prealgebra. Round to the nearest whole percent.

49. The number of violent crimes in a city decreased from 675 to 534. Find the percent of decrease. Round to the nearest tenth percent.

50. The current charge for dumping waste in a local landfill is \$16 per cubic foot. To cover new environmental costs, the charge will increase to \$33 per cubic foot. Find the percent of increase.

51. A local union negotiated a new contract that increases the hourly pay 15% over last year's pay. The old hourly rate was \$11.50. Find the new hourly rate rounded to the nearest cent.

52. This year the fund drive for a charity collected \$215,000. Next year, a 4% decrease is expected. Find how much is expected to be collected in next year's drive.

(7.5) Solve.

53. If the sales tax rate is 9.5%, what is the total amount charged for a \$250 coat?

54. Find the sales tax paid on a \$25.50 purchase if the sales tax rate is 8.5%. Round to the nearest cent.

55. Russ James is a sales representative for a chemical company and is paid a commission rate of 5% on all sales. Find his commission if he sold \$100,000 worth of chemicals last month.

56. Carol Sell is a sales clerk in a clothing store. She receives a commission of 7.5% on all sales. Find her commission for the week if her sales for the week were \$4005. Round to the nearest cent.

57. A \$3000 mink coat is on sale for 30% off. Find the discount and the sale price.

58. A \$90 calculator is on sale for 10% off. Find the discount and the sale price.

(7.6) *Solve.*

59. Find the simple interest due on \$4000 loaned for 4 months with a 12% interest rate.
60. Find the simple interest due on \$6500 loaned for 3 months with a 20% interest rate.
61. Find the total amount in an account if \$5500 is compounded annually at 12% for 15 years.
62. Find the total amount in an account if \$6000 is compounded semiannually at 11% for 10 years.
63. Find the total amount in an account if \$100 is compounded quarterly at 12% for 5 years.
64. Find the total amount in an account if \$1000 is compounded quarterly at 18% for 20 years.

Mixed Review

Write each percent as a decimal.

65. 3.8%
66. 124.5%

Write each decimal as a percent.

67. 0.54
68. 95.2

Write each percent as a fraction or mixed number in simplest form.

69. 47%
70. 5.6%

Write each fraction or mixed number as a percent.

71. $\frac{1}{8}$
72. $\frac{6}{5}$

Translating *Translate each into an equation and solve.*

73. 43 is 16% of what number?
74. 27.5 is what percent of 25?
75. What number is 36% of 1968?
76. 67 is what percent of 50?

Translate each into a proportion and solve.

77. 75 is what percent of 25?
78. What number is 16% of 240?
79. 28 is 5% of what number?
80. 52 is what percent of 16?

Solve.

81. The total number of cans in a soft drink machine is 300. If 78 soft drinks have been sold, find the percent of soft drink cans that have been sold.
82. A home valued at \$96,950 last year has lost 7% of its value this year. Find the loss in value.
83. A dinette set sells for \$568.00. If the sales tax rate is 8.75%, find the total price of the dinette set.
84. The original price of a video game is \$23.00. It is on sale for 15% off. What is the amount of the discount?
85. A candy salesman makes a commission of \$1.60 from each case of candy he sells. If a case of candy costs \$12.80, what is his rate of commission?
86. Find the total amount due on a 6-month loan of \$1400 at a simple interest rate of 13%.
87. Find the total amount due on a loan of \$5500 for 9 years at 12.5% simple interest.

MULTIPLE CHOICE All of the exercises below are **Multiple Choice**. Choose the correct letter.

- 1. Since “percent” means “per hundred,” choose the number that does *not* equal 12%.

A. $\frac{12}{100}$ B. 0.12 C. $\frac{3}{25}$ D. 1.2

- 2. Choose the number that does not equal 100%.

A. 10 B. 1 C. 1.00 D. $\frac{100}{100}$

- 3. Choose the number that does not equal 50%.

A. 0.5 B. 5 C. $\frac{1}{2}$ D. $\frac{50}{100}$

- 4. Choose the number that does *not* equal 80%.

A. 0.80 B. 0.8 C. 8 D. $\frac{4}{5}$

Use the information below for Exercises 5 through 8.

- 100% of a number is that original number.
- 50% of a number is half that number.
- 25% of a number is $\frac{1}{4}$ of that number.
- 10% of a number is $\frac{1}{10}$ of that number.

For Exercises 5 through 8, choose the letter that correctly fills in each blank.

A. 100% B. 50% C. 25% D. 10%

- 5. _____ of 70 is 35.

- 6. _____ of 88 is 8.8.

- 7. _____ of 47 is 47.

- 8. _____ of 28 is 7.

For Exercise 9, choose the correct letter.

- 9. Your bill at a restaurant is \$24.86. If you want to leave a 20% tip, which tip amount is closest to 20%?

A. \$2.48 B. \$20 C. \$5 D. \$10

For Exercises 10 and 11, an amount of \$150 is to be discounted by 10%.

- 10. Find the amount of discount.

A. \$15 B. \$45 C. \$30 D. \$10

- 11. Find the new discounted price (original price – discount).

A. \$120 B. \$10 C. \$140 D. \$135

For Exercise 12, choose the correct letter.

- 12. If the original price of a pair of shoes is \$40 and the shoe price is to be discounted by 25% at the register, choose the closest amount for your shoes before tax.

A. \$10 B. \$20 C. \$30 D. \$40

Write each percent as a decimal.

▶ 1. 85%

▶ 2. 500%

▶ 3. 0.6%

Write each decimal as a percent.

▶ 4. 0.056

▶ 5. 6.1

▶ 6. 0.35

Write each percent as a fraction or a mixed number in simplest form.

▶ 7. 120%

▶ 8. 38.5%

▶ 9. 0.2%

Write each fraction or mixed number as a percent.

▶ 10. $\frac{11}{20}$

▶ 11. $\frac{3}{8}$

▶ 12. $1\frac{3}{4}$

- ▶ 13. In the past decade, Americans have increased their consumption of bottled water by $\frac{1}{4}$. Write $\frac{1}{4}$ as a percent.
(Source: Bottled Water Organization)

- ▶ 14. As of 2016, 45% of all working-age households have no retirement plan in place. Write 45% as a fraction.
(Source: National Institute on Retirement Security)

Solve.

▶ 15. What number is 42% of 80?

▶ 16. 0.6% of what number is 7.5?

▶ 17. 567 is what percent of 756?

Answers

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

13. _____

14. _____

15. _____

16. _____

17. _____

18. *Solve. If necessary, round percents to the nearest tenth, dollar amounts to the nearest cent, and all other numbers to the nearest whole.*

- 19.
- 20.
- 21.
- 22.
- 23.
- 24.
- 25.
- 26.
- 27.
- 28.
- ▶ 18. An alloy is 12% copper. How much copper is contained in 320 pounds of this alloy?
- ▶ 19. A farmer in Nebraska estimates that 20% of his potential crop, or \$11,350 worth, has been lost to a hard freeze. Find the total value of his potential crop.
- ▶ 20. If the local sales tax rate is 8.25%, find the total amount charged for a stereo system priced at \$354.
- ▶ 21. A town's population increased from 25,200 to 26,460. Find the percent of increase.
- ▶ 22. A \$120 framed picture is on sale for 15% off. Find the discount and the sale price.
- ▶ 23. Randy Nguyen is paid a commission rate of 4% on all sales. Find Randy's commission if his sales were \$9875.
- ▶ 24. A sales tax of \$13.77 is added to an item's price of \$152.99. Find the sales tax rate. Round to the nearest whole percent.
- ▶ 25. Find the simple interest earned on \$2000 saved for $3\frac{1}{2}$ years at an interest rate of 9.25%.
- ▶ 26. \$1365 is compounded annually at 8%. Find the total amount in the account after 5 years.
- ▶ 27. A couple borrowed \$400 from a bank at 13.5% simple interest for 6 months for car repairs. Find the total amount due the bank at the end of the 6-month period.
- ▶ 28. In a recent 5-year period, the number of a category of crimes reported in New York City decreased from 51,209 to 50,008. Find the percent of decrease. (*Source: New York State Division of Criminal Justice Services*)

1. Multiply: 236×86

2. Multiply: 409×76

3. Subtract 7 from -3 .

4. Subtract -2 from 8.

5. Solve: $x - 2 = -1$

6. Solve: $x + 4 = 3$

7. Solve: $3(2x - 6) + 6 = 0$

8. Solve: $5(x - 2) = 3x$

9. Write an equivalent fraction with the given denominator.

$$3 = \frac{\quad}{7}$$

10. Write an equivalent fraction with the given denominator.

$$8 = \frac{\quad}{5}$$

11. Simplify: $-\frac{10}{27}$

12. Simplify: $\frac{10y}{32}$

13. Divide: $-\frac{7}{12} \div -\frac{5}{6}$

14. Divide: $-\frac{2}{5} \div \frac{7}{10}$

15. Evaluate $y - x$ if $x = -\frac{3}{10}$ and $y = -\frac{8}{10}$.

16. Evaluate $2x + 3y$ if $x = \frac{2}{5}$ and $y = -\frac{1}{5}$.

17. Find: $-\frac{3}{4} - \frac{1}{14} + \frac{6}{7}$

18. Find: $\frac{2}{9} + \frac{7}{15} - \frac{1}{3}$

19. Simplify: $\frac{\frac{1}{2} + \frac{3}{8}}{\frac{3}{4} - \frac{1}{6}}$

20. Simplify: $\frac{\frac{2}{3} + \frac{1}{6}}{\frac{3}{4} - \frac{3}{5}}$

21. Solve: $\frac{x}{2} = \frac{x}{3} + \frac{1}{2}$

22. Solve: $\frac{x}{2} + \frac{1}{5} = 3 - \frac{x}{5}$

Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____

23. a.

b.

24. a.

b.

25.

26.

27.

28.

29.

30.

31.

32.

33.

34.

35.

36.

37.

38.

39.

40.

41.

42.

43.

44.

45.

46.

47.

48.

49.

50.

23. Write each mixed number as an improper fraction.

a. $4\frac{2}{9}$

b. $1\frac{8}{11}$

24. Write each mixed number as an improper fraction.

a. $3\frac{2}{5}$

b. $6\frac{2}{7}$

Write each decimal as a fraction or mixed number. Write your answer in simplest form.

25. 0.125

26. 0.85

27. -105.083

28. 17015

29. Subtract: $85 - 17.31$. Check your answer.

30. Subtract: $38 - 10.06$. Check your answer.

Multiply.

31. 7.68×10

32. 12.483×100

33. $(-76.3)(1000)$

34. -853.75×10

35. Evaluate $x \div y$ for $x = 2.5$ and $y = 0.05$.

36. Is 470 a solution of the equation $\frac{x}{100} = 4.75$?

37. Find the median of the scores: 67, 91, 75, 86, 55, 91

38. Find the mean of 36, 40, 86, and 30.

39. Write the ratio of 2.5 to 3.15 as a fraction in simplest form.

40. Write the ratio of 5.8 to 7.6 as a fraction in simplest form.

41. A store charges \$3.36 for a 16-ounce jar of picante sauce. What is the unit price in dollars per ounce?

42. Flooring tiles cost \$90 for a box with 40 tiles. Each tile is 1 square foot. Find the unit price in dollars per square foot.

43. Is $\frac{4.1}{7} = \frac{2.9}{5}$ a true proportion?

44. Is $\frac{6.3}{9} = \frac{3.5}{5}$ a true proportion?

45. On a chamber of commerce map of Abita Springs, 5 miles corresponds to 2 inches. How many miles correspond to 7 inches?

46. A student doing math homework can complete 7 problems in about 6 minutes. At this rate, how many problems can be completed in 30 minutes?

Write each percent as a fraction or mixed number in simplest form.

47. 1.9%

48. 2.3%

49. $33\frac{1}{3}\%$

50. 108%

Graphing and Introduction to Statistics and Probability

8



BEV, or Battery Electric Vehicle



HEV, or Hybrid Electric Vehicle



PHEV, or Plug-in Hybrid Electric Vehicle

What is a BEV, HEV, and PHEV?

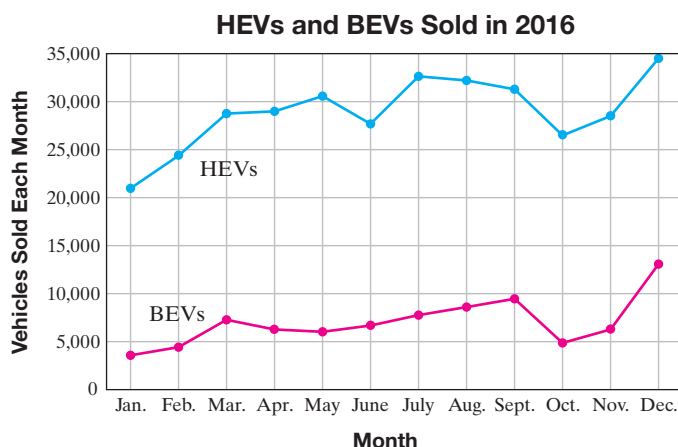
The car industry is getting complicated. In this Chapter Opener, we study the popularity of electric vehicles (EV). In general, all of the above are versions of electric vehicles (EV). Let's simplify the differences below:

BEV, or Battery Electric Vehicle: These are vehicles that operate solely on battery power that can be charged by plugging into an outlet or charging station. They are sometimes called “all electric vehicles.”

HEV, or Hybrid Electric Vehicle: These vehicles run on battery power (electricity) and an engine (usually gasoline-powered). We can call these non-plug-in hybrids.

PHEV, or Plug-in Hybrid Electric Vehicle: These are also hybrid vehicles, but their batteries can only be charged by plugging in to an outlet or charging station.

The graph below is a double line graph of hybrid electric vehicles (HEVs) and battery electric vehicles (BEVs) sold in 2016 in the United States. A graph like this helps us visually see trends and make predictions. In Section 8.4, Exercises 55 through 60, we use this graph for such applications.



Source: Electric Drive Transportation Association

We often need to make decisions based on known statistics or the probability of an event occurring. For example, we decide whether or not to bring an umbrella to work based on the probability of rain. We interpret numerical data, often presented in graph form, from newspaper and magazine articles and in television news reports. We can predict which football team will win based on the trend in its previous wins and losses. This chapter reviews presenting data in a usable form on a graph and the basic ideas of probability.

Sections

- 8.1** Pictographs, Bar Graphs, Histograms, Line Graphs, and Introduction to Statistics
- 8.2** Circle Graphs
- 8.3** The Rectangular Coordinate System and Paired Data
- Integrated Review—** Reading Graphs
- 8.4** Graphing Linear Equations in Two Variables
- 8.5** Counting and Introduction to Probability

Check Your Progress

Vocabulary Check
Chapter Highlights
Chapter Review
Getting Ready for the Test
Chapter Test
Cumulative Review

8.1 Pictographs, Bar Graphs, Histograms, Line Graphs, and Introduction to Statistics

Objectives

- A** Read Pictographs.
- B** Read and Construct Bar Graphs.
- C** Read and Construct Histograms (or Frequency Distribution Graphs).
- D** Read Line Graphs.
- E** Calculate range, mean, median and mode from a frequency distribution table or graph.

Practice 1

Use the pictograph shown in Example 1 to answer the following questions:

- a. Approximate the number of people who primarily speak Spanish.
- b. Approximate how many more people primarily speak Spanish than Arabic.

Answers

- 1. a. 400 million people
- b. 175 million people

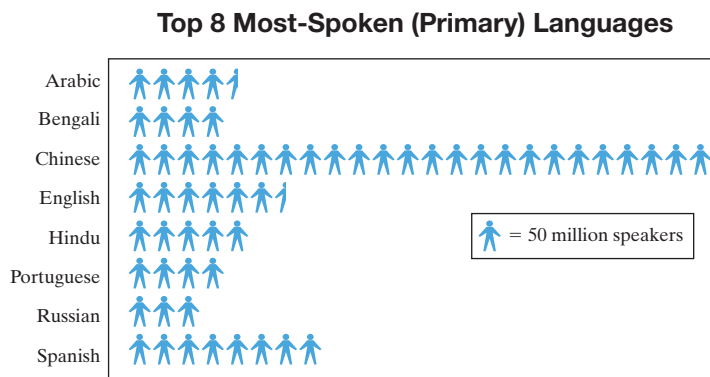
Often data are presented visually in a graph. In this section, we practice reading several kinds of graphs including pictographs, bar graphs, and line graphs.

Objective A Reading Pictographs

A **pictograph** such as the one below is a graph in which pictures or symbols are used. This type of graph contains a key that explains the meaning of the symbol used. An advantage of using a pictograph to display information is that comparisons can easily be made. A disadvantage of using a pictograph is that it is often hard to tell what fractional part of a symbol is shown. For example, in the pictograph below, Arabic shows a part of a symbol, but it's hard to read with any accuracy what fractional part of a symbol is shown.

Example 1 Calculating Languages Spoken

The following pictograph shows the top eight most-spoken (primary) languages. Use this pictograph to answer the questions.



- a. Approximate the number of people who primarily speak Russian.
- b. Approximate how many more people primarily speak English than Russian.

Solution:

- a. Russian corresponds to 3 symbols, and each symbol represents 50 million speakers. This means that the number of people who primarily speak Russian is approximately $3 \cdot (50 \text{ million})$ or 150 million people.
- b. English shows $3\frac{1}{2}$ more symbols than Russian. This means that $3\frac{1}{2} \cdot (50 \text{ million})$ or 175 million more people primarily speak English than Russian.

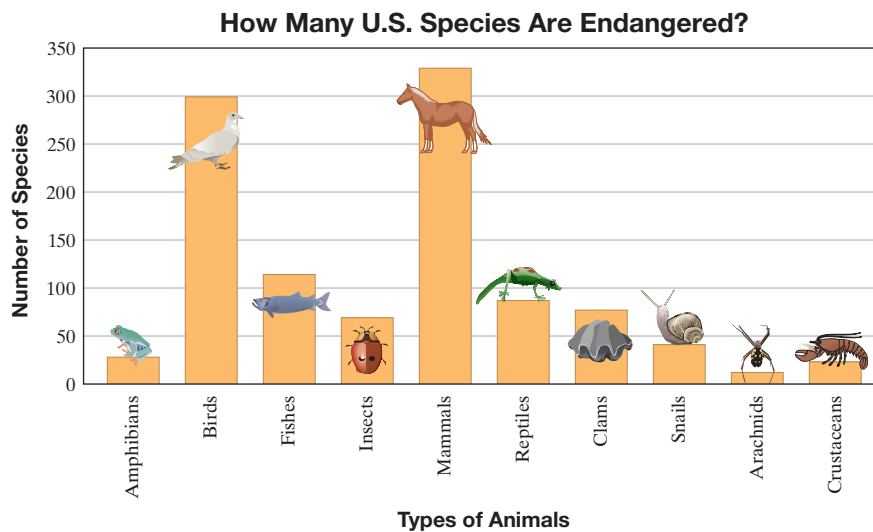
Work Practice 1

Objective B Reading and Constructing Bar Graphs

Another way to visually present data is with a **bar graph**. Bar graphs can appear with vertical bars or horizontal bars. Although we have studied bar graphs in previous sections, we now practice reading the height or length of the bars contained in a bar graph. An advantage to using bar graphs is that a scale is usually included for greater accuracy. Care must be taken when reading bar graphs, as well as other types of graphs—they may be misleading, as shown later in this section.

Example 2 Finding the Number of Endangered Species

The following bar graph shows the number of endangered species in the United States in 2016. Use this graph to answer the questions.

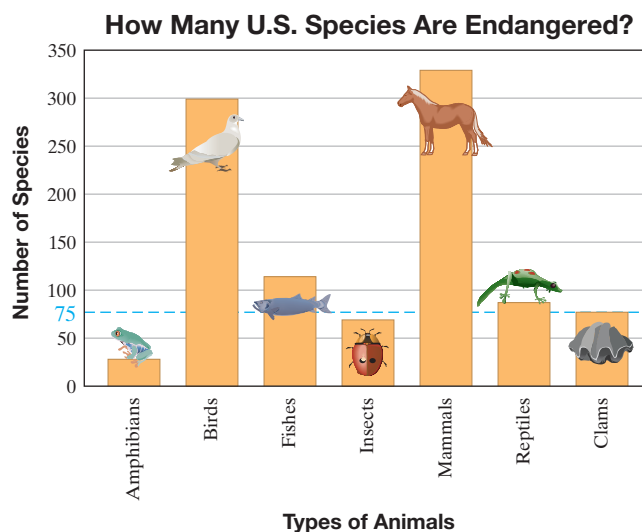


Source: U.S. Fish and Wildlife Service

- Approximate the number of endangered species that are clams.
- Which category has the most endangered species?

Solution:

- To approximate the number of endangered species that are clams, we go to the top of the bar that represents clams. From the top of this bar, we move horizontally to the left until the scale is reached. We read the height of the bar on the scale as approximately 75. There are approximately 75 clam species that are endangered, as shown.



Source: U.S. Fish and Wildlife Service

- The most endangered species is represented by the tallest (longest) bar. The tallest bar corresponds to mammals.

Practice 2

Use the bar graph in Example 2 to answer the following questions:

- Approximate the number of endangered species that are fishes.
- Which category shows the fewest endangered species?

Work Practice 2**Answers**

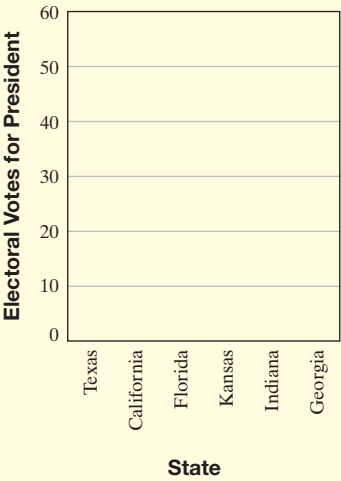
2. a. 115 b. arachnids

Practice 3

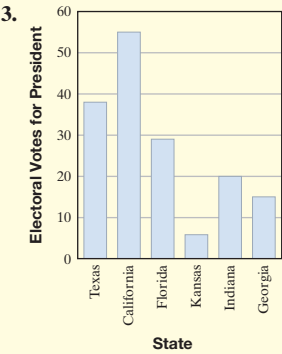
Draw a vertical bar graph using the information in the table about electoral votes for selected states.

Total Electoral Votes by Selected States	
State	Electoral Votes
Texas	38
California	55
Florida	29
Kansas	6
Indiana	20
Georgia	15

(Source: World Almanac)



Answer



Next, we practice constructing a bar graph.

Example 3

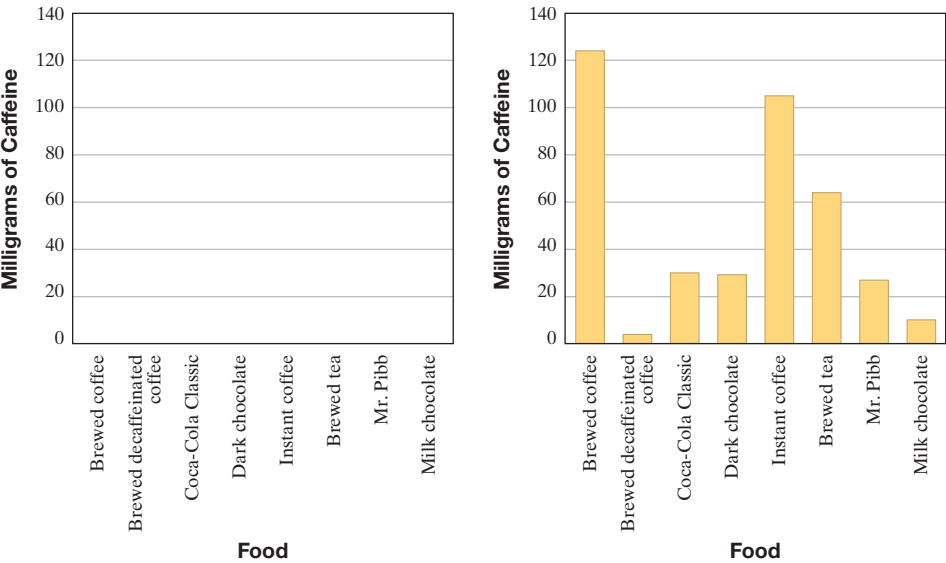
Draw a vertical bar graph using the information in the table below, which gives the caffeine content of selected foods.

Average Caffeine Content of Selected Foods			
Food	Milligrams	Food	Milligrams
Brewed coffee (percolator, 8 ounces)	124	Instant coffee (8 ounces)	104
Brewed decaffeinated coffee (8 ounces)	3	Brewed tea (U.S. brands, 8 ounces)	64
Coca-Cola Classic (8 ounces)	31	Mr. Pibb (8 ounces)	27
Dark chocolate (semisweet, $1\frac{1}{2}$ ounces)	30	Milk chocolate (8 ounces)	9

(Sources: International Food Information Council and the Coca-Cola Company)

Solution: We draw and label a vertical line and a horizontal line as shown below on the left. These lines are also called axes. We place the different food categories along the horizontal axis. Along the vertical axis, we place a scale.

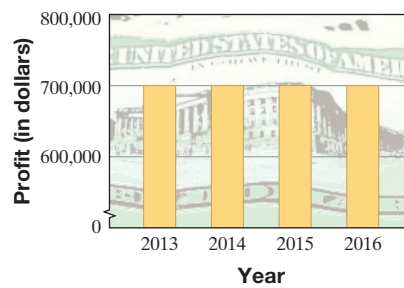
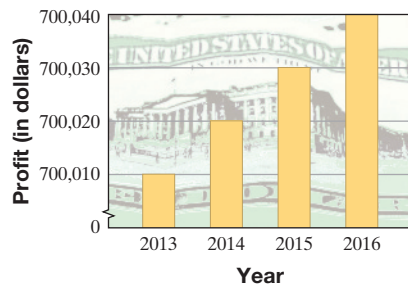
There are many choices of scales that would be appropriate. Notice that the milligrams range from a low of 3 to a high of 124. From this information, we use a scale that starts at 0 and then shows multiples of 20 so that the scale is not too cluttered. The scale stops at 140, the smallest multiple of 20 that will allow all milligrams to be graphed. It may also be helpful to draw horizontal lines along the scale markings to help draw the vertical bars at the correct height. The finished bar graph is shown below on the right.



Work Practice 3

As mentioned previously, graphs can be misleading. Both graphs on the next page show the same information, but with different scales. Special care should be taken when forming conclusions from the appearance of a graph.

Notice the $\frac{1}{2}$ symbol on each vertical scale on the graphs below. This symbol alerts us that numbers are missing from that scale



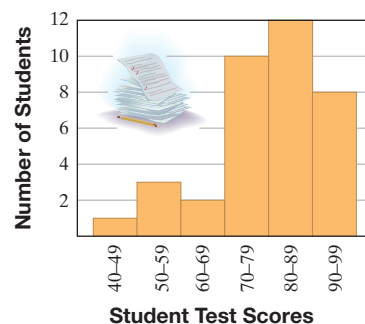
Are profits shown in the graphs above greatly increasing, or are they remaining about the same?

Objective C Reading and Constructing Histograms

Suppose that the test scores of 36 students are summarized in the table below: We call this table a **frequency distribution table** since one column gives the frequency or number of times the event in the other column occurred.

Student Scores	Frequency (Number of Students)
40–49	1
50–59	3
60–69	2
70–79	10
80–89	12
90–99	8

The results in this frequency distribution table can be displayed in a histogram. A **histogram** is a special bar graph. The width of each bar represents a range of numbers called a **class interval**. The height of each bar corresponds to how many times a number in the class interval occurs and is called the **class frequency**. The bars in a histogram lie side by side with no space between them. Note: Another name for this histogram is a **frequency distribution graph**.



Example 4 Reading a Histogram on Student Test Scores

Use the preceding histogram to determine how many students scored 50–59 on the test.

Solution: We find the bar representing 50–59. The height of this bar is 3, which means 3 students scored 50–59 on the test.

Work Practice 4

Practice 4

Use the histogram above Example 4 to determine how many students scored 80–89 on the test.

Answer

4. 12

Practice 5

Use the histogram above Example 4 to determine how many students scored less than 80 on the test.

Practice 6

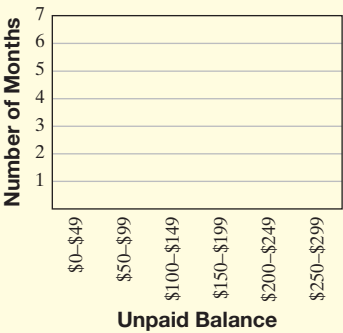
Complete the frequency distribution table for the data below. Each number represents a credit card owner’s unpaid balance for one month.

0	53	89	125
265	161	37	76
62	201	136	42

Class Intervals (Credit Card Balances)	Tally	Class Frequency (Number of Months)
\$0–\$49	_____	_____
\$50–\$99	_____	_____
\$100–\$149	_____	_____
\$150–\$199	_____	_____
\$200–\$249	_____	_____
\$250–\$299	_____	_____

Practice 7

Construct a histogram from the frequency distribution table for Practice 6.



Answers

5. 16

6.

Tally	Class Frequency (Number Months)	Tally	Class Frequency (Number Months)
	3		1
	4		1
	2		1

7.

Example 5 Reading a Histogram on Student Test Scores

Use the histogram above Example 4 to determine how many students scored 80 or above on the test.

Solution: We see that two different bars fit this description. There are 12 students who scored 80–89 and 8 students who scored 90–99. The sum of these two categories is 12 + 8 or 20 students. Thus, 20 students scored 80 or above on the test.

Work Practice 5

Now we will look at a way to construct histograms.

The daily high temperatures for 1 month in New Orleans, Louisiana, are recorded in the following list:

85°	90°	95°	89°	88°	94°
87°	90°	95°	92°	95°	94°
82°	92°	96°	91°	94°	92°
89°	89°	90°	93°	95°	91°
88°	90°	88°	86°	93°	89°

The data in this list have not been organized and can be hard to interpret. One way to organize the data is to place them in a **frequency distribution table**. We will do this in Example 6.

Example 6 Completing a Frequency Distribution on Temperature

Complete the frequency distribution table for the preceding temperature data.

Solution: Go through the data and place a tally mark in the second column of the table next to the class interval. Then count the tally marks and write each total in the third column of the table.

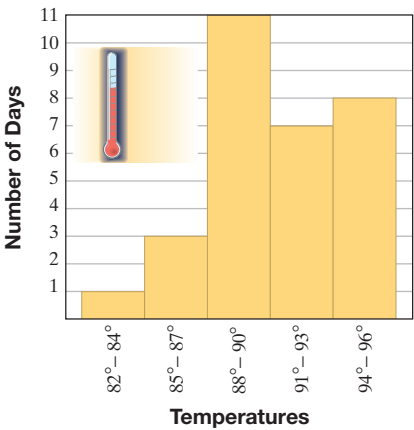
Work Practice 6

Class Intervals (Temperatures)	Tally	Class Frequency (Number of Days)
82°–84°		1
85°–87°		3
88°–90°		11
91°–93°		7
94°–96°		8

Example 7 Constructing a Histogram

Construct a histogram from the frequency distribution table in Example 6.

Solution:



Work Practice 7

✓ **Concept Check** Which of the following sets of data is better suited to representation by a histogram? Explain.

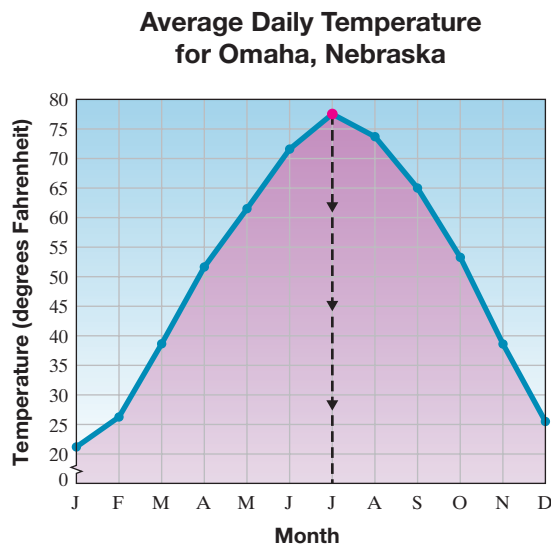
Set 1		Set 2	
Grade on Final	# of Students	Section Number	Avg. Grade on Final
51–60	12	150	78
61–70	18	151	83
71–80	29	152	87
81–90	23	153	73
91–100	25		

Objective D Reading Line Graphs

Another common way to display information with a graph is by using a **line graph**. An advantage of a line graph is that it can be used to visualize relationships between two quantities. A line graph can also be very useful in showing changes over time.

Example 8 Reading Temperatures from a Line Graph

The following line graph shows the average daily temperature for each month in Omaha, Nebraska. Use this graph to answer the questions below.



Source: National Climatic Data Center

- During what month is the average daily temperature the highest?
- During what month, from July through December, is the average daily temperature 65°F?
- During what months is the average daily temperature less than 30°F?

Solution:

- The month with the highest temperature corresponds to the highest point. This is the red point shown on the graph above. We follow this highest point downward to the horizontal month scale and see that this point corresponds to July.

(Continued on next page)

Practice 8

Use the temperature graph in Example 8 to answer the following questions:

- During what month is the average daily temperature the lowest?
- During what month is the average daily temperature 25°F?
- During what months is the average daily temperature greater than 70°F?

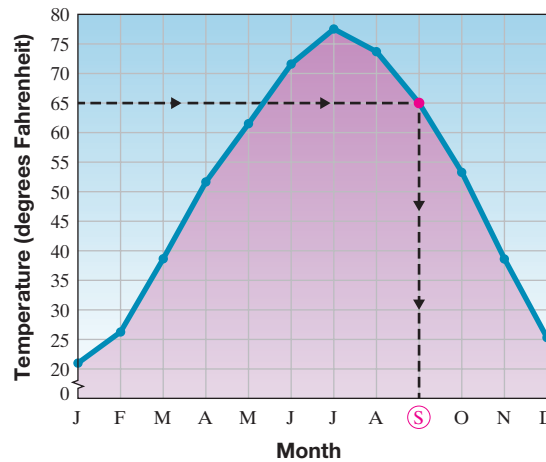
Answers

8. a. January b. December
c. June, July, and August

✓ Concept Check Answer

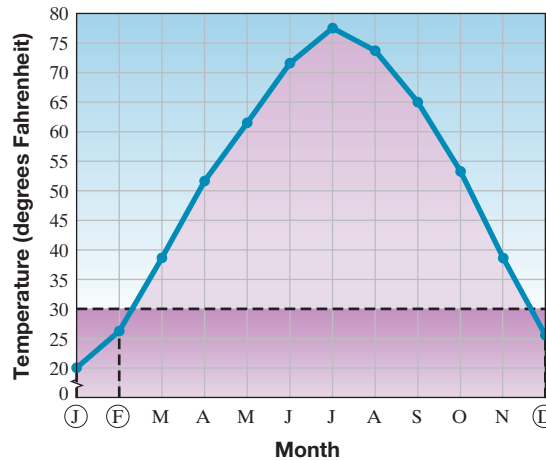
Set 1; the grades are arranged in ranges of scores.

- b. The months July through December correspond to the right side of the graph. We find the 65°F mark on the vertical temperature scale and move to the right until a point on the right side of the graph is reached. From that point, we move downward to the horizontal month scale and read the corresponding month. During the month of September, the average daily temperature is 65°F .



Source: National Climatic Data Center

- c. To see what months the temperature is less than 30°F , we find what months correspond to points that fall below the 30°F mark on the vertical scale. These months are January, February, and December.



Source: National Climatic Data Center

Work Practice 8

Objective E Finding the Range of a Data Set and Reviewing Mean, Median, and Mode

In Section 5.7, we studied three measures of central tendency—the mean, the median, and the mode. All three of these can be used to calculate a number that might best describe a data set.

In this objective, we study one way to describe the dispersion of a data set, and we review mean, median, and mode. What is dispersion? In statistics, **dispersion** is a way to describe the degree to which the data values are scattered.

Range

The range of a data set is the difference between the largest data value and the smallest data value.

$$\text{range} = \text{largest data value} - \text{smallest data value}$$

Example 9

The following pulse rates (for 1 minute) were recorded for a group of 15 students. Find the range.

78, 80, 66, 68, 71, 64, 82, 71, 70, 65, 70, 75, 77, 86, 72.

Solution: range = largest data value – smallest data value
 $= 86 - 64$
 $= 22$

The range of this data set is 22.

Work Practice 9

Before we review mean, median, and mode, let's introduce a formula for finding the position of the median.

- Recall that the **median** of a set of numbers in numerical order is the middle number.
- If the number of items is odd, the median is the middle number.
- If the number of items is even, the median is the *mean* (average) of the two middle numbers.

For a long list of data items, this formula gives us the **position** of the median.

Position of the Median

For n data items in order from smallest to largest, the median is the item in the

$$\frac{n + 1}{2} \text{ position.}$$

Note:

If n is an even number, then the position formula, $\frac{n + 1}{2}$, will not be a whole number.

In this case, simply find the average of the two data items whose positions are closest to, but before and after $\frac{n + 1}{2}$.

Helpful Hint

The formula above, $\frac{n + 1}{2}$ does not give the *value* of the median, just the **position of the median**.

Practice 9

The table lists the rounded salary of 10 staff members. Find the range.

Staff	Salary (in thousands)
A	\$32
B	\$34
C	\$46
D	\$38
E	\$42
F	\$95
G	\$102
H	\$50
J	\$42

Answer

9. \$70 thousand

Practice 10

One state with a young retirement age is Michigan. The table below is from a poll of retirement ages from that state.

Age	Frequency
50	1
59	3
60	3
62	5
63	2
67	1

Find the (a) range, (b) mean, (c) median, and (d) mode of these data. If needed, round answers to 1 decimal place.

Helpful Hint

Since there are three 60's, for example, we can either use:

$60 + 60 + 60$ or $3 \cdot 60$
(or $60 \cdot 3$).

Answers

10. a. range: 17 b. mean: 60.7
c. median: 62, d. mode: 62

Example 10

Find the (a) range, (b) mean, (c) median, and (d) mode from this frequency distribution table of retirement ages. If needed, round answers to 1 decimal place.

Age	Frequency
60	3
61	1
62	1
63	2
64	2
65	2

Solution: Study the table for a moment. From the frequency column, we see that there are 11 data items ($3 + 1 + 1 + 2 + 2 + 2$).

a. range = largest data value – smallest data value

$$= 65 - 60$$

$$= 5$$

The range of this data set is 5.

b. To find the mean, we use our mean formula:

$$\begin{aligned} \text{mean} &= \frac{\text{sum of items}}{\text{number of items}} = \frac{3 \cdot 60 + 61 + 62 + 2 \cdot 63 + 2 \cdot 64 + 2 \cdot 65}{11} \\ &= \frac{687}{11} \approx 62.5 \end{aligned}$$

The mean of the data set is approximately 62.5.

c. Since there are 11 data items and the items are arranged in numerical order in the table, we find $\frac{n+1}{2}$ to locate the middle item. This is $\frac{11+1}{2} = \frac{12}{2} = 6$, or the sixth item.

The median is the sixth number, or 63.

d. The mode has the greatest frequency, so the mode is 60.

Work Practice 10**Vocabulary, Readiness & Video Check**

Fill in each blank with one of the choices below. Not all choices will be used.

pictograph	bar	class frequency	range	mean
histogram	line	class interval	median	mode

- A _____ graph presents data using vertical or horizontal bars.
- A _____ is a graph in which pictures or symbols are used to visually present data.
- A _____ graph displays information with a line that connects data points.
- A _____ is a special bar graph in which the width of each bar represents a _____ and the height of each bar represents the _____.
- _____ = greatest data value – smallest data value
- The formula $\frac{n+1}{2}$ can be used to find the position of the _____.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



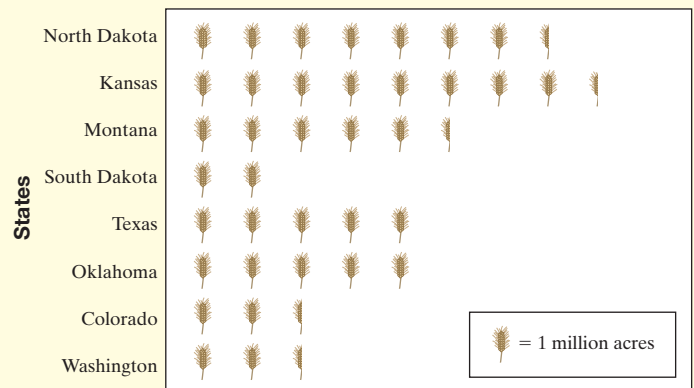
See Video 8.1

- Objective A** 7. From the pictograph in Example 1, how would you approximate the number of wildfires for any given year?
- Objective B** 8. What is one advantage of displaying data in a bar graph?
- Objective C** 9. Complete this statement based on the lecture before Example 6: A histogram is a special kind of _____.
- Objective D** 10. From the line graph in Examples 10–13, what year averaged the greatest number of goals per game and what was this average?
- Objective E** 11. Based on the results of Example 15, give an example of a data set of four data items whose range is 0.

8.1 Exercise Set MyLab Math

Objective A The following pictograph shows the number of acres devoted to wheat production in 2016 in selected states. Use this graph to answer Exercises 1 through 8. See Example 1. (Source: U.S. Department of Agriculture)

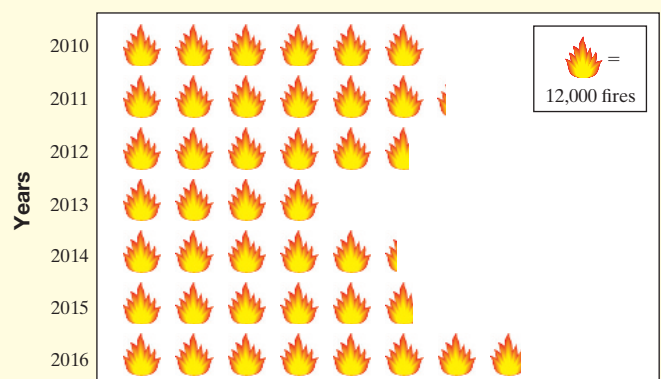
- Which state plants the greatest acreage in wheat?
- Which of the states shown plant the least amount of wheat acreage?
- Approximate the number of acres of wheat planted in Montana.
- Approximate the number of acres of wheat planted in Kansas.
- Which state(s) plant less than 3,000,000 acres of wheat?
- Which state(s) plant more than 7,000,000 acres of wheat?
- Which state plants more wheat: Montana or Oklahoma?

Annual Wheat Acreage in Selected Top States

- Which states plant about the same amount of acreage?

The following pictograph shows the average number of wildfires in the United States between 2010 and 2016. Use this graph to answer Exercises 9 through 16. See Example 1. (Source: National Interagency Fire Center)

- Approximate the number of wildfires in 2013.
- Approximately how many wildfires were there in 2012?
- Which year, of the years shown, had the most wildfires?
- In what years were the number of wildfires greater than 72,000?

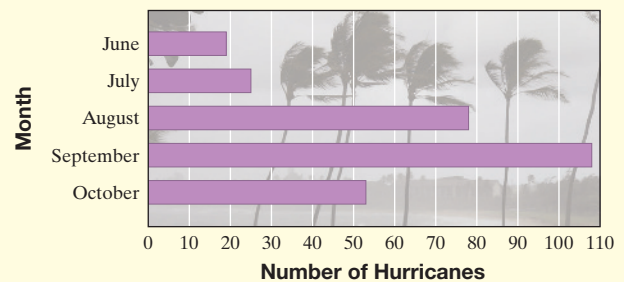
Wildfires in the United States

13. What was the amount of decrease in wildfires from 2012 to 2013?
14. What was the amount of increase in wildfires from 2015 to 2016?
15. What was the average annual number of wildfires from 2013 to 2015? (*Hint: How do you calculate the average?*)
16. Give a possible explanation for the sharp increase in the number of wildfires in 2016.

Objective B *The National Weather Service has exacting definitions for hurricanes; they are tropical storms with winds in excess of 74 mph. The following bar graph shows the number of hurricanes, by month, that have made landfall on the mainland United States between 1851 and 2016. Use this graph to answer Exercises 17 through 22. See Example 2. (Source: National Weather Service: National Hurricane Center)*

17. In which month did the most hurricanes make landfall in the United States?
18. In which month did the fewest hurricanes make landfall in the United States?
19. Approximate the number of hurricanes that made landfall in the United States during the month of August.
20. Approximate the number of hurricanes that made landfall in the United States in September.
21. In 2008 alone, two hurricanes made landfall during the month of August. What fraction of all the 78 hurricanes that made landfall during August is this?

Hurricanes Making Landfall in the United States, by Month, 1851–2016

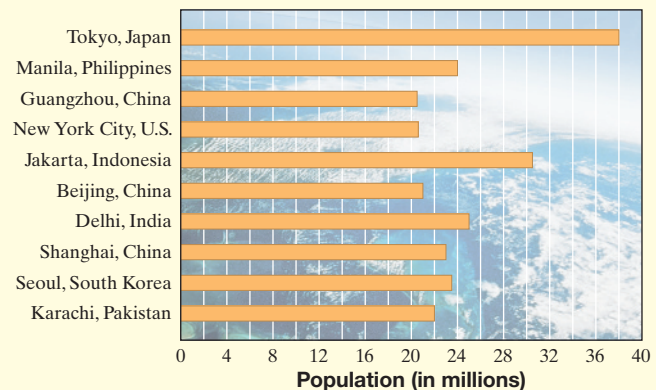


22. In 2007, only one hurricane made landfall on the United States during the entire season, in the month of September. If there have been 108 hurricanes to make landfall in the month of September since 1851, approximately what percent of these arrived in 2007?

The following horizontal bar graph shows the approximate 2016 population of the world's largest cities (including their suburbs). Use this graph to answer Exercises 23 through 28. See Example 2. (Source: CityPopulation)

23. Name the city with the largest population, and estimate its population.
24. Name the city whose population is between 23 million and 24 million.
25. Name the city in the United States with the largest population, and estimate its population.
26. Name the two cities that have approximately the same population.
27. How much larger (in terms of population) is Manila, Philippines, than Beijing, China?

World's Largest Cities (including Suburbs)

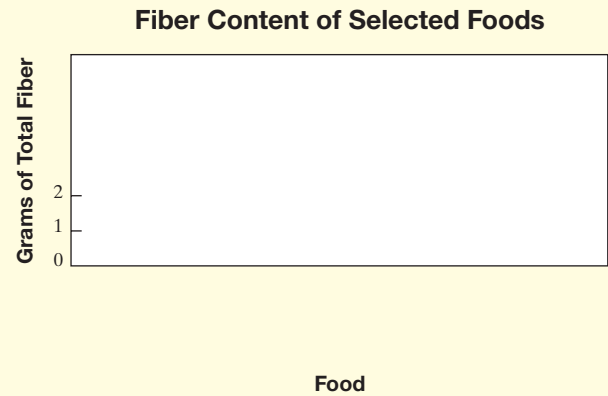


28. How much larger (in terms of population) is Jakarta, Indonesia, than Seoul, South Korea?

Use the information given to draw a vertical bar graph. Clearly label the bars. See Example 3.

▶ 29.

Fiber Content of Selected Foods	
Food	Grams of Total Fiber
Kidney beans $\left(\frac{1}{2} \text{ c}\right)$	4.5
Oatmeal, cooked $\left(\frac{3}{4} \text{ c}\right)$	3.0
Peanut butter, chunky (2 tbsp)	1.5
Popcorn (1 c)	1.0
Potato, baked with skin (1 med)	4.0
Whole wheat bread (1 slice)	2.5
(Sources: American Dietetic Association and National Center for Nutrition and Dietetics)	



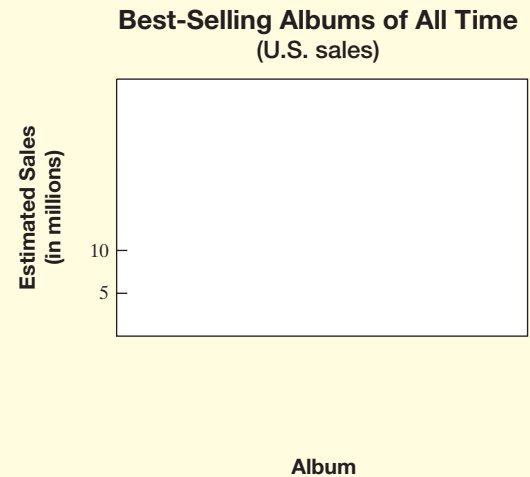
30.

U.S. Annual Food Sales	
Year	Sales in Billions of Dollars
2013	1410
2014	1462
2015	1512
2016	1584
(Source: U.S. Department of Agriculture)	



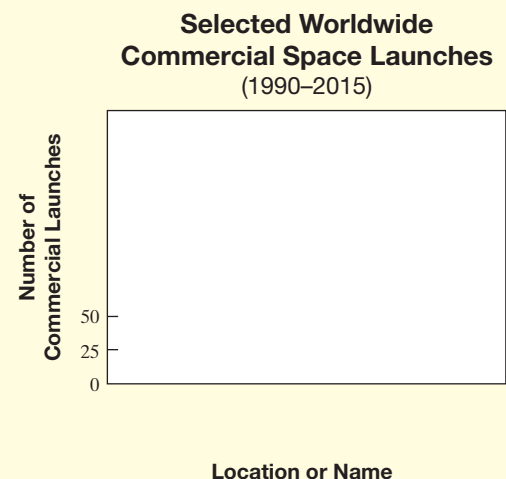
31.

Best-Selling Albums of All Time (U.S. Sales)	
Album	Estimated Sales (in millions)
Pink Floyd: <i>The Wall</i> (1979)	23
Michael Jackson: <i>Thriller</i> (1982)	29
Billy Joel: <i>Greatest Hits Volumes I&II</i> (1985)	23
Eagles: <i>Their Greatest Hits</i> (1976)	29
Led Zeppelin: <i>Led Zeppelin IV</i> (1971)	23
(Source: Recording Industry Association of America)	



32.

Selected Worldwide Commercial Space Launches	
Location or Name	Total Commercial Space Launches 1990–2015
United States	181
Europe	169
Russia	162
China	23
Sea Launch*	41
*Sea Launch is an international venture involving four countries that uses its own launch facility outside national borders.	
(Source: Bureau of Transportation Statistics)	



Objective C The following histogram shows the number of miles that each adult, from a survey of 100 adults, drives per week. Use this histogram to answer Exercises 33 through 42. See Examples 4 and 5.

33. How many adults drive 100–149 miles per week?

34. How many adults drive 200–249 miles per week?

▶ 35. How many adults drive fewer than 150 miles per week?

36. How many adults drive 200 miles or more per week?

37. How many adults drive 100–199 miles per week?

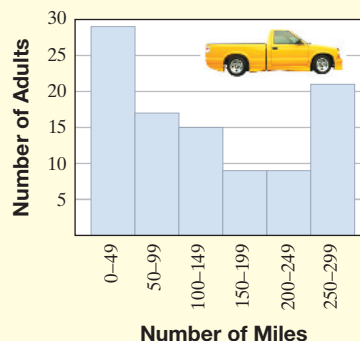
▶ 39. How many more adults drive 250–299 miles per week than 200–249 miles per week?

41. What is the ratio of adults who drive 150–199 miles per week to the total number of adults surveyed?

38. How many adults drive 150–249 miles per week?

40. How many more adults drive 0–49 miles per week than 50–99 miles per week?

42. What is the ratio of adults who drive 50–99 miles per week to the total number of adults surveyed?



The following histogram shows the ages of householders for the year 2015. Use this histogram to answer Exercises 43 through 50. For Exercises 45 through 50, estimate to the nearest whole million. See Examples 4 and 5.

43. The most householders were in what age range?

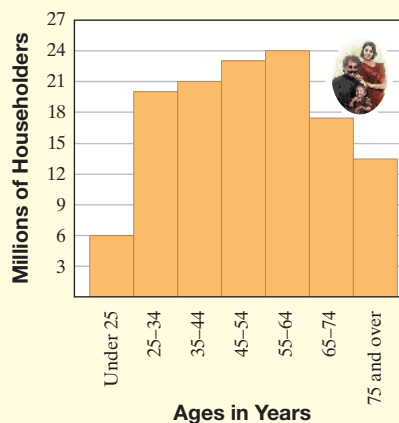
44. The least number of householders were in what age range?

45. How many householders were 55–64 years old?

46. How many householders were 35–44 years old?

47. How many householders were 44 years old or younger?

49. How many more householders were 45–54 years old than 35–44 years old?



Source: U.S. Bureau of the Census, Current Population Reports

48. How many householders were 55 years old or older?

50. How many more householders were 45–54 years old than 75 and over?

The following list shows the golf scores for an amateur golfer. Use this list to complete the frequency distribution table to the right. See Example 6.

78 84 91 93 97
97 95 85 95 96
101 89 92 89 100

- ▶ 51.
- ▶ 52.
- ▶ 53.
- ▶ 54.

Class Intervals (Scores)	Tally	Class Frequency (Number of Games)
70–79		
80–89		
90–99		
100–109		

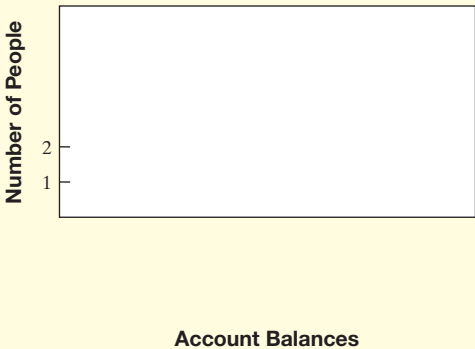
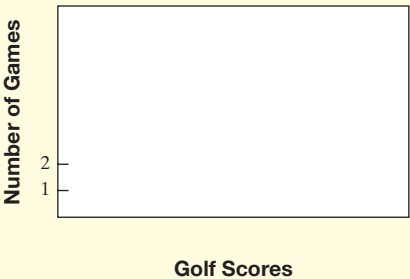
Twenty-five people in a survey were asked to give their current checking account balances. Use the balances shown in the following list to complete the frequency distribution table to the right. See Example 6.

\$53 \$105 \$162 \$443 \$109
\$468 \$47 \$259 \$316 \$228
\$207 \$357 \$15 \$301 \$75
\$86 \$77 \$512 \$219 \$100
\$192 \$288 \$352 \$166 \$292

- 55.
- 56.
- 57.
- 58.
- 59.
- 60.

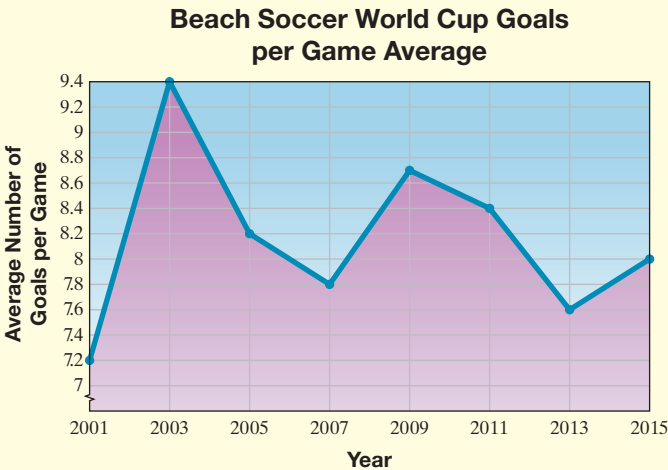
Class Intervals (Account Balances)	Tally	Class Frequency (Number of People)
\$0–\$99		
\$100–\$199		
\$200–\$299		
\$300–\$399		
\$400–\$499		
\$500–\$599		

- ▶ 61. Use the frequency distribution table from Exercises 51 through 54 to construct a histogram. See Example 7.
- 62. Use the frequency distribution table from Exercises 55 through 60 to construct a histogram. See Example 7.



Objective D Beach Soccer World Cup is now held every two years. The following line graph shows the World Cup goals per game average for beach soccer during the years shown. Use this graph to answer Exercises 63 through 70. See Example 8.

- ▶ 63. Find the average number of goals per game in 2015.
- 64. Find the average number of goals per game in 2013.
- ▶ 65. During what year shown was the average number of goals per game the highest?
- 66. During what year shown was the average number of goals per game the lowest?



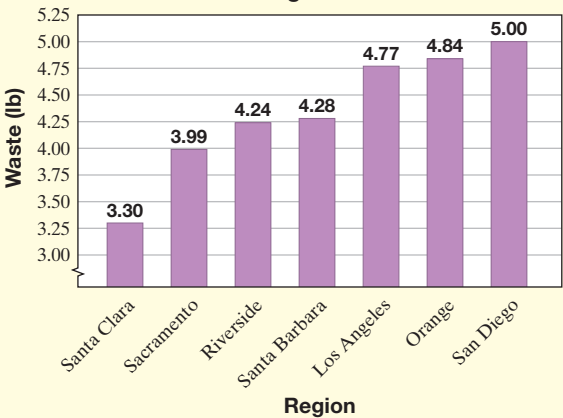
Source: Wikipedia

67. From 2013 to 2015, did the average number of goals per game increase or decrease?
68. From 2011 to 2013, did the average number of goals per game increase or decrease?
69. During what year(s) shown were the average goals per game less than 8?
70. During what year(s) shown were the average goals per game greater than 8?

Objective E Find the range for each data set. See Example 9.

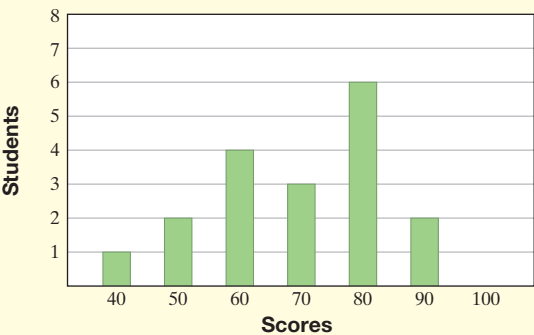
71. 14, 16, 8, 10, 20
72. 25, 15, 11, 40, 37
73. 206, 206, 555, 556
74. 129, 188, 188, 276
75. 9, 9, 9, 9, 11
76. 7, 7, 7, 7, 10

77. Average Waste Disposed per Person per Day
Selected Regions of California



Sources: Equinox Center, 2013; Calrecycle, 2013

78. Scores on a Basic Math Test



Use each frequency distribution table to find the **a.** mean, **b.** median, and **c.** mode. If needed, round the mean to 1 decimal place. See Example 10.

79.

Data Item	Frequency
5	1
6	1
7	2
8	5
9	6
10	2

80.

Data Item	Frequency
3	2
4	1
5	4
6	7
7	2
8	1

81.

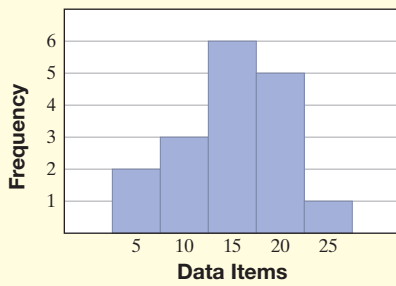
Data Item	Frequency
2	5
3	7
4	4
5	7
6	8
7	8
8	8
9	6
10	5

82.

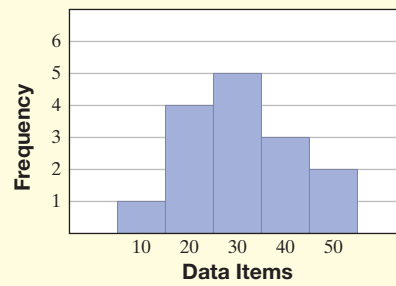
Data Item	Frequency
4	3
5	8
6	5
7	8
8	2

Use each graph of data items to find the **a. mean**, **b. median**, and **c. mode**. If needed round the mean to one decimal place.

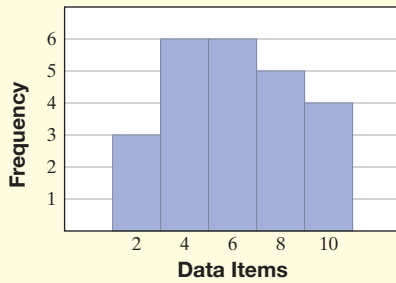
83.



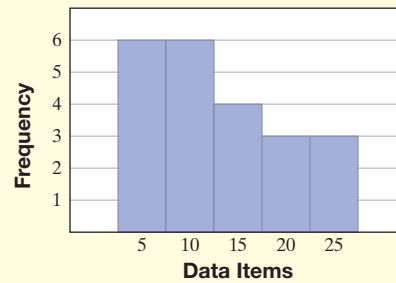
84.



85.



86.



Review

Find each percent. See Sections 7.2 or 7.3.

87. 30% of 12

88. 45% of 120

89. 10% of 62

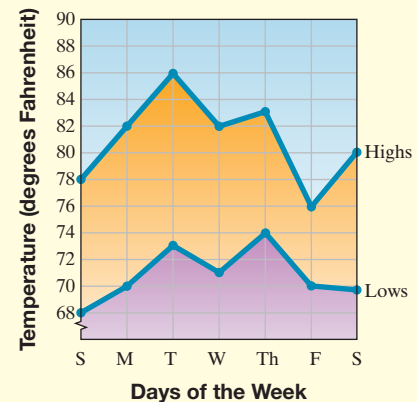
90. 95% of 50

Write each fraction as a percent. See Section 7.1.

91. $\frac{1}{4}$ 92. $\frac{2}{5}$ 93. $\frac{17}{50}$ 94. $\frac{9}{10}$

Concept Extensions

The following double line graph shows temperature highs and lows for a week. Use this graph to answer Exercises 95 through 100.



95. What was the high temperature reading on Thursday?

96. What was the low temperature reading on Thursday?

97. What day was the temperature the lowest? What was this low temperature?

98. What day of the week was the temperature the highest? What was this high temperature?

99. On what day of the week was the difference between the high temperature and the low temperature the greatest? What was this difference in temperature?

100. On what day of the week was the difference between the high temperature and the low temperature the least? What was this difference in temperature?

101. True or false? With a bar graph, the width of the bar is just as important as the height of the bar. Explain your answer.

102. Kansas plants about 17% of the wheat acreage in the United States. About how many acres of wheat are planted in the United States, according to the pictograph for Exercises 1 through 8? Round to the nearest million acres.

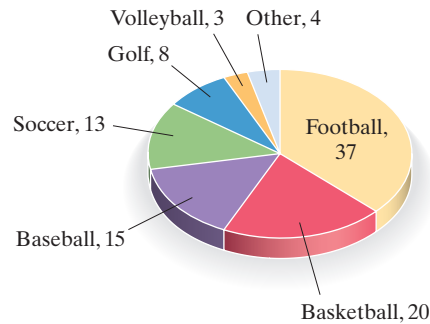
8.2 Circle Graphs

Objectives

- A** Read Circle Graphs.
- B** Draw Circle Graphs.

Objective A Reading Circle Graphs

In Exercise Set 7.1, the following **circle graph** was shown. This particular graph shows the favorite sport for 100 adults.



Each sector of the graph (shaped like a piece of pie) shows a category and the relative size of the category. In other words, the most popular sport is football, and it is represented by the largest sector.

Practice 1

Find the ratio of adults preferring golf to total adults. Write the ratio as a fraction in simplest form.

Example 1

Find the ratio of adults preferring basketball to total adults. Write the ratio as a fraction in simplest form.

Solution: The ratio is

$$\frac{\text{people preferring basketball}}{\text{total adults}} = \frac{20}{100} = \frac{1}{5}$$

Work Practice 1

A circle graph is often used to show percents in different categories, with the whole circle representing 100%.

Practice 2

Using the circle graph shown in Example 2, determine the percent of visitors to the United States that came from Europe, Asia, or South America.

Example 2 Using a Circle Graph

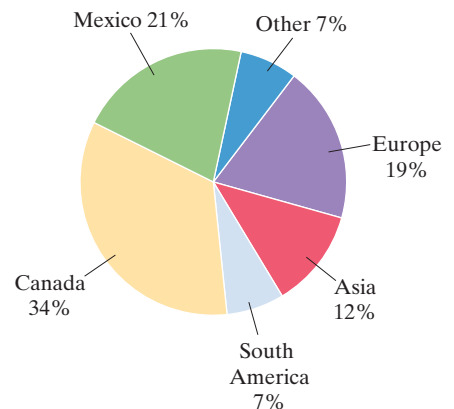
The following graph shows the percent of visitors to the United States in a recent year by various regions. Using the circle graph shown, determine the percent of visitors who came to the United States from Mexico or Canada.

Solution: To find this percent, we add the percents corresponding to Mexico and Canada. The percent of visitors to the United States that came from Mexico or Canada is

$$34\% + 21\% = 55\%$$

Work Practice 2

Visitors to U.S. by Region



Source: Office of Travel and Tourism Industries, 2012

Answers

1. $\frac{2}{25}$
2. 38%

Helpful Hint

Since a circle graph represents a whole, the percents should add to 100% or 1. Notice this is true for Example 2.

Example 3 Finding Percent of Population

The U.S. Department of Commerce forecasts 81 million international visitors to the United States in 2017. Use the circle graph from Example 2 and predict the number of tourists that might be from Europe.

Solution: We use the percent equation.

$$\begin{aligned}\text{amount} &= \text{percent} \cdot \text{base} \\ \text{amount} &= 0.19 \cdot 81,000,000 \\ &= 0.19(81,000,000) \\ &= 15,390,000\end{aligned}$$

Thus, 15,390,000 tourists might come from Europe in 2017.

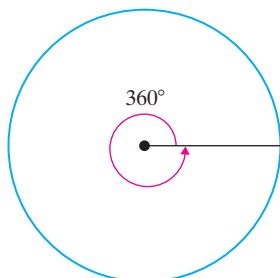
Work Practice 3

✓Concept Check Can the following data be represented by a circle graph? Why or why not?

Responses to the Question "In Which Activities Are You Involved?"	
Intramural sports	60%
On-campus job	42%
Fraternity/sorority	27%
Academic clubs	21%
Music programs	14%

Objective B Drawing Circle Graphs

To draw a circle graph, we use the fact that a whole circle contains 360° (degrees).

**Practice 3**

Use the information in Example 3 and the circle graph from Example 2 to predict the number of tourists from Mexico in 2017.

Answer

3. 17,010,000 tourists from Mexico

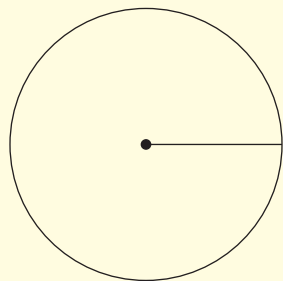
✓Concept Check Answer

no; the percents add up to more than 100%

Practice 4

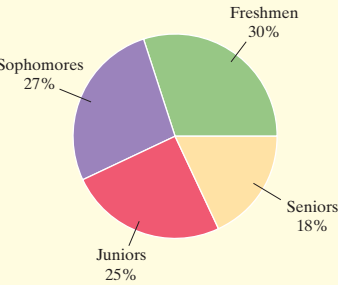
Use the data shown to draw a circle graph.

Freshmen	30%
Sophomores	27%
Juniors	25%
Seniors	18%



Answer

4.



Example 4 Drawing a Circle Graph for U.S. Armed Forces Personnel

The following table shows the percent of U.S. armed forces personnel that were in each branch of service in 2016. (Source: U.S. Department of Defense)

Branch of Service	Percent
Army	37
Navy	25
Marine Corps	14
Air Force	24
(Note: The Coast Guard is now under the Department of Homeland Security.)	

Draw a circle graph showing this data.

Solution: First we find the number of degrees in each sector representing each branch of service. Remember that the whole circle contains 360°. (We will round degrees to the nearest whole degree.)

Sector	Degrees in Each Sector
Army	$37\% \times 360^\circ = 0.37 \times 360^\circ = 133.2^\circ \approx 133^\circ$
Navy	$25\% \times 360^\circ = 0.25 \times 360^\circ = 90^\circ$
Marine Corps	$14\% \times 360^\circ = 0.14 \times 360^\circ = 50.4^\circ \approx 50^\circ$
Air Force	$24\% \times 360^\circ = 0.24 \times 360^\circ = 86.4^\circ \approx 86^\circ$

Helpful Hint

Check your calculations by finding the sum of the degrees.

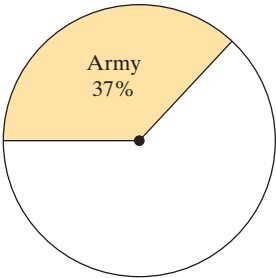
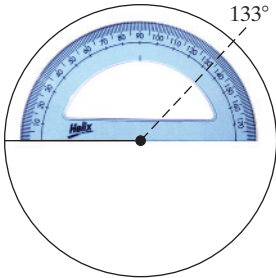
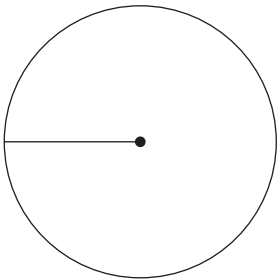
$133^\circ + 90^\circ + 50^\circ + 86^\circ = 359^\circ$

The sum should be 360°. (Our sum varies slightly because of rounding.)

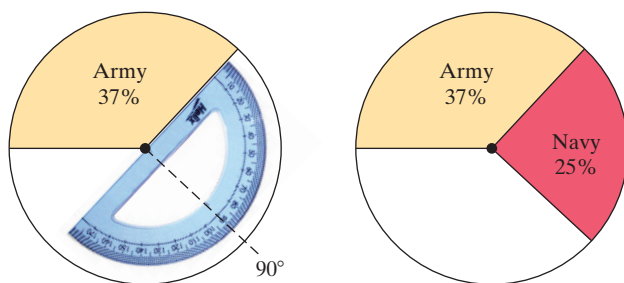
Next we draw a circle and mark its center. Then we draw a line from the center of the circle to the circle itself.

To construct the sectors, we will use a **protractor**. A protractor measures the number of degrees in an angle. We place the hole in the protractor over the center of the circle. Then we adjust the protractor so that 0° on the protractor is aligned with the line that we drew.

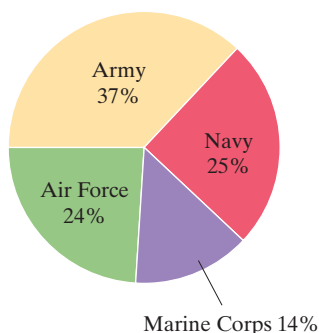
It makes no difference which sector we draw first. To construct the “Army” sector, we find 133° on the protractor and mark our circle. Then we remove the protractor and use this mark to draw a second line from the center to the circle itself.



To construct the “Navy” sector, we follow the same procedure as above, except that we line up 0° with the second line we drew and mark the protractor at 90° .



We continue in this manner until the circle graph is complete.



Work Practice 4

✓ Concept Check True or false? The larger a sector in a circle graph, the larger the percent of the total it represents. Explain your answer.

✓ Concept Check Answer
true

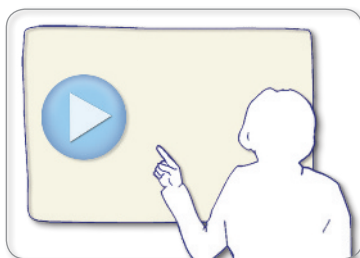
Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

sector circle 100 360

1. In a _____ graph, each section (shaped like a piece of pie) shows a category and the relative size of the category.
2. A circle graph contains pie-shaped sections, each called a _____.
3. The number of degrees in a whole circle is _____.
4. If a circle graph has percent labels, the percents should add up to _____.

Martin-Gay Interactive Videos



See Video 8.2

Watch the section lecture video and answer the following questions.

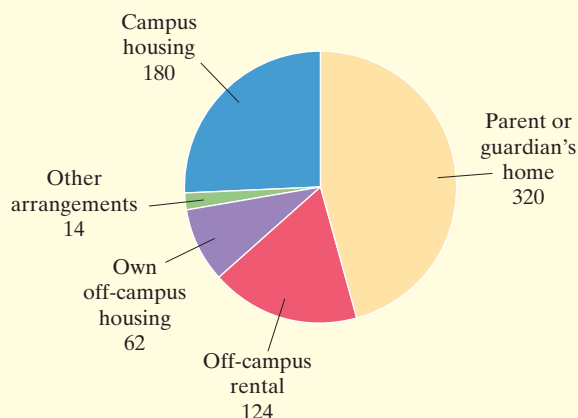
Objective A 5. From Example 3, when a circle graph shows different parts or percents of some whole category, what is the sum of the percents in the whole circle graph?

Objective B 6. From Example 6, when looking at the sector degree measures of a circle graph, the whole circle graph corresponds to what degree measure?

8.2 Exercise Set MyLab Math

Objective A The following circle graph is a result of surveying 700 college students. They were asked where they live while attending college. Use this graph to answer Exercises 1 through 6. Write all ratios as fractions in simplest form. See Example 1.

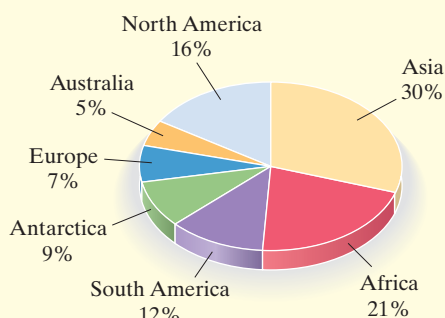
- ▶ 1. Where do most of these college students live?
2. Besides the category “Other arrangements,” where do the fewest of these college students live?
- ▶ 3. Find the ratio of students living in campus housing to total students.
4. Find the ratio of students living in off-campus rentals to total students.
5. Find the ratio of students living in campus housing to students living in a parent or guardian’s home.



6. Find the ratio of students living in off-campus rentals to students living in a parent or guardian’s home.

The following circle graph shows the percent of the land area of the continents of Earth. Use this graph for Exercises 7 through 14. See Example 2.

7. Which continent is the largest?
8. Which continent is the smallest?
9. What percent of the land on Earth is accounted for by Asia and Europe together?
10. What percent of the land on Earth is accounted for by North and South America?



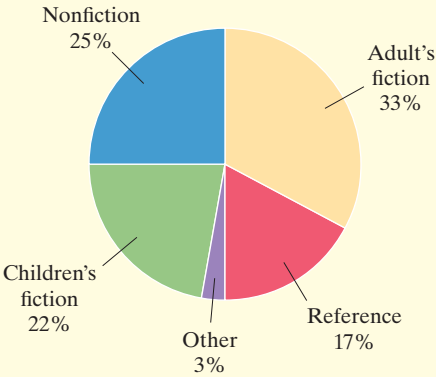
Source: National Geographic Society

The total amount of land from the continents is approximately 57,000,000 square miles. Use the graph to find the area of the continents given in Exercises 11 through 14. See Example 3.

11. Asia
12. South America
13. Australia
14. Europe

The following circle graph shows the percent of the types of books available at Midway Memorial Library. Use this graph for Exercises 15 through 24. See Example 2.

- ▶ 15. What percent of books are classified as some type of fiction?
- 16. What percent of books are nonfiction or reference?
- ▶ 17. What is the second-largest category of books?
- 18. What is the third-largest category of books?



If this library has 125,600 books, find how many books are in each category given in Exercises 19 through 24. See Example 3.

- ▶ 19. Nonfiction

20. Reference

21. Children's fiction
22. Adult's fiction

23. Reference or other

24. Nonfiction or other

Objective B Fill in each table. Round to the nearest degree. Then draw a circle graph to represent the information given in each table. (Remember: The total of “Degrees in Sector” column should equal 360° or very close to 360° because of rounding.) See Example 4.

▶ 25. **Types of Apples Grown in Washington State**

Type of Apple	Percent	Degrees in Sector
Red Delicious	37%	
Golden Delicious	13%	
Fuji	14%	
Gala	15%	
Granny Smith	12%	
Other varieties	6%	
Braeburn	3%	

(Source: U.S. Apple Association)

26. **Color Distribution of M&M's Milk Chocolate**

Color	Percent	Degrees in Sector
Blue	24%	
Orange	20%	
Green	16%	
Yellow	14%	
Red	13%	
Brown	13%	

(Source: M&M Mars)

27.

Distribution of Large Dams by Continent		
Continent	Percent	Degrees in Sector
Europe	19%	
North America	32%	
South America	3%	
Asia	39%	
Africa	5%	
Australia	2%	

(Source: International Commission on Large Dams)

28.

Distribution of Department of the Interior Public Lands by Management		
Bureau of Management	Percent	Degrees in Sector
Bureau of Indian Affairs	9%	
National Park Service	11%	
Fish and Wildlife	20%	
U.S. Forest Service	31%	
Bureau of Land Management	29%	

(Source: Department of the Interior: National Park Service)

Review

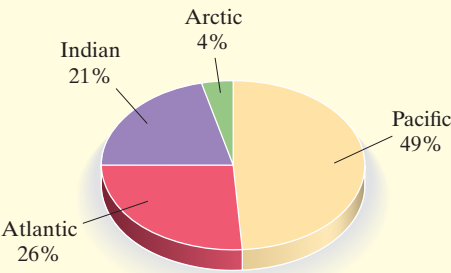
Write the prime factorization of each number. See Section 4.2.

29. 20
30. 25
31. 40
32. 16
33. 85
34. 105

Concept Extensions

The following circle graph shows the relative sizes of the great oceans.

35. Without calculating, determine which ocean is the largest. How can you answer this question by looking at the circle graph?
36. Without calculating, determine which ocean is the smallest. How can you answer this question by looking at the circle graph?



Source: Philip’s World Atlas

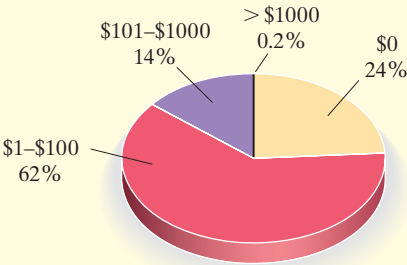
These oceans together make up 264,489,800 square kilometers of Earth’s surface. Find the square kilometers for each ocean.

37. Pacific Ocean
38. Atlantic Ocean
39. Indian Ocean
40. Arctic Ocean

The following circle graph summarizes the results of online spending in America. Let's use these results to make predictions about the online spending behavior of a community of 2800 Internet users age 18 and over. Use this graph for Exercises 41 through 46. Round to the nearest whole. (Note: Because of rounding, these percents do not have a sum of 100%.)

- 41. How many of the survey respondents said that they spend \$0 online each month?
- 42. How many of the survey respondents said that they spend \$1–\$100 online each month?
- 43. How many of the survey respondents said that they spend \$0 to \$100 online each month?
- 44. How many of the survey respondents said that they spend \$1 to \$1000 online each month?

Online Spending per Month



Source: The Digital Future Report, 2013

- 45. Find the ratio of *number* of respondents who spend \$0 online to *number* of respondents who spend \$1–\$100 online. Write the ratio as a fraction. Simplify the fraction if possible.
- 46. Find the ratio of *percent* of respondents who spend \$101–\$1000 online to *percent* of those who spend \$1–\$100. Write the ratio as a fraction with integers in the numerator and denominator. Simplify the fraction if possible.

See the Concept Checks in this section.

- 47. Can the data below be represented by a circle graph? Why or why not?

Responses to the Question “What Classes Are You Taking?”	
Math	80%
English	72%
History	37%
Biology	21%
Chemistry	14%

- 48. True or false? The smaller a sector in a circle graph, the smaller the percent of the total it represents. Explain why.

8.3 The Rectangular Coordinate System and Paired Data

Objectives

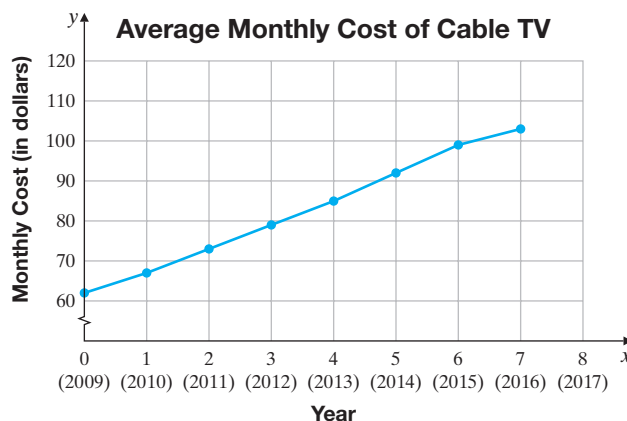
- A** Plot Points on a Rectangular Coordinate System.
- B** Determine Whether Ordered Pairs Are Solutions of Equations.
- C** Complete Ordered Pair Solutions of Equations.

Objective A Plotting Points

In Section 8.1, we saw how bar and line graphs can be used to show relationships between items listed on horizontal and vertical axes.

The table and line graph below both show the U.S. average monthly spending on basic enhanced cable for the years shown. (*Source: NBC News*)

Average Monthly Spending on Cable TV	
Year	Average Cost per Month
2009	\$62
2010	\$67
2011	\$73
2012	\$79
2013	\$85
2014	\$92
2015	\$99
2016	\$103

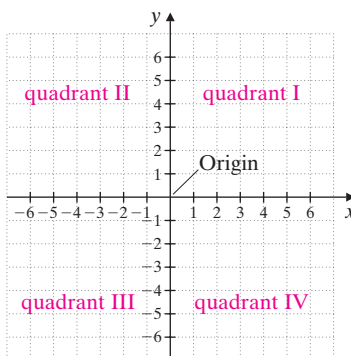


Both the table and the line graph show the relationship between two quantities (year and money), called **paired data**. These data are called paired data because each year corresponds to, or is paired with, a specific amount of money. In other words, the year 2015 is paired with \$99. Notice that the dots, or points, on the line graph visually show the same pairing. For the year 2015, the height of the dot is 99, representing \$99.

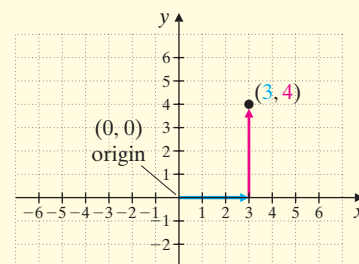
We can call the “year” axis the horizontal axis and the “money” axis the vertical axis.

In general, we can use this same horizontal and vertical axis idea to describe the location of points in a plane. A **plane** is a flat surface that extends indefinitely. Surfaces like a plane (except that they don’t extend indefinitely) are a classroom floor, a blackboard or whiteboard, and this sheet of paper.

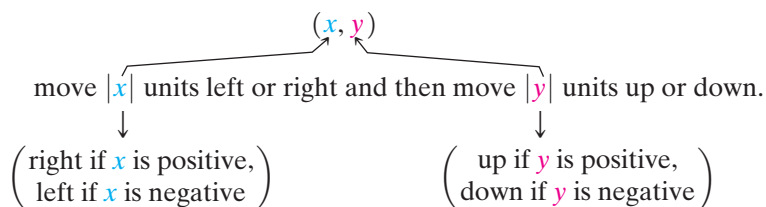
The system that we use to describe the locations of points in a plane is called the **rectangular coordinate system**. It consists of two number lines, one horizontal and one vertical, intersecting at the point 0 on each number line. This point of intersection is called the **origin**. We call the horizontal number line the **x-axis** and the vertical number line the **y-axis**. Notice that the axes divide the plane into four regions, called **quadrants**. They are numbered as shown.



Every point in the rectangular coordinate system corresponds to an **ordered pair of numbers**, such as $(3, 4)$. The first number, 3, of an ordered pair is associated with the x -axis and is called the **x -coordinate** or **x -value**. The second number, 4, is associated with the y -axis and is called the **y -coordinate** or **y -value**. To find the **single point** on the rectangular coordinate system corresponding to the ordered pair $(3, 4)$, start at the origin. Move 3 units in the positive direction along the x -axis. From there, move 4 units in the positive direction parallel to the y -axis as shown to the left. This process of locating a point on the rectangular coordinate system is called **plotting the point**. Since the origin is located at 0 on the x -axis and 0 on the y -axis, the origin corresponds to the ordered pair $(0, 0)$.



In general, to plot the ordered pair (x, y) , start at the origin. Next,

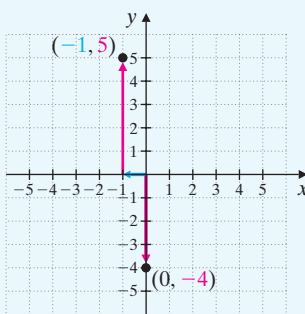


Helpful Hint

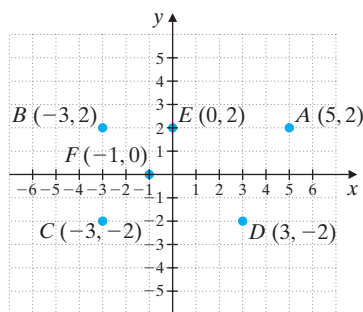
Since the first number, or x -coordinate, of an ordered pair is associated with the x -axis, it tells how many units to move left or right. Similarly, the second number, or y -coordinate, tells how many units to move up or down.

To plot $(-1, 5)$, start at the origin. Move 1 unit left (because the x -coordinate is negative) and then 5 units up and draw a dot at that point. This dot is the graph of the point that corresponds to the ordered pair $(-1, 5)$.

To plot $(0, -4)$, start at the origin, move 0 units (left or right), then 4 units down, and then draw a dot.



Below are some more plotted points with their corresponding ordered pairs. Another name for the rectangular coordinate system is the Cartesian coordinate system. This name comes from French scientist, philosopher, and mathematician René Descartes (1596–1650), who introduced the system.

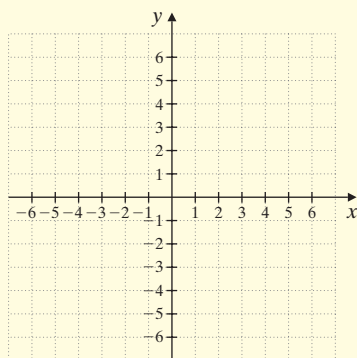


Helpful Hint

Remember that **each point** in the rectangular coordinate system corresponds to exactly **one ordered pair** and that **each ordered pair** corresponds to exactly **one point**.

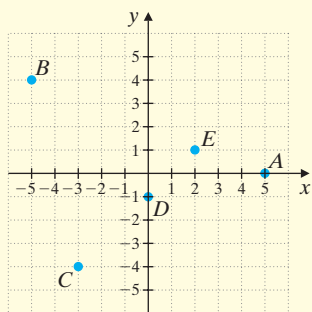
Practice 1

Plot each point corresponding to the ordered pairs on the same set of axes. $(2, 3)$, $(-4, 2)$, $(-6, -5)$, $(2, -2)$, $(5, 0)$, $(0, -3)$, $(-\frac{1}{2}, 4)$



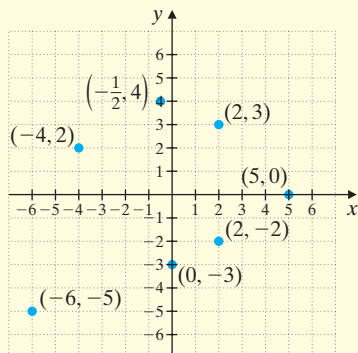
Practice 2

Find the ordered pair corresponding to each point plotted on the rectangular coordinate system.



Answers

1.



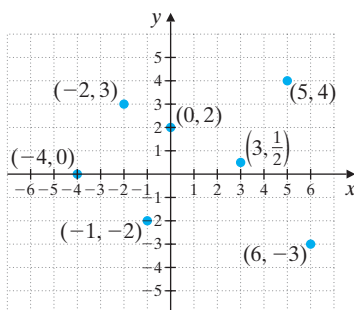
2. $A(5, 0)$; $B(-5, 4)$; $C(-3, -4)$; $D(0, -1)$; $E(2, 1)$

Example 1

Plot each point corresponding to the ordered pairs on the same set of axes.

$$(5, 4), (-2, 3), (-1, -2), (6, -3), (0, 2), (-4, 0), \left(3, \frac{1}{2}\right)$$

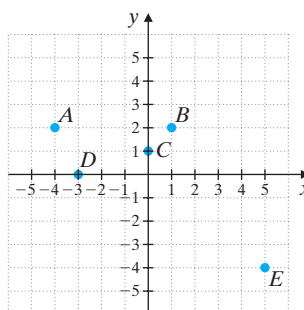
Solution:



Work Practice 1

Example 2

Find the ordered pair corresponding to each point plotted below.



Solution:

Point A has coordinates $(-4, 2)$.

Point B has coordinates $(1, 2)$.

Point C has coordinates $(0, 1)$.

Point D has coordinates $(-3, 0)$.

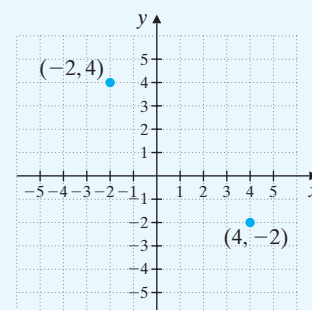
Point E has coordinates $(5, -4)$.

Work Practice 2

Have you noticed a pattern from the preceding examples? **If an ordered pair has a y -coordinate of 0, its graph lies on the x -axis. If an ordered pair has an x -coordinate of 0, its graph lies on the y -axis.**

Helpful Hint

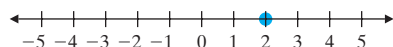
Order is the key word in “ordered pair.” The first value always corresponds to the x -value and the second value always corresponds to the y -value. For example, $(-2, 4)$ does not describe the same point as $(4, -2)$.



Objective B Determining Whether an Ordered Pair Is a Solution

Let's see how we can use ordered pairs of numbers to record solutions of equations containing two variables.

Recall that an equation with one variable, such as $3 + x = 5$, has one solution—in this case, 2 because 2 is the only value that can be substituted for x so that the resulting equation $3 + 2 = 5$ is a true statement. We can graph this solution on a number line.



An equation with two variables such as $x + y = 7$ has many solutions. Each solution is a pair of numbers, one for each variable, that makes the equation a true statement. For example, $x = 4$ and $y = 3$ is a solution of the equation $x + y = 7$ because, when x is replaced with 4 and y is replaced with 3, $x + y = 7$ becomes $4 + 3 = 7$, which is a true statement. We can write the solution $x = 4$ and $y = 3$ as the ordered pair $(4, 3)$. Study the chart below for more solutions.

Replacement Values	$x + y = 7$	True or False	Ordered Pair Solution
$x = 4, y = 3$	$4 + 3 = 7$	True	$(4, 3)$
$x = 0, y = 7$	$0 + 7 = 7$	True	$(0, 7)$
$x = 7, y = 1$	$7 + 1 = 7$	False	No
$x = 11, y = -4$	$11 + (-4) = 7$	True	$(11, -4)$

In general, we say that an ordered pair of numbers is a solution of an equation if the equation is a true statement when the variables are replaced by the coordinates of the ordered pair.

Let's practice deciding whether an ordered pair of numbers is a solution of an equation in two variables.

Example 3 Is $(-1, 6)$ a solution of the equation $2x + y = 4$?

Solution: Replace x with -1 and y with 6 in the equation $2x + y = 4$ to see if the result is a true statement.

$$\begin{array}{rcl}
 2x + y & = & 4 \quad \text{Original equation} \\
 2(-1) + 6 & \stackrel{?}{=} & 4 \quad \text{Replace } x \text{ with } -1 \text{ and } y \text{ with } 6. \\
 -2 + 6 & \stackrel{?}{=} & 4 \quad \text{Multiply.} \\
 4 & = & 4 \quad \text{True}
 \end{array}$$

Since $4 = 4$ is true, $(-1, 6)$ is a solution of the equation $2x + y = 4$.

Work Practice 3

Unlike equations with one variable, equations with two variables, such as $x + y = 7$, have so many solutions we cannot simply list them. Instead, to “see” all the solutions of a two-variable equation, we draw a graph of its solutions.

Practice 3

Is $(0, -4)$ a solution of the equation $x + 3y = -12$?

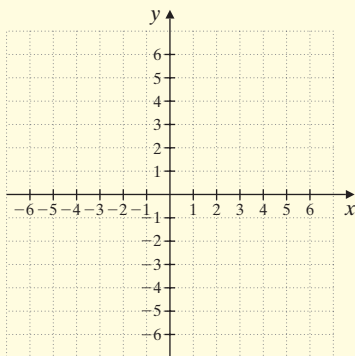
Answer

3. yes

Practice 4

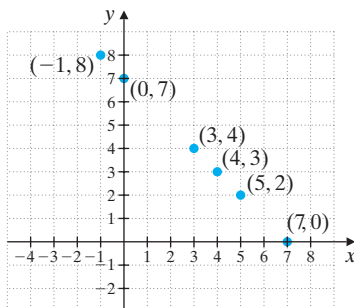
Each ordered pair listed is a solution of the equation $x - y = 5$. Plot them on the same set of axes.

- a. (6, 1) b. (5, 0)
 c. (0, -5) d. (7, 2)
 e. (-1, -6) f. (2, -3)

**Example 4**

Each ordered pair listed is a solution of the equation $x + y = 7$. Plot them on the same set of axes.

- a. (3, 4) b. (0, 7) c. (-1, 8)
 d. (7, 0) e. (5, 2) f. (4, 3)

Solution:**Work Practice 4**

Notice that the points in Example 4 all seem to lie on the same line. We will discuss this more in the next section.

An equation such as $x + y = 7$ is called a **linear** or **first-degree equation in two variables**. It is called a **first-degree equation** because the exponent on x and y is an understood 1. The equation is called an equation **in two variables** because it contains two different variables, x and y .

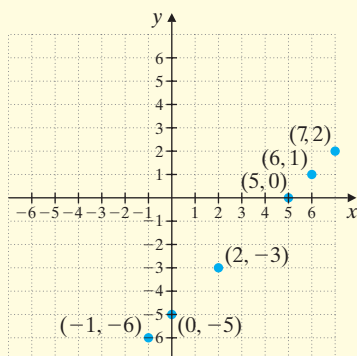
Objective C Completing Ordered Pair Solutions

In the next section, we will graph linear equations in two variables. Before that, we need to practice finding ordered pair solutions of these equations. If one coordinate of an ordered pair solution is known, the other coordinate can be determined. To find the unknown coordinate, replace the appropriate variable with the known coordinate in the equation. Doing so results in an equation with one variable that we can solve.

Practice 5

Complete each ordered pair solution of the equation $y = -5x$.

- a. (5,) b. (, 0)
 c. (-3,)

Answers**4.**

5. a. (5, -25) b. (0, 0) c. (-3, 15)

Example 5

Complete each ordered pair solution of the equation $y = 2x$.

- a. (3,) b. (, 0) c. (-2,)

Solution:

- a. In the ordered pair (3,), the x -value is 3. To find the corresponding y -value, let $x = 3$ in the equation $y = 2x$ and calculate the value of y .

$$y = 2x \quad \text{Original equation}$$

$$y = 2(3) \quad \text{Replace } x \text{ with } 3.$$

$$y = 6 \quad \text{Multiply.}$$

Check: To check, replace x with 3 and y with 6 in the original equation to see that a true statement results.

$$y = 2x \quad \text{Original equation}$$

$$6 \stackrel{?}{=} 2(3) \quad \text{Let } x = 3 \text{ and } y = 6.$$

$$6 = 6 \quad \text{True}$$

The ordered pair solution is (3, 6).

- b. In the ordered pair $(\quad, 0)$, the y -value is 0. To find the corresponding x -value, replace y with 0 in the equation and solve for x .

$$y = 2x \quad \text{Original equation}$$

$$0 = 2x \quad \text{Replace } y \text{ with } 0.$$

$$\frac{0}{2} = \frac{2x}{2} \quad \text{Divide both sides by 2.}$$

$$0 = x \quad \text{Simplify.}$$

$$\text{Check: } y = 2x \quad \text{Original equation}$$

$$0 \stackrel{?}{=} 2 \cdot 0 \quad \text{Let } x = 0 \text{ and } y = 0.$$

$$0 = 0 \quad \text{True}$$

The ordered pair solution is $(0, 0)$, the origin.

- c. In the ordered pair $(-2, \quad)$, the x -value is -2 . To find the corresponding y -value, replace x with -2 in the equation and calculate the value for y .

$$y = 2x \quad \text{Original equation}$$

$$y = 2(-2) \quad \text{Replace } x \text{ with } -2.$$

$$y = -4 \quad \text{Multiply.}$$

$$\text{Check: } y = 2x \quad \text{Original equation}$$

$$-4 \stackrel{?}{=} 2(-2) \quad \text{Let } x = -2 \text{ and } y = -4.$$

$$-4 = -4 \quad \text{True}$$

The ordered pair solution is $(-2, -4)$.

Work Practice 5

Example 6 Complete each ordered pair solution of the equation $y = 3x - 4$.

a. $(2, \quad)$

b. $(\quad, -16)$

Solution:

- a. Replace x with 2 in the equation and calculate the value for y .

$$y = 3x - 4 \quad \text{Original equation}$$

$$y = 3 \cdot 2 - 4 \quad \text{Replace } x \text{ with } 2.$$

$$y = 2 \quad \text{Simplify.}$$

The ordered pair solution is $(2, 2)$.

- b. Replace y with -16 in the equation and solve for x .

$$y = 3x - 4 \quad \text{Original equation}$$

$$-16 = 3x - 4 \quad \text{Replace } y \text{ with } -16.$$

$$-16 + 4 = 3x - 4 + 4 \quad \text{Add 4 to both sides.}$$

$$-12 = 3x \quad \text{Simplify.}$$

$$\frac{-12}{3} = \frac{3x}{3} \quad \text{Divide both sides by 3.}$$

$$-4 = x \quad \text{Simplify.}$$

The ordered pair solution is $(-4, -16)$.

Work Practice 6

Practice 6

Complete each ordered pair solution of the equation $y = 5x + 2$.

a. $(0, \quad)$

b. $(\quad, -3)$

Answers

6. a. $(0, 2)$ b. $(-1, -3)$

Vocabulary, Readiness & Video Check

Use the words and phrases below to fill in each blank. The questions and statements all have to do with the rectangular coordinate system.

plotting	x	one	$(0, 0)$	ordered pair of numbers
origin	y	four	plane	

- In the ordered pair $(-1, 2)$, the _____-value is -1 and the _____-value is 2.
- In the rectangular coordinate system, the point of intersection of the x -axis and the y -axis is called the _____.

3. The axes divide the plane into _____ regions, called quadrants.
4. Every ordered pair of numbers, such as $\left(\frac{1}{2}, -3\right)$, corresponds to how many points in the plane? _____
5. The process of locating a point on the rectangular coordinate system is called _____ the point.
6. The origin corresponds to the ordered pair _____.
7. A(n) _____ is a flat surface that extends indefinitely in all directions.
8. Every point in the rectangular coordinate system corresponds to a(n) _____.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



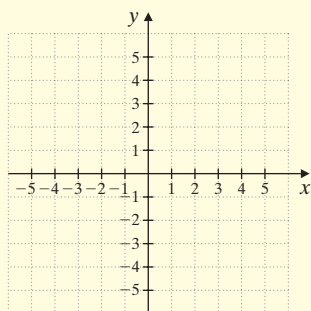
See Video 8.3

- Objective A** 9. Complete this statement based on the lecture before Example 1: In a rectangular coordinate system, the x -axis and the y -axis number lines intersect at a(n) _____ angle and at the zero _____ of each line. ▶
- Objective B** 10. In Example 2, why are we reminded that order is important in ordered pairs? ▶
- Objective C** 11. In Example 3, what value of x completes the ordered pair solution when the y -value is 5? ▶

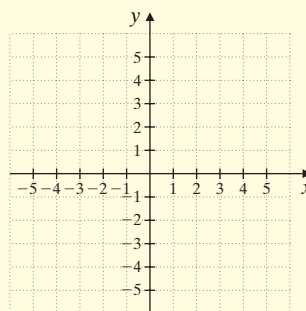
8.3 Exercise Set MyLab Math

Objective A Use the rectangular coordinate system below each exercise to plot the points corresponding to the ordered pairs. See Example 1.

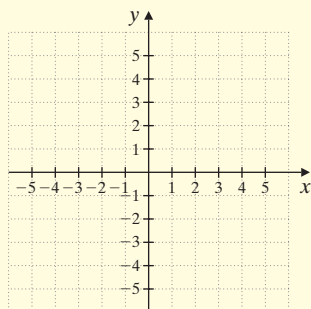
1. $(1, 3), (-2, 4), (0, 2), (-5, 0), (-3, -3), (5, -5)$



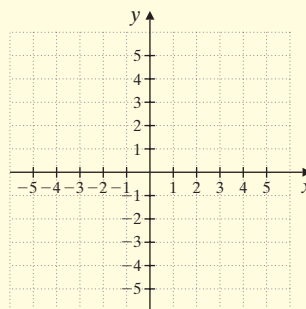
2. $(5, 2), (3, -4), (-1, -1), (0, -1), (4, 0), (-2, 4)$



3. $\left(2\frac{1}{2}, 3\right), (0, -3), (-4, -6), \left(-1, 5\frac{1}{2}\right), (1, 0), (3, -5)$

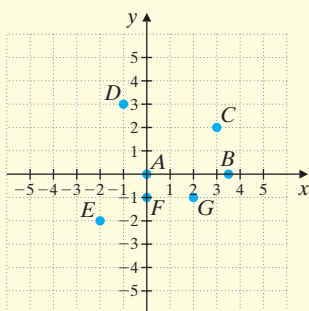


4. $\left(5, \frac{1}{2}\right), \left(-3\frac{1}{2}, 0\right), (-1, 4), (4, -1), (0, 2), (-5, -5)$

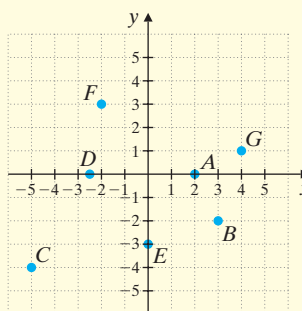


Find the x - and y -coordinates of each labeled point. See Example 2.

5.



6.

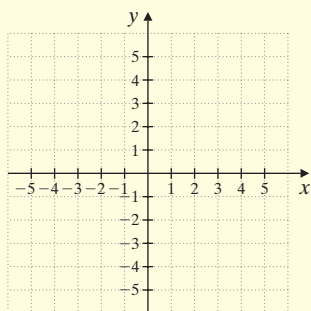


Objective B Determine whether each ordered pair is a solution of the given linear equation. See Example 3.

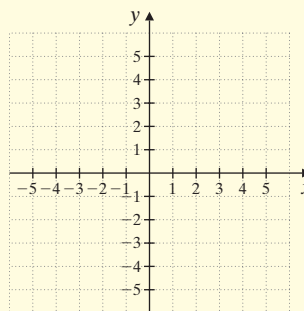
7. $(0, 0); y = -20x$ 8. $(1, 14); x = 14y$ 9. $(1, 2); x - y = 3$
10. $(-1, 9); x + y = 8$ 11. $(-2, -3); y = 2x + 1$ 12. $(1, 1); y = -x$
13. $(6, -2); x = -3y$ 14. $(-9, 1); x = -9y$ 15. $(5, 0); 3y + 2x = 10$
16. $(1, 1); -5y + 4x = -1$ 17. $(3, 1); x - 5y = -1$ 18. $(0, 2); x - 7y = -15$

Use the rectangular coordinate system below each exercise to plot the three ordered pair solutions of the given equation. See Example 4.

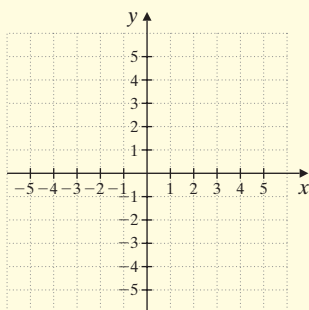
19. $2x + y = 5; (1, 3), (0, 5), (3, -1)$



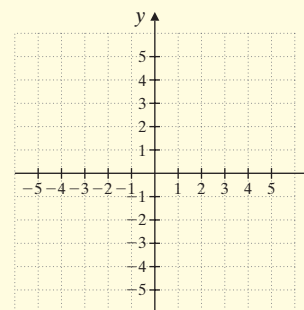
20. $3x + y = 5; (1, 2), (0, 5), (2, -1)$



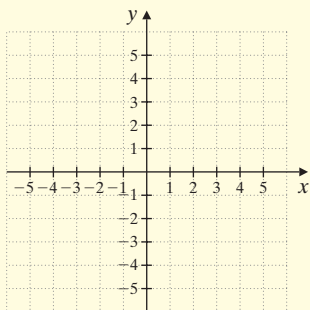
21. $x = 5y; (5, 1), (0, 0), (-5, -1)$



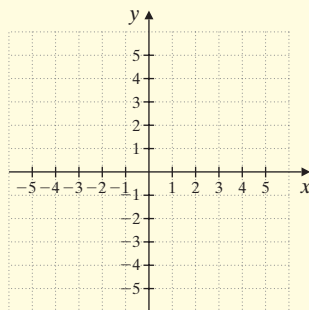
22. $y = -3x; (1, -3), (2, -6), (-1, 3)$



23. $y = -2x + 3$; $(4, -5)$, $(2, -1)$, $(0, 3)$



24. $y = 3x + 1$; $(0, 1)$, $(1, 4)$, $(-1, -2)$



Objective C Complete each ordered pair solution of the given equations. See Examples 5 and 6.

25. $y = -9x$; $(1, \quad)$, $(0, \quad)$, $(\quad, -18)$

26. $x = -7y$; $(\quad, 2)$, $(14, \quad)$, $(\quad, -1)$

27. $x - y = 14$; $(2, \quad)$, $(\quad, -8)$, $(0, \quad)$

28. $x - y = 8$; $(0, \quad)$, $(\quad, 0)$, $(5, \quad)$

▶ 29. $x + y = -2$; $(-2, \quad)$, $(1, \quad)$, $(\quad, 5)$

30. $x + y = -3$; $(\quad, 0)$, $(0, \quad)$, $(4, \quad)$

31. $x = y - 4$; $(\quad, -12)$, $(\quad, 3)$, $(100, \quad)$

32. $y = x + 5$; $(1, \quad)$, $(\quad, 7)$, $(3, \quad)$

▶ 33. $y = 3x - 5$; $(1, \quad)$, $(2, \quad)$, $(\quad, 4)$

34. $x = -12y + 5$; $(\quad, -1)$, $(\quad, 1)$, $(41, \quad)$

35. $x = -y$; $(\quad, 0)$, $(3, \quad)$, $(\quad, -9)$

36. $y = -x$; $(\quad, 0)$, $(2, \quad)$, $(\quad, 2)$

37. $x + 2y = -8$; $(4, \quad)$, $(\quad, -3)$, $(0, \quad)$

38. $-5x + y = 1$; $(0, \quad)$, $(-1, \quad)$, $(2, \quad)$

Review

Perform each indicated operation on decimals. See Sections 5.2 through 5.4.

39. $5.6 - 3.9$

40. $5 + 2.54 + 8.7$

41. $5.6 \cdot 3.9$

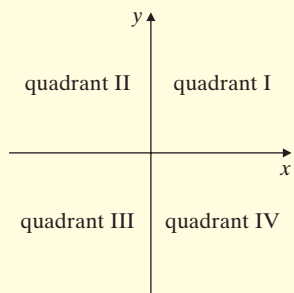
42. $0.56 \div 0.8$

43. $(0.236)(-100)$

44. $44.72 \div 100$

Concept Extensions

Recall that the axes divide the plane into four quadrants as shown. For Exercises 45 through 52, if a and b are both positive numbers, determine whether each statement is true or false.

45. (a, b) lies in quadrant I.46. $(-a, -b)$ lies in quadrant IV.47. $(0, b)$ lies on the y -axis.48. $(a, 0)$ lies on the x -axis.49. $(0, -b)$ lies on the x -axis.50. $(-a, 0)$ lies on the y -axis.51. $(-a, b)$ lies in quadrant III.52. $(a, -b)$ lies in quadrant IV.

 53. Is the ordered pair $(4, -3)$ plotted to the left or right of the y -axis? Explain.

 54. Is the ordered pair $(6, -2)$ plotted above or below the x -axis? Explain.

Plot the points $A(4, 3)$, $B(-2, 3)$, $C(-2, -1)$, and $D(4, -1)$ on a rectangular coordinate system.

55. Draw a line segment connecting A and B , B and C , C and D , and D and A . Name the figure drawn.

56. Find the length and the width of the figure.

57. Find the perimeter of the figure.

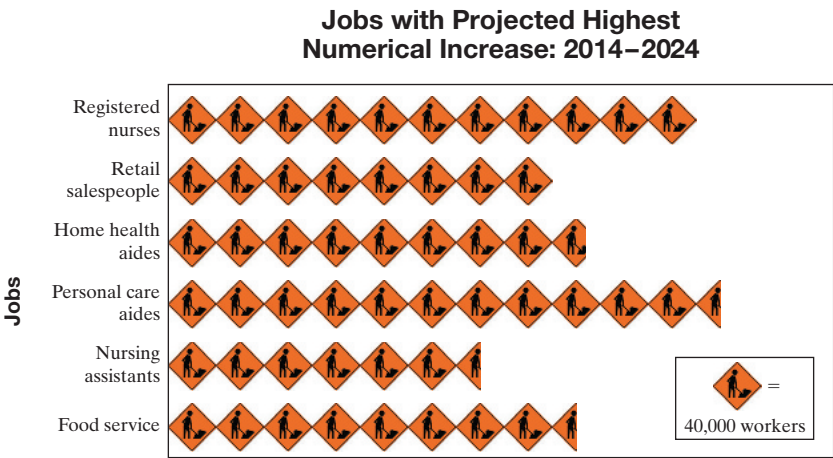
58. Find the area of the figure.

Answers

1.
2.
3.
4.
5.
6.
7.
8.

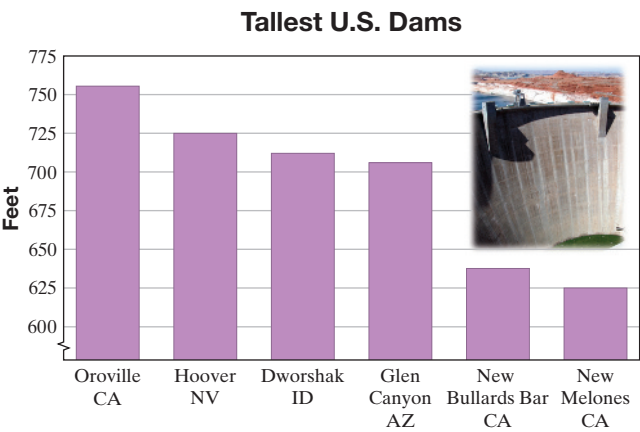
Reading Graphs

The following pictograph shows the six occupations with the largest estimated numerical increase (not salary increase) in employment in the United States between 2014 and 2024. Use this graph to answer Exercises 1 through 4.



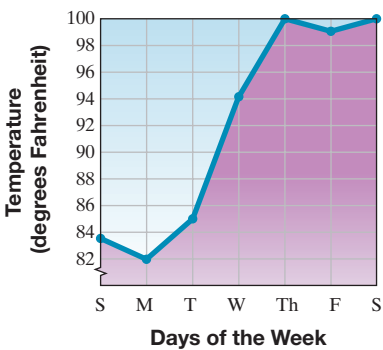
1. Approximate the increase in the number of retail salespeople from 2014 to 2024.
2. Approximate the increase in the number of registered nurses from 2014 to 2024.
3. Which occupation is expected to show the greatest increase in number of employees between the years shown?
4. Which of the listed occupations is expected to show the smallest increase in number of employees between the years shown?

The following bar graph shows the tallest U.S. dams. Use this graph to answer Exercises 5 through 8.



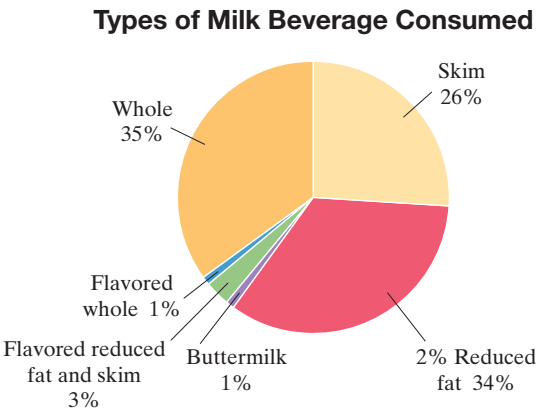
5. Name the U.S. dam with the greatest height and estimate its height.
6. Name the U.S. dam whose height is between 625 and 650 feet and estimate its height.
7. Estimate how much taller the Hoover Dam is than the Glen Canyon Dam.
8. How many U.S. dams have heights over 700 feet?

The following line graph shows the daily high temperatures for one week in Annapolis, Maryland. Use this graph to answer Exercises 9 through 12.



- 9. Name the day(s) of the week with the highest temperature and give that high temperature.
- 10. Name the day(s) of the week with the lowest temperature and give that low temperature.
- 11. On what days of the week was the temperature less than 90° Fahrenheit?
- 12. On what days of the week was the temperature greater than 90° Fahrenheit?

The following circle graph shows the types of milk beverages consumed in the United States. Use this graph for Exercises 13 through 16. If a store in Kerrville, Texas, sells 200 quart containers of milk per week, estimate how many quart containers are sold in each category below.



Source: U.S. Department of Agriculture

- 13. Whole milk
- 14. Skim milk
- 15. Buttermilk
- 16. Flavored reduced fat and skim milk

The following list shows weekly quiz scores for a student in prealgebra. Use this list to complete the frequency distribution table below.

50 80 80 90 88 97
67 89 71 78 93 99
75 53 93 83 86

	Class Intervals (Scores)	Tally	Class Frequency (Number of Quizzes)
17.	50–59		
18.	60–69		
19.	70–79		
20.	80–89		
21.	90–99		

- 9.
- 10.
- 11.
- 12.
- 13.
- 14.
- 15.
- 16.
- 17.
- 18.
- 19.
- 20.
- 21.

22. _____

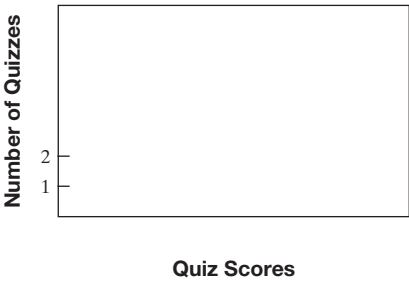
23. _____

24. _____

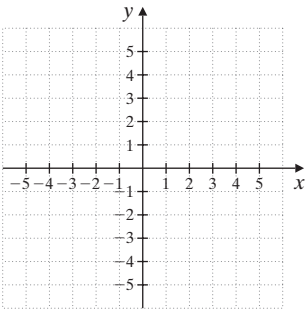
25. _____

26. _____

22. Use the frequency distribution table from Exercises 17 through 21 to construct a histogram.



23. Plot the points corresponding to the ordered pairs on the same set of axes. $(0, 2)$, $(-1, 4)$, $(2, 1)$, $(-3, 0)$, $(-3, -5)$, $(4, -1)$



24. Determine whether the ordered pair $(1, 3)$ is a solution of the linear equation $x = 3y$.

25. Determine whether the ordered pair $(-2, -4)$ is a solution of the linear equation $x + y = -6$.

26. Complete each ordered pair solution of the equation $x - y = 6$. $(0, \quad)$, $(\quad, 0)$, $(2, \quad)$

8.4 Graphing Linear Equations in Two Variables

Now that we know how to plot points in a rectangular coordinate system and how to find ordered pair solutions, we are ready to graph linear equations in two variables. First, we give a formal definition.

Linear Equation in Two Variables

A linear equation in two variables is an equation that can be written in the form

$$ax + by = c$$

where a , b , and c are numbers,
and a and b are not both 0.

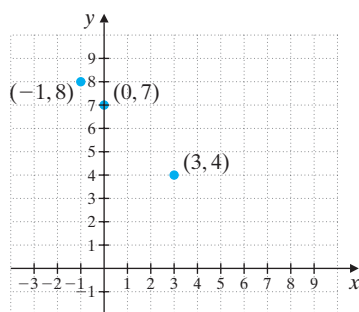
$$\text{Examples: } \begin{cases} 2x + 3y = 6 \\ 2x = 5 \\ y = 3 \end{cases}$$

In the last section, we discovered that a linear equation in two variables has many solutions. For the linear equation $x + y = 7$, we listed, for example, the solutions $(3, 4)$, $(0, 7)$, $(-1, 8)$, $(7, 0)$, $(5, 2)$, and $(4, 3)$. Are these all the solutions? No. There are infinitely many solutions of the equation $x + y = 7$ since there are infinitely many pairs of numbers whose sum is 7. Every linear equation in two variables has infinitely many ordered pair solutions. Since it is impossible to list every solution, we graph the solutions instead.

Objective A Graphing Linear Equations in Two Variables by Plotting Points

Fortunately, the pattern described by the solutions of a linear equation makes “seeing” the solutions possible by graphing. This is so because **all the solutions of a linear equation in two variables correspond to points on a single straight line**. If we plot just a few of these points and draw the straight line connecting them, we have a complete graph of all the solutions.

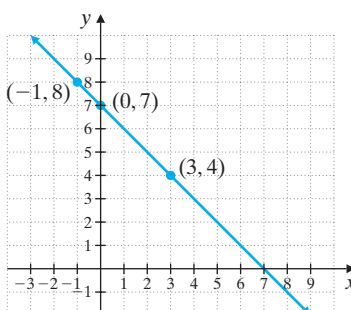
To graph the equation $x + y = 7$, then, we plot a few ordered pair solutions, say $(3, 4)$, $(0, 7)$, and $(-1, 8)$.



Objective

A Graph Linear Equations by Plotting Points.

Now we connect these three points by drawing a straight line through them. The arrows at both ends of the line indicate that the line goes on forever in both directions.

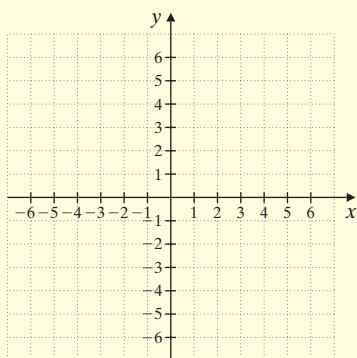


Every point on this line corresponds to an ordered pair solution of the equation $x + y = 7$. Also, every ordered pair solution of the equation $x + y = 7$ corresponds to a point on this line. In other words, this line is the graph of the equation $x + y = 7$. Although a line can be drawn using just two points, we will graph a third solution to check our work.

Practice 1

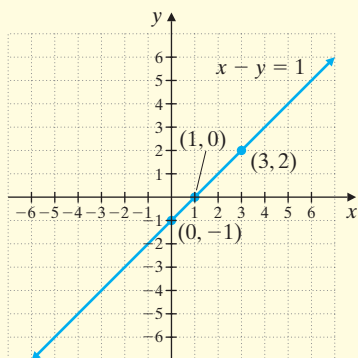
Graph the equation $x - y = 1$ by plotting the following points, which satisfy the equation, and drawing a line through the points.

$$(3, 2), (0, -1), (1, 0)$$



Answer

1.

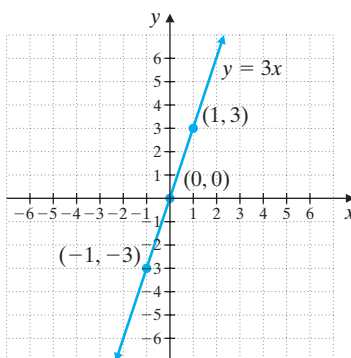


Example 1

Graph the equation $y = 3x$ by plotting the following points, which satisfy the equation, and drawing a line through the points.

$$(1, 3), (0, 0), (-1, -3)$$

Solution: Plot the points and draw a line through them. The line is the graph of the linear equation $y = 3x$.



Work Practice 1

Graphing Linear Equations and Finding Ordered Pair Solutions

- To graph a linear equation in two variables, find three ordered pair solutions, graph the solutions, and draw a line through the plotted points.
- To find an ordered pair solution of an equation, choose either an x -value or a y -value of the ordered pair and complete the ordered pair as we did in the previous section.

Example 2 Graph: $x - y = 6$

Solution: Find any three ordered pair solutions. For each solution, we choose a value for x or y , and replace x or y by its chosen value in the equation. Then we solve the resulting equation for the other variable.

For example, let $x = 3$.

$$x - y = 6 \quad \text{Original equation}$$

$$3 - y = 6 \quad \text{Let } x = 3.$$

$$-y = 6 - 3 \quad \text{Subtract 3 from both sides.}$$

$$-y = 3 \quad \text{Simplify.}$$

$$\frac{-y}{-1} = \frac{3}{-1} \quad \text{Divide both sides by } -1.$$

$$y = -3 \quad \text{Simplify.}$$

Also, let $y = 0$.

$$x - y = 6 \quad \text{Original equation}$$

$$x - 0 = 6 \quad \text{Let } y = 0.$$

$$x = 6 \quad \text{Simplify.}$$

The ordered pair is $(6, 0)$.

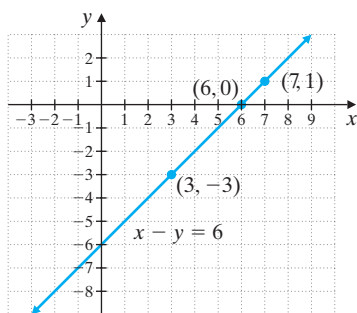
The ordered pair is $(3, -3)$.

If we let $x = 7$, and solve the resulting equation for y , we will see that $y = 1$.

A third ordered pair solution is, then, $(7, 1)$.

Plot the three ordered pair solutions on a rectangular coordinate system and then draw a line through the three points. This line is the graph of $x - y = 6$. (For organization purposes, the ordered pairs are entered in the table below.)

x	y
3	-3
6	0
7	1



Work Practice 2

Helpful Hint

All three points should fall on the same straight line. If they do not, check your ordered pair solutions for a mistake, since the graph of every linear equation is a line.

✓Concept Check In Example 2, is the point $(9, -3)$ on the graph of $x - y = 6$?

Example 3 Graph: $y = 5x + 1$

Solution: For this equation, we will choose three x -values and find the corresponding y -values. We choose $x = 0$, $x = 1$, and $x = 2$. The first table on the next page shows our chosen x -values.

If $x = 0$, then

$$y = 5x + 1 \text{ becomes}$$

$$y = 5 \cdot 0 + 1 \text{ or}$$

$$y = 1$$

If $x = 1$, then

$$y = 5x + 1 \text{ becomes}$$

$$y = 5 \cdot 1 + 1 \text{ or}$$

$$y = 6$$

If $x = 2$, then

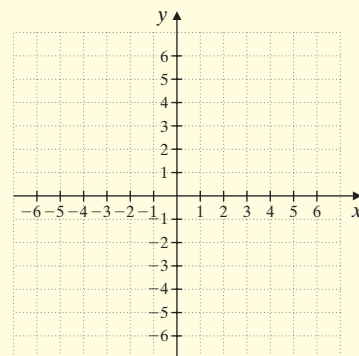
$$y = 5x + 1 \text{ becomes}$$

$$y = 5 \cdot 2 + 1 \text{ or}$$

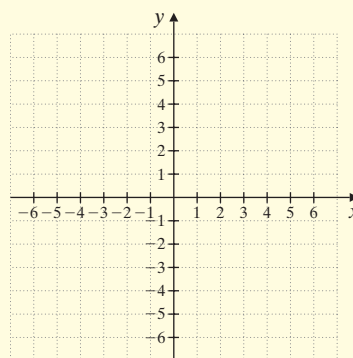
$$y = 11$$

Practice 2

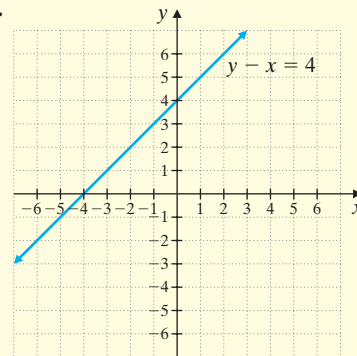
Graph: $y - x = 4$

**Practice 3**

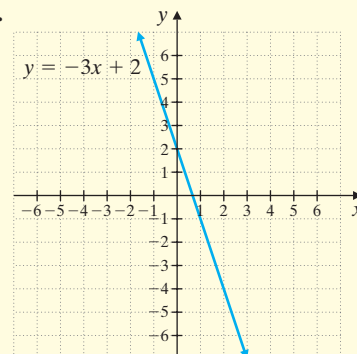
Graph: $y = -3x + 2$

**Answers**

2.



3.

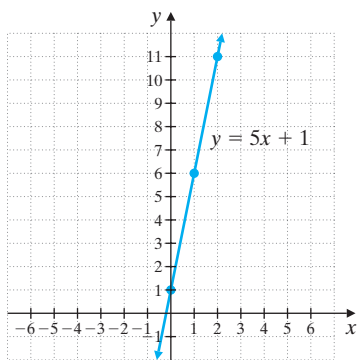


✓Concept Check Answer
no

Now we complete the table of values, plot the ordered pair solutions, and graph the equation $y = 5x + 1$.

x	y
0	
1	
2	

x	y
0	1
1	6
2	11

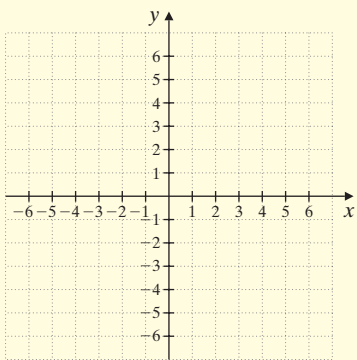


Work Practice 3

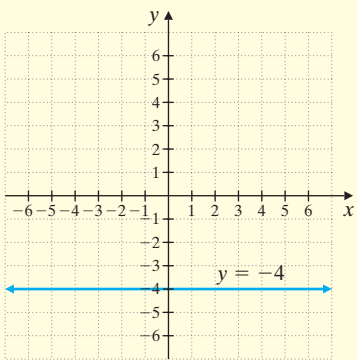
Next, we will graph a few special linear equations.

Practice 4

Graph: $y = -4$



Answer
4.



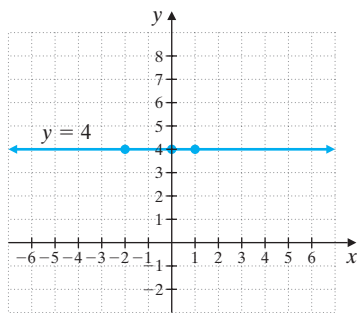
Example 4 Graph: $y = 4$

Solution: The equation $y = 4$ can be written as $0x + y = 4$. When the equation is written in this form, notice that no matter what value we choose for x , y is always 4.

$$\begin{aligned} 0 \cdot x + y &= 4 \\ 0 \cdot (\text{any number}) + y &= 4 \\ 0 + y &= 4 \\ y &= 4 \end{aligned}$$

Fill in a table listing ordered pair solutions of $y = 4$. Choose any three x -values. The y -values are always 4. Plot the ordered pair solutions and graph $y = 4$.

x	y
0	4
1	4
-2	4

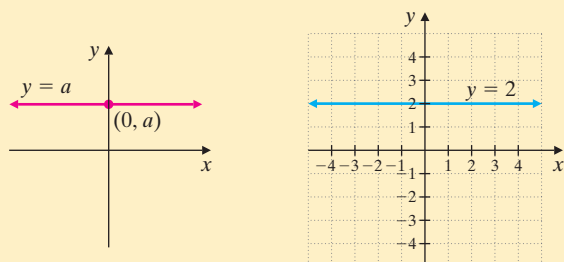


The graph is a horizontal line that crosses the y -axis at 4.

Work Practice 4

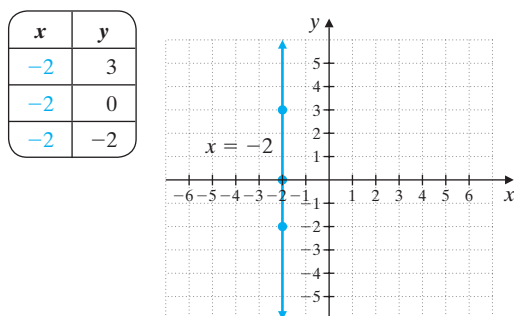
Horizontal Lines

If a is a number, then the graph of $y = a$ is a *horizontal line* that crosses the y -axis at a . For example, the graph of $y = 2$ is a horizontal line that crosses the y -axis at 2.



Example 5 Graph: $x = -2$

Solution: The equation $x = -2$ can be written as $x + 0y = -2$. No matter what y -value we choose, x is always -2 . Fill in a table listing ordered pair solutions of $x = -2$. Choose any three y -values. The x -values are always -2 . Plot the ordered pair solutions and graph $x = -2$.

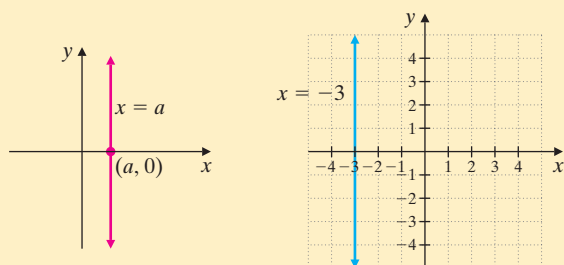


The graph is a vertical line that crosses the x -axis at -2 .

Work Practice 5

Vertical Lines

If a is a number, then the graph of $x = a$ is a *vertical line* that crosses the x -axis at a . For example, the graph of $x = -3$ is a vertical line that crosses the x -axis at -3 .

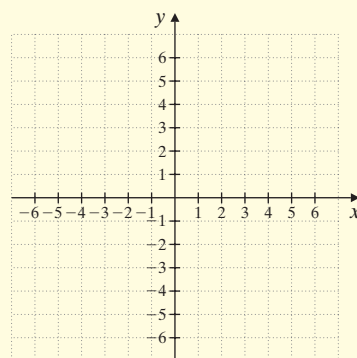


✓Concept Check Determine whether the following equations represent vertical lines, horizontal lines, or neither.

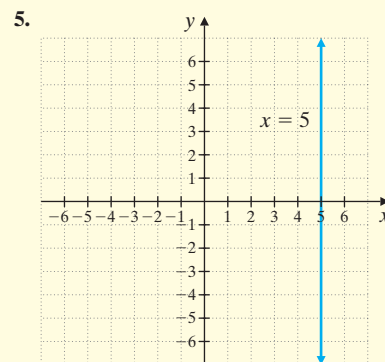
- a. $y = -6$ b. $x + y = 7$ c. $x = \frac{1}{2}$ d. $x = \frac{1}{2}y$

Practice 5

Graph: $x = 5$



Answer



✓Concept Check Answers

- a. horizontal line b. neither
c. vertical line d. neither

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

horizontal line
vertical linear

1. A _____ equation in two variables can be written in the form $ax + by = c$.
2. The graph of the equation $x = 5$ is a _____ line.
3. The graph of the equation $y = -2$ is a _____ line.
4. The graph of the equation $3x - 2y = 6$ is a _____ that is not vertical or horizontal.

Martin-Gay Interactive Videos Watch the section lecture video and answer the following question.



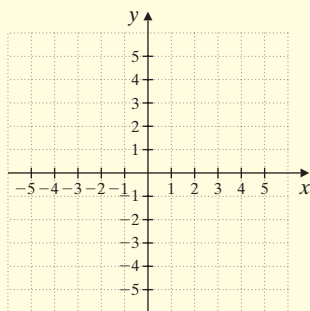
See Video 8.4

- Objective A** 5. Complete this statement based on the lecture before
- Example 1: There are a(n) _____ number of ordered pair solutions to a linear equation in two variables.
6. In Example 1 we're told we only need two points to draw a line, so why is a third point found and graphed?
 7. In Example 2, why is it easier to pick values for x ?

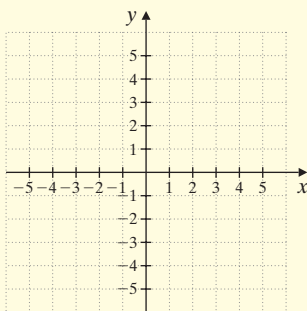
8.4 Exercise Set MyLab Math

Objective A Graph each equation. See Examples 1 through 3.

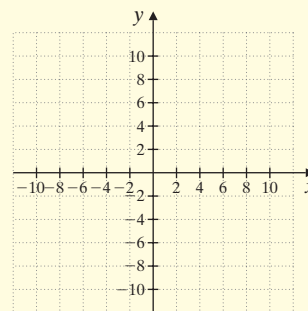
1. $x + y = 4$



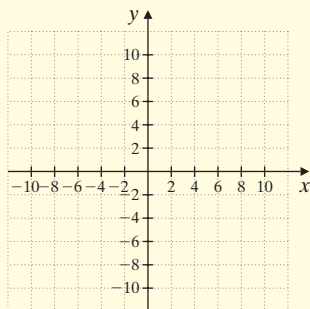
2. $x + y = 2$



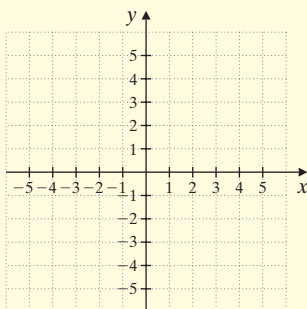
3. $x - y = -6$



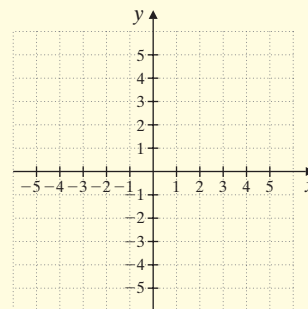
4. $y - x = 6$



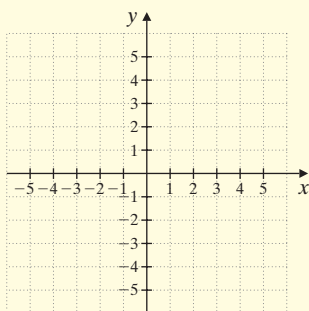
5. $y = 4x$



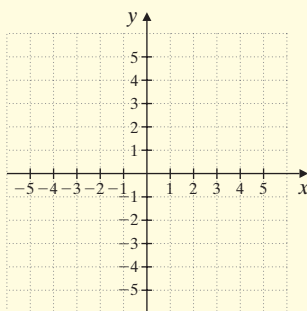
6. $x = 2y$



7. $y = 2x - 1$

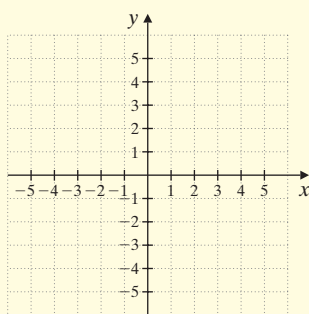


8. $y = 2x + 5$

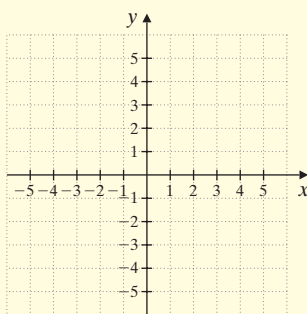


Graph each equation. See Examples 4 and 5.

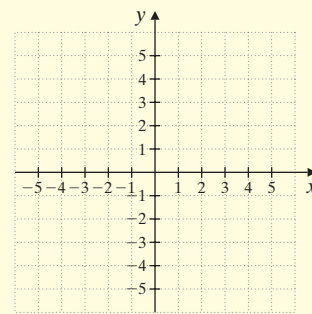
9. $x = -3$



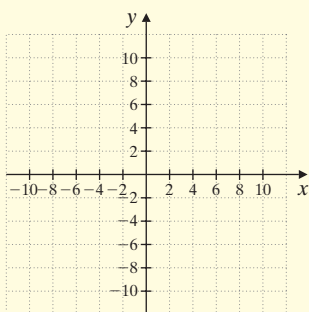
10. $y = 1$



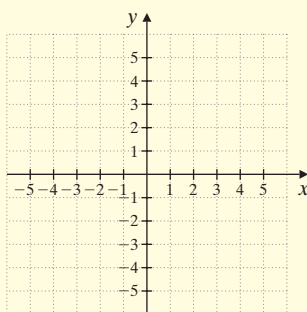
11. $y = -3$



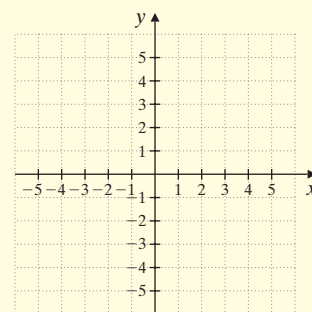
12. $x = -7$



13. $x = 0$

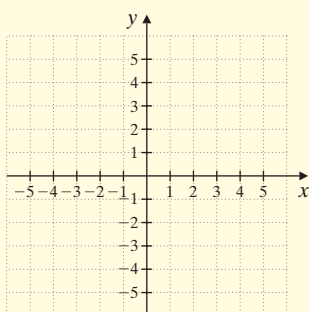


14. $y = 0$

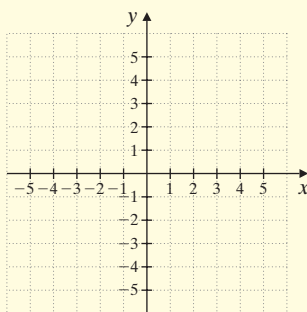


Mixed Practice Graph each equation. See Examples 1 through 5.

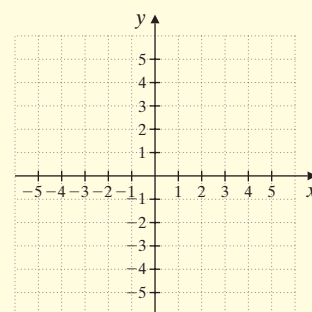
15. $y = -2x$



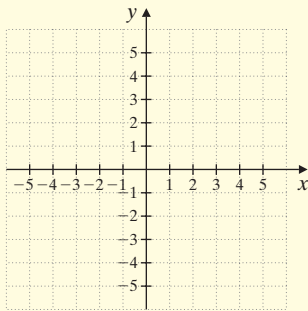
16. $x = y$



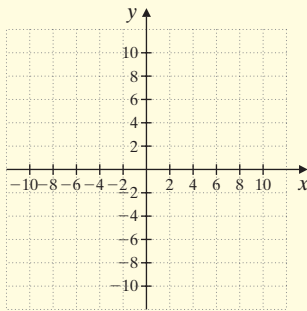
17. $y = -2$



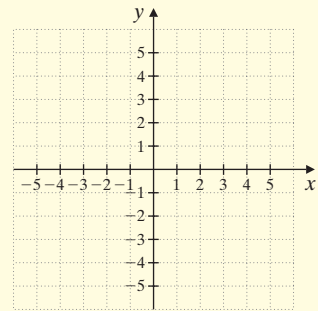
18. $x = 1$



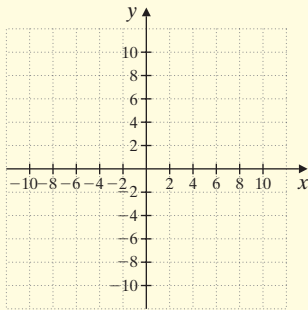
19. $x + 2y = 12$



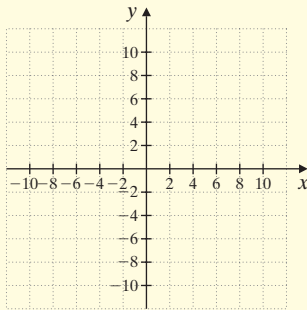
20. $3x - y = 3$



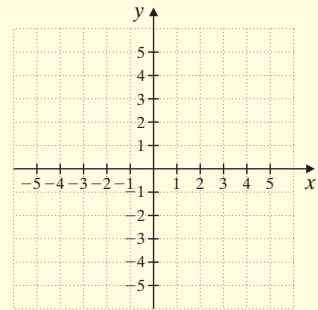
21. $x = 6$



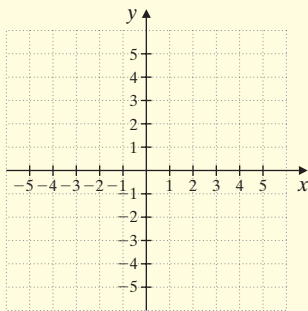
22. $y = 7$



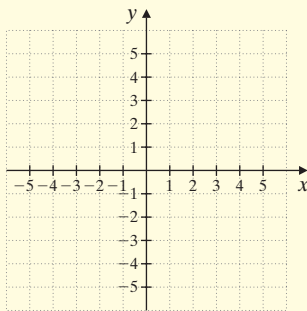
23. $y = x - 3$



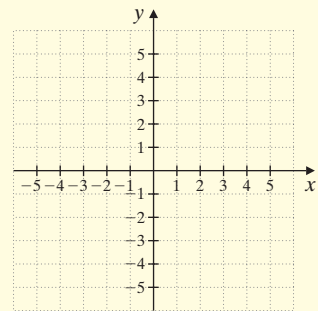
24. $y = x - 1$



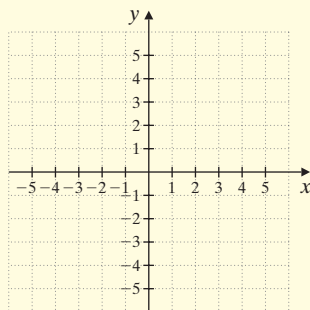
25. $x = y - 4$



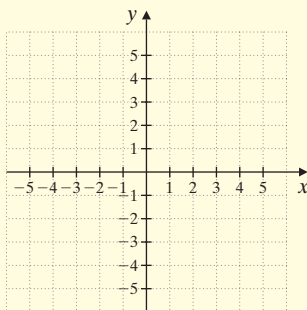
26. $x = 5 - y$



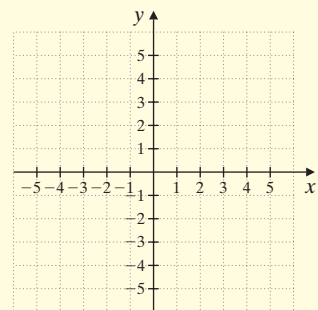
27. $x + 3 = 0$



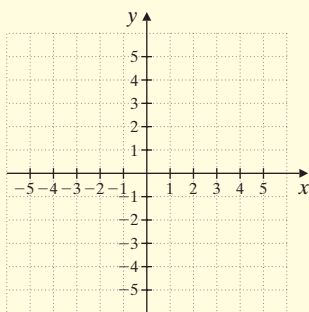
28. $y + 4 = 0$



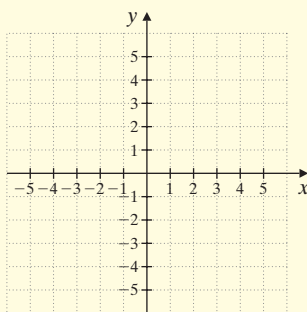
29. $y = -\frac{1}{4}x$



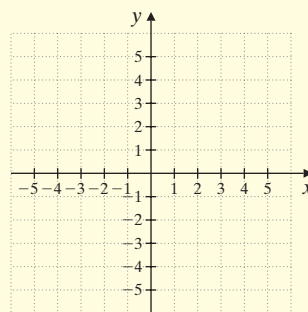
30. $y = \frac{1}{4}x$



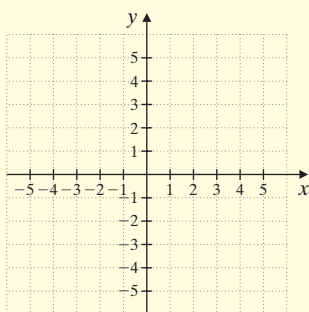
31. $y = \frac{1}{3}x$



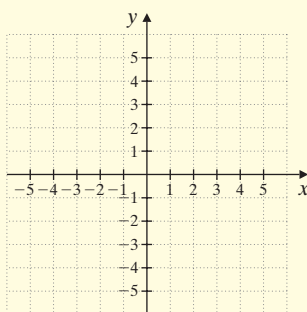
32. $y = -\frac{1}{3}x$



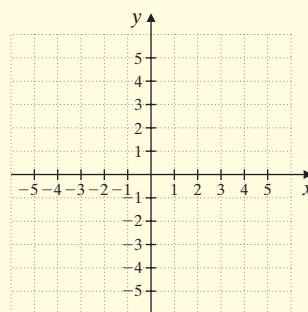
33. $y = 4x + 2$



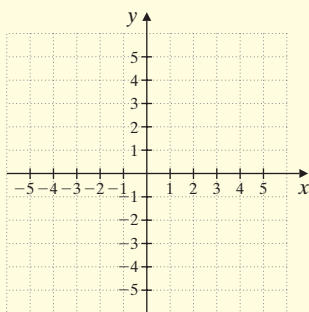
34. $y = -2x + 3$



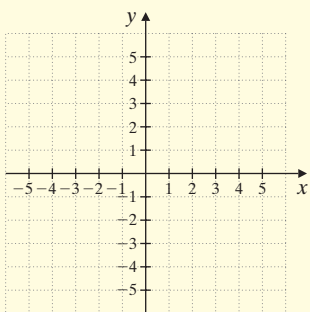
35. $2x + 3y = 6$



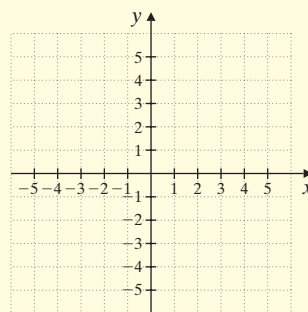
36. $5x - 2y = 10$



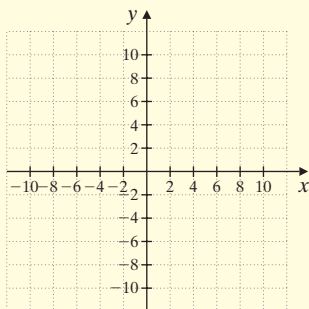
37. $x = -3.5$



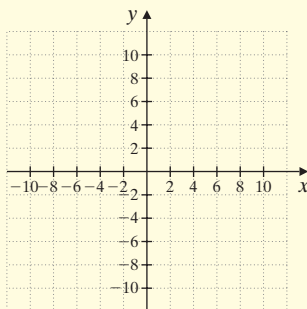
38. $y = 5.5$



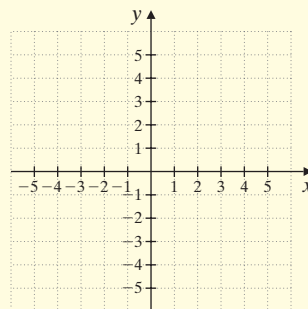
39. $3x - 4y = 24$



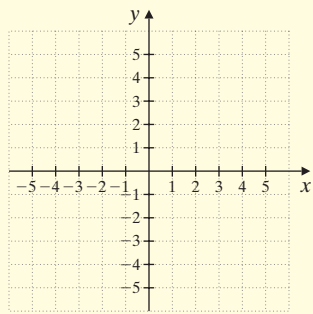
40. $4x + 2y = 16$



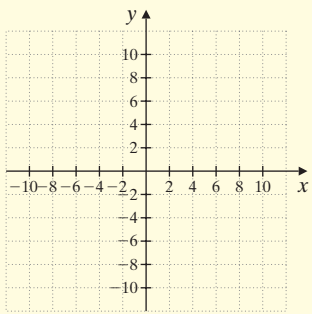
41. $y = \frac{1}{2}$



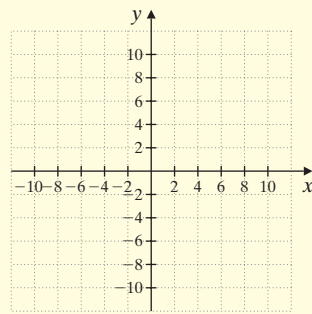
42. $x = -\frac{1}{2}$



43. $y = \frac{1}{3}x - 2$



44. $x = \frac{1}{3}y - 2$



Review

Perform the indicated operations. Give the result as a fraction in simplest form. See Sections 4.3 through 4.5 and 5.5.

45. $(0.5)\left(-\frac{1}{8}\right)$

46. $\frac{3}{4} - \frac{19}{20}$

47. $\frac{3}{4} \div \left(-\frac{19}{20}\right)$

48. $0.75 + \frac{3}{10}$

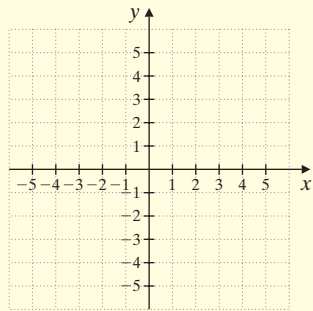
49. $\frac{2}{11} - \frac{x}{11}$

50. $\frac{2}{11}\left(-\frac{x}{11}\right)$

Concept Extensions

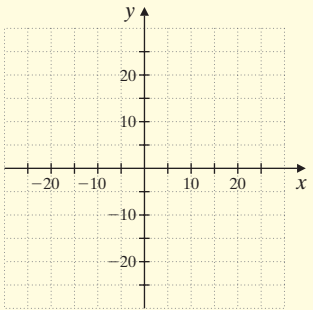
51. Fill in the table and use it to graph $y = |x|$ on the coordinate system.

x	y
-3	
-2	
-1	
0	
1	
2	
3	



52. Graph: $15x - 18y = 270$
To do so, complete the table and find at least 2 additional ordered pair solutions.

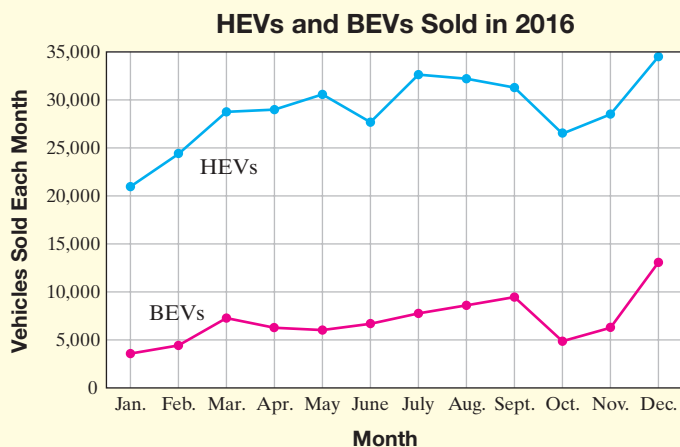
x	y
0	
	0



53. In your own words, explain how to graph a linear equation in two variables.

54. Suppose that a classmate tries to graph a linear equation in two variables and plots three ordered pair solutions. If the three points do not all lie on the same line, what should the student do next?

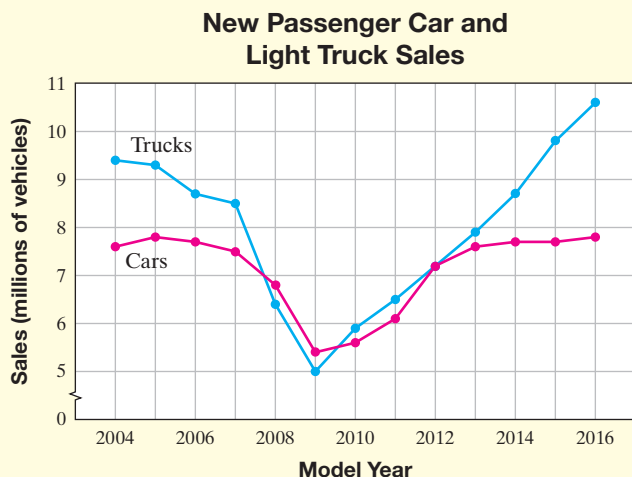
The graph below is called a double line graph (or a double broken-line graph.) Use this graph for Exercises 55 through 60. (See the Chapter Opener.)



Source: Electric Drive Transportation Association

55. Overall, is the number of HEVs sold in the United States increasing or decreasing?
56. Overall, is the number of BEVs sold in the United States increasing or decreasing?
57. Estimate the number of HEVs sold during May 2016.
58. Estimate the number of BEVs sold during May 2016.
59. Place a straight edge along the points for HEVs sold in January 2016 and October 2016. Draw a line using these two points and use this line to estimate the number of HEVs sold in February 2017.
60. Place a straight edge along the points for BEVs sold in January 2016 and October 2016. Draw a line using these two points and use this line to estimate the number of BEVs sold in February 2017.

Use this double line graph to answer Exercises 61 through 66. (Source: Bureau of Transportation Statistics)



Source: Bureau of Transportation Statistics

61. From 2006 to 2009, were passenger car sales increasing or decreasing?
62. In which of the model years shown were truck sales at a maximum?
63. Estimate the number of light trucks sold in the United States in 2013.
64. Estimate the number of passenger cars sold in 2011.
65. Describe any trends you see in this graph.
66. In which two years was the number of passenger cars sold greater than the number of light trucks sold?

8.5 Counting and Introduction to Probability

Objectives

- A** Use a Tree Diagram to Count Outcomes.
- B** Find the Probability of an Event.

Objective A Using a Tree Diagram

In our daily conversations, we often talk about the likelihood or **probability** of a given result occurring. For example:

The *chance* of thundershowers is 70 percent.

What are the *odds* that the New Orleans Saints will go to the Super Bowl?

What is the *probability* that you will finish cleaning your room today?

Each of these chance happenings—thundershowers, the New Orleans Saints playing in the Super Bowl, and finishing cleaning your room today—is called an **experiment**. The possible results of an experiment are called **outcomes**. For example, flipping a coin is an experiment, and the possible outcomes are heads (H) or tails (T).

One way to picture the outcomes of an experiment is to draw a **tree diagram**. Each outcome is shown on a separate branch. For example, the outcomes of flipping a coin are



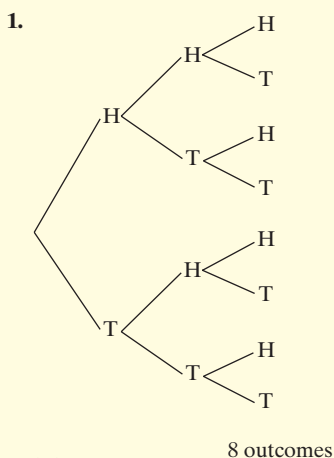
Practice 1

Draw a tree diagram for tossing a coin three times. Then use the diagram to find the number of possible outcomes.

Practice 2

Draw a tree diagram for an experiment consisting of tossing a coin and then rolling a die. Then use the diagram to find the number of possible outcomes. (Answer appears on the next page.)

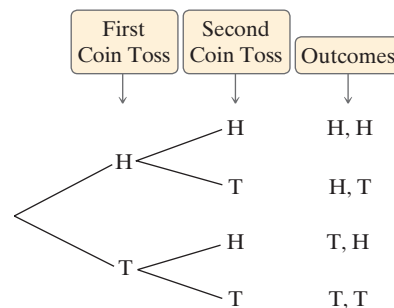
Answer



Example 1

Draw a tree diagram for tossing a coin twice. Then use the diagram to find the number of possible outcomes.

Solution:



There are 4 possible outcomes when tossing a coin twice.

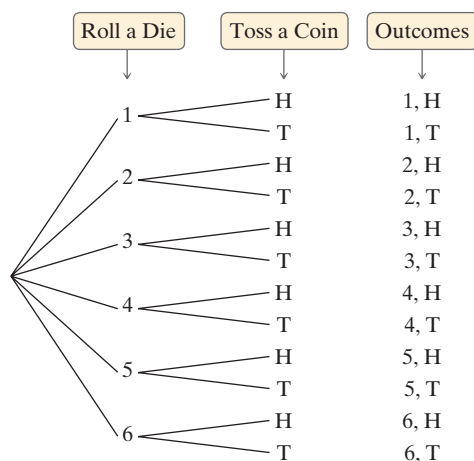
Work Practice 1

Example 2

Draw a tree diagram for an experiment consisting of rolling a die and then tossing a coin. Then use the diagram to find the number of possible outcomes.



Solution: Recall that a die has six sides and that each side represents a number, 1 through 6.



There are 12 possible outcomes for rolling a die and then tossing a coin.

Work Practice 2

Any number of outcomes considered together is called an **event**. For example, when tossing a coin twice, H, H is an event. The event is tossing heads first and tossing heads second. Another event would be tossing tails first and then heads (T, H), and so on.

Objective B Finding the Probability of an Event

As we mentioned earlier, the **probability of an event is a measure of the chance or likelihood of it occurring**. For example, if a coin is tossed, what is the probability that heads occurs? Since one of two equally likely possible outcomes is heads, the probability is $\frac{1}{2}$.

The Probability of an Event

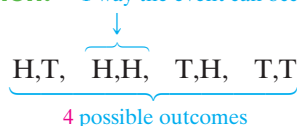
$$\text{probability of an event} = \frac{\text{number of ways that the event can occur}}{\text{number of possible outcomes}}$$

Note from the definition of probability that the probability of an event is always between 0 and 1, inclusive (i.e., including 0 and 1). A probability of 0 means that an event won't occur, and a probability of 1 means that an event is certain to occur.

Example 3

If a coin is tossed twice, find the probability of tossing heads on the first toss and then heads again on the second toss (H, H).

Solution: 1 way the event can occur



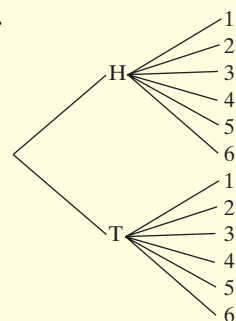
(Continued on next page)

Practice 3

If a coin is tossed three times, find the probability of tossing tails, then heads, then tails (T, H, T).

Answers

2.



12 outcomes

3. $\frac{1}{8}$

$$\text{probability} = \frac{1}{4} \begin{array}{l} \text{Number of ways the event can occur} \\ \text{Number of possible outcomes} \end{array}$$

The probability of tossing heads and then heads is $\frac{1}{4}$.

Work Practice 3

Practice 4

If a die is rolled one time, find the probability of rolling a 2 or a 5.

Example 4

If a die is rolled one time, find the probability of rolling a 3 or a 4.

Solution: Recall that there are 6 possible outcomes when rolling a die.

$$\begin{array}{l} \text{possible outcomes: } 1, 2, 3, 4, 5, 6 \\ \text{probability of a 3 or a 4} = \frac{2}{6} \begin{array}{l} \text{Number of ways the event can occur} \\ \text{Number of possible outcomes} \end{array} \\ = \frac{1}{3} \text{ Simplest form} \end{array}$$

Work Practice 4

✓ **Concept Check** Suppose you have calculated a probability of $\frac{11}{9}$. How do you know that you have made an error in your calculation?

Practice 5

Use the diagram and information in Example 5 and find the probability of choosing a blue marble from the box.

Answers

4. $\frac{1}{3}$ 5. $\frac{1}{2}$

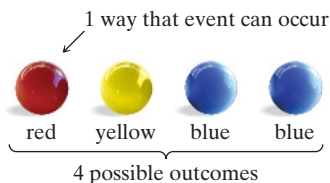
✓ Concept Check Answer

The number of ways an event can occur can't be larger than the number of possible outcomes.

Example 5

Find the probability of choosing a red marble from a box containing 1 red, 1 yellow, and 2 blue marbles.

Solution:



$$\text{probability} = \frac{1}{4}$$

Work Practice 5

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Choices may be used more than once.

0	probability	tree diagram
1	outcome	

- A possible result of an experiment is called a(n) _____.
- A(n) _____ shows each outcome of an experiment as a separate branch.
- The _____ of an event is a measure of the likelihood of it occurring.

4. _____ is calculated by the number of ways that an event can occur divided by the number of possible outcomes.
5. A probability of _____ means that an event won't occur.
6. A probability of _____ means that an event is certain to occur.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 8.5 

- Objective A** 7. In  Example 1, how was the possible number of outcomes for the experiment determined from the tree diagram? 
- Objective B** 8. In  Example 2, what is the probability of getting a 7? Explain this result. 

8.5 Exercise Set MyLab Math

Objective A Draw a tree diagram for each experiment. Then use the diagram to find the number of possible outcomes. See Examples 1 and 2.

1. Choosing a letter in the word MATH and then a number (1, 2, or 3)
2. Choosing a number (1 or 2) and then a vowel (a, e, i, o, or u)



Spinner A



Spinner B

3. Spinning Spinner A once

4. Spinning Spinner B once

5. Spinning Spinner B twice

6. Spinning Spinner A twice

7. Spinning Spinner A and then Spinner B

8. Spinning Spinner B and then Spinner A

9. Tossing a coin and then spinning Spinner B

10. Spinning Spinner A and then tossing a coin

Objective B *If a single die is tossed once, find the probability of each event. See Examples 3 through 5.*

▶ 11. A 5

12. A 9

13. A 1 or a 6

14. A 2 or a 3

▶ 15. An even number

16. An odd number

17. A number greater than 2

18. A number less than 6

Suppose the spinner shown is spun once. Find the probability of each event. See Examples 3 through 5.



▶ 19. The result of the spin is 2.

20. The result of the spin is 3.

21. The result of the spin is 1, 2, or 3.

22. The result of the spin is not 3.

23. The result of the spin is an odd number.

24. The result of the spin is an even number.

If a single choice is made from the bag of marbles shown, find the probability of each event. See Examples 3 through 5.



25. A red marble is chosen.

26. A blue marble is chosen.

27. A yellow marble is chosen.

28. A green marble is chosen.

29. A green or red marble is chosen.

30. A blue or yellow marble is chosen.

A new drug is being tested that is supposed to lower blood pressure. This drug was given to 200 people, and the results are shown below.

Lower Blood Pressure	Higher Blood Pressure	Blood Pressure Not Changed
152	38	10

31. If a person is testing this drug, what is the probability that his or her blood pressure will be higher?

32. If a person is testing this drug, what is the probability that his or her blood pressure will be lower?

33. If a person is testing this drug, what is the probability that his or her blood pressure will not change?

✎ 34. What is the sum of the answers to Exercises 31, 32, and 33? In your own words, explain why.

Review

Perform each indicated operation. See Sections 4.3 and 4.5.

35. $\frac{1}{2} + \frac{1}{3}$

36. $\frac{7}{10} - \frac{2}{5}$

37. $\frac{1}{2} \cdot \frac{1}{3}$

38. $\frac{7}{10} \div \frac{2}{5}$

39. $5 \div \frac{3}{4}$

40. $\frac{3}{5} \cdot 10$

Concept Extensions

Recall that a deck of cards contains 52 cards. These cards consist of four suits (hearts, spades, clubs, and diamonds) of each of the following: 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king, and ace. If a card is chosen from a deck of cards, find the probability of each event.

41. The king of hearts

42. The 10 of spades

43. A king

44. A 10



45. A heart

46. A club

47. A card in black ink

48. A queen or ace

Two dice are tossed. Find the probability of each sum of the dice. (Hint: Draw a tree diagram of the possibilities of two tosses of a die, and then find the sum of the numbers on each branch.)

49. A sum of 6

50. A sum of 10

51. A sum of 13

52. A sum of 2

Solve. See the Concept Check in this section.

 53. In your own words, explain why the probability of an event cannot be greater than 1.

 54. In your own words, explain when the probability of an event is 0.

Chapter 8 Group Activity

Scatter Diagrams

Sections 8.1 and 8.3

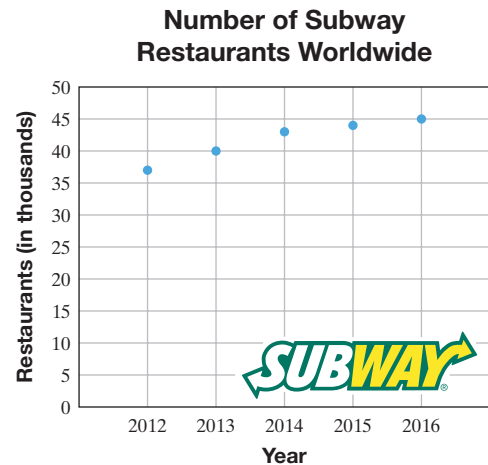
In Section 8.1, we learned about presenting data visually in a graph, such as a pictograph, bar graph, or line graph. Now we learn about another type of graph, based on ordered pairs, that can be used to present data.

Data that can be represented as an ordered pair are called paired data. Many types of data collected from the real world are paired data. For instance, the total amount of rainfall a location receives each year can be written as an ordered pair of the form (year, total rainfall in inches) and is paired data. The graph of paired data as points in the rectangular coordinate system is called a **scatter**

diagram, or scatter plot. Scatter diagrams can be used to look for patterns and trends in paired data.

For example, the data shown in the table are paired data that can be written as a set of ordered pairs of the form (year, number of restaurants in thousands), such as (2012, 37) and (2015, 44). A scatter diagram of the paired data is shown below. Notice that the horizontal axis is labeled “Year” to describe the x -coordinates in the ordered pairs. The vertical axis is labeled “Restaurants (in thousands)” to describe the y -coordinates in the ordered pairs.

Number of Subway Restaurants	
Year	Number of Restaurants (in thousands)
2012	37
2013	40
2014	43
2015	44
2016	45
(Source: Subway)	



The table to the right gives the number of Wal-Mart Sam's Clubs each year. Use the table to answer the following questions.

- Write this paired data as a set of ordered pairs of the form (year, number in hundreds).
- Create a scatter diagram of the paired data.

Number of Wal-Mart Sam's Clubs	
Year	Number of Sam's Clubs
2012	611
2013	620
2014	632
2015	647
2016	655
(Source: Wal-Mart Corporation)	

- What trend in the paired data does the scatter diagram show?
- Find or collect your own paired data and present them in a scatter diagram. What does the graph show?

Chapter 8 Vocabulary Check

Fill in each blank with one of the words or phrases listed below.

- outcomes
- quadrants
- line
- y
- class frequency
- probability
- pictograph
- plotting
- origin
- experiment
- linear
- class interval
- histogram
- bar
- x
- circle
- tree diagram

1. A(n) _____ graph presents data using vertical or horizontal bars.
2. The possible results of an experiment are the _____.
3. A(n) _____ is a graph in which pictures or symbols are used to visually present data.
4. A(n) _____ graph displays information with a line that connects data points.
5. In the ordered pair (a, b) the _____-value is a and the _____-value is b .
6. A(n) _____ is one way to picture and count outcomes.
7. A(n) _____ is an activity being considered, such as tossing a coin or rolling a die.
8. In a(n) _____ graph, each section (shaped like a piece of pie) shows a category and the relative size of the category.
9. The _____ of an event is $\frac{\text{number of ways that the event can occur}}{\text{number of possible outcomes}}$.
10. A(n) _____ is a special bar graph in which the width of each bar represents a(n) _____ and the height of each bar represents the _____.
11. The point of intersection of the x -axis and the y -axis in the rectangular coordinate system is called the _____.
12. The axes divide the plane into four regions called _____.
13. The process of locating a point on the rectangular coordinate system is called _____ the point.
14. A(n) _____ equation in two variables can be written in the form $ax + by = c$.

Helpful Hint

▶ Are you preparing for your test? To help, don't forget to take these:

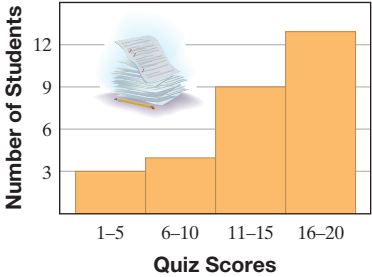
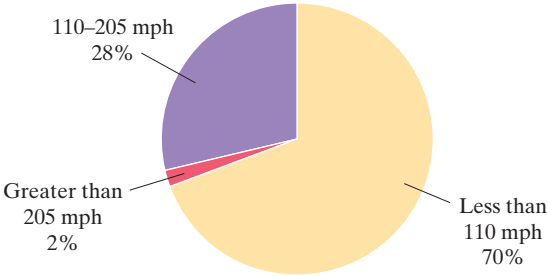
- Chapter 8 Getting Ready for the Test on page 607
- Chapter 8 Test on page 609

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

8

Chapter Highlights

Definitions and Concepts	Examples														
Section 8.1 Pictographs, Bar Graphs, Histograms, Line Graphs, and Introduction to Statistics															
A pictograph is a graph in which pictures or symbols are used to visually present data. A line graph displays information with a line that connects data points. A bar graph presents data using vertical or horizontal bars. The bar graph on the right shows the number of acres of corn harvested in a recent year for selected states.	Corn Production, Selected States <table border="1"><caption>Corn Production, Selected States</caption><thead><tr><th>State</th><th>Acres Harvested (in millions)</th></tr></thead><tbody><tr><td>IA</td><td>13</td></tr><tr><td>KS</td><td>4</td></tr><tr><td>TX</td><td>2</td></tr><tr><td>IL</td><td>12</td></tr><tr><td>MN</td><td>7</td></tr><tr><td>NE</td><td>8</td></tr></tbody></table> Source: U.S. Department of Agriculture	State	Acres Harvested (in millions)	IA	13	KS	4	TX	2	IL	12	MN	7	NE	8
State	Acres Harvested (in millions)														
IA	13														
KS	4														
TX	2														
IL	12														
MN	7														
NE	8														

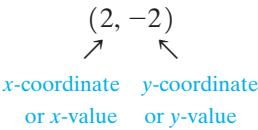
Definitions and Concepts	Examples
Section 8.1 Pictographs, Bar Graphs, Histograms, Line Graphs, and Introduction to Statistics <i>(continued)</i> A histogram is a special bar graph in which the width of each bar represents a class interval and the height of each bar represents the class frequency. The histogram on the right shows student quiz scores. Range The range of a data set is the difference between the largest data value and the smallest data value. $\text{range} = \text{largest data value} - \text{smallest data value}$	1. Approximately how many acres of corn were harvested in Iowa? 12,800,000 acres 2. About how many more acres of corn were harvested in Illinois than Nebraska? 12 million – 8.5 million 3.5 million or 3,500,000 acres  1. How many students received a score of 6–10? 4 students 2. How many students received a score of 11–20? $9 + 13 = 22$ students The range of data set 5, 7, 9, 11, 21, 21 is: $\text{range} = 21 - 5 = 16$
Section 8.2 Circle Graphs	
In a circle graph, each section (shaped like a piece of pie) shows a category and the relative size of the category. The circle graph on the right classifies tornadoes by wind speed.	Tornado Wind Speeds  Source: National Oceanic and Atmospheric Administration 1. What percent of tornadoes have wind speeds of 110 mph or greater? $28\% + 2\% = 30\%$ 2. If there were 939 tornadoes in the United States in 2012, how many of these might we expect to have had wind speeds less than 110 mph? Find 70% of 939. $70\%(939) = 0.70(939) = 657.3 \approx 657$ Around 657 tornadoes would be expected to have had wind speeds of less than 110 mph.

Definitions and Concepts

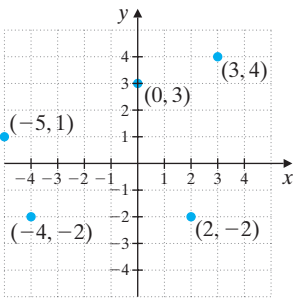
Examples

Section 8.3 The Rectangular Coordinate System and Paired Data

The **rectangular coordinate system** consists of two number lines intersecting at the point 0 on each number line. The horizontal number line is called the **x-axis** and the vertical number line is called the **y-axis**. Every point in the rectangular coordinate system corresponds to an **ordered pair of numbers** such as



An ordered pair is a **solution** of an equation if the equation is a true statement when the variables are replaced by the coordinates of the ordered pair.



Is $(2, -1)$ a solution of $5x - y = 11$?

$5x - y = 11$

$5(2) - (-1) \stackrel{?}{=} 11$ Replace x with 2 and y with -1 .

$10 + 1 \stackrel{?}{=} 11$ Multiply.

$11 = 11$ True

Yes, $(2, -1)$ is a solution of $5x - y = 11$.

Section 8.4 Graphing Linear Equations in Two Variables

Linear Equations in Two Variables
A linear equation in two variables is an equation that can be written in the form
 $ax + by = c$
where a , b , and c are numbers, and a and b are not both 0. **To graph a linear equation in two variables**, find and plot three ordered pair solutions and draw a line through the plotted points.

The graph of $y = a$ is a **horizontal line** that crosses the y -axis at a .
The graph of $x = a$ is a **vertical line** that crosses the x -axis at a .

Graph: $y = 4x$

Let $x = 1$.

Let $x = -1$.

Let $x = 0$.

$y = 4(1)$

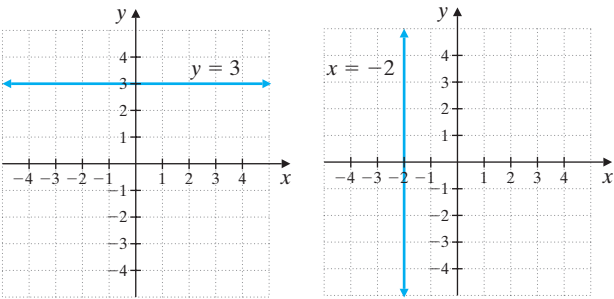
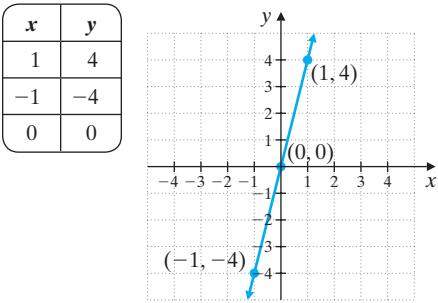
$y = 4(-1)$

$y = 4(0)$

$y = 4$

$y = -4$

$y = 0$

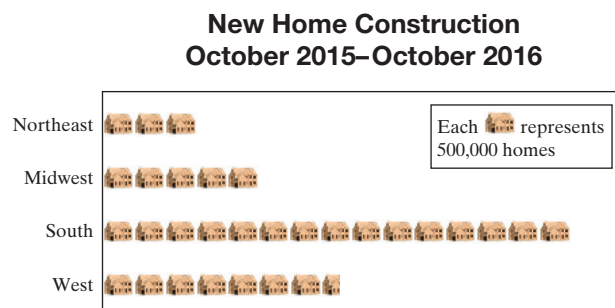


Definitions and Concepts	Examples
Section 8.5 Counting and Introduction to Probability	
An experiment is an activity being considered, such as tossing a coin or rolling a die. The possible results of an experiment are the outcomes. A tree diagram is one way to picture and count outcomes.	Draw a tree diagram for tossing a coin and then choosing a number from 1 to 4.
Any number of outcomes considered together is called an event. The probability of an event is a measure of the chance or likelihood of it occurring. $\text{probability of an event} = \frac{\text{number of ways that the event can occur}}{\text{number of possible outcomes}}$	Find the probability of tossing a coin twice and tails occurring each time. 1 way the event can occur $(H, H), (H, T), (T, H), (T, T)$ 4 possible outcomes $\text{probability} = \frac{1}{4}$

Chapter 8

Review

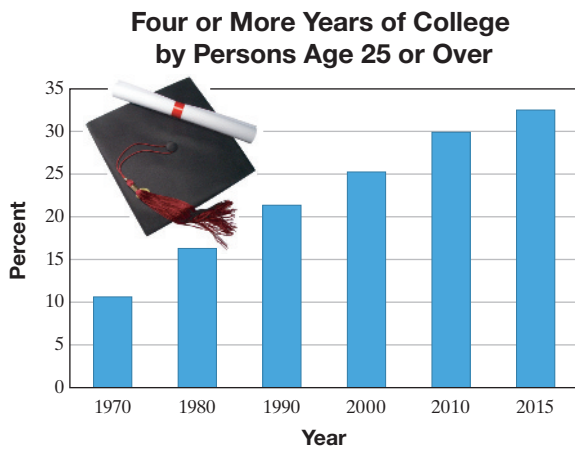
(8.1) The following pictograph shows the number of new homes constructed from October 2015 to October 2016, by region. Use this graph to answer Exercises 1 through 6.



Source: U.S. Census Bureau

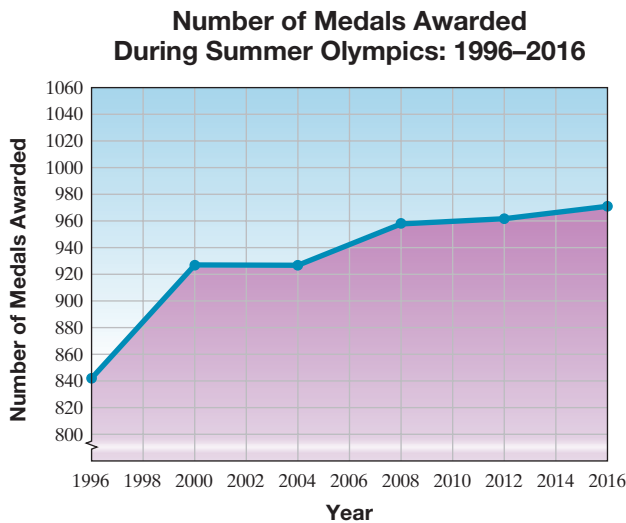
- How many new homes were constructed in the Midwest during the given year?
- How many new homes were constructed in the South during the given year?
- Which region had the most new homes constructed?
- Which region had the fewest new homes constructed?
- Which region(s) had 3,000,000 or more new homes constructed?
- Which region(s) had fewer than 3,000,000 new homes constructed?

The following bar graph shows the percent of persons age 25 or over who completed four or more years of college. Use this graph to answer Exercises 7 through 10.



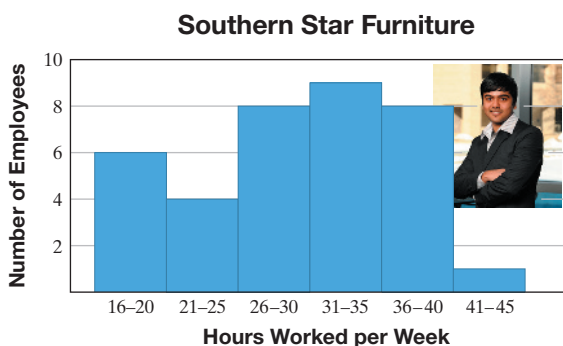
Source: U.S. Census Bureau

The following line graph shows the total number of Olympic medals awarded during the Summer Olympics since 1996. Use this graph to answer Exercises 11 through 16.



Source: International Olympic Committee

The following histogram shows the hours worked per week by the employees of Southern Star Furniture. Use this histogram to answer Exercises 17 through 20.



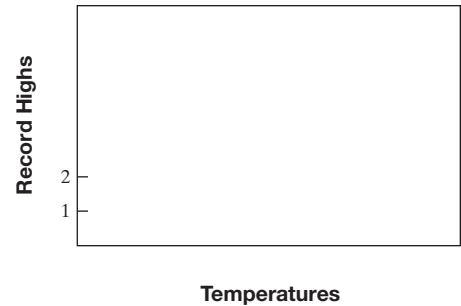
7. Approximate the percent of persons who had completed four or more years of college by 2010.
8. What year shown had the greatest percent of persons completing four or more years of college?
9. What years shown had 20% or more of persons completing four or more years of college?
10. Describe any patterns you notice in this graph.
11. Approximate the number of medals awarded during the Summer Olympics of 2012.
12. Approximate the number of medals awarded during the Summer Olympics of 2000.
13. Approximate the number of medals awarded during the Summer Olympics of 2016.
14. Approximate the number of medals awarded during the Summer Olympics of 1996.
15. How many more medals were awarded at the Summer Olympics of 2008 than at the Summer Olympics of 2004?
16. The number of medals awarded at the Summer Olympics of 2016 is the greatest number of medals awarded at previous Summer Olympics. Why do you think this is so? Give your explanation in complete sentences.
17. How many employees work 41–45 hours per week?
18. How many employees work 21–25 hours per week?
19. How many employees work 30 hours or less per week?
20. How many employees work 36 hours or more per week?

Following is a list of monthly record high temperatures for New Orleans, Louisiana. Use this list to complete the frequency distribution table below.

83	96	101	92
85	100	92	102
89	101	87	84

	Class Intervals (Temperatures)	Tally	Class Frequency (Number of Months)
21.	80°-89°		
22.	90°-99°		
23.	100°-109°		

24. Use the table from Exercises 21–23 to draw a histogram.



Find the range for each data set.

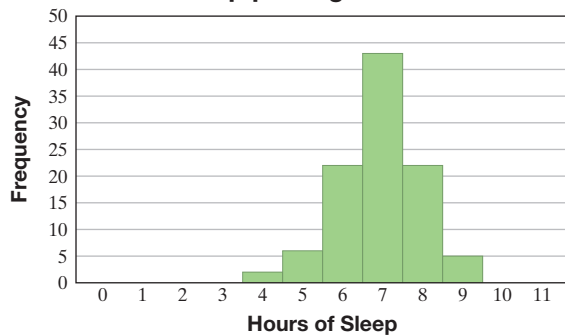
25. 4, 4, 4, 3, 3, 5, 6

26. 1, 1, 1, 8, 8

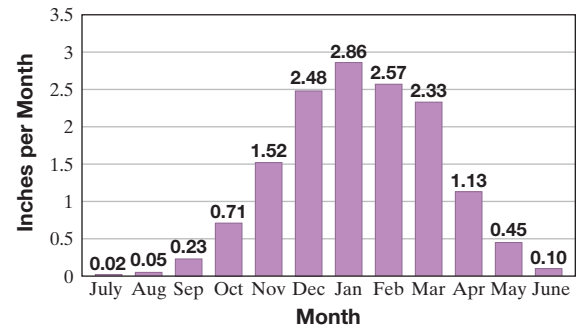
27. 70, 75, 95, 60, 88

28. 80, 87, 97, 99, 85

29. Hours of Sleep per Night for 100 Students



30. Monthly Average Precipitation for a City in California



Use each frequency distribution table to find the **a.** mean, **b.** median, and **c.** mode. If needed, round the mean to 1 decimal place.

31.

Data Item	Frequency
60	2
61	10
62	5
63	11
64	3

32.

Data Item	Frequency
60	5
61	7
62	8
63	10
64	15

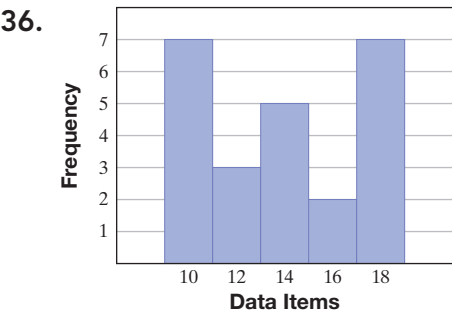
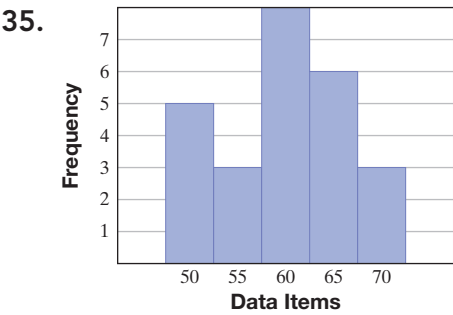
33.

Data Item	Frequency
11	5
12	5
13	3
14	1
16	3
17	1

34.

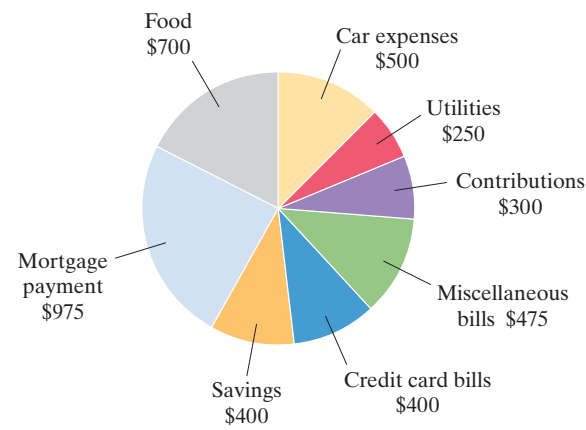
Data Item	Frequency
15	3
16	2
17	1
18	6
19	4
20	6

Use each graph of data items to find the **a.** mean, **b.** median, and **c.** mode. If needed round the mean to 1 decimal place.



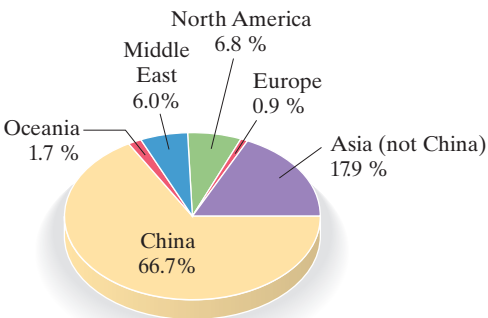
(8.2) The following circle graph shows a family’s \$4000 monthly budget. Use this graph to answer Exercises 37 through 42. Write all ratios as fractions in simplest form.

- 37. What is the largest budget item?
- 38. What is the smallest budget item?
- 39. How much money is budgeted for the mortgage payment and utilities?
- 40. How much money is budgeted for savings and contributions?
- 41. Find the ratio of the mortgage payment to the total monthly budget.
- 42. Find the ratio of food to the total monthly budget.



In 2016, there were approximately 117 buildings 200 meters or taller completed in the world. The following circle graph shows the percent of these buildings by region. Use this graph to answer Exercises 43 through 46. Round each answer to the nearest whole.

**Percent of Tall Buildings Completed in 2016
200 Meters or Taller by Region**

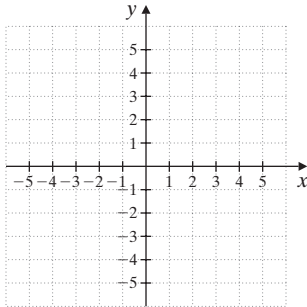


Source: Council on Tall Buildings and Urban Habitats

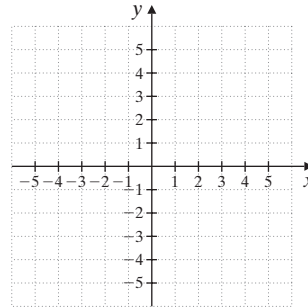
- 43. How many completed tall buildings were located in China?
- 44. How many completed tall buildings were located in the rest of Asia?
- 45. How many completed tall buildings were located in Oceania?
- 46. How many completed tall buildings were located in the Middle East?

(8.3) Complete and graph the ordered pair solutions of each given equation.

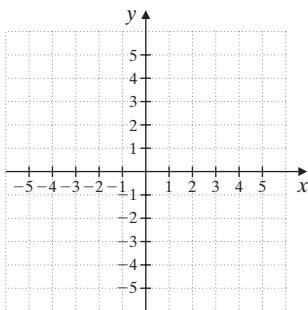
47. $x = -6y$; $(0, \quad)$, $(\quad, -1)$, $(-6, \quad)$



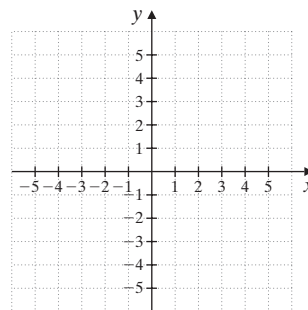
48. $y = 3x - 2$; $(0, \quad)$, $(-1, \quad)$, $(2, \quad)$



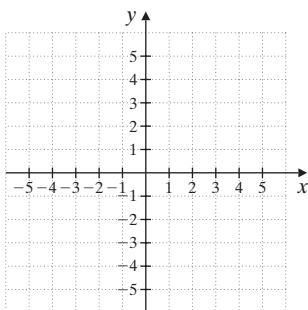
49. $x + y = -4$; $(-1, \quad)$, $(\quad, 0)$, $(-5, \quad)$



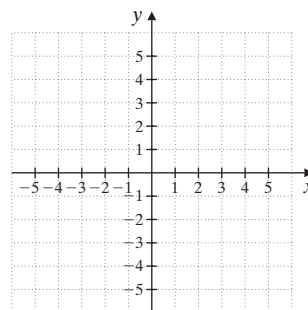
50. $x - y = 3$; $(4, \quad)$, $(0, \quad)$, $(\quad, 3)$



51. $y = 3x$; $(1, \quad)$, $(-2, \quad)$, $(\quad, 0)$

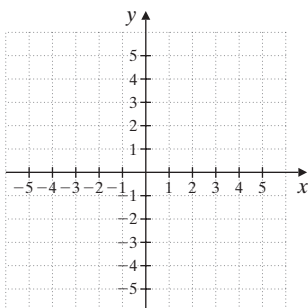


52. $x = y + 6$; $(1, \quad)$, $(6, \quad)$, $(\quad, -4)$

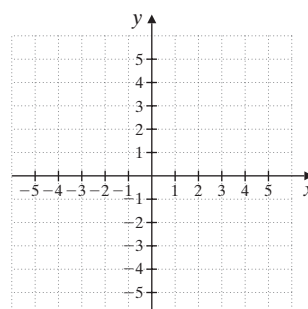


(8.4) Graph each linear equation.

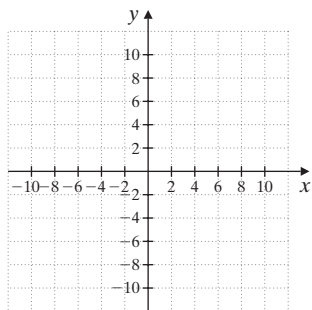
53. $x = -5$



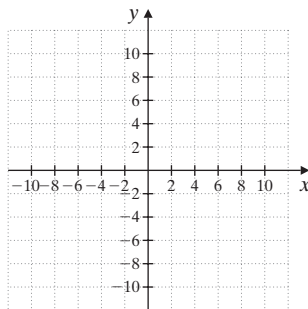
54. $y = \frac{3}{2}$



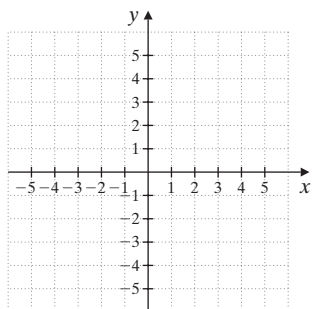
55. $x + y = 11$



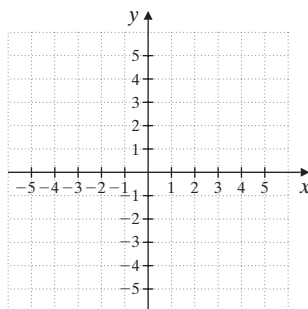
56. $x - y = 11$



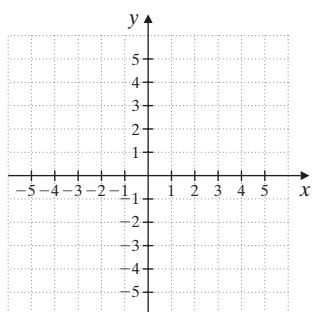
57. $y = 4x - 2$



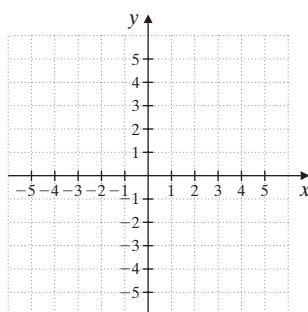
58. $y = 5x$



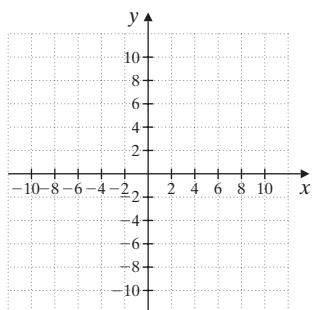
59. $x = -2y$



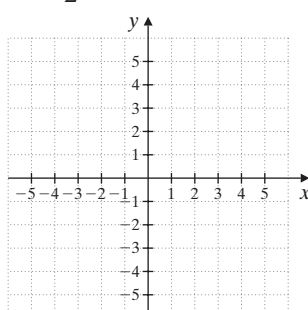
60. $x + y = -1$



61. $2x - 3y = 12$



62. $x = \frac{1}{2}y$



(8.5) Draw a tree diagram for each experiment. Then use the diagram to determine the number of outcomes.



Spinner 1



Spinner 2

63. Tossing a coin and then spinning Spinner 1

64. Spinning Spinner 2 and then tossing a coin

65. Spinning Spinner 1 twice

66. Spinning Spinner 2 twice

67. Spinning Spinner 1 and then Spinner 2

Find the probability of each event.



Die

68. Rolling a 4 on a die

69. Rolling a 3 on a die

70. Spinning a 4 on Spinner 1

71. Spinning a 3 on Spinner 1

72. Spinning either a 1, 3, or 5 on Spinner 1

73. Spinning either a 2 or a 4 on Spinner 1

Mixed Review

Given a bag containing 2 red marbles, 2 blue marbles, 3 yellow marbles, and 1 green marble, find the following:

74. The probability of choosing a blue marble from the bag

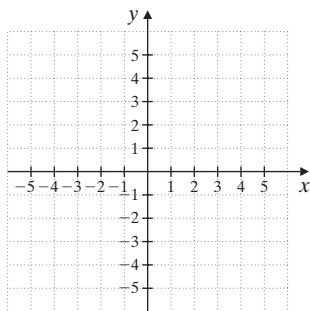
75. The probability of choosing a yellow marble from the bag

76. The probability of choosing a red marble from the bag

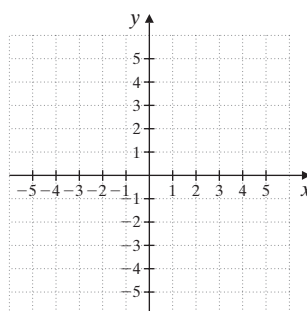
77. The probability of choosing a green marble from the bag

Graph each equation.

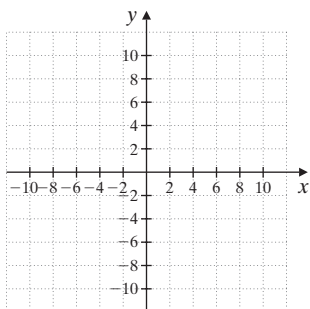
78. $x = -4$



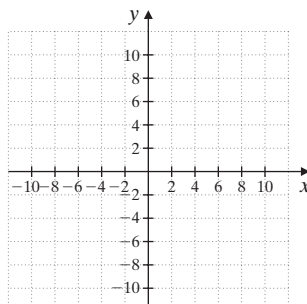
79. $y = 3$



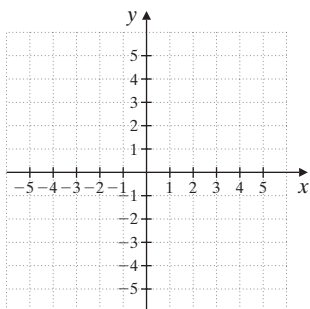
80. $x - 2y = 8$



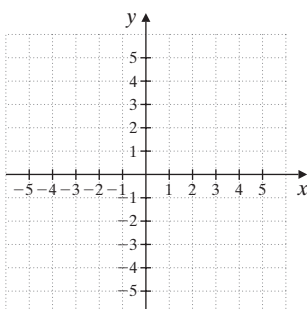
81. $2x + y = 6$



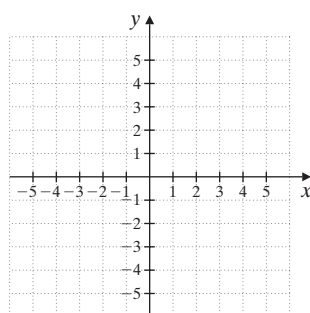
82. $x = y + 3$



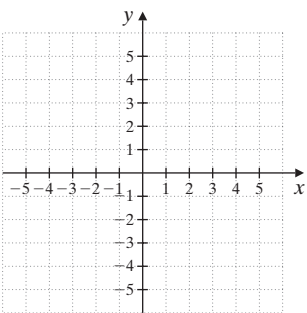
83. $x + y = -4$



84. $y = \frac{3}{4}x$



85. $y = -\frac{3}{4}x$



For each graph, calculate parts **a.**–**d.** Round the mean to two decimal places.

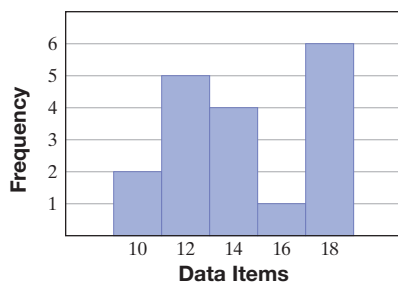
a. mean

b. median

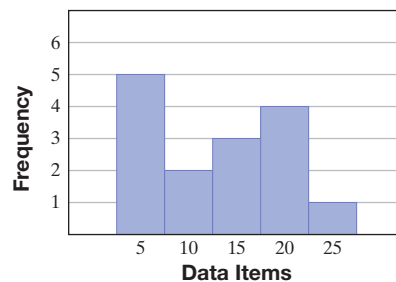
c. mode

d. range

86.

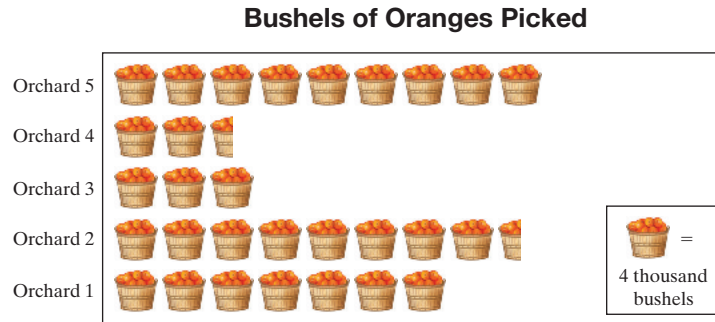


87.



MULTIPLE CHOICE Exercises 1 through 18 are **Multiple Choice**. Choose the correct letter.

For Exercises 1 through 4, use the graph below.



- ▶ 1. How many bushels of oranges did Orchard 1 produce?

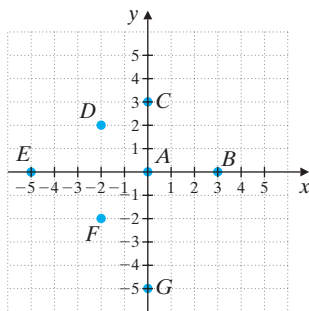
A. 7 bushels
B. 28 bushels
C. 28,000 bushels
- ▶ 2. Which orchard above produced the most bushels?

A. Orchard 1
B. Orchard 2
C. Orchard 5
D. Orchards 2 and 5 produced the same.
- ▶ 3. How many bushels of oranges did Orchard 4 produce?

A. 3 bushels
B. $2\frac{1}{2}$ bushels
C. 10 bushels
D. 10 thousand bushels
- ▶ 4. How many more bushels did Orchard 2 produce than Orchard 4?

A. 24 bushels
B. $6\frac{1}{2}$ bushels
C. 24,000 bushels
D. 6,000 bushels

For Exercises 5 through 10, match each ordered pair with the letter of its graph. Not all letters will be used.



- | | |
|-----------------|--------------|
| ▶ 5. $(-2, -2)$ | 6. $(0, 3)$ |
| ▶ 7. $(3, 0)$ | 8. $(-5, 0)$ |
| ▶ 9. $(0, -5)$ | 10. $(0, 0)$ |

For Exercises 11 through 15 choose the correct letter.

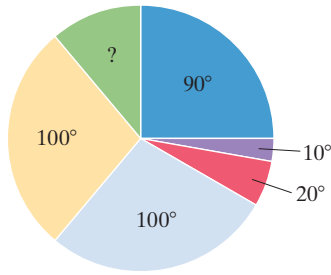
- ▶ 11. Choose the equation whose graph is a vertical line.

A. $y = 2$
B. $x = 5$

C. $x + y = 0$
D. $x - y = 0$
- ▶ 12. Choose the ordered pair that is *not* a solution of $x - y = 10$.

A. $(0, 10)$
B. $(12, 2)$
C. $(10, 0)$
D. $(5, -5)$

- 13. Choose the degrees in the unknown sector of this circle graph.



- A. 60° B. 50° C. 40° D. 80°
- 14. The five colored marbles are placed in a bag. What is the probability of choosing a red marble?
- A. $\frac{1}{5}$ B. $\frac{2}{5}$ C. $\frac{3}{5}$ D. $\frac{4}{5}$
- 15. For the marbles in Exercise 14, What is the probability of choosing a green marble?
- A. 0 B. 1 C. $\frac{2}{5}$ D. $\frac{3}{5}$



For Exercises 16 through 18, use the data sets and choices below to answer.

- A. data set: 10, 10, 10, 10, 10 B. data set: 6, 8, 10, 12, 14
 C. data set: 8, 9, 10, 11, 12 D. they are the same
- 16. Which data set has the greatest range?
- 17. Which data set has the greatest median?
- 18. Which data set has the greatest mean?

MATCHING Choose **two** data sets (two letters) from the column on the right that make each exercise statement true. Data sets may be used more than once or not at all.

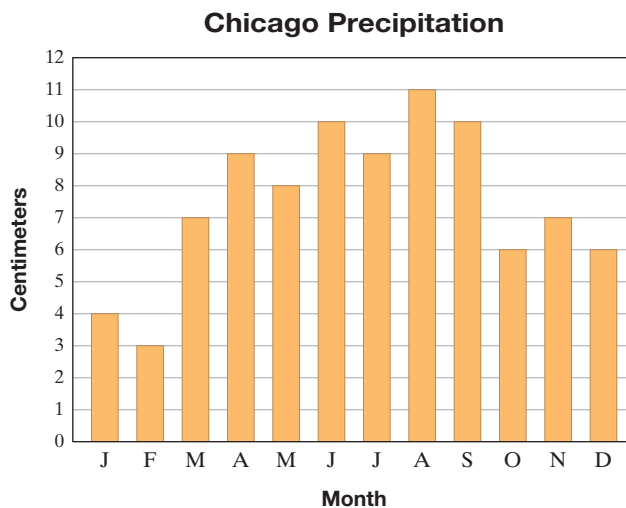
- 19. equal means A. 10, 20, 30, 30, 40, 50
 ► 20. equal number of modes B. 9, 11, 11, 30, 48
 ► 21. equal ranges C. 20, 25, 30, 35, 40
 ► 22. equal medians D. 50, 55, 60, 65, 70

The following pictograph shows the money collected each week from a wrapping paper fundraiser. Use this graph to answer Exercises 1 through 3.



- ▶ 1. How much money was collected during the second week?
- ▶ 2. During which week was the most money collected? How much money was collected during that week?
- ▶ 3. What was the total amount of money collected for the fundraiser?

The bar graph shows the normal monthly precipitation in centimeters for Chicago, Illinois. Use this graph to answer Exercises 4 through 6.



Source: U.S. National Oceanic and Atmospheric Administration, *Climatology of the United States*, No. 81

- ▶ 4. During which month(s) does Chicago normally have more than 9 centimeters of precipitation?
- ▶ 5. During which month does Chicago normally have the least amount of precipitation? How much precipitation occurs during that month?

Answers

1. _____

2. _____

3. _____

4. _____

5. _____

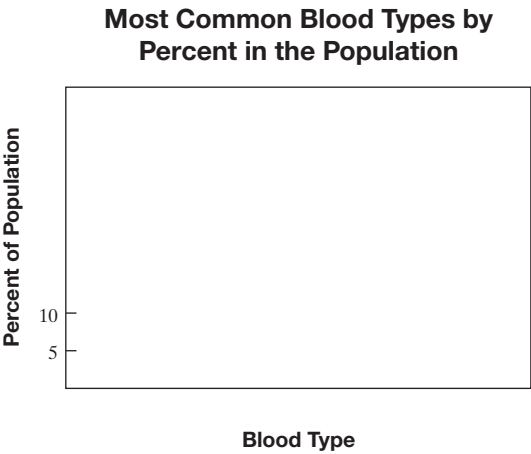
6. _____

▶ 6. During which month(s) does 7 centimeters of precipitation normally occur?

7. _____

▶ 7. Use the information in the table to draw a bar graph. Clearly label each bar.

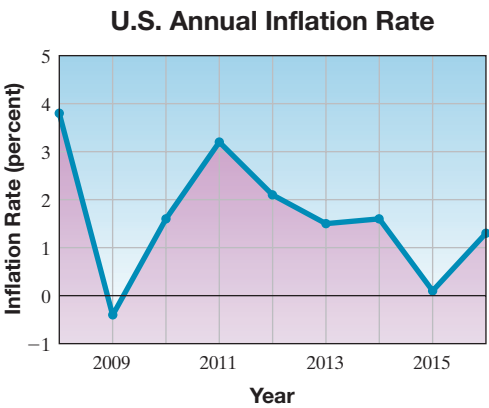
Most Common Blood Types	
Blood Type	% of Population with This Blood Type
O+	38%
A+	34%
B+	9%
O−	7%
A−	6%
AB+	3%
B−	2%
AB−	1%



8. _____

The following line graph shows the annual inflation rate in the United States for the years 2008–2016. Use this graph to answer Exercises 8 through 10.

9. _____



▶ 8. Approximate the annual inflation rate in 2014.

▶ 9. During which of the years shown was the inflation rate greater than 3%?

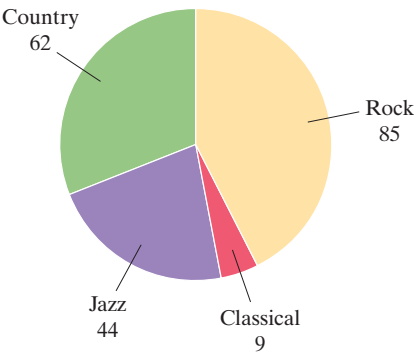
10. _____

▶ 10. During which sets of years was the inflation rate decreasing?

Source: Bureau of Labor Statistics

11. _____

The result of a survey of 200 people is shown in the following circle graph. Each person was asked to tell his or her favorite type of music. Use this graph to answer Exercises 11 and 12.



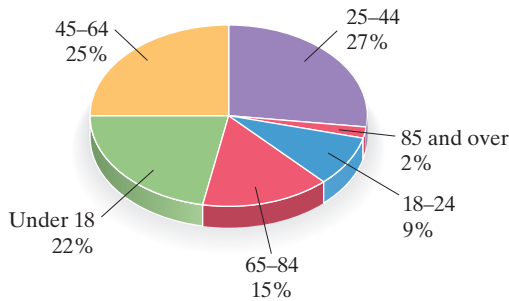
▶ 11. Find the ratio of those who prefer rock music to the total number surveyed.

▶ 12. Find the ratio of those who prefer country music to those who prefer jazz.

12. _____

The following circle graph shows the projected age distribution of the population of the United States in 2020. There are projected to be 335 million people in the United States in 2020. Use the graph to find how many people are expected to be in the age groups given.

U.S. Population in 2020 by Age Groups



Source: U.S. Census Bureau

13. _____

14. _____

15. _____

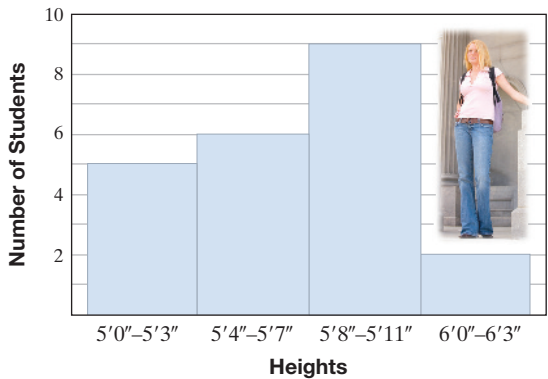
16. _____

17. _____

- ▶ 13. Under 18 (Round to nearest whole million.)
- ▶ 14. 25-44 (Round to nearest whole million.)

A professor measures the heights of the students in her class. The results are shown in the following histogram. Use this histogram to answer Exercises 15 and 16.

Student Heights



- ▶ 15. How many students are 5'8"-5'11" tall?
- ▶ 16. How many students are 5'7" or shorter?

- ▶ 17. The history test scores of 25 students are shown below. Use these scores to complete the frequency distribution table.

70	86	81	65	92
43	72	85	69	97
82	51	75	50	68
88	83	85	77	99
77	63	59	84	90

Class Intervals (Scores)	Tally	Class Frequency (Number of Students)
40-49		
50-59		
60-69		
70-79		
80-89		
90-99		

18. _____

▶ 18. Use the results of Exercise 17 to draw a histogram.



19. _____

Given the data items:

10, 18, 13, 16, 13

20. _____

- ▶ 19. Find the range.
- ▶ 20. Use the data for Exercise 17 and find the range.
- ▶ 21. Use the frequency distribution table to find the following: Round the mean to 1 decimal place.

- a. mean
- b. median
- c. mode
- d. range

Data Item	Frequency
90	3
91	1
92	2
93	8
94	8

21. a. _____

b. _____

c. _____

d. _____

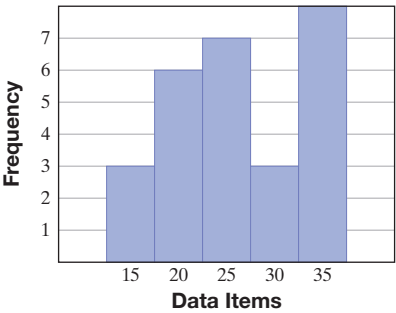
22. a. _____

b. _____

c. _____

d. _____

- ▶ 22. Use the graph of data items to find the following: Round the mean to 1 decimal place.
- a. mean
- b. median
- c. mode
- d. range

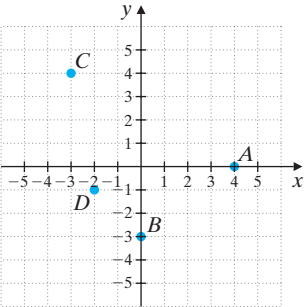


23. _____

24. _____

Find the coordinates of each point in the graph below. Round the mean to 1 decimal place.

- ▶ 23. A
- ▶ 24. B
- ▶ 25. C
- ▶ 26. D

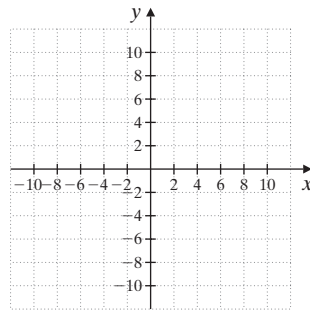
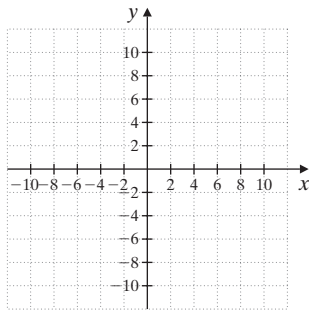


25. _____

26. _____

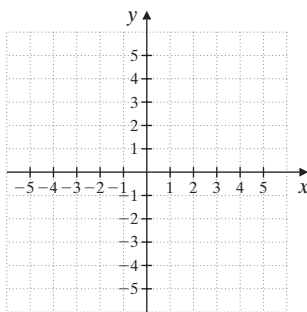
Complete and graph the ordered pair solutions of each given equation.

► 27. $x = -6y$; $(0, \quad)$, $(\quad, 1)$, $(12, \quad)$ ► 28. $y = 7x - 4$; $(2, \quad)$, $(-1, \quad)$, $(0, \quad)$

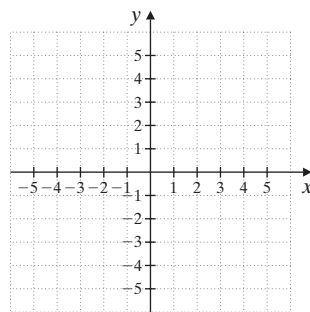


Graph each linear equation.

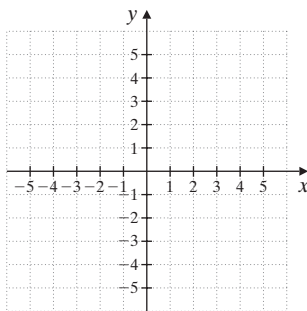
► 29. $y + x = -4$



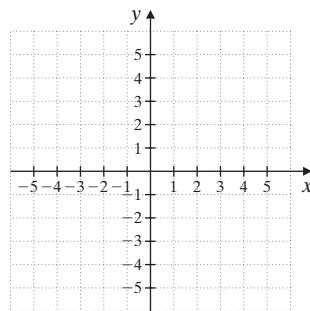
► 30. $y = -4$



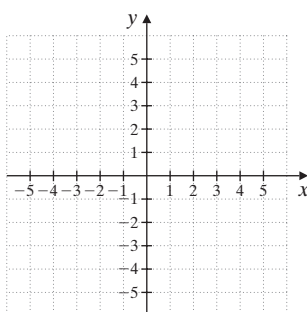
► 31. $y = 3x - 5$



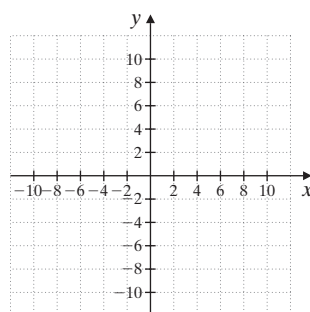
► 32. $x = 5$



► 33. $y = -\frac{1}{2}x$



► 34. $3x - 2y = 12$



27. _____

28. _____

29. _____

30. _____

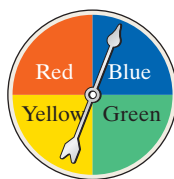
31. _____

32. _____

33. _____

34. _____

- ▶ **35.** Draw a tree diagram for the experiment of spinning the spinner twice. Then use the diagram to determine the number of outcomes.



- ▶ **36.** Draw a tree diagram for the experiment of tossing a coin twice. Then use the diagram to determine the number of outcomes.

Suppose that the numbers 1 through 10 are each written on same-size pieces of paper and placed in a bag. You then select one piece of paper from the bag.

- ▶ **37.** What is the probability of choosing a 6 from the bag?
- ▶ **38.** What is the probability of choosing a 3 or a 4 from the bag?

35.

36.

37.

38.

1. Simplify: $4^3 + [3^2 - (10 \div 2)] - 7 \cdot 3$
2. $7^2 + [5^3 - (6 \div 3)] + 4 \cdot 2$
3. Evaluate $x - y$ for $x = -3$ and $y = 9$.
4. Evaluate $x - y$ for $x = 7$ and $y = -2$.
5. Solve: $3y - 7y = 12$
6. Solve: $2x - 6x = 24$
7. Solve: $\frac{x}{6} + 1 = \frac{4}{3}$
8. Solve: $\frac{7}{2} + \frac{a}{4} = 1$
9. Add: $2\frac{1}{3} + 5\frac{3}{8}$
10. Add: $3\frac{2}{5} + 4\frac{3}{4}$
11. Write 5.9 as a mixed number.
12. Write 2.8 as a mixed number.
13. Subtract: $3.5 - 0.068$. Check your answer.
14. Subtract: $7.4 - 0.073$. Check your answer.
15. Multiply: 0.0531×16
16. Multiply: 0.147×0.2
17. Divide: $-5.98 \div 115$
18. Divide: $27.88 \div 205$
19. Simplify: $(-1.3)^2 + 2.4$
20. Simplify: $(-2.7)^2$
21. Write $\frac{1}{4}$ as a decimal.
22. Write $\frac{3}{8}$ as a decimal.
23. Solve: $5(x - 0.36) = -x + 2.4$
24. Solve: $4(0.35 - x) = x - 7$
25. Use the appendix or a calculator to approximate $\sqrt{80}$ to the nearest thousandth.
26. Use the appendix or a calculator to approximate $\sqrt{60}$ to the nearest thousandth.

Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. _____
24. _____
25. _____
26. _____

27.

28.

29.

30.

31.

32.

33.

34.

35.

36.

37.

38.

39.

40.

41.

42.

43.

44.

45.

46.

47.

48.

49.

50.

27. Write the ratio of 21 to 29 using fractional notation.

28. Write the ratio of 7 to 15 using fractional notation.

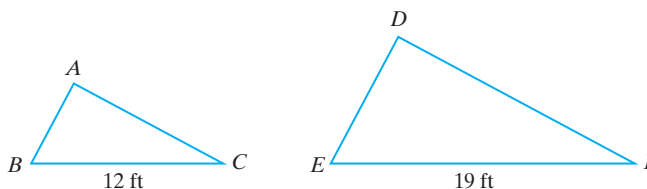
29. Write as a unit rate: 337.5 miles every 15 gallons of gas

30. Write as a unit price: \$1.59 for 3 ounces

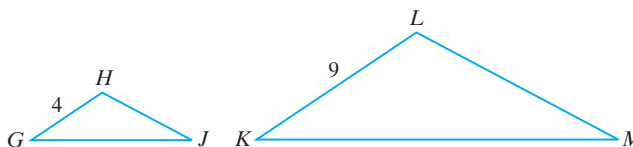
31. Solve $\frac{51}{34} = \frac{-3}{x}$ for x .

32. Solve $\frac{8}{5} = \frac{x}{10}$ for x .

△ 33. Find the ratio of corresponding sides for the similar triangles ABC and DEF .



△ 34. Find the ratio of corresponding sides for the similar triangles GHJ and KLM .



Write each percent as a decimal.

35. 4.6%

36. 32%

37. 0.74%

38. 2.7%

39. What number is 35% of 60?

40. What number is 40% of 36?

41. 20.8 is 40% of what number?

42. 9.5 is 25% of what number?

43. The sales tax on a \$310 Sony flat-screen digital 32-inch television is \$26.35. Find the sales tax rate.

44. The sales tax on a \$2.00 yo-yo is \$0.13. Find the sales tax rate.

45. \$4000 is invested at 5.3% compounded quarterly for 10 years. Find the total amount at the end of 10 years.

46. Linda Bonnett borrows \$1600 for 1 year. If the interest is \$128.60, find the monthly payment. (Hint: Find the total amount due and divide by the number of months in a year.)

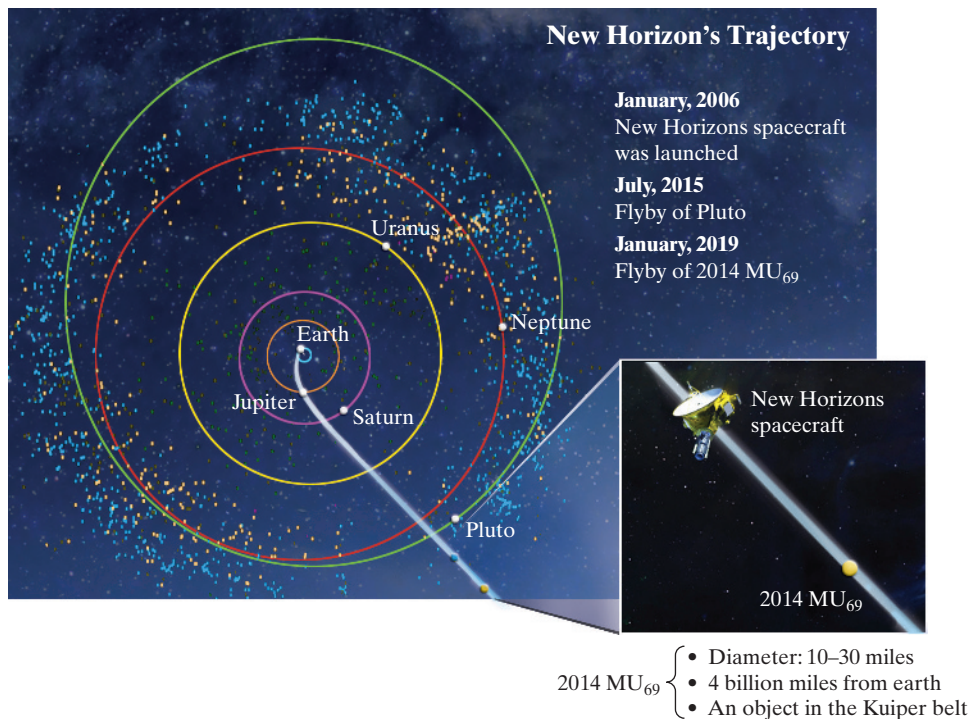
47. Find the median of the list of numbers: 25, 54, 56, 57, 60, 71, 98

48. Find the median of the list of numbers: 43, 46, 47, 50, 52, 83

49. If a die is rolled one time, find the probability of rolling a 3 or a 4.

50. If a die is rolled one time, find the probability of rolling an even number.

Geometry and Measurement



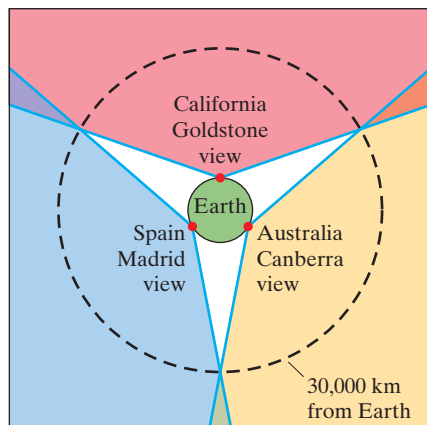
Source: Minor Planet Center, www.cfeeps.net and others

Where Is New Horizons Spacecraft Now, and How Do We Receive Data Collected?

New Horizons is NASA's robotic spacecraft mission. This spacecraft is about the size and shape of a grand piano with a satellite dish attached. It was launched in January 2006 and is now headed for a January 1, 2019, flyby past 2014 MU₆₉, a small object in the Kuiper asteroid belt. How do we receive the images of this spacecraft when it is so far into deep space?

The Deep Space Network (DSN) is a worldwide network of large antennas and communication facilities. When a mission is in deep space, fewer sites are needed for sending and receiving transmissions; thus, the DSN uses only three sites shown below. The diagram below shows an overview of Earth from the vantage point of the North Pole and the location of these three sites.

We study some geometry of the DSN in Section 9.1, Exercises 65 and 66, and Section 9.7, Exercises 23 through 26.



9

The word *geometry* is formed from the Greek words *geo*, meaning "Earth," and *metron*, meaning "measure." Geometry literally means to measure the Earth. In this chapter we learn about various geometric figures and their properties, such as perimeter, area, and volume. Knowledge of geometry can help us solve practical problems in all types of real-life situations.

Sections

- 9.1** Lines and Angles
- 9.2** Perimeter
- 9.3** Area, Volume, and Surface Area
- Integrated Review—**
Geometry Concepts
- 9.4** Linear Measurement
- 9.5** Weight and Mass
- 9.6** Capacity
- 9.7** Temperature and
Conversions Between the
U.S. and Metric Systems

Check Your Progress

- Vocabulary Check
- Chapter Highlights
- Chapter Review
- Getting Ready for the Test
- Chapter Test
- Cumulative Review

9.1 Lines and Angles

Objectives

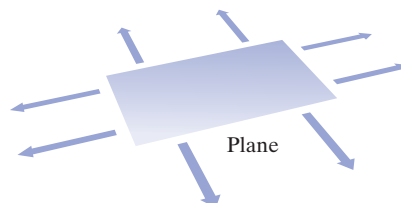
- A** Identify Lines, Line Segments, Rays, and Angles.
- B** Classify Angles as Acute, Right, Obtuse, or Straight.
- C** Identify Complementary and Supplementary Angles.
- D** Find Measures of Angles.

Objective A Identifying Lines, Line Segments, Rays, and Angles

Let's begin with a review of two important concepts—space and plane.

Space extends in all directions indefinitely. Examples of objects in space are houses, grains of salt, bushes, your *Prealgebra* textbook, and you.

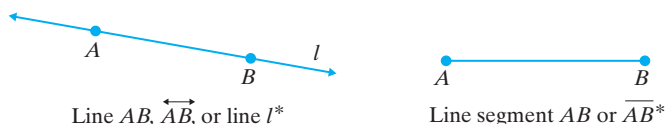
A **plane** is a flat surface that extends indefinitely. Surfaces like a plane are a classroom floor or a blackboard or whiteboard.



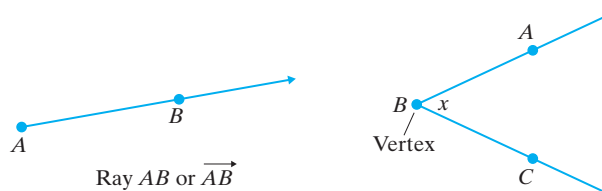
The most basic concept of geometry is the idea of a point in space. A **point** has no length, no width, and no height, but it does have location. We represent a point by a dot, and we usually label points with capital letters.



A **line** is a set of points extending indefinitely in two directions. A line has no width or height, but it does have length. We can name a line by any two of its points or by a single lowercase letter. A **line segment** is a piece of a line with two endpoints.*



A **ray** is a part of a line with one endpoint. A ray extends indefinitely in one direction. An **angle** is made up of two rays that share the same endpoint. The common endpoint is called the **vertex**.



The angle in the figure above can be named

$\angle ABC$ $\angle CBA$ $\angle B$ or $\angle x$
 ↑ ↑
 The vertex is the middle point.

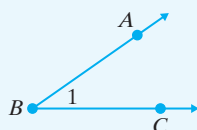
Rays BA and BC are **sides** of the angle.

*Although line l is also line BA or $\overleftrightarrow{BA}$, we will use only one order of points to name lines or line segments.

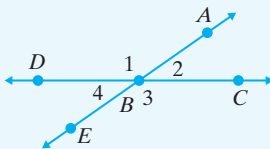
Helpful Hint

Naming an Angle

When there is no confusion as to what angle is being named, you may use the vertex alone.



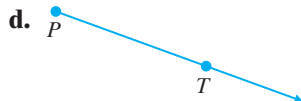
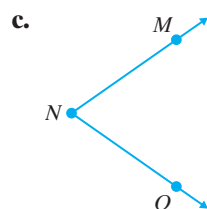
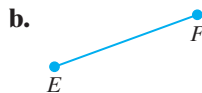
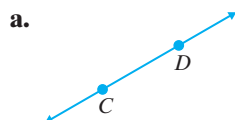
Name of $\angle B$ is all right.
There is no confusion. $\angle B$ means $\angle 1$.



Name of $\angle B$ is *not* all right.
There is confusion. Does $\angle B$ mean $\angle 1$, $\angle 2$, $\angle 3$, or $\angle 4$?

Example 1

Identify each figure as a line, a ray, a line segment, or an angle. Then name the figure using the given points.



Solution:

Figure (a) extends indefinitely in two directions. It is line CD or $\overleftrightarrow{CD}$.

Figure (b) has two endpoints. It is line segment EF or $\overline{EF}$.

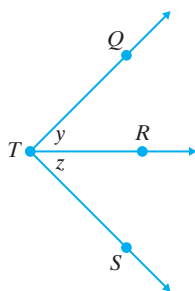
Figure (c) has two rays with a common endpoint. It is $\angle MNO$, $\angle ONM$, or $\angle N$.

Figure (d) is part of a line with one endpoint. It is ray PT or $\overrightarrow{PT}$.

Work Practice 1

Example 2

List other ways to name $\angle y$.

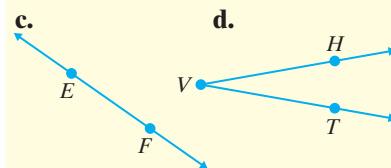
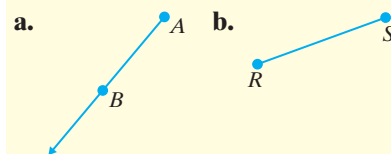


Solution: Two other ways to name $\angle y$ are $\angle QTR$ and $\angle RTQ$. We may *not* use the vertex alone to name this angle because three different angles have T as their vertex.

Work Practice 2

Practice 1

Identify each figure as a line, a ray, a line segment, or an angle. Then name the figure using the given points.



Practice 2

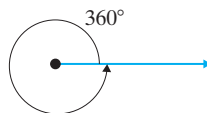
Use the figure in Example 2 to list other ways to name $\angle z$.

Answers

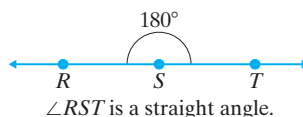
1. a. ray; ray AB or $\overrightarrow{AB}$
- b. line segment; line segment RS or $\overline{RS}$
- c. line; line EF or $\overleftrightarrow{EF}$
- d. angle; $\angle TVH$ or $\angle HVT$ or $\angle V$
2. $\angle RTS$, $\angle STR$

Objective B Classifying Angles as Acute, Right, Obtuse, or Straight

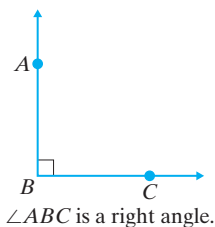
An angle can be measured in **degrees**. The symbol for degrees is a small, raised circle, $^\circ$. There are 360° in a full revolution, or a full circle.



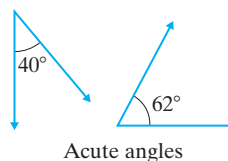
$\frac{1}{2}$ of a revolution measures $\frac{1}{2}(360^\circ) = 180^\circ$. An angle that measures 180° is called a **straight angle**.



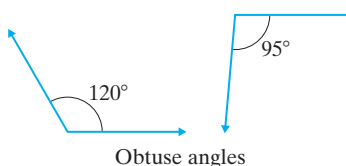
$\frac{1}{4}$ of a revolution measures $\frac{1}{4}(360^\circ) = 90^\circ$. An angle that measures 90° is called a **right angle**. The symbol $\perp$ is used to denote a right angle.



An angle whose measure is between 0° and 90° is called an **acute angle**.

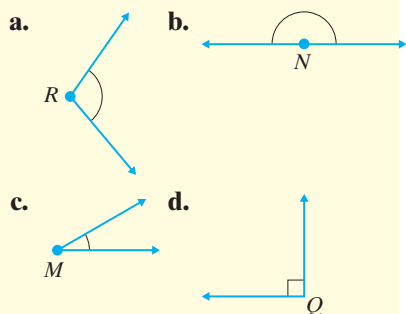


An angle whose measure is between 90° and 180° is called an **obtuse angle**.



Practice 3

Classify each angle as acute, right, obtuse, or straight.

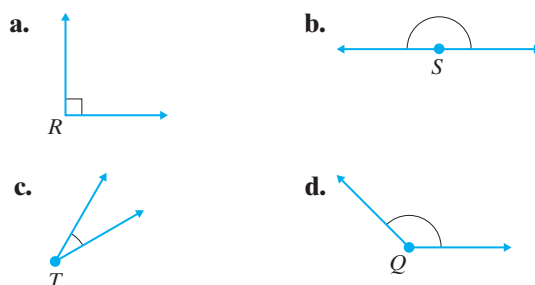


Answers

3. a. obtuse b. straight c. acute
d. right

Example 3

Classify each angle as acute, right, obtuse, or straight.

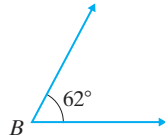


Solution:

- a. $\angle R$ is a **right** angle, denoted by $\perp$. It measures 90° .
- b. $\angle S$ is a **straight** angle. It measures 180° .
- c. $\angle T$ is an **acute** angle. It measures between 0° and 90° .
- d. $\angle Q$ is an **obtuse** angle. It measures between 90° and 180° .

Work Practice 3

Let's look at $\angle B$ below, whose measure is 62° .



There is a shorthand notation for writing the measure of this angle. To write "The measure of $\angle B$ is 62° ", we can write,

$$m\angle B = 62^\circ.$$

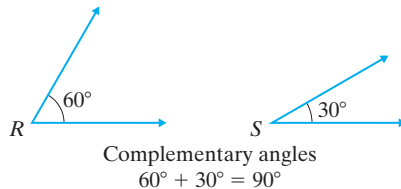
By the way, note that $\angle B$ is an acute angle because $m\angle B$ is between 0° and 90° .

Objective C Identifying Complementary and Supplementary Angles

Two angles that have a sum of 90° are called **complementary angles**. We say that each angle is the **complement** of the other.

$\angle R$ and $\angle S$ are **complementary angles** because

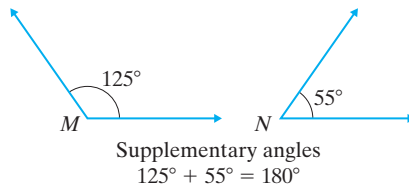
$$m\angle R + m\angle S = 60^\circ + 30^\circ = 90^\circ$$



Two angles that have a sum of 180° are called **supplementary angles**. We say that each angle is the **supplement** of the other.

$\angle M$ and $\angle N$ are **supplementary angles** because

$$m\angle M + m\angle N = 125^\circ + 55^\circ = 180^\circ$$



Example 4 Find the complement of a 48° angle.

Solution: Two angles that have a sum of 90° are complementary. This means that the **complement** of an angle that measures 48° is an angle that measures $90^\circ - 48^\circ = 42^\circ$.

Work Practice 4**Practice 4**

Find the complement of a 29° angle.

Answer

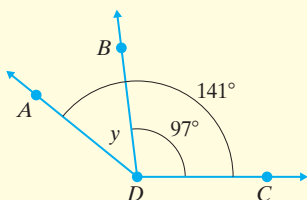
4. 61°

Practice 5

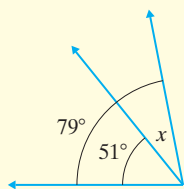
Find the supplement of a 67° angle.

Practice 6

- a. Find the measure of $\angle y$.



- b. Find the measure of $\angle x$.



- c. Classify $\angle x$ and $\angle y$ as acute, obtuse, or right angles.

Example 5

Find the supplement of a 107° angle.

Solution: Two angles that have a sum of 180° are supplementary. This means that the **supplement** of an angle that measures 107° is an angle that measures $180^\circ - 107^\circ = 73^\circ$.

Work Practice 5

✓ **Concept Check** True or false? The supplement of a 48° angle is 42° . Explain.

Objective D Finding Measures of Angles

Measures of angles can be added or subtracted to find measures of related angles.

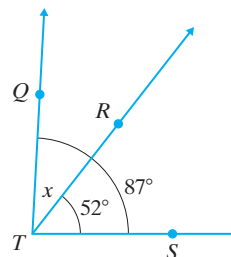
Example 6

Find the measure of $\angle x$. Then classify $\angle x$ as an acute, obtuse, or right angle.

Solution: $m\angle x = m\angle QTS - m\angle RTS$
 $= 87^\circ - 52^\circ$
 $= 35^\circ$

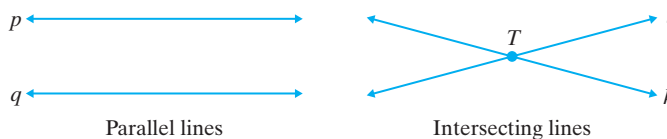
Thus, the measure of $\angle x$ ($m\angle x$) is 35° .

Since $\angle x$ measures between 0° and 90° , it is an acute angle.

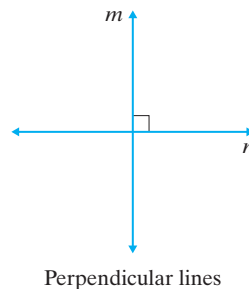


Work Practice 6

Two lines in a plane can be either parallel or intersecting. **Parallel lines** never meet. **Intersecting lines** meet at a point. The symbol $\parallel$ is used to indicate “is parallel to.” For example, in the figure, $p \parallel q$.

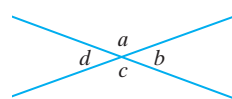


Some intersecting lines are perpendicular. Two lines are **perpendicular** if they form right angles when they intersect. The symbol $\perp$ is used to denote “is perpendicular to.” For example, in the figure below, $n \perp m$.



When two lines intersect, four angles are formed. Two angles that are opposite each other are called **vertical angles**. Vertical angles have the same measure.

Two angles that share a common side are called **adjacent angles**. Adjacent angles formed by intersecting lines are supplementary. That is, they have a sum of 180° .



Vertical angles:
 $\angle a$ and $\angle c$
 $\angle d$ and $\angle b$

Adjacent angles:
 $\angle a$ and $\angle b$
 $\angle b$ and $\angle c$
 $\angle c$ and $\angle d$
 $\angle d$ and $\angle a$

Answers

5. 113° 6. a. 44° b. 28° c. acute

✓ Concept Check Answer

false; the *complement* of a 48° angle is 42° ; the *supplement* of a 48° angle is 132°

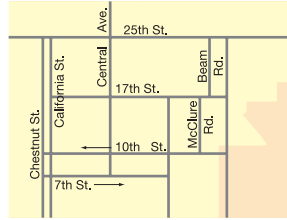
Here are a few real-life examples of the lines we just discussed.



Parallel lines

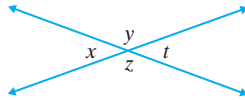


Vertical angles



Perpendicular lines

Example 7 Find the measure of $\angle x$, $\angle y$, and $\angle z$ if the measure of $\angle t$ is 42° .



Solution: Since $\angle t$ and $\angle x$ are vertical angles, they have the same measure, so $\angle x$ measures 42° .

Since $\angle t$ and $\angle y$ are adjacent angles, their measures have a sum of 180° . So $\angle y$ measures $180^\circ - 42^\circ = 138^\circ$.

Since $\angle y$ and $\angle z$ are vertical angles, they have the same measure. So $\angle z$ measures 138° .

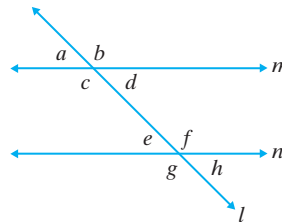
Work Practice 7

A line that intersects two or more lines at different points is called a **transversal**. Line l is a transversal that intersects lines m and n . The eight angles formed have special names. Some of these names are:

Corresponding angles: $\angle a$ and $\angle e$, $\angle c$ and $\angle g$, $\angle b$ and $\angle f$, $\angle d$ and $\angle h$

Alternate interior angles: $\angle c$ and $\angle f$, $\angle d$ and $\angle e$

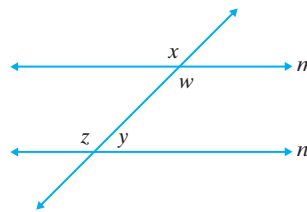
When two lines cut by a transversal are *parallel*, the following statement is true:



Parallel Lines Cut by a Transversal

If two parallel lines are cut by a transversal, then the measures of **corresponding angles are equal** and the measures of the **alternate interior angles are equal**.

Example 8 Given that $m \parallel n$ and that the measure of $\angle w$ is 100° , find the measures of $\angle x$, $\angle y$, and $\angle z$.



Solution:

$$m\angle x = 100^\circ$$

$\angle x$ and $\angle w$ are vertical angles.

$$m\angle z = 100^\circ$$

$\angle x$ and $\angle z$ are corresponding angles.

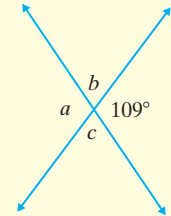
$$m\angle y = 180^\circ - 100^\circ = 80^\circ$$

$\angle z$ and $\angle y$ are supplementary angles.

Work Practice 8

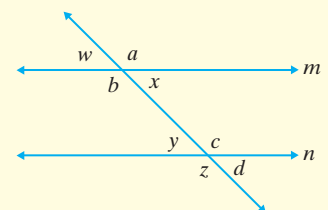
Practice 7

Find the measure of $\angle a$, $\angle b$, and $\angle c$.



Practice 8

Given that $m \parallel n$ and that the measure of $\angle w = 45^\circ$, find the measures of all the angles shown.



Answers

7. $m\angle a = 109^\circ$; $m\angle b = 71^\circ$;
 $m\angle c = 71^\circ$

8. $m\angle x = 45^\circ$; $m\angle y = 45^\circ$;
 $m\angle z = 135^\circ$; $m\angle a = 135^\circ$;
 $m\angle b = 135^\circ$; $m\angle c = 135^\circ$;
 $m\angle d = 45^\circ$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

acute	straight	degrees	adjacent	parallel	intersecting
obtuse	space	plane	point	vertical	vertex
right	angle	ray	line	perpendicular	transversal

- A(n) _____ is a flat surface that extends indefinitely.
- A(n) _____ has no length, no width, and no height.
- _____ extends in all directions indefinitely.
- A(n) _____ is a set of points extending indefinitely in two directions.
- A(n) _____ is part of a line with one endpoint.
- A(n) _____ is made up of two rays that share a common endpoint. The common endpoint is called the _____.
- A(n) _____ angle measures 180° .
- A(n) _____ angle measures 90° .
- A(n) _____ angle measures between 0° and 90° .
- A(n) _____ angle measures between 90° and 180° .
- _____ lines never meet and _____ lines meet at a point.
- Two intersecting lines are _____ if they form right angles when they intersect.
- An angle can be measured in _____.
- A line that intersects two or more lines at different points is called a(n) _____.
- When two lines intersect, four angles are formed, called _____ angles.
- Two angles that share a common side are called _____ angles.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 9.1

Objective A 17. In the lecture after Example 2, what are the four ways we can name the angle shown? ▶

Objective B 18. In the lecture before Example 3, what type of angle forms a line? What is its measure? ▶

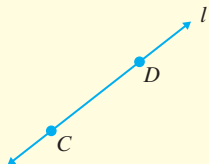
Objective C 19. What calculation is used to find the answer to Example 6? ▶

Objective D 20. In the lecture before Example 7, two lines in a plane that aren't parallel must what? ▶

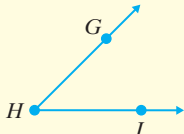
9.1 Exercise Set MyLab Math

Objective A Identify each figure as a line, a ray, a line segment, or an angle. Then name the figure using the given points. See Examples 1 and 2.

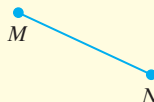
▶ 1.



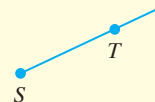
2.

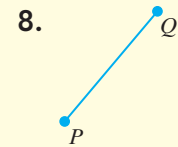
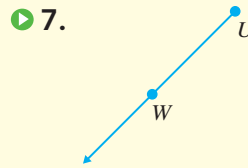
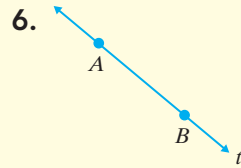
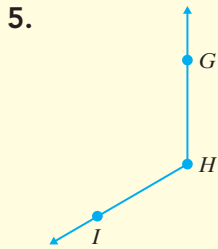


3.



4.





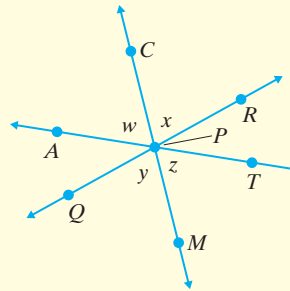
List two other ways to name each angle. See Example 2.

9. $\angle x$

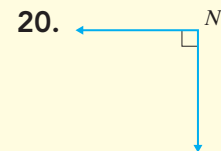
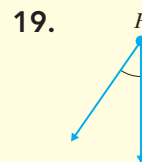
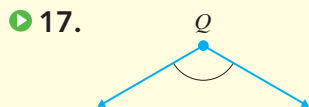
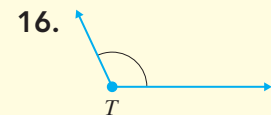
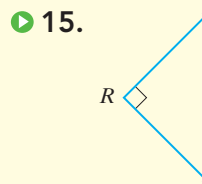
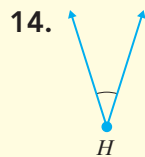
10. $\angle w$

11. $\angle z$

12. $\angle y$



Objective B Classify each angle as acute, right, obtuse, or straight. See Example 3.



Objective C Find each complementary or supplementary angle as indicated. See Examples 4 and 5.

21. Find the complement of a 23° angle.

22. Find the complement of a 77° angle.

23. Find the supplement of a 17° angle.

24. Find the supplement of a 77° angle.

25. Find the complement of a 58° angle.

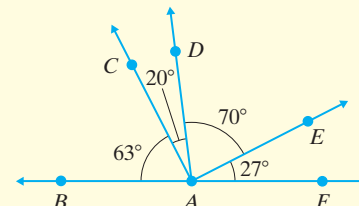
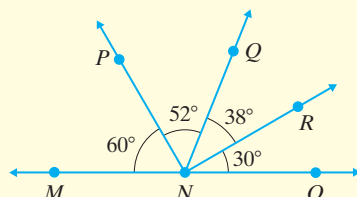
26. Find the complement of a 22° angle.

27. Find the supplement of a 150° angle.

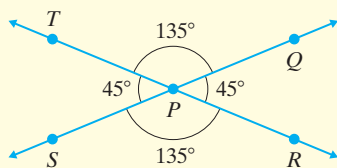
28. Find the supplement of a 130° angle.

29. Identify the pairs of complementary angles.

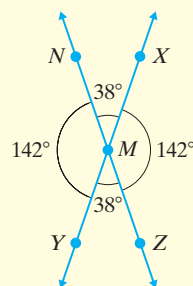
30. Identify the pairs of complementary angles.



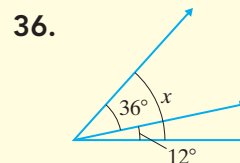
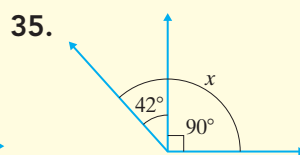
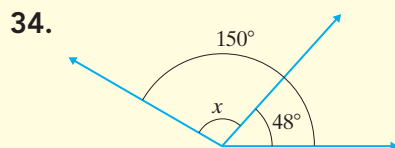
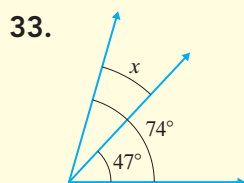
31. Identify the pairs of supplementary angles.



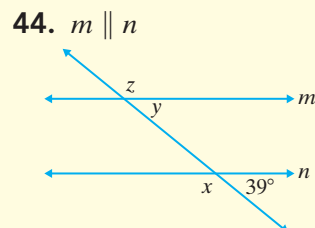
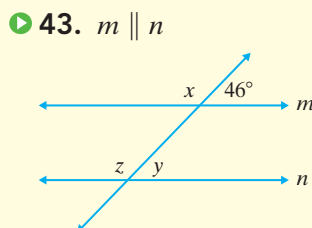
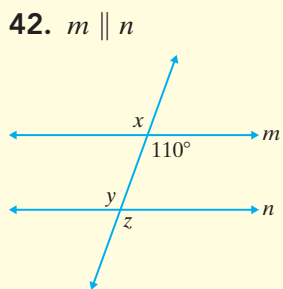
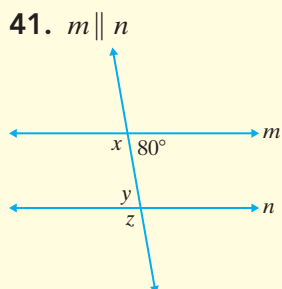
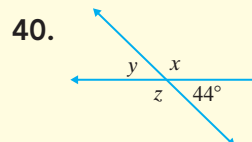
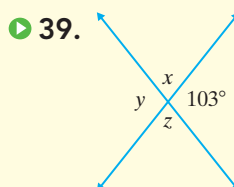
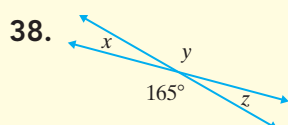
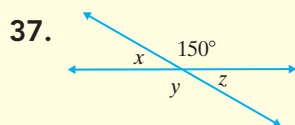
32. Identify the pairs of supplementary angles.



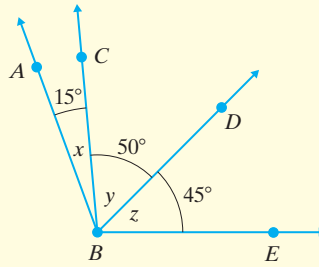
Objective D Find the measure of $\angle x$ in each figure. See Example 6.



Find the measures of angles x , y , and z in each figure. See Examples 7 and 8.



Objectives **A D** **Mixed Practice** Find two other ways of naming each angle. See Example 2.



45. $\angle x$

46. $\angle y$

47. $\angle z$

48. $\angle ABE$ (just name one other way)

Find the measure of each angle in the figure above. See Example 6.

▶ 49. $\angle ABC$

50. $\angle EBD$

51. $\angle CBD$

52. $\angle CBA$

▶ 53. $\angle DBA$

54. $\angle EBC$

55. $\angle CBE$

56. $\angle ABE$

Review

Perform each indicated operation. See Sections 4.3, 4.5, and 4.7

57. $\frac{7}{8} + \frac{1}{4}$

58. $\frac{7}{8} - \frac{1}{4}$

59. $\frac{7}{8} \cdot \frac{1}{4}$

60. $\frac{7}{8} \div \frac{1}{4}$

61. $3\frac{1}{3} - 2\frac{1}{2}$

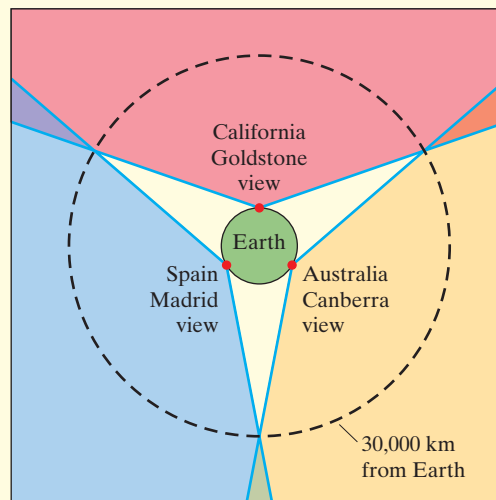
62. $3\frac{1}{3} + 2\frac{1}{2}$

63. $3\frac{1}{3} \div 2\frac{1}{2}$

64. $3\frac{1}{3} \cdot 2\frac{1}{2}$

Concept Extensions

Use this North Pole overhead view of the three sites of the Deep Space Network to answer Exercises 65 and 66. (See the Chapter Opener.)



65. How many degrees are there around the Earth at the equator?

66. If the three sites of the Deep Space Network (red dots shown) are about the same number of degrees apart, how many degrees apart are they?

67. One great pyramid at Chichen Itza, Mexico, was the Temple of Kukulcan. The four faces of this pyramid have protruding stairways that rise at a 45° angle. Find the complement of this angle.



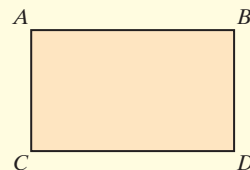
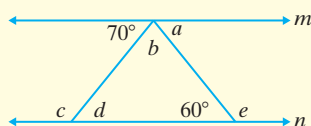
68. The UNESCO World Heritage Site of the Cahokia Mounds is located in Cahokia, Illinois. This site was once populated by the Mississippian People of North America and was built up on a series of mounds. These mounds were completely constructed of silt and dirt. The largest and best known of these is the Monk's Mound, which is considered a truncated pyramid, with the top being a flat base rather than extending to a point. On one side of this mound, modern archeologists have measured a 35° angle. Find the complement of this angle.



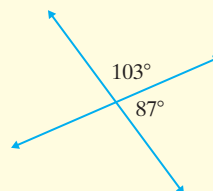
69. The angle between the two walls of the Vietnam Veterans Memorial in Washington, D.C., is 125.2° . Find the supplement of this angle. (Source: National Park Service)
70. The faces of Khafre's Pyramid at Giza, Egypt, are inclined at an angle of 53.13° . Find the complement of this angle. (Source: PBS NOVA Online)

Answer true or false for Exercises 71 through 74. See the Concept Check in this section. If false, explain why.

71. The complement of a 100° angle is an 80° angle.
72. It is possible to find the complement of a 120° angle.
73. It is possible to find the supplement of a 120° angle.
74. The supplement of a 5° angle is a 175° angle.
75. If lines m and n are parallel, find the measures of angles a through e .
76. Below is a rectangle. List which segments, if extended, would be parallel lines.



77. Can two supplementary angles both be acute? Explain why or why not.
78. In your own words, describe how to find the complement and the supplement of a given angle.
79. Find two complementary angles with the same measure.
80. Is the figure below possible? Why or why not?

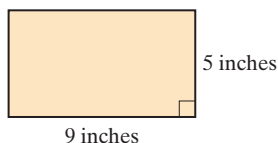


9.2 Perimeter

Objective A Using Formulas to Find Perimeters

Recall from Section 1.3 that the perimeter of a polygon is the distance around the polygon. This means that the perimeter of a polygon is the sum of the lengths of its sides.

Example 1 Find the perimeter of the rectangle below.



Solution:

$$\begin{aligned}\text{perimeter} &= 9 \text{ inches} + 9 \text{ inches} + 5 \text{ inches} + 5 \text{ inches} \\ &= 28 \text{ inches}\end{aligned}$$

Work Practice 1

Notice that the perimeter of the rectangle in Example 1 can be written as $2 \cdot (9 \text{ inches}) + 2 \cdot (5 \text{ inches})$.

↑ ↑
length width

In general, we can say that the perimeter of a rectangle is always

$$2 \cdot \text{length} + 2 \cdot \text{width}$$

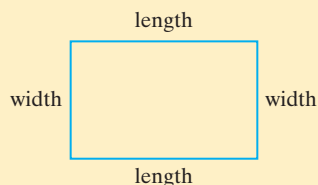
As we have just seen, the perimeters of some special figures such as rectangles form patterns. These patterns are given as **formulas**. The formula for the perimeter of a rectangle is shown next:

Perimeter of a Rectangle

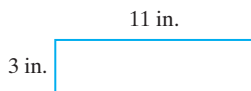
$$\text{Perimeter} = 2 \cdot \text{length} + 2 \cdot \text{width}$$

In symbols, this can be written as

$$P = 2l + 2w$$



Example 2 Find the perimeter of a rectangle with a length of 11 inches and a width of 3 inches.



Solution: We use the formula for perimeter and replace the letters by their known lengths.

$$\begin{aligned}P &= 2l + 2w \\ &= 2 \cdot 11 \text{ in.} + 2 \cdot 3 \text{ in.} \quad \text{Replace } l \text{ with } 11 \text{ in. and } w \text{ with } 3 \text{ in.} \\ &= 22 \text{ in.} + 6 \text{ in.} \\ &= 28 \text{ in.}\end{aligned}$$

The perimeter is 28 inches.

Work Practice 2

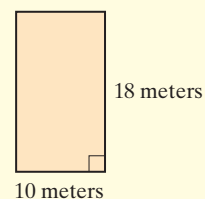
Objectives

A Use Formulas to Find Perimeters.

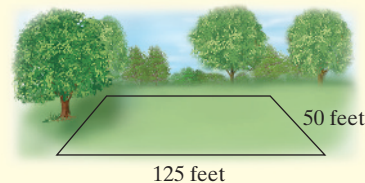
B Use Formulas to Find Circumferences.

Practice 1

a. Find the perimeter of the rectangle.



b. Find the perimeter of the rectangular lot shown below:



Practice 2

Find the perimeter of a rectangle with a length of 32 centimeters and a width of 15 centimeters.

Answers

1. **a.** 56 m **b.** 350 ft **2.** 94 cm

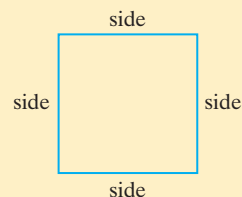
Recall that a square is a special rectangle with all four sides the same length. The formula for the perimeter of a square is shown next:

Perimeter of a Square

$$\begin{aligned}\text{Perimeter} &= \text{side} + \text{side} + \text{side} + \text{side} \\ &= 4 \cdot \text{side}\end{aligned}$$

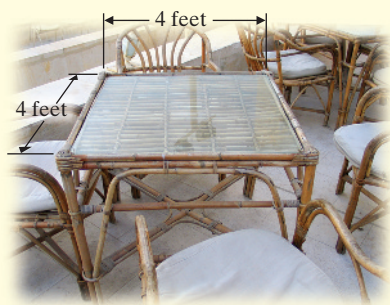
In symbols,

$$P = 4s$$



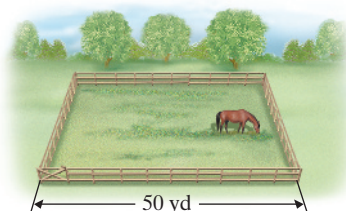
Practice 3

Find the perimeter of a square tabletop if each side is 4 feet long.



Example 3 Finding the Perimeter of a Field

How much fencing is needed to enclose a square field 50 yards on a side?



Solution: To find the amount of fencing needed, we find the distance around, or perimeter. The formula for the perimeter of a square is $P = 4s$. We use this formula and replace s by 50 yards.

$$\begin{aligned}P &= 4s \\ &= 4 \cdot 50 \text{ yd} \\ &= 200 \text{ yd}\end{aligned}$$

The amount of fencing needed is **200 yards**.

Work Practice 3

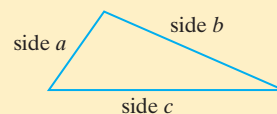
The formula for the perimeter of a triangle with sides of lengths a , b , and c is given next:

Perimeter of a Triangle

$$\text{Perimeter} = \text{side } a + \text{side } b + \text{side } c$$

In symbols,

$$P = a + b + c$$



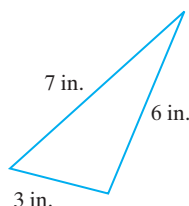
Practice 4

Find the perimeter of a triangle if the sides are 6 centimeters, 10 centimeters, and 8 centimeters in length.

Answers

3. 16 ft 4. 24 cm

Example 4 Find the perimeter of a triangle if the sides are 3 inches, 7 inches, and 6 inches.



Solution: The formula for the perimeter is $P = a + b + c$, where a , b , and c are the lengths of the sides. Thus,

$$\begin{aligned} P &= a + b + c \\ &= 3 \text{ in.} + 7 \text{ in.} + 6 \text{ in.} \\ &= 16 \text{ in.} \end{aligned}$$

The perimeter of the triangle is **16 inches**.

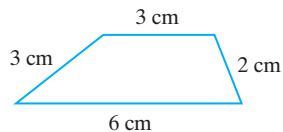
Work Practice 4

The method for finding the perimeter of any polygon is given next:

Perimeter of a Polygon

The perimeter of a polygon is the sum of the lengths of its sides.

Example 5 Find the perimeter of the trapezoid shown below:



Solution: To find the perimeter, we find the sum of the lengths of its sides.

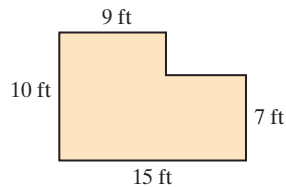
$$\text{perimeter} = 3 \text{ cm} + 2 \text{ cm} + 6 \text{ cm} + 3 \text{ cm} = 14 \text{ cm}$$

The perimeter is **14 centimeters**.

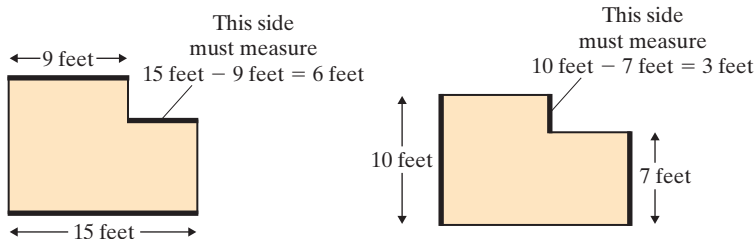
Work Practice 5

Example 6 Finding the Perimeter of a Room

Find the perimeter of the room shown below:

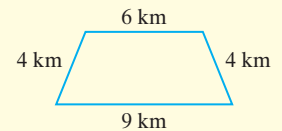


Solution: To find the perimeter of the room, we first need to find the lengths of all sides of the room.



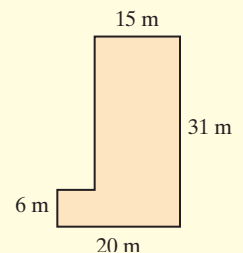
Practice 5

Find the perimeter of the trapezoid shown.



Practice 6

Find the perimeter of the room shown.

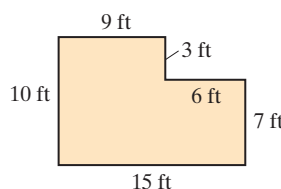


Answers

5. 23 km 6. 102 m

(Continued on next page)

Now that we know the measures of all sides of the room, we can add the measures to find the perimeter.



$$\begin{aligned}\text{perimeter} &= 10 \text{ ft} + 9 \text{ ft} + 3 \text{ ft} + 6 \text{ ft} + 7 \text{ ft} + 15 \text{ ft} \\ &= 50 \text{ ft}\end{aligned}$$

The perimeter of the room is 50 feet.

Work Practice 6

Practice 7

A rectangular lot measures 60 feet by 120 feet. Find the cost to install fencing around the lot if the cost of fencing is \$1.90 per foot.

Example 7 Calculating the Cost of Wallpaper Border

A rectangular room measures 10 feet by 12 feet. Find the cost to hang a wallpaper border on the walls close to the ceiling if the cost of the wallpaper border is \$1.09 per foot.

Solution: First we find the perimeter of the room.

$$\begin{aligned}P &= 2l + 2w \\ &= 2 \cdot 12 \text{ ft} + 2 \cdot 10 \text{ ft} \quad \text{Replace } l \text{ with 12 feet and } w \text{ with 10 feet.} \\ &= 24 \text{ ft} + 20 \text{ ft} \\ &= 44 \text{ ft}\end{aligned}$$

The cost of the wallpaper is

$$\text{cost} = \$1.09 \cdot 44 \text{ ft} = \$47.96$$

The cost of the wallpaper is \$47.96.

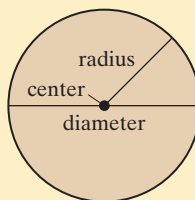
Work Practice 7

Objective B Using Formulas to Find Circumferences

Recall from Section 5.3 that the distance around a circle is called the **circumference**. This distance depends on the radius or the diameter of the circle.

The formulas for circumference are shown next:

Circumference of a Circle



Circumference = $2 \cdot \pi \cdot \text{radius}$ or Circumference = $\pi \cdot \text{diameter}$
In symbols,

$$C = 2\pi r \quad \text{or} \quad C = \pi d$$

$$\text{where } \pi \approx 3.14 \quad \text{or} \quad \pi \approx \frac{22}{7}.$$

To better understand circumference and π (pi), try the following experiment. Take any can and measure its circumference and its diameter.



The can in the figure above has a circumference of 23.5 centimeters and a diameter of 7.5 centimeters. Now divide the circumference by the diameter.

$$\frac{\text{circumference}}{\text{diameter}} = \frac{23.5 \text{ cm}}{7.5 \text{ cm}} \approx 3.13$$

Try this with other sizes of cylinders and circles—you should always get a number close to 3.1. The exact ratio of circumference to diameter is π . (Recall that $\pi \approx 3.14$ or $\approx \frac{22}{7}$.)

Example 8 Finding Circumference of a Circular Spa

A homeowner plans to install a border of new tiling around the circumference of her circular spa. If her spa has a diameter of 14 feet, find its exact circumference. Then use the approximation 3.14 for π to approximate the circumference.



Solution: Because we are given the diameter, we use the formula $C = \pi d$.

$$\begin{aligned} C &= \pi d \\ &= \pi \cdot 14 \text{ ft} \quad \text{Replace } d \text{ with 14 feet.} \\ &= 14\pi \text{ ft} \end{aligned}$$

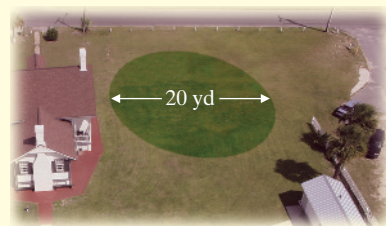
The circumference of the spa is *exactly* 14π feet. By replacing π with the *approximation* 3.14, we find that the circumference is *approximately* $14 \text{ feet} \cdot 3.14 = 43.96$ feet.

Work Practice 8

✓Concept Check The distance around which figure is greater: a square with side length 5 inches or a circle with radius 3 inches?

Practice 8

An irrigation device waters a circular region with a diameter of 20 yards. Find the exact circumference of the watered region, then use $\pi \approx 3.14$ to give an approximation.



Answer

8. exactly 20π yd ≈ 62.8 yd

✓Concept Check Answer
a square with side length 5 in.

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

circumference	radius	π	$\frac{22}{7}$
diameter	perimeter	3.14	

- The _____ of a polygon is the sum of the lengths of its sides.
- The distance around a circle is called the _____.
- The exact ratio of circumference to diameter is _____.
- The diameter of a circle is double its _____.
- Both _____ and _____ are approximations for π .
- The radius of a circle is half its _____.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



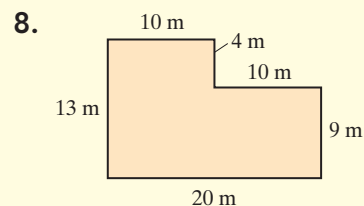
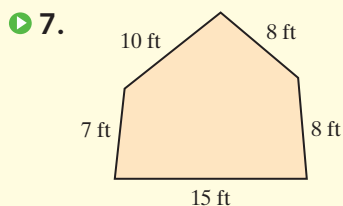
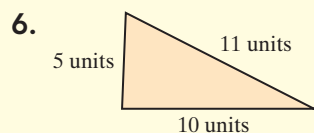
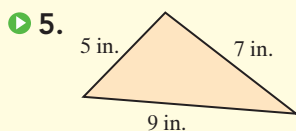
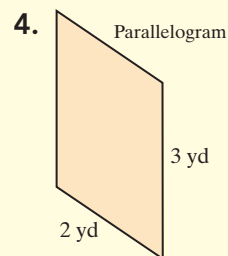
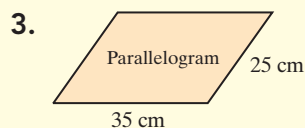
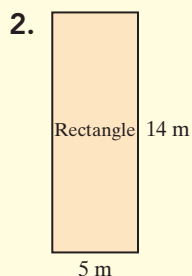
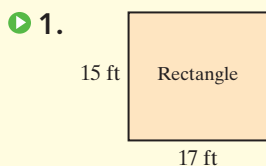
See Video 9.2

Objective A 7. In Example 1, how can the perimeter be found if we forget the formula?

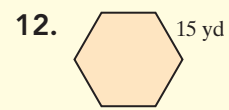
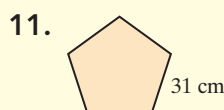
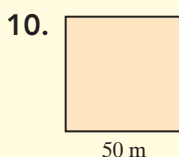
Objective B 8. From the lecture before Example 6, circumference is a special name for what?

9.2 Exercise Set MyLab Math

Objective A Find the perimeter of each figure. (See Appendix A.1 for any unknown geometric figures.) See Examples 1 through 6.



Find the perimeter of each regular polygon. (The sides of a regular polygon have the same length.)



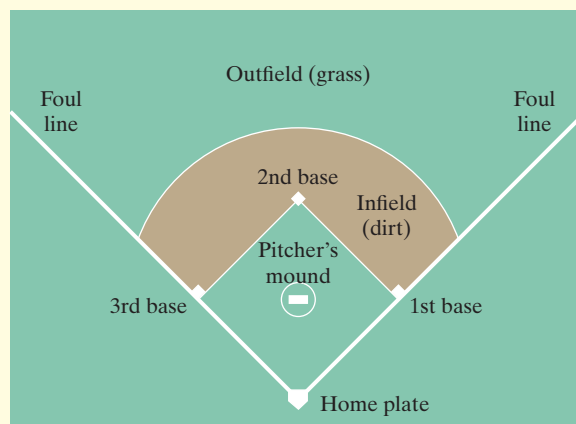
Solve. See Examples 1 through 7.

13. A polygon has sides of length 5 feet, 3 feet, 2 feet, 7 feet, and 4 feet. Find its perimeter.

14. A triangle has sides of length 8 inches, 12 inches, and 10 inches. Find its perimeter.

15. A line-marking machine lays down lime powder to mark both foul lines on a baseball field. If each foul line for this field measures 312 feet, how many feet of lime powder will be deposited?

16. A baseball diamond has 4 sides, with each side length 90 feet. If a baseball player hits a home run, how far does the player run (home plate, around the bases, then back to home plate)?



17. If a football field is 53 yards wide and 120 yards long, what is the perimeter?



18. A stop sign has eight equal sides of length 12 inches. Find its perimeter.



19. A metal strip is being installed around a workbench that is 8 feet long and 3 feet wide. Find how much stripping is needed for this project.

20. Find how much fencing is needed to enclose a rectangular garden 70 feet by 21 feet.

21. If the stripping in Exercise 19 costs \$2.50 per foot, find the total cost of the stripping.

22. If the fencing in Exercise 20 costs \$2 per foot, find the total cost of the fencing.

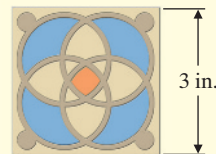
23. A regular octagon has a side length of 9 inches.
a. How many sides does an octagon have?
b. Find its perimeter.

24. A regular pentagon has a side length of 14 meters.
a. How many sides does a pentagon have?
b. Find its perimeter.

25. Find the perimeter of the top of a square compact disc case if the length of one side is 7 inches.

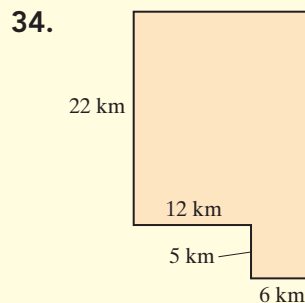
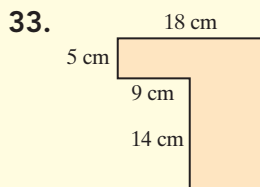
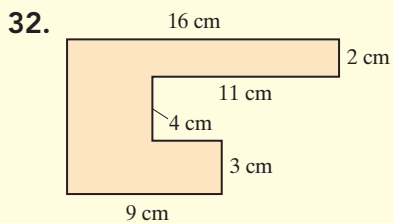
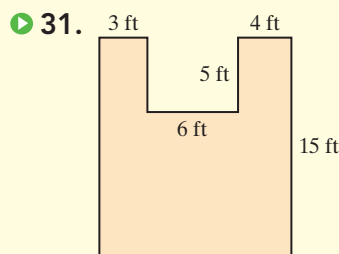
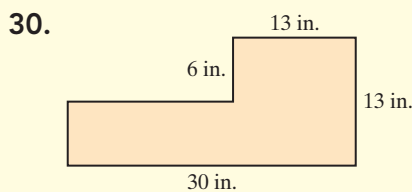
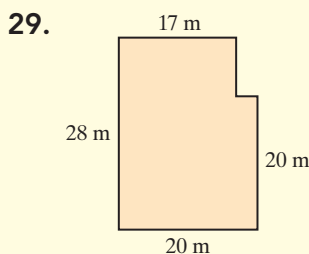


26. Find the perimeter of a square ceramic tile with a side of length 3 inches.

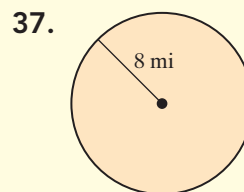
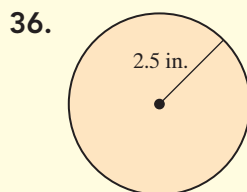
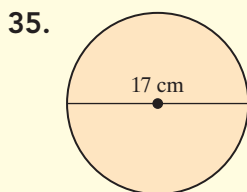


27. A rectangular room measures 10 feet by 11 feet. Find the cost of installing a strip of wallpaper around the room if the wallpaper costs \$0.86 per foot.
28. A rectangular house measures 85 feet by 70 feet. Find the cost of installing gutters around the house if the cost is \$2.36 per foot.

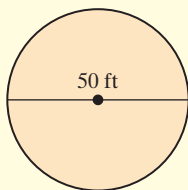
Find the perimeter of each figure. See Example 6.



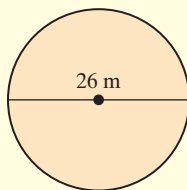
Objective B Find the circumference of each circle. Give the exact circumference and then an approximation. Use $\pi \approx 3.14$. See Example 8.



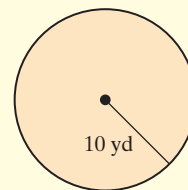
38.



▶ 39.

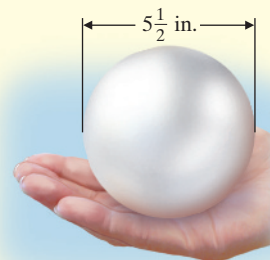


40.



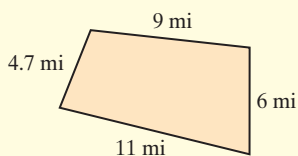
Solve.

41. Wyley Robinson just bought a trampoline for his children to use. The trampoline has a diameter of 15 feet. If Wyley wishes to buy netting to go around the outside of the trampoline, how many feet of netting does he need? Use 3.14 for π .
42. The largest round barn in the world is located at the Marshfield Fairgrounds in Wisconsin. The barn has a diameter of 150 ft. What is the circumference of the barn? Use 3.14 for π . (Source: *The Milwaukee Journal Sentinel*)
43. Meteor Crater, near Winslow, Arizona, is 4000 feet in diameter. Approximate the distance around the crater. Use 3.14 for π . (Source: *The Handy Science Answer Book*)
44. The *Pearl of Lao-tze* has a diameter of $5\frac{1}{2}$ inches. Approximate the distance around the pearl. Use $\frac{22}{7}$ for π . (Source: *The Guinness World Records*)

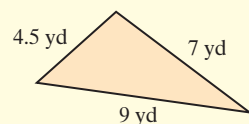


Objectives A B Mixed Practice Find the distance around each figure. For circles, give the exact circumference and then an approximation. Use $\pi \approx 3.14$. See Examples 1 through 8.

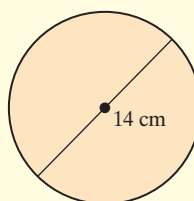
45.



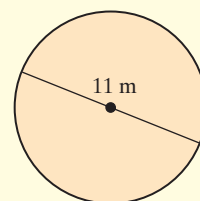
46.



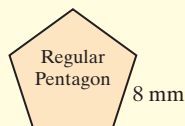
47.



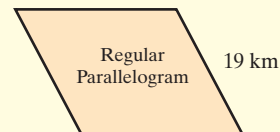
48.



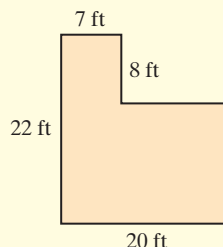
49.



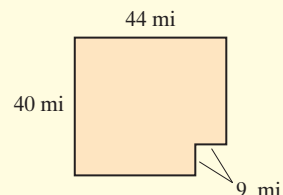
50.



51.



52.



Review

Simplify. See Section 1.7.

53. $5 + 6 \cdot 3$
54. $25 - 3 \cdot 7$
55. $(20 - 16) \div 4$
56. $6 \cdot (8 + 2)$
57. $72 \div (2 \cdot 6)$
58. $(72 \div 2) \cdot 6$
59. $(18 + 8) - (12 + 4)$
60. $4^1 \cdot (2^3 - 8)$

Concept Extensions

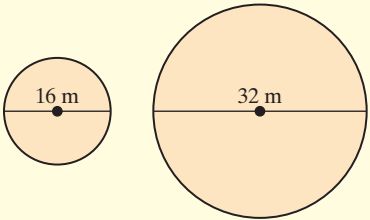
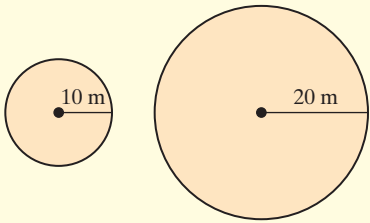
There are a number of factors that determine the dimensions of a rectangular soccer field. Use the table below to answer Exercises 61 and 62.

Soccer Field Width and Length		
Age	Width Min–Max	Length Min–Max
Under 6/7:	15–20 yards	25–30 yards
Under 8:	20–25 yards	30–40 yards
Under 9:	30–35 yards	40–50 yards
Under 10:	40–50 yards	60–70 yards
Under 11:	40–50 yards	70–80 yards
Under 12:	40–55 yards	100–105 yards
Under 13:	50–60 yards	100–110 yards
International:	70–80 yards	110–120 yards

61. a. Find the minimum length and width of a soccer field for 8-year-old children. (Carefully consider the age.)
b. Find the perimeter of this field.
62. a. Find the maximum length and width of a soccer field for 12-year-old children.
b. Find the perimeter of this field.

Solve. See the Concept Check in this section. Choose the figure that has the greater distance around.

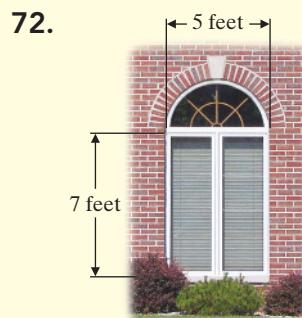
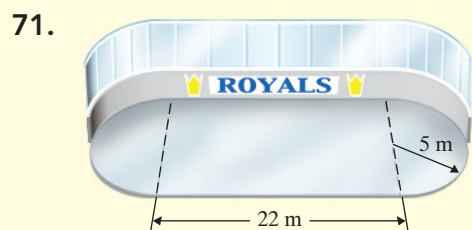
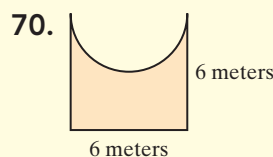
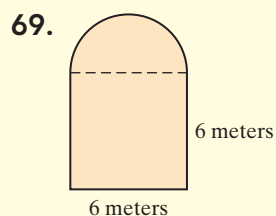
63. a. A square with side length 3 inches
b. A circle with diameter 4 inches
64. a. A circle with diameter 7 inches
b. A square with side length 7 inches
65. a. Find the circumference of each circle.
Approximate the circumference by using 3.14 for π .
66. a. Find the circumference of each circle.
Approximate the circumference by using 3.14 for π .



- b. If the radius of a circle is doubled, is its corresponding circumference doubled?
- b. If the diameter of a circle is doubled, is its corresponding circumference doubled?

67. In your own words, explain how to find the perimeter of any polygon.
68. In your own words, explain how perimeter and circumference are the same and how they are different.

Find the perimeter. Round your results to the nearest tenth.



9.3 Area, Volume, and Surface Area

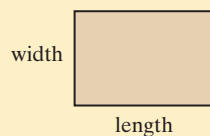
Objective A Finding Area of Plane Regions

Recall that area measures the number of square units that cover the surface of a plane region; that is, a region that lies in a plane. Thus far, we know how to find the areas of a rectangle and a square. These formulas, as well as formulas for finding the areas of other common geometric figures, are given next.

Area Formulas of Common Geometric Figures

Geometric Figure

RECTANGLE



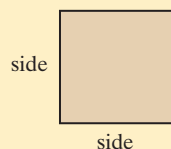
Area Formula

Area of a rectangle:

$$\text{Area} = \text{length} \cdot \text{width}$$

$$A = lw$$

SQUARE



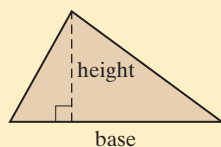
Area of a square:

$$\text{Area} = \text{side} \cdot \text{side}$$

$$A = s \cdot s = s^2$$

Objectives

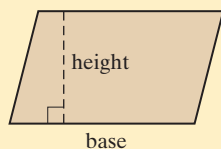
- A** Find the Area of Plane Regions. ▶
- B** Find the Volume and Surface Area of Solids. ▶

Geometric Figure**TRIANGLE****Area Formula**

Area of a triangle:

$$\text{Area} = \frac{1}{2} \cdot \text{base} \cdot \text{height}$$

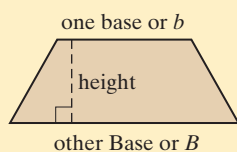
$$A = \frac{1}{2}bh$$

PARALLELOGRAM

Area of a parallelogram:

$$\text{Area} = \text{base} \cdot \text{height}$$

$$A = bh$$

TRAPEZOID

Area of a trapezoid:

$$\text{Area} = \frac{1}{2} \cdot (\text{one base} + \text{other Base}) \cdot \text{height}$$

$$A = \frac{1}{2}(b + B)h$$

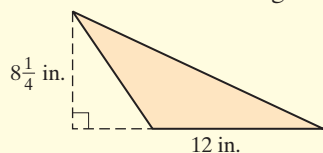
Use these formulas for the following examples.

Helpful Hint

Area is always measured in square units.

Practice 1

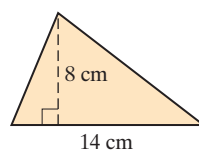
Find the area of the triangle.

**Answer**

1. $49\frac{1}{2}$ sq in.

Example 1

Find the area of the triangle.

**Solution:**

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2} \cdot 14 \text{ cm} \cdot 8 \text{ cm}$$

Replace b , base, with 14 cm and h , height, with 8 cm.

$$= \frac{2 \cdot 7 \cdot 8}{2} \text{ square centimeters}$$

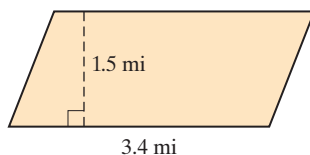
Write 14 as $2 \cdot 7$.

$$= 56 \text{ square centimeters}$$

The area is 56 square centimeters.

Work Practice 1

Example 2 Find the area of the parallelogram.



Solution:

$$\begin{aligned} A &= bh \\ &= 3.4 \text{ miles} \cdot 1.5 \text{ miles} && \text{Replace } b, \text{ base, with 3.4 miles and } h, \text{ height, with 1.5 miles.} \\ &= 5.1 \text{ square miles} \end{aligned}$$

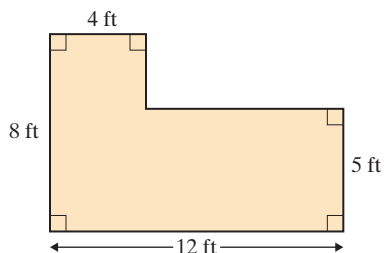
The area is 5.1 square miles.

Work Practice 2

Helpful Hint

When finding the area of figures, check to make sure that all measurements are in the same units before calculations are made.

Example 3 Find the area of the figure.



Solution: Split the figure into two rectangles. To find the area of the figure, we find the sum of the areas of the two rectangles.

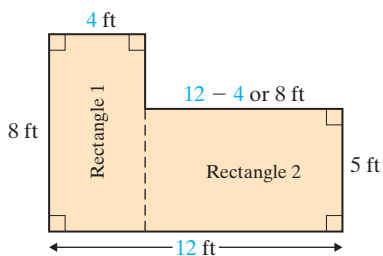
$$\begin{aligned} \text{area of Rectangle 1} &= lw \\ &= 8 \text{ feet} \cdot 4 \text{ feet} \\ &= 32 \text{ square feet} \end{aligned}$$

Notice that the length of Rectangle 2 is 12 feet $-$ 4 feet or 8 feet.

$$\begin{aligned} \text{area of Rectangle 2} &= lw \\ &= 8 \text{ feet} \cdot 5 \text{ feet} \\ &= 40 \text{ square feet} \end{aligned}$$

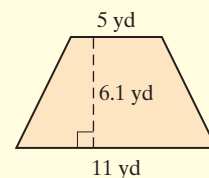
$$\begin{aligned} \text{area of the figure} &= \text{area of Rectangle 1} + \text{area of Rectangle 2} \\ &= 32 \text{ square feet} + 40 \text{ square feet} \\ &= 72 \text{ square feet} \end{aligned}$$

Work Practice 3



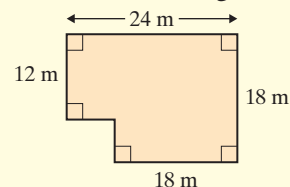
Practice 2

Find the area of the trapezoid.



Practice 3

Find the area of the figure.

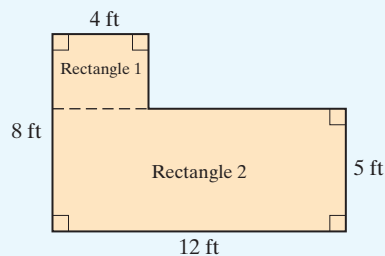


Answers

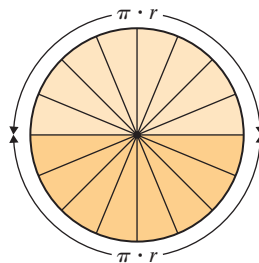
2. 48.8 sq yd 3. 396 sq m

Helpful Hint

The figure in Example 3 could also be split into two rectangles as shown.

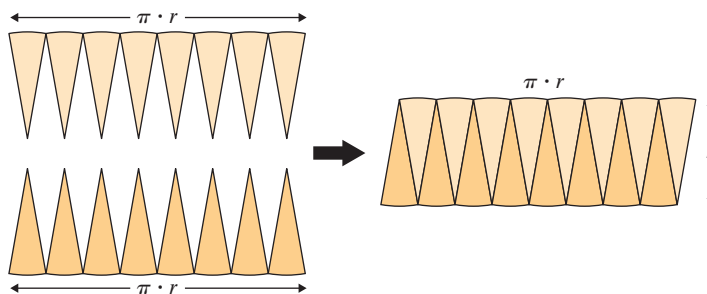


To better understand the formula for area of a circle, try the following. Cut a circle into many pieces, as shown.



The circumference of a circle is $2\pi r$. This means that the circumference of half a circle is half of $2\pi r$, or πr .

Then unfold the two halves of the circle and place them together, as shown.



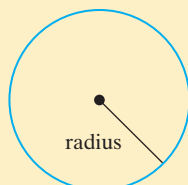
The figure on the right is almost a parallelogram with a base of πr and a height of r . The area is

$$\begin{aligned} A &= \text{base} \cdot \text{height} \\ &= (\pi r) \cdot r \\ &= \pi r^2 \end{aligned}$$

This is the formula for the area of a circle.

Area Formula of a Circle

Circle



Area of a circle

$$\text{Area} = \pi \cdot (\text{radius})^2$$

$$A = \pi r^2$$

(A fraction approximation for π is $\frac{22}{7}$.)

(A decimal approximation for π is 3.14.)

Example 4 Find the area of a circle with a radius of 3 feet. Find the exact area and an approximation. Use 3.14 as an approximation for π .

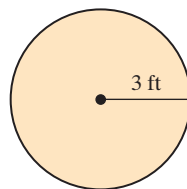
Solution: We let $r = 3$ feet and use the formula

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \cdot (3 \text{ feet})^2 && \text{Replace } r \text{ with 3 feet.} \\ &= 9 \cdot \pi \text{ square feet} && \text{Replace } (3 \text{ feet})^2 \text{ with 9 sq ft.} \end{aligned}$$

To approximate this area, we substitute 3.14 for π .

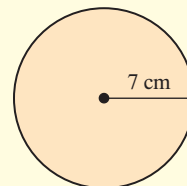
$$\begin{aligned} 9 \cdot \pi \text{ square feet} &\approx 9 \cdot 3.14 \text{ square feet} \\ &= 28.26 \text{ square feet} \end{aligned}$$

The *exact* area of the circle is 9π square feet, which is *approximately* 28.26 square feet.



Practice 4

Find the area of the given circle. Find the exact area and an approximation. Use 3.14 as an approximation for π .

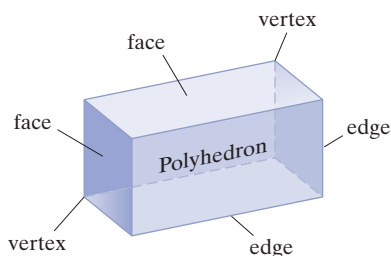


Work Practice 4

✓ Concept Check Use estimation to decide which figure would have a larger area: a circle of diameter 10 in. or a square 10 in. long on each side.

Objective B Finding Volume and Surface Area of Solids

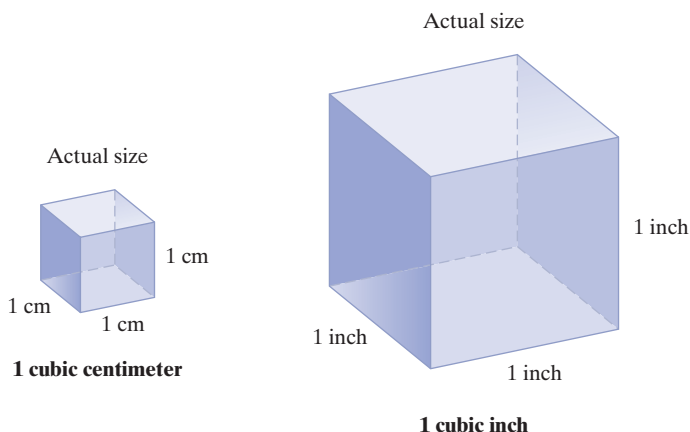
A **convex solid** is a set of points, S , not all in one plane, such that for any two points A and B in S , all points between A and B are also in S . In this section, we will find the volume and surface area of special types of solids called polyhedrons. A solid formed by the intersection of a finite number of planes is called a **polyhedron**. The box to the right is an example of a polyhedron.



Each of the plane regions of a polyhedron is called a **face** of the polyhedron. If the intersection of two faces is a line segment, this line segment is an **edge** of the polyhedron. The intersections of the edges are the **vertices** of the polyhedron.

Volume is a measure of the space of a solid. The volume of a box or can, for example, is the amount of space inside. Volume can be used to describe the amount of juice in a pitcher or the amount of concrete needed to pour a foundation for a house.

The volume of a solid is the number of **cubic units** in the solid. A cubic centimeter and a cubic inch are illustrated.



Answer

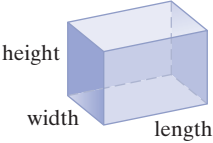
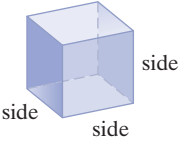
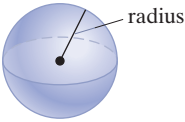
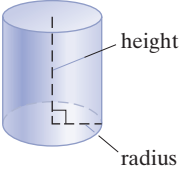
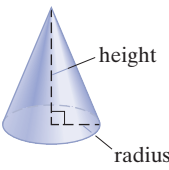
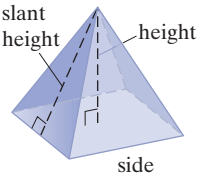
4. 49π sq cm $\approx$ 153.86 sq cm

✓ Concept Check Answer

A square 10 in. long on each side would have a larger area.

The **surface area** of a polyhedron is the sum of the areas of the faces of the polyhedron. For example, each face of the cube to the left on the previous page has an area of 1 square centimeter. Since there are 6 faces of the cube, the sum of the areas of the faces is 6 square centimeters. Surface area can be used to describe the amount of material needed to cover a solid. Surface area is measured in square units.

Formulas for finding the volumes, V , and surface areas, SA , of some common solids are given next. (Note: Spheres, circular cylinders, and cones are not polyhedrons, but they are solids and we will calculate surface areas and volumes of these solids.)

Volume and Surface Area Formulas of Common Solids	
Solid	Formulas
RECTANGULAR SOLID 	$V = lwh$ $SA = 2lh + 2wh + 2lw$ where h = height, w = width, l = length
CUBE 	$V = s^3$ $SA = 6s^2$ where s = side
SPHERE 	$V = \frac{4}{3}\pi r^3$ $SA = 4\pi r^2$ where r = radius
CIRCULAR CYLINDER 	$V = \pi r^2 h$ $SA = 2\pi rh + 2\pi r^2$ where h = height, r = radius
CONE 	$V = \frac{1}{3}\pi r^2 h$ $SA = \pi r\sqrt{r^2 + h^2} + \pi r^2$ where h = height, r = radius
SQUARE-BASED PYRAMID 	$V = \frac{1}{3}s^2 h$ $SA = B + \frac{1}{2}pl$ where B = area of base, p = perimeter of base, h = height, s = side, l = slant height

Example 5

Find the volume and surface area of a rectangular box that is 12 inches long, 6 inches wide, and 3 inches high.



Solution: Let $h = 3$ in., $l = 12$ in., and $w = 6$ in.

$$V = lwh$$

$$V = 12 \text{ inches} \cdot 6 \text{ inches} \cdot 3 \text{ inches} = 216 \text{ cubic inches}$$

The volume of the rectangular box is 216 cubic inches.

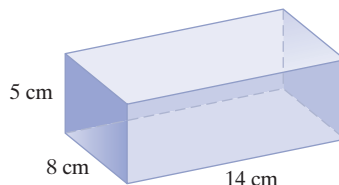
$$\begin{aligned} SA &= 2lh + 2wh + 2lw \\ &= 2(12 \text{ in.})(3 \text{ in.}) + 2(6 \text{ in.})(3 \text{ in.}) + 2(12 \text{ in.})(6 \text{ in.}) \\ &= 72 \text{ sq in.} + 36 \text{ sq in.} + 144 \text{ sq in.} \\ &= 252 \text{ sq in.} \end{aligned}$$

The surface area of the rectangular box is 252 square inches.

Work Practice 5

✓ Concept Check Juan is calculating the volume of the following rectangular solid. Find the error in his calculation.

~~$$\begin{aligned} \text{Volume} &= l + w + h \\ &= 14 \text{ cm} + 8 \text{ cm} + 5 \text{ cm} \\ &= 27 \text{ cu cm} \end{aligned}$$~~

**Example 6**

Find the volume and surface area of a ball of radius 2 inches. Give the exact volume and surface area. Then use the approximation $\frac{22}{7}$ for π .



Solution:

$$V = \frac{4}{3}\pi r^3$$

Formula for volume of a sphere

$$V = \frac{4}{3} \cdot \pi (2 \text{ in.})^3$$

Let $r = 2$ inches.

$$= \frac{32}{3}\pi \text{ cu in.}$$

Exact volume

$$\approx \frac{32}{3} \cdot \frac{22}{7} \text{ cu in.}$$

Approximate π with $\frac{22}{7}$.

$$= \frac{704}{21} \text{ or } 33\frac{11}{21} \text{ cu in.}$$

Approximate volume

(Continued on next page)

Practice 5

Find the volume and surface area of a rectangular box that is 7 feet long, 3 feet wide, and 4 feet high.

Practice 6

Find the volume and surface area of a ball of radius $\frac{1}{2}$ centimeter. Give the exact volume and surface area. Then use $\frac{22}{7}$ for π and approximate the values.

Answers

5. $V = 84$ cu ft; $SA = 122$ sq ft

6. $V = \frac{1}{6}\pi$ cu cm $\approx \frac{11}{21}$ cu cm;

$$SA = \pi \text{ sq cm} \approx 3\frac{1}{7} \text{ sq cm}$$

✓ Concept Check Answer

$$\begin{aligned} \text{Volume} &= lwh \\ &= 14 \text{ cm} \cdot 8 \text{ cm} \cdot 5 \text{ cm} \\ &= 560 \text{ cu cm} \end{aligned}$$

The volume of the sphere is exactly $\frac{32}{3}\pi$ cubic inches or approximately $33\frac{11}{21}$ cubic inches.

$$SA = 4\pi r^2$$

Formula for surface area

$$SA = 4 \cdot \pi (2 \text{ in.})^2$$

Let $r = 2$ inches.

$$= 16\pi \text{ sq in.}$$

Exact surface area

$$\approx 16 \cdot \frac{22}{7} \text{ sq in.}$$

Approximate π with $\frac{22}{7}$.

$$= \frac{352}{7} \text{ or } 50\frac{2}{7} \text{ sq in.}$$

Approximate surface area

The surface area of the sphere is exactly 16π square inches or approximately $50\frac{2}{7}$ square inches.

Work Practice 6

Practice 7

Find the volume of a cylinder of radius 5 inches and height 9 inches. Give the exact answer and an approximate answer. Use 3.14 for π .

Example 7

Find the volume of a can that has a $3\frac{1}{2}$ -inch radius and a height of 6 inches. Give the exact volume and an approximate volume. Use $\frac{22}{7}$ for π .



Solution: Using the formula for a circular cylinder, we have

$$\begin{aligned} V &= \pi \cdot r^2 \cdot h \\ &= \pi \cdot \left(\frac{7}{2} \text{ in.}\right)^2 \cdot 6 \text{ in.} \\ &= \pi \cdot \frac{49}{4} \text{ sq in.} \cdot 6 \text{ in.} \\ &= \frac{\pi \cdot 49 \cdot \cancel{2} \cdot 3}{\cancel{2} \cdot 2} \text{ cu in.} \\ &= 73\frac{1}{2}\pi \text{ cu in. or } 73.5\pi \text{ cu in.} \end{aligned}$$

This is the exact volume. To approximate the volume, use the approximation $\frac{22}{7}$ for π .

$$\begin{aligned} V &= 73\frac{1}{2}\pi \approx \frac{147}{2} \cdot \frac{22}{7} \text{ cu in.} \\ &= \frac{21 \cdot \cancel{7} \cdot \cancel{2} \cdot 11}{\cancel{2} \cdot \cancel{7}} \text{ cu in.} \\ &= 231 \text{ cu in.} \end{aligned}$$

The volume is approximately 231 cubic inches.

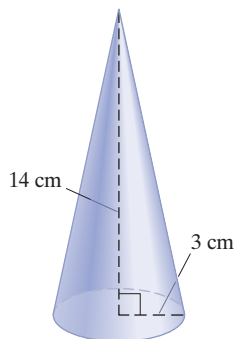
Work Practice 7

Answer

7. 225π cu in. ≈ 706.5 cu in.

Example 8

Find the volume of a cone that has a height of 14 centimeters and a radius of 3 centimeters. Give the exact answer and an approximate answer. Use 3.14 for π .



Solution: Using the formula for volume of a cone, we have

$$\begin{aligned} V &= \frac{1}{3} \cdot \pi \cdot r^2 \cdot h \\ &= \frac{1}{3} \cdot \pi \cdot (3 \text{ cm})^2 \cdot 14 \text{ cm} \quad \text{Replace } r \text{ with 3 cm and } h \text{ with 14 cm.} \\ &= 42\pi \text{ cu cm} \end{aligned}$$

Thus, 42π cubic centimeters is the exact volume. To approximate the volume, use the approximation 3.14 for π .

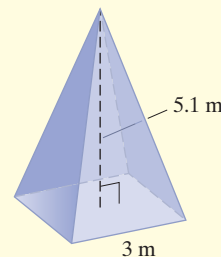
$$\begin{aligned} V &\approx 42 \cdot 3.14 \text{ cu cm} \quad \text{Replace } \pi \text{ with 3.14.} \\ &= 131.88 \text{ cu cm} \end{aligned}$$

The volume is approximately **131.88 cubic centimeters**.

 Work Practice 8

Practice 8

Find the volume of a square-based pyramid that has a 3-meter side and a height of 5.1 meters.



Answer

8. 15.3 cu m

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

area	surface area	cubic
volume	square	

- The _____ of a polyhedron is the sum of the areas of its faces.
- The measure of the amount of space inside a solid is its _____.
- _____ measures the amount of surface enclosed by a region.
- Volume is measured in _____ units.
- Area is measured in _____ units.
- Surface area is measured in _____ units.

Martin-Gay Interactive Videos Watch the section lecture video and answer the following questions.



See Video 9.3

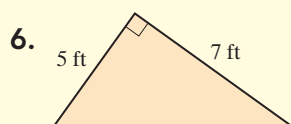
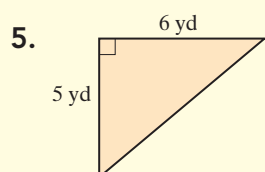
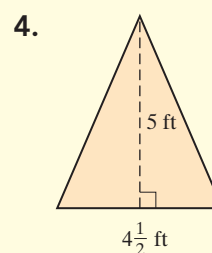
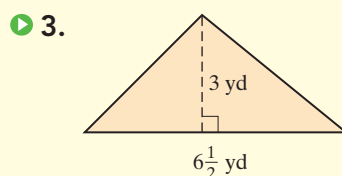
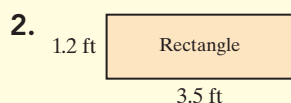
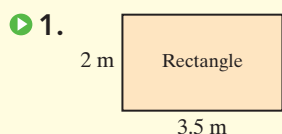
Objective A 7. Explain why we need to use a formula twice to solve

Example 3.

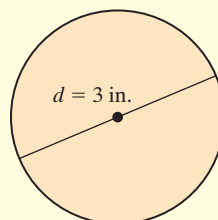
Objective B 8. In Examples 7 and 8, explain the difference in the two volume answers found for each.

9.3 Exercise Set MyLab Math

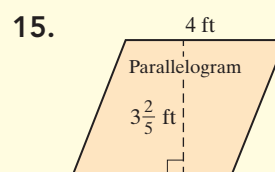
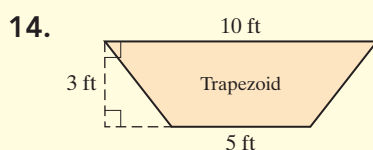
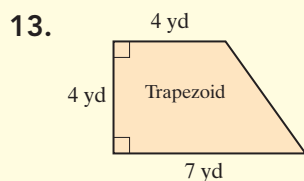
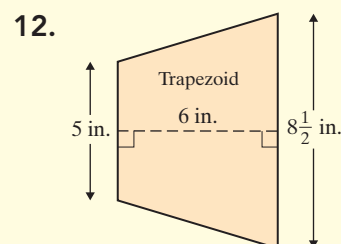
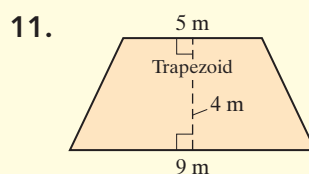
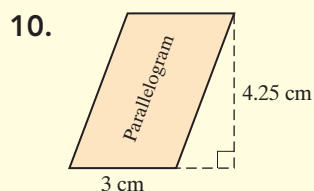
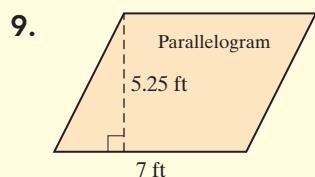
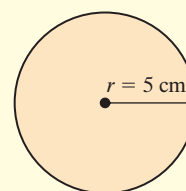
Objective A Find the area of each geometric figure. If the figure is a circle, give the exact area and then use the given approximation for π to approximate the area. See Examples 1 through 4.

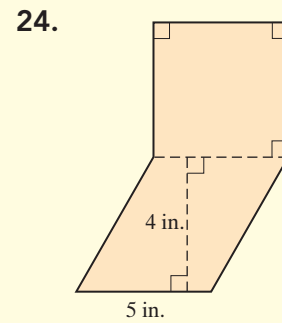
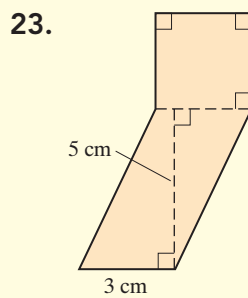
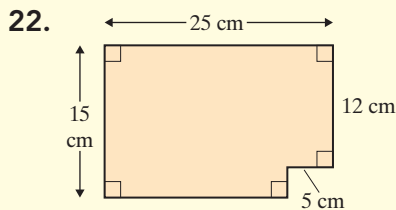
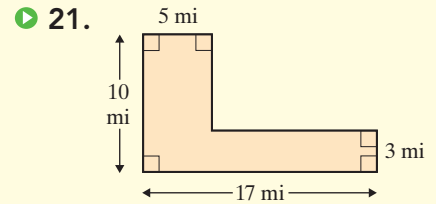
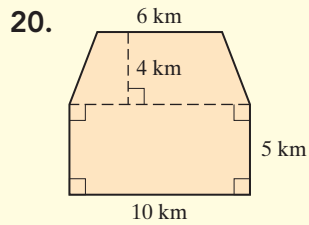
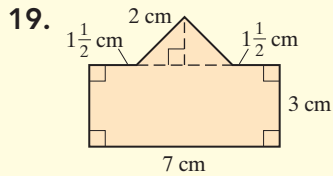
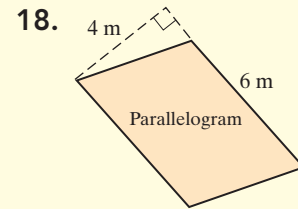
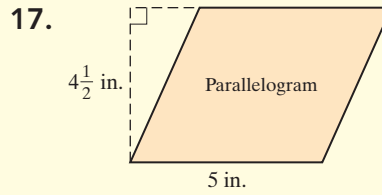
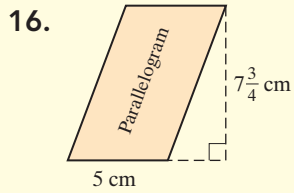


7. Use 3.14 for π .

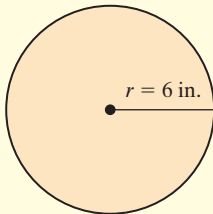


8. Use $\frac{22}{7}$ for π .

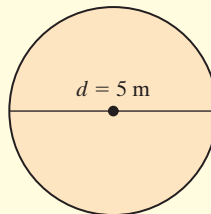




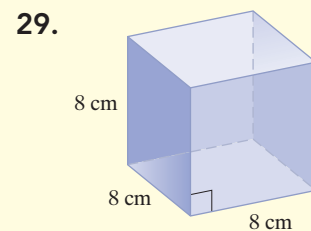
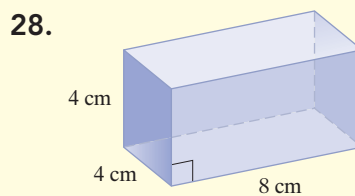
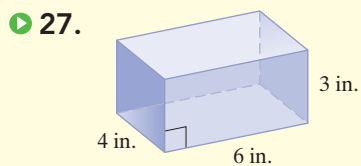
25. Use $\frac{22}{7}$ for π .



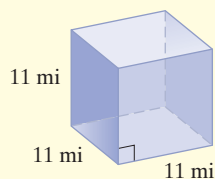
26. Use 3.14 for π .



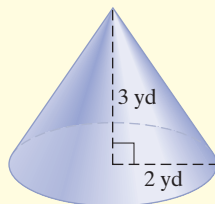
Objective B Find the volume and surface area of each solid. See Examples 5 through 8. For formulas containing π , give the exact answer and then approximate using $\frac{22}{7}$ for π .



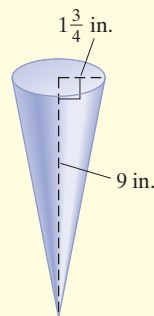
30.



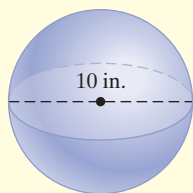
31. For surface area, use $\pi = 3.14$ and round to the nearest hundredth.



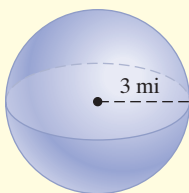
32. For surface area, use $\pi = 3.14$ and round to the nearest hundredth.



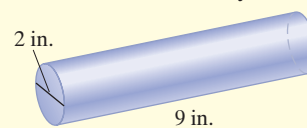
▶ 33.



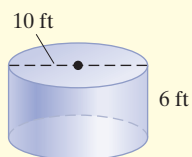
34.



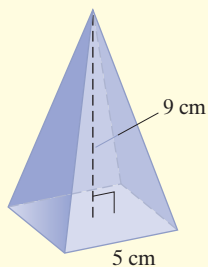
▶ 35. Find the volume only.



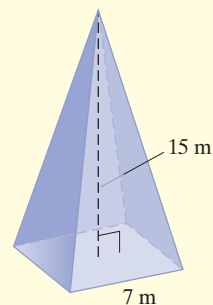
36. Find the volume only.



37. Find the volume only.

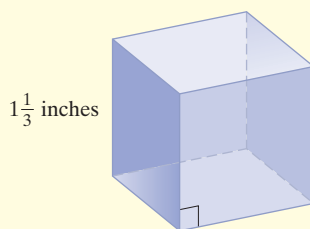


38. Find the volume only.

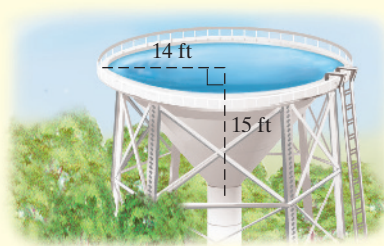


Objectives A B Mixed Practice Solve. See Examples 1 through 8.

39. Find the volume of a cube with edges of $1\frac{1}{3}$ inches.

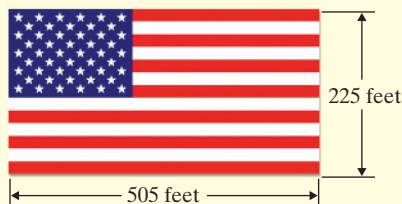


40. A water storage tank is in the shape of a cone with the pointed end down. If the radius is 14 ft and the depth of the tank is 15 ft, approximate the volume of the tank in cubic feet. Use $\frac{22}{7}$ for π .



41. Find the volume and surface area of a rectangular box 2 ft by 1.4 ft by 3 ft.

43. The largest American flag measures 505 feet by 225 feet. It's the U.S. "Super flag" owned by "Ski" Demski of Long Beach, California. Find its area. (Source: Guinness World Records)



42. Find the volume and surface area of a box in the shape of a cube that is 5 ft on each side.

44. The largest indoor illuminated sign is a billboard at Dubai Airport. It measures 28 meters by 6.2 meters. Find its area. (Source: The Guinness Book of World Records)



- ▶ 45. A drapery panel measures 6 ft by 7 ft. Find how many square feet of material are needed for *four* panels.

47. A paperweight is in the shape of a square-based pyramid 20 centimeters tall. If an edge of the base is 12 centimeters, find the volume of the paperweight.

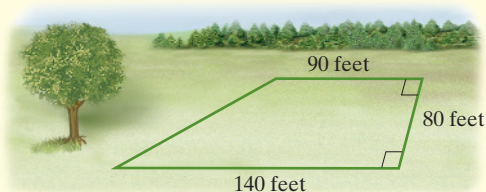


46. A page in this book measures 27.6 cm by 21.5 cm. Find its area.

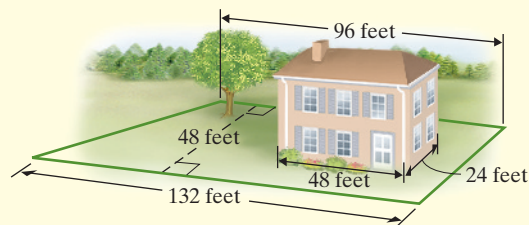
48. A birdbath is made in the shape of a hemisphere (half-sphere). If its radius is 10 inches, approximate its volume. Use $\frac{22}{7}$ for π .



49. Find how many square feet of land are in the following plot:



50. For Gerald Gomez to determine how much grass seed he needs to buy, he must know the size of his yard. Use the drawing to determine how many square feet are in his yard.

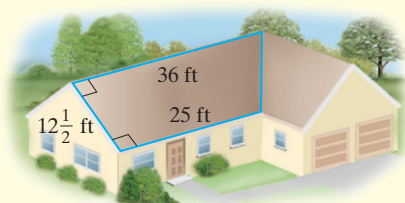


51. Find the exact volume and surface area of a sphere with a radius of 7 inches.

52. A tank is in the shape of a cylinder 8 feet tall and 3 feet in radius. Find the exact volume and surface area of the tank.

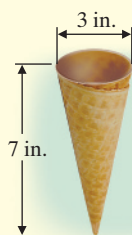
53. The outlined part of the roof shown is in the shape of a trapezoid and needs to be shingled. The number of shingles to buy depends on the area.

a. Use the dimensions given to find the area of the outlined part of the roof to the nearest whole square foot.



b. Shingles are packaged in a unit called a “square.” If a “square” covers 100 square feet, how many whole squares need to be purchased to shingle this part of the roof?

55. Find the exact volume of a waffle ice cream cone with a 3-in. diameter and a height of 7 inches.



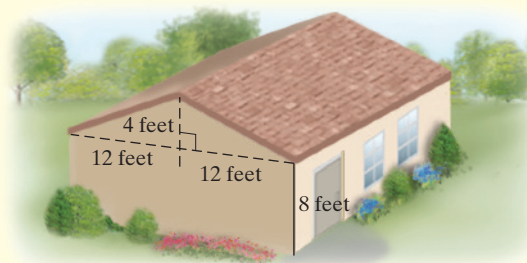
57. Paul Revere’s Pizza in the USA will bake and deliver a round pizza with a 4-foot diameter. This pizza is called the “Ultimate Party Pizza” and its current price is \$99.99. Find the exact area of the top of the pizza and an approximation. Use 3.14 as an approximation for π .

59. Zorbing is an extreme sport invented by two New Zealanders who joke that they were looking for a way to walk on water. A Zorb is a large sphere inside a second sphere with the space between the spheres pumped full of air. There is a tunnel-like opening so a person can crawl into the inner sphere. You are strapped in and sent down a Zorbing hill. A standard Zorb is approximately 3 m in diameter. Find the exact volume of a Zorb, and approximate the volume using 3.14 for π .



54. The entire side of the building shaded in the drawing is to be bricked. The number of bricks to buy depends on the area.

a. Find the area.



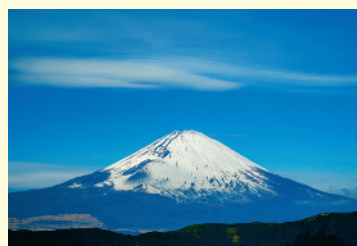
b. If the side area of each brick (including mortar room) is $\frac{1}{6}$ square ft, find the number of bricks that are needed to brick the end of the building.

56. A snow globe has a diameter of 6 inches. Find its exact volume. Then approximate its volume using 3.14 for π .



58. The face of a circular watch has a diameter of 2 centimeters. What is its area? Find the exact area and an approximation. Use 3.14 as an approximation for π .

60. Mount Fuji, in Japan, is considered the most beautiful composite volcano in the world. The mountain is in the shape of a cone whose height is about 3.5 kilometers and whose base radius is about 3 kilometers. Approximate the volume of Mt. Fuji in cubic kilometers. Use $\frac{22}{7}$ for π .



61. A $10\frac{1}{2}$ -foot by 16-foot concrete wall is to be built using concrete blocks. Find the area of the wall.
62. The floor of Terry's attic is 24 feet by 35 feet. Find how many square feet of insulation are needed to cover the attic floor.
63. Find the volume of a pyramid with a square base 5 inches on a side and a height of $1\frac{3}{10}$ inches.
64. Approximate to the nearest hundredth the volume of a sphere with a radius of 2 centimeters. Use 3.14 for π .

The Space Cube is supposed to be the world's smallest computer, with dimensions of 2 inches by 2 inches by 2.2 inches.

65. Find the volume of the Space Cube.
66. Find the surface area of the Space Cube.

Review

Evaluate. See Section 1.7

- | | | | |
|-----------------|-----------------|-----------------|-----------------|
| 67. 5^2 | 68. 7^2 | 69. 3^2 | 70. 20^2 |
| 71. $1^2 + 2^2$ | 72. $5^2 + 3^2$ | 73. $4^2 + 2^2$ | 74. $1^2 + 6^2$ |

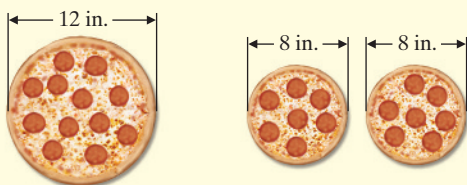
Concept Extensions

Given the following situations, tell whether you are more likely to be concerned with area or perimeter.

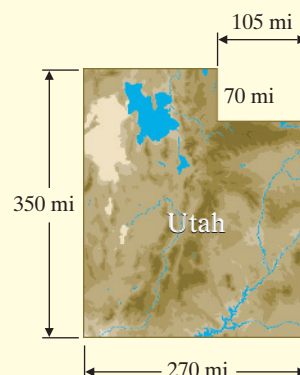
- | | |
|--|--|
| 75. ordering fencing to fence a yard | 76. ordering grass seed to plant in a yard |
| 77. buying carpet to install in a room | 78. buying gutters to install on a house |
| 79. ordering paint to paint a wall | 80. ordering baseboards to install in a room |
| 81. buying a wallpaper border to go on the walls around a room | 82. buying fertilizer for your yard |

Solve.

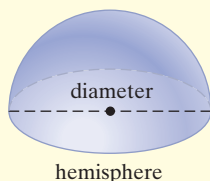
83. A pizza restaurant recently advertised two specials. The first special was a 12-inch pizza for \$10. The second special was two 8-inch pizzas for \$9. Determine the better buy. (*Hint: First compare the areas of the two specials and then find a price per square inch for both specials.*)



84. Find the approximate area of the state of Utah.



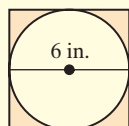
85. The Hayden Planetarium, at the Museum of Natural History in New York City, boasts a dome that has a diameter of 20 m. The dome is a hemisphere, or half a sphere. What is the volume enclosed by the dome at the Hayden Planetarium? Use 3.14 for π and round to the nearest hundredth. (Source: Hayden Planetarium)



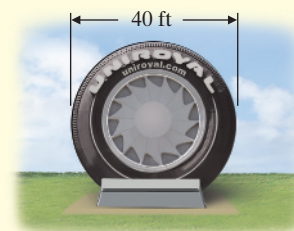
86. The Adler Museum in Chicago has a planetarium, its StarRider Theater, that has a diameter of 55 feet. Find the surface area of its hemispheric (half a sphere) dome. Use 3.14 for π . (Source: The Adler Museum)

87. Can you compute the volume of a rectangle? Why or why not?
88. In your own words, explain why perimeter is measured in units and area is measured in square units. (Hint: See Section 1.5 for an introduction to the meaning of area.)

89. Find the area of the shaded region. Use the approximation 3.14 for π .

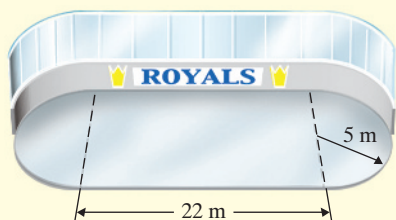


90. Michigan is the home of the Uniroyal Giant Tire sculpture, which was built originally as a Ferris wheel for the 1965 New York World's Fair. The diameter of this eight-story sculpture is 40 ft. Find the exact area of the face of the tire and an approximation. Use $\pi \approx 3.14$. (Source: Guinness Book of World Records)

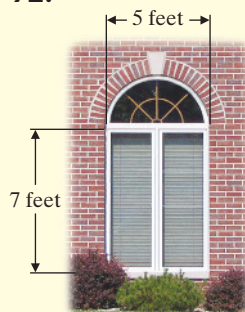


Find the area of each figure. If needed, use $\pi \approx 3.14$ and round results to the nearest tenth.

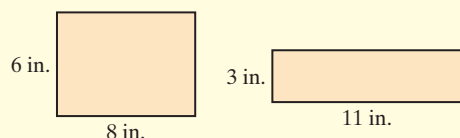
91. Find the skating area.



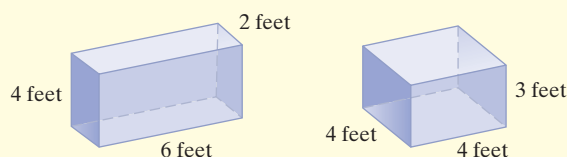
- 92.



93. Do two rectangles with the same perimeter have the same area? To see, find the perimeter and the area of each rectangle.



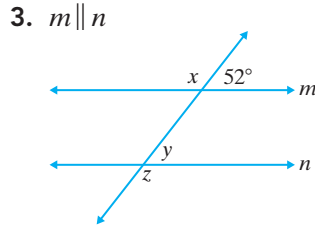
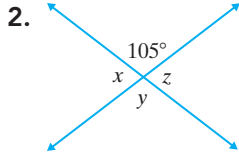
94. Do two rectangular solids with the same volume have the same surface area? To see, find the volume and surface area of each rectangular solid.



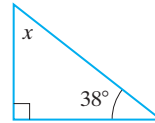
Geometry Concepts

1. Find the supplement and the complement of a 27° angle.

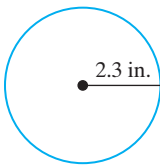
Find the measures of angles x , y , and z in each figure in Exercises 2 and 3.



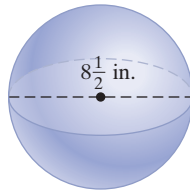
4. Find the measure of $\angle x$. (Hint: The sum of the angle measures of a triangle is 180° .)



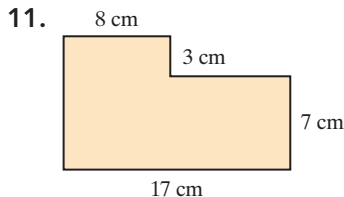
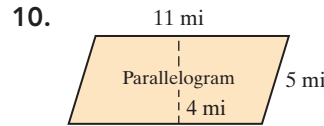
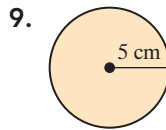
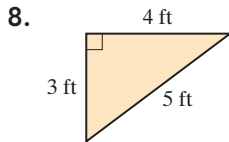
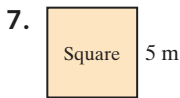
5. Find the diameter.



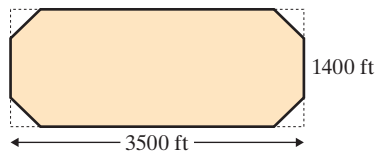
6. Find the radius.



For Exercises 7 through 11, find the perimeter (or circumference) and area of each figure. For the circle, give the exact circumference and area. Then use $\pi \approx 3.14$ to approximate each. Don't forget to attach correct units.



12. The smallest cathedral is in High-landville, Missouri. The rectangular floor of the cathedral measures 14 feet by 17 feet. Find its perimeter and its area. (Source: *The Guinness Book of World Records*)



13. A new Tesla Giga factory is being constructed in Nevada. Although the factory will be in the shape of an emerald, find the perimeter and area of a rectangle with length 3500 feet and width 1400 feet.

Find the volume of each solid. Don't forget to attach correct units. For Exercises 14 and 15, find the surface area also.

14. A cube with edges of 4 inches each

15. A rectangular box 2 feet by 3 feet by 5.1 feet

16. A pyramid with a square base 10 centimeters on a side and a height of 12 centimeters

17. A sphere with a diameter of 3 miles. Give the exact volume and then use $\pi \approx \frac{22}{7}$ to approximate.

Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____

9.4 Linear Measurement

Objectives

- A** Define U.S. Units of Length and Convert from One Unit to Another. 
- B** Use Mixed U.S. Units of Length. 
- C** Perform Arithmetic Operations on U.S. Units of Length. 
- D** Define Metric Units of Length and Convert from One Unit to Another. 
- E** Perform Arithmetic Operations on Metric Units of Length. 

Objective A Defining and Converting U.S. System Units of Length

In the United States, two systems of measurement are commonly used. They are the **United States (U.S.), or English, measurement system** and the **metric system**. The U.S. measurement system is familiar to most Americans. Units such as feet, miles, ounces, and gallons are used. However, the metric system is also commonly used in fields such as medicine, sports, international marketing, and certain physical sciences. We are accustomed to buying 2-liter bottles of soft drinks, watching televised coverage of the 100-meter dash at the Olympic Games, or taking a 200-milligram dose of pain reliever.

The U.S. system of measurement uses the **inch, foot, yard, and mile** to measure **length**. The following is a summary of equivalencies between units of length:

U.S. Units of Length

$$\begin{aligned}12 \text{ inches (in.)} &= 1 \text{ foot (ft)} \\3 \text{ feet} &= 1 \text{ yard (yd)} \\36 \text{ inches} &= 1 \text{ yard} \\5280 \text{ feet} &= 1 \text{ mile (mi)}\end{aligned}$$

To convert from one unit of length to another, we will use **unit fractions**. We define a unit fraction to be a fraction that is equivalent to 1. Examples of unit fractions are as follows:

Unit Fractions

$$\begin{aligned}\frac{12 \text{ in.}}{1 \text{ ft}} &= 1 \text{ or } \frac{1 \text{ ft}}{12 \text{ in.}} = 1 \text{ (since } 12 \text{ in.} = 1 \text{ ft)} \\ \frac{3 \text{ ft}}{1 \text{ yd}} &= 1 \text{ or } \frac{1 \text{ yd}}{3 \text{ ft}} = 1 \text{ (since } 3 \text{ ft} = 1 \text{ yd)} \\ \frac{5280 \text{ ft}}{1 \text{ mi}} &= 1 \text{ or } \frac{1 \text{ mi}}{5280 \text{ ft}} = 1 \text{ (since } 5280 \text{ ft} = 1 \text{ mi)}\end{aligned}$$

Remember that multiplying a number by 1 does not change the value of the number.

Example 1 Convert 8 feet to inches.

Solution: We multiply 8 feet by a unit fraction that uses the equality $12 \text{ inches} = 1 \text{ foot}$. The unit fraction should be in the form $\frac{\text{units to convert to}}{\text{original units}}$ or, in this case, $\frac{12 \text{ inches}}{1 \text{ foot}}$. We do this so that like units will divide out to 1, as shown.

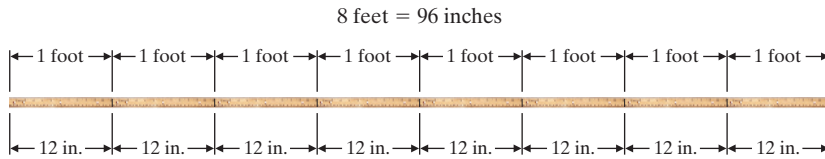
$$\begin{aligned}8 \text{ ft} &= \frac{8 \text{ ft}}{1} \cdot 1 \\&= \frac{8 \cancel{\text{ft}}}{1} \cdot \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} \quad \text{Multiply by 1 in the form of } \frac{12 \text{ in.}}{1 \text{ ft}}. \\&= 8 \cdot 12 \text{ in.} \\&= 96 \text{ in.} \quad \text{Multiply.}\end{aligned}$$

Practice 1

Convert 6 feet to inches.

Answer
1. 72 in.

Thus, $8 \text{ ft} = 96 \text{ in.}$, as shown in the diagram:



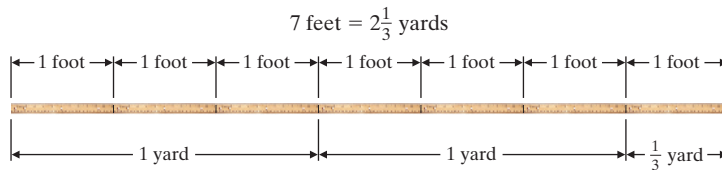
Work Practice 1

Example 2 Convert 7 feet to yards.

Solution: We multiply by a unit fraction that compares 1 yard to 3 feet.

$$\begin{aligned}
 7 \text{ ft} &= \frac{7 \text{ ft}}{1} \cdot 1 \\
 &= \frac{7 \text{ ft}}{1} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} && \leftarrow \text{Units to convert to} \\
 &&& \leftarrow \text{Original units} \\
 &= \frac{7}{3} \text{ yd} \\
 &= 2\frac{1}{3} \text{ yd} && \text{Divide.}
 \end{aligned}$$

Thus, $7 \text{ ft} = 2\frac{1}{3} \text{ yd}$, as shown in the diagram.



Work Practice 2

Example 3 Finding the Length of a Pelican's Bill

The Australian pelican has the longest bill, measuring from 13 to 18.5 inches long. The pelican in the photo has a 15-inch bill. Convert 15 inches to feet, using decimals in your final answer.

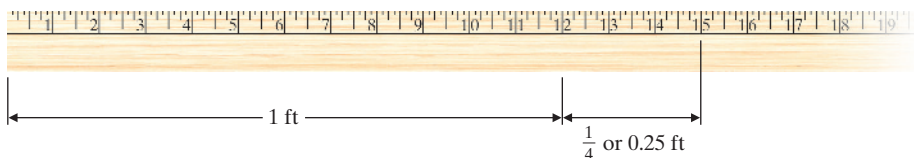
Solution:

$$\begin{aligned}
 15 \text{ in.} &= \frac{15 \text{ in.}}{1} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} && \leftarrow \text{Units to convert to} \\
 &&& \leftarrow \text{Original units} \\
 &= \frac{15}{12} \text{ ft} \\
 &= \frac{5}{4} \text{ ft} && \text{Simplify } \frac{15}{12}. \\
 &= 1.25 \text{ ft} && \text{Divide.}
 \end{aligned}$$



Thus, $15 \text{ in.} = 1.25 \text{ ft}$, as shown in the diagram.

$15 \text{ inches} = 1.25 \text{ ft}$



Work Practice 3

Practice 2

Convert 8 yards to feet.

Helpful Hint

When converting from one unit to another, select a unit fraction with the properties below:

units you are converting to
original units

By using this unit fraction, the original units will divide out, as wanted.

Practice 3

Suppose the pelican's bill in the photo measures 18 inches. Convert 18 inches to feet, using decimals.

Answers

2. 24 ft 3. 1.5 ft

Objective B Using Mixed U.S. System Units of Length

Sometimes it is more meaningful to express a measurement of length with mixed units such as 1 ft and 5 in. We usually condense this and write 1 ft 5 in.

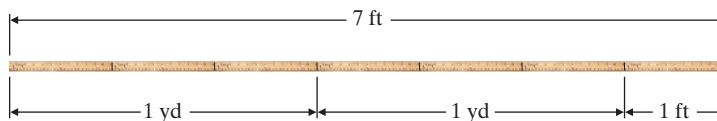
In Example 2, we found that 7 feet is the same as $2\frac{1}{3}$ yards. The measurement can also be written as a mixture of yards and feet. That is,

$$7 \text{ ft} = \underline{\hspace{1cm}} \text{ yd } \underline{\hspace{1cm}} \text{ ft}$$

Because 3 ft = 1 yd, we divide 3 into 7 to see how many whole yards are in 7 feet. The quotient is the number of yards, and the remainder is the number of feet.

$$\begin{array}{r} 2 \text{ yd } 1 \text{ ft} \\ 3 \overline{)7} \\ \underline{-6} \\ 1 \end{array}$$

Thus, 7 ft = 2 yd 1 ft, as seen in the diagram:

**Practice 4**

Convert:

$$68 \text{ in.} = \underline{\hspace{1cm}} \text{ ft } \underline{\hspace{1cm}} \text{ in.}$$

Example 4 Convert: 134 in. = ft in.

Solution: Because 12 in. = 1 ft, we divide 12 into 134. The quotient is the number of feet. The remainder is the number of inches. To see why we divide 12 into 134, notice that

$$134 \text{ in.} = \frac{134 \cancel{\text{in.}}}{1} \cdot \frac{1 \text{ ft}}{12 \cancel{\text{in.}}} = \frac{134}{12} \text{ ft}$$

$$\begin{array}{r} 11 \text{ ft } 2 \text{ in.} \\ 12 \overline{)134} \\ \underline{-12} \\ 14 \\ \underline{-12} \\ 2 \end{array}$$

Thus, 134 in. = 11 ft 2 in.

Work Practice 4**Practice 5**

Convert 5 yards 2 feet to feet.

Example 5 Convert 3 feet 7 inches to inches.

Solution: First, we convert 3 feet to inches. Then we add 7 inches.

$$3 \text{ ft} = \frac{3 \cancel{\text{ft}}}{1} \cdot \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} = 36 \text{ in.}$$

Then

$$3 \text{ ft } 7 \text{ in.} = 36 \text{ in.} + 7 \text{ in.} = 43 \text{ in.}$$

Work Practice 5

Answers

4. 5 ft 8 in. 5. 17 ft

Objective C Performing Operations on U.S. System Units of Length

Finding sums or differences of measurements often involves converting units, as shown in the next example. Just remember that, as usual, only like units can be added or subtracted.

Example 6 Add 3 ft 2 in. and 5 ft 11 in.

Solution: To add, we line up the similar units.

$$\begin{array}{r} 3 \text{ ft } 2 \text{ in.} \\ + 5 \text{ ft } 11 \text{ in.} \\ \hline 8 \text{ ft } 13 \text{ in.} \end{array}$$

Since 13 inches is the same as 1 ft 1 in., we have

$$\begin{aligned} 8 \text{ ft } 13 \text{ in.} &= 8 \text{ ft} + \overbrace{1 \text{ ft } 1 \text{ in.}} \\ &= 9 \text{ ft } 1 \text{ in.} \end{aligned}$$

Work Practice 6

✓Concept Check How could you estimate the following sum?

$$\begin{array}{r} 7 \text{ yd } 4 \text{ in.} \\ + 3 \text{ yd } 27 \text{ in.} \\ \hline \end{array}$$

Example 7 Multiply 8 ft 9 in. by 3.

Solution: By the distributive property, we multiply 8 ft by 3 and 9 in. by 3.

$$\begin{array}{r} 8 \text{ ft } 9 \text{ in.} \\ \times \quad 3 \\ \hline 24 \text{ ft } 27 \text{ in.} \end{array}$$

Since 27 in. is the same as 2 ft 3 in., we simplify the product as

$$\begin{aligned} 24 \text{ ft } 27 \text{ in.} &= 24 \text{ ft} + \overbrace{2 \text{ ft } 3 \text{ in.}} \\ &= 26 \text{ ft } 3 \text{ in.} \end{aligned}$$

Work Practice 7

We divide in a similar manner as above.

Example 8 Finding the Length of a Piece of Rope

A rope of length 6 yd 1 ft has 2 yd 2 ft cut from one end. Find the length of the remaining rope.

Solution: Subtract 2 yd 2 ft from 6 yd 1 ft.

$$\begin{array}{rcl} \text{beginning length} & \rightarrow & 6 \text{ yd } 1 \text{ ft} \\ - \text{amount cut} & \rightarrow & - 2 \text{ yd } 2 \text{ ft} \\ \hline \text{remaining length} & & \end{array}$$

We cannot subtract 2 ft from 1 ft, so we borrow 1 yd from the 6 yd. One yard is converted to 3 ft and combined with the 1 ft already there.

(Continued on next page)

Practice 6

Add 4 ft 8 in. to 8 ft 11 in.

Practice 7

Multiply 4 ft 7 in. by 4.

Practice 8

A carpenter cuts 1 ft 9 in. from a board of length 5 ft 8 in. Find the remaining length of the board.

Answers

6. 13 ft 7 in. 7. 18 ft 4 in.
8. 3 ft 11 in.

✓Concept Check Answer

round each to the nearest yard:
 $7 \text{ yd} + 4 \text{ yd} = 11 \text{ yd}$

Borrow 1 yd = 3 ft

$$5 \text{ yd} + \textcircled{1 \text{ yd}} \textcircled{3 \text{ ft}}$$

$$6 \text{ yd } 1 \text{ ft} = 5 \text{ yd } 4 \text{ ft}$$

$$\begin{array}{r} -2 \text{ yd } 2 \text{ ft} \\ \hline 3 \text{ yd } 2 \text{ ft} \end{array}$$

The remaining rope is 3 yd 2 ft long.

Work Practice 8**Objective D** Defining and Converting Metric System Units of Length

The basic unit of length in the metric system is the **meter**. A meter is slightly longer than a yard. It is approximately 39.37 inches long. Recall that a yard is 36 inches long.

$$1 \text{ yard} = 36 \text{ inches}$$

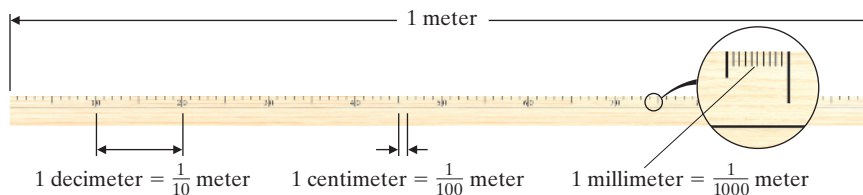


$$1 \text{ meter} \approx 39.37 \text{ inches}$$

All units of length in the metric system are based on the meter. The following is a summary of the prefixes used in the metric system. Also shown are equivalencies between units of length. Like the decimal system, the metric system uses powers of 10 to define units.

Metric Units of Length	
1 kilo meter (km)	= 1000 meters (m)
1 hecto meter (hm)	= 100 m
1 deka meter (dam)	= 10 m
1 meter (m)	= 1 m
1 deci meter (dm)	= 1/10 m or 0.1 m
1 centi meter (cm)	= 1/100 m or 0.01 m
1 milli meter (mm)	= 1/1000 m or 0.001 m

The figure below will help you with decimeters, centimeters, and millimeters.

**Helpful Hint**

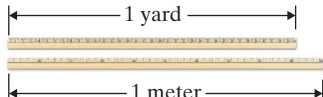
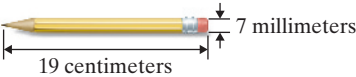
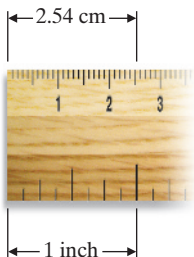
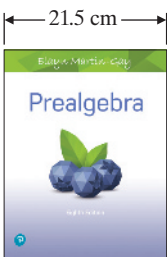
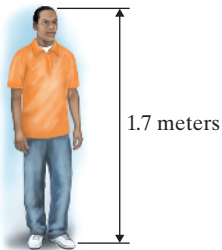
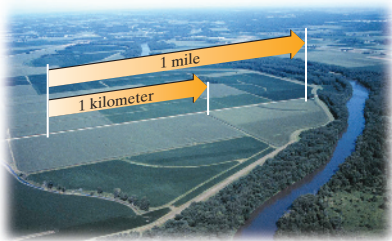
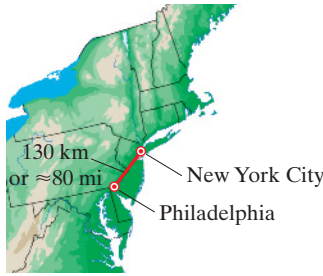
Study the figure above for other equivalencies between metric units of length.

$$\begin{array}{ll} 10 \text{ decimeters} = 1 \text{ meter} & 10 \text{ millimeters} = 1 \text{ centimeter} \\ 100 \text{ centimeters} = 1 \text{ meter} & 10 \text{ centimeters} = 1 \text{ decimeter} \\ 1000 \text{ millimeters} = 1 \text{ meter} & \end{array}$$

These same prefixes are used in the metric system for mass and capacity. The most commonly used measurements of length in the metric system are the **meter**, **millimeter**, **centimeter**, and **kilometer**.

✓Concept Check Is this statement reasonable? “The screen of a home television set has a 30-meter diagonal.” Why or why not?

Being comfortable with the metric units of length means gaining a “feeling” for metric lengths, just as you have a “feeling” for the lengths of an inch, a foot, and a mile. To help you accomplish this, study the following examples:

Examples of Metric System – Length		
 • A millimeter is about the thickness of a large paper clip. • A centimeter is about the width of a large paper clip.	 • A meter is slightly longer than a yard.	
	 • Pencil dimensions	
 • 1 in. = 2.54 cm	 • The width of this book is approximately 21.5 centimeters.	 • Man's height
 • A kilometer is about two-thirds of a mile.	 • The distance between New York City and Philadelphia is about 130 kilometers by air.	

As with the U.S. system of measurement, unit fractions may be used to convert from one unit of length to another. For example, let's convert 1200 meters to kilometers. To do so, we will multiply by 1 in the form of the unit fraction

$$\begin{array}{l}
 \frac{1 \text{ km}}{1000 \text{ m}} \quad \leftarrow \text{Units to convert to} \\
 \quad \quad \quad \leftarrow \text{Original units}
 \end{array}$$

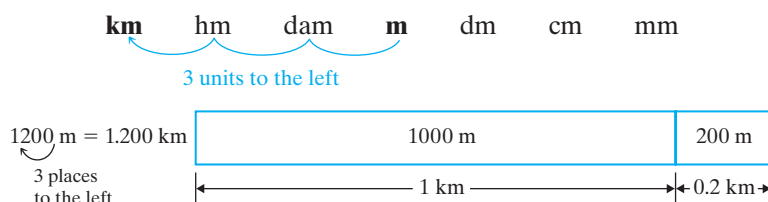
$$1200 \text{ m} = \frac{1200 \text{ m}}{1} \cdot 1 = \frac{1200 \text{ m}}{1} \cdot \frac{1 \text{ km}}{1000 \text{ m}} = \frac{1200 \text{ km}}{1000} = 1.2 \text{ km}$$

Unit fraction

The metric system does, however, have a distinct advantage over the U.S. system of measurement: the ease of converting from one unit of length to another. Since all units of length are powers of 10 of the meter, converting from one unit of length to another is as simple as moving the decimal point. Listing units of length in

order from largest to smallest helps to keep track of how many places to move the decimal point when converting.

Let's again convert 1200 meters to kilometers. This time, to convert from meters to kilometers, we move along the chart shown, 3 units to the left, from meters to kilometers. This means that we move the decimal point 3 places to the left.



Thus, 1200 m = 1.2 km, as shown in the diagram.

Practice 9

Convert 2.5 m to millimeters.

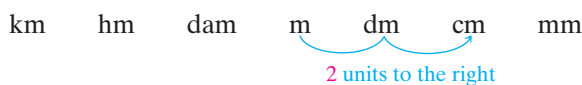
Example 9

Convert 2.3 m to centimeters.

Solution: First we will convert by using a unit fraction.

$$2.3 \text{ m} = \frac{2.3 \text{ m}}{1} \cdot \frac{100 \text{ cm}}{1 \text{ m}} = 230 \text{ cm}$$

Now we will convert by listing the units of length in order from left to right and moving from meters to centimeters.



$$2.30 \text{ m} = 230 \text{ cm}$$

2 places to the right

With either method, we get 230 cm.

Work Practice 9

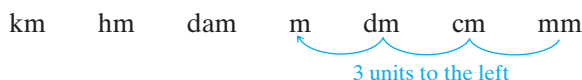
Practice 10

Convert 3500 m to kilometers.

Example 10

Convert 450,000 mm to meters.

Solution: We list the units of length in order from left to right and move from millimeters to meters.



Thus, move the decimal point 3 places to the left.

$$450,000 \text{ mm} = 450.000 \text{ m or } 450 \text{ m}$$

Work Practice 10

✓ **Concept Check** What is wrong with the following conversion of 150 cm to meters?

$$150.00 \text{ cm} = 15,000 \text{ m}$$

Objective E Performing Operations on Metric System Units of Length

To add, subtract, multiply, or divide with metric measurements of length, we write all numbers using the same unit of length and then add, subtract, multiply, or divide as with decimals.

Answers

9. 2500 mm 10. 3.5 km

✓ Concept Check Answer

decimal point should be moved two places to the left: 1.5 m

Example 11 Subtract 430 m from 1.3 km.**Solution:** First we convert both measurements to kilometers or both to meters.

$$\begin{array}{rcl}
 430 \text{ m} = 0.43 \text{ km} & \text{or} & 1.3 \text{ km} = 1300 \text{ m} \\
 \begin{array}{r} 1.30 \text{ km} \\ - 0.43 \text{ km} \\ \hline 0.87 \text{ km} \end{array} & & \begin{array}{r} 1300 \text{ m} \\ - 430 \text{ m} \\ \hline 870 \text{ m} \end{array}
 \end{array}$$

The difference is 0.87 km or 870 m.

Work Practice 11**Example 12** Multiply 5.7 mm by 4.**Solution:** Here we simply multiply the two numbers. Note that the unit of measurement remains the same.

$$\begin{array}{r}
 5.7 \text{ mm} \\
 \times 4 \\
 \hline
 22.8 \text{ mm}
 \end{array}$$

Work Practice 12**Example 13** Finding a Person's Height

Fritz Martinson was 1.2 meters tall on his last birthday. Since then, he has grown 14 centimeters. Find his current height in meters.

Solution:

$$\begin{array}{rcl}
 \text{original height} & \rightarrow & 1.20 \text{ m} \\
 + \text{ height grown} & \rightarrow & + 0.14 \text{ m} \quad (\text{Since } 14 \text{ cm} = 0.14 \text{ m}) \\
 \hline
 \text{current height} & & 1.34 \text{ m}
 \end{array}$$

Fritz is now 1.34 meters tall.

Work Practice 13**Practice 11**

Subtract 640 m from 2.1 km.

Practice 12

Multiply 18.3 hm by 5.

Practice 13

Doris Blackwell is knitting a scarf that is currently 0.8 meter long. If she knits an additional 45 centimeters, how long will the scarf be?

Answers

- 11.** 1.46 km or 1460 m **12.** 91.5 hm
13. 125 cm or 1.25 m

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

inches yard unit fraction
 feet meter

- The basic unit of length in the metric system is the _____.
- The expression $\frac{1 \text{ foot}}{12 \text{ inches}}$ is an example of a(n) _____.
- A meter is slightly longer than a(n) _____.
- One foot equals 12 _____.
- One yard equals 3 _____.
- One yard equals 36 _____.
- One mile equals 5280 _____.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 9.4

Objective A 8. In Example 3, what units are used in the denominator of the unit fraction and why was this decided?

Objective B 9. In Example 4, how is a mixed unit similar to a mixed number? Use examples in your answer.

Objective C 10. In Example 5, why is the sum of the addition problem not the final answer? Reference the sum in your answer.

Objective D 11. In the lecture before Example 6, why is it easier to convert metric units than U.S. units?

Objective E 12. What two answers did we get for Example 8? Explain why both answers are correct.

9.4 Exercise Set MyLab Math

Objective A Convert each measurement as indicated. See Examples 1 through 3.

- ▶ 1. 60 in. to feet
2. 84 in. to feet
3. 12 yd to feet
4. 18 yd to feet
5. 42,240 ft to miles
6. 36,960 ft to miles
- ▶ 7. $8\frac{1}{2}$ ft to inches
8. $12\frac{1}{2}$ ft to inches
- ▶ 9. 10 ft to yards
10. 25 ft to yards
11. 6.4 mi to feet
12. 3.8 mi to feet
13. 162 in. to yd (Write answer as a decimal.)
14. 7216 yd to mi (Write answer as a decimal.)
15. 3 in. to ft (Write answer as a decimal.)
16. 129 in. to ft (Write answer as a decimal.)

Objective B Convert each measurement as indicated. See Examples 4 and 5.

17. 40 ft = ____ yd ____ ft
18. 100 ft = ____ yd ____ ft
19. 85 in. = ____ ft ____ in.
20. 59 in. = ____ ft ____ in.
21. 10,000 ft = ____ mi ____ ft
22. 25,000 ft = ____ mi ____ ft
- ▶ 23. 5 ft 2 in. = ____ in.
24. 6 ft 10 in. = ____ in.
25. 8 yd 2 ft = ____ ft
26. 4 yd 1 ft = ____ ft
27. 2 yd 1 ft = ____ in.
28. 1 yd 2 ft = ____ in.

Objective C Perform each indicated operation. Simplify the result if possible. See Examples 6 through 8.

29. $3 \text{ ft } 10 \text{ in.} + 7 \text{ ft } 4 \text{ in.}$

30. $12 \text{ ft } 7 \text{ in.} + 9 \text{ ft } 11 \text{ in.}$

▶ 31. $12 \text{ yd } 2 \text{ ft} + 9 \text{ yd } 2 \text{ ft}$

32. $16 \text{ yd } 2 \text{ ft} + 8 \text{ yd } 2 \text{ ft}$

33. $22 \text{ ft } 8 \text{ in.} - 16 \text{ ft } 3 \text{ in.}$

34. $15 \text{ ft } 5 \text{ in.} - 8 \text{ ft } 2 \text{ in.}$

35. $18 \text{ ft } 3 \text{ in.} - 10 \text{ ft } 9 \text{ in.}$

36. $14 \text{ ft } 8 \text{ in.} - 3 \text{ ft } 11 \text{ in.}$

37. $28 \text{ ft } 8 \text{ in.} \div 2$

38. $34 \text{ ft } 6 \text{ in.} \div 2$

39. $16 \text{ yd } 2 \text{ ft} \times 5$

40. $15 \text{ yd } 1 \text{ ft} \times 8$

Objective D Convert as indicated. See Examples 9 and 10.

41. 60 m to centimeters

42. 46 m to centimeters

43. 40 mm to centimeters

44. 14 mm to centimeters

45. 500 m to kilometers

46. 400 m to kilometers

47. 1700 mm to meters

48. 6400 mm to meters

▶ 49. 1500 cm to meters

50. 6400 cm to meters

51. 0.42 km to centimeters

52. 0.95 km to centimeters

53. 7 km to meters

54. 5 km to meters

55. 8.3 cm to millimeters

56. 4.6 cm to millimeters

57. 20.1 mm to decimeters

58. 140.2 mm to decimeters

▶ 59. 0.04 m to millimeters

60. 0.2 m to millimeters

Objective E Perform each indicated operation. Remember to insert units when writing your answers. See Examples 11 through 13.

61. $8.6 \text{ m} + 0.34 \text{ m}$

62. $14.1 \text{ cm} + 3.96 \text{ cm}$

63. $2.9 \text{ m} + 40 \text{ mm}$

64. $30 \text{ cm} + 8.9 \text{ m}$

▶ 65. $24.8 \text{ mm} - 1.19 \text{ cm}$

66. $45.3 \text{ m} - 2.16 \text{ dam}$

67. $15 \text{ km} - 2360 \text{ m}$

68. $14 \text{ cm} - 15 \text{ mm}$

69. $18.3 \text{ m} \times 3$

70. $14.1 \text{ m} \times 4$

71. $6.2 \text{ km} \div 4$

72. $9.6 \text{ m} \div 5$

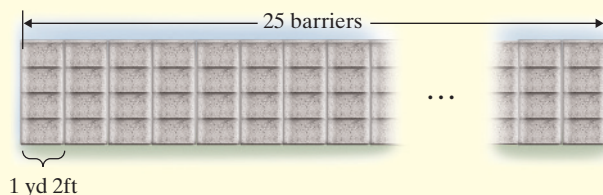
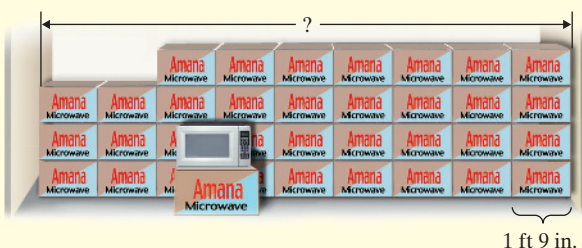
Objectives A C D E Mixed Practice Solve. Remember to insert units when writing your answers. For Exercises 73 through 82, complete the charts. See Examples 1 through 13.

		Yards	Feet	Inches
73.	Chrysler Building in New York City		1046	
74.	4-story building			792
75.	Python length		35	
76.	Ostrich height			108

		Meters	Millimeters	Kilometers	Centimeters
77.	Length of elephant	5			
78.	Height of grizzly bear	3			
79.	Tennis ball diameter				6.5
80.	Golf ball diameter				4.6
81.	Distance from London to Paris			342	
82.	Distance from Houston to Dallas			396	

83. The National Zoo maintains a small patch of bamboo, which it grows as a food supply for its pandas. Two weeks ago, the bamboo was 6 ft 10 in. tall. Since then, the bamboo has grown 3 ft 8 in. How tall is the bamboo now?
84. While exploring in the Marianas Trench, a submarine probe was lowered to a point 1 mile 1400 feet below the ocean's surface. Later it was lowered an additional 1 mile 4000 feet below this point. How far was the probe below the surface of the Pacific?
85. At its deepest point, the Grand Canyon of the Colorado River in Arizona is about 6000 ft. The Grand Canyon of the Yellowstone River, which is in Yellowstone National Park in Wyoming, is at most 900 feet deep. How much deeper is the Grand Canyon of the Colorado River than the Grand Canyon of the Yellowstone River? (Source: National Park Service)
86. The Grand Canyon of the Gunnison River, in Colorado, is often called the Black Canyon of the Gunnison because it is so steep that light rarely penetrates the depth of the canyon. The Black Canyon of the Gunnison is only 1150 ft wide at its narrowest point. At its narrowest, the Grand Canyon of the Yellowstone is $\frac{1}{2}$ mile wide. Find the difference in width between the Grand Canyon of the Yellowstone and the Black Canyon of the Gunnison. (Note: Notice that the dimensions are different.) (Source: National Park Service)
87. The tallest man in the world is recorded as Robert Pershing Wadlow of Alton, Illinois. Born in 1918, he measured 8 ft 11 in. at his tallest. The shortest man in the world is Chandra Bahadur Dangi of Nepal, who measures 21.5 in. How many times taller than Chandra is Robert? Round to one decimal place. (Source: Guinness World Records)
88. A 3.4-m rope is attached to a 5.8-m rope. However, when the ropes are tied, 8 cm of length is lost to form the knot. What is the length of the tied ropes?

89. The ice on a pond is 5.33 cm thick. For safe skating, the owner of the pond insists that it be 80 mm thick. How much thicker must the ice be before skating is allowed?
90. The sediment on the bottom of the Towamencin Creek is normally 14 cm thick, but a recent flood washed away 22 mm of sediment. How thick is it now?
91. The Amana Corporation stacks up its microwave ovens in a distribution warehouse. Each stack is 1 ft 9 in. wide. How far from the wall would 9 of these stacks extend?
92. The highway commission is installing concrete sound barriers along a highway. Each barrier is 1 yd 2 ft long. Find the total length of 25 barriers placed end to end.



93. A logging firm needs to cut a 67-m-long redwood log into 20 equal pieces before loading it onto a truck for shipment. How long will each piece be?
94. A 112.5-foot-tall dead pinewood tree is removed by starting at the top and cutting off 9-foot-long sections. How many whole sections are removed?
95. The giant Coca-Cola billboard in Times Square is lit by more than 2.6 million LEDs, which are completely powered by wind energy. This huge advertisement measures approximately 44 feet by 65 feet. Find the perimeter of the sign in yards. (Source: Coca-Cola Company)
96. The longest truck in the world is operated by Gould Transport in Australia, and is the 182-ft Road Train. How many yards long are 2 of these trucks? (Source: Guinness World Records)



Review

Write each decimal as a fraction and each fraction as a decimal. See Sections 5.1 and 5.5.

97. 0.21

98. 0.86

99. $\frac{13}{100}$

100. $\frac{47}{100}$

101. $\frac{1}{4}$

102. $\frac{3}{20}$

Concept Extensions

Determine whether the measurement in each statement is reasonable.

103. The width of a twin-size bed is 20 meters.

104. A window measures 1 meter by 0.5 meter.

105. A drinking glass is made of glass 2 millimeters thick.

106. A paper clip is 4 kilometers long.

107. The distance across the Colorado River is 50 kilometers.

108. A model's hair is 30 centimeters long.

Estimate each sum or difference. See the first Concept Check in this section.

109.
$$\begin{array}{r} 5 \text{ yd } 2 \text{ in.} \\ + 7 \text{ yd } 30 \text{ in.} \\ \hline \end{array}$$

110.
$$\begin{array}{r} 45 \text{ ft } 1 \text{ in.} \\ - 10 \text{ ft } 11 \text{ in.} \\ \hline \end{array}$$

111. Using a unit other than the foot, write a length that is equivalent to 4 feet. (*Hint:* There are many possibilities.)

112. Using a unit other than the meter, write a length that is equivalent to 7 meters. (*Hint:* There are many possibilities.)

 113. To convert from meters to centimeters, the decimal point is moved two places to the right. Explain how this relates to the fact that the prefix *centi* means $\frac{1}{100}$.

 114. Explain why conversions in the metric system are easier to make than conversions in the U.S. system of measurement.

115. An advertisement sign outside Fenway Park in Boston measures 18.3 m by 18.3 m. What is the area of this sign?

9.5 Weight and Mass

Objective A Defining and Converting U.S. System Units of Weight

Whenever we talk about how heavy an object is, we are concerned with the object's **weight**. We discuss weight when we refer to a 12-ounce box of Rice Krispies, a 15-pound tabby cat, or a barge hauling 24 tons of garbage.



12 ounces



15 pounds



24 tons of garbage

The most common units of weight in the U.S. measurement system are the **ounce**, the **pound**, and the **ton**. The following is a summary of equivalencies between units of weight:

U.S. Units of Weight

$$16 \text{ ounces (oz)} = 1 \text{ pound (lb)}$$

$$2000 \text{ pounds} = 1 \text{ ton}$$

Unit Fractions

$$\frac{16 \text{ oz}}{1 \text{ lb}} = \frac{1 \text{ lb}}{16 \text{ oz}} = 1$$

$$\frac{2000 \text{ lb}}{1 \text{ ton}} = \frac{1 \text{ ton}}{2000 \text{ lb}} = 1$$

✓Concept Check If you were describing the weight of a fully loaded semi-trailer, which type of unit would you use: ounce, pound, or ton? Why?

Unit fractions that equal 1 are used to convert between units of weight in the U.S. system. When converting using unit fractions, recall that the numerator of a unit fraction should contain the units we are converting to and the denominator should contain the original units.

Example 1 Convert 9000 pounds to tons.

Solution: We multiply 9000 lb by a unit fraction that uses the equality 2000 pounds = 1 ton.

Remember, the unit fraction should be $\frac{\text{units to convert to}}{\text{original units}}$ or $\frac{1 \text{ ton}}{2000 \text{ lb}}$.

$$9000 \text{ lb} = \frac{9000 \text{ lb}}{1} \cdot 1 = \frac{9000 \cancel{\text{lb}}}{1} \cdot \frac{1 \text{ ton}}{2000 \cancel{\text{lb}}} = \frac{9000 \text{ tons}}{2000} = \frac{9}{2} \text{ tons or } 4 \frac{1}{2} \text{ tons}$$

(Continued on next page)

Objectives

- A** Define U.S. Units of Weight and Convert from One Unit to Another.
- B** Perform Arithmetic Operations on U.S. Units of Weight.
- C** Define Metric Units of Mass and Convert from One Unit to Another.
- D** Perform Arithmetic Operations on Metric Units of Mass.

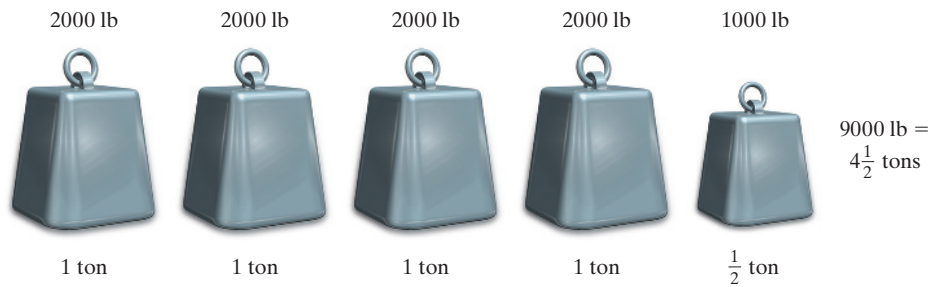
Practice 1

Convert 6500 pounds to tons.

Answer

$$1. 3 \frac{1}{4} \text{ tons}$$

✓Concept Check Answer
ton



Work Practice 1

Practice 2

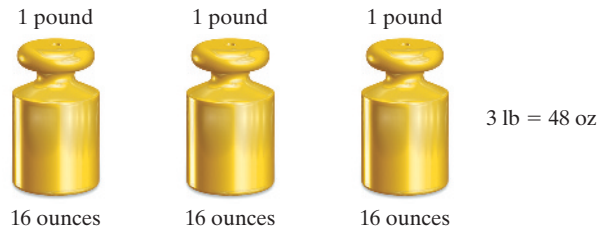
Convert 72 ounces to pounds.

Example 2

Convert 3 pounds to ounces.

Solution: We multiply by the unit fraction $\frac{16 \text{ oz}}{1 \text{ lb}}$ to convert from pounds to ounces.

$$3 \text{ lb} = \frac{3 \text{ lb}}{1} \cdot 1 = \frac{3 \cancel{\text{lb}}}{1} \cdot \frac{16 \text{ oz}}{1 \cancel{\text{lb}}} = 3 \cdot 16 \text{ oz} = 48 \text{ oz}$$



Work Practice 2

As with length, it is sometimes useful to simplify a measurement of weight by writing it in terms of mixed units.

Practice 3

Convert:

47 ounces = _____ lb _____ oz

Example 3

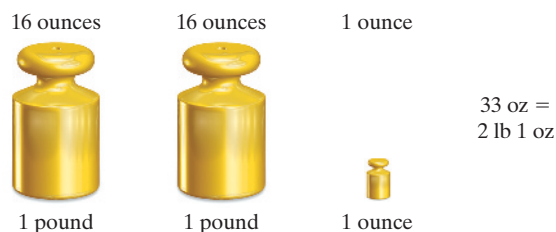
Convert: 33 ounces = _____ lb _____ oz

Solution: Because $16 \text{ oz} = 1 \text{ lb}$, divide 16 into 33 to see how many pounds are in 33 ounces. The quotient is the number of pounds, and the remainder is the number of ounces. To see why we divide 16 into 33, notice that

$$33 \text{ oz} = 33 \cancel{\text{oz}} \cdot \frac{1 \text{ lb}}{16 \cancel{\text{oz}}} = \frac{33}{16} \text{ lb}$$

$$\begin{array}{r} 2 \text{ lb } 1 \text{ oz} \\ 16 \overline{)33} \\ \underline{-32} \\ 1 \end{array}$$

Thus, 33 ounces is the same as **2 lb 1 oz**.



Work Practice 3

Answers

2. $4 \frac{1}{2} \text{ lb}$ 3. 2 lb 15 oz

Objective B Performing Operations on U.S. System Units of Weight

Performing arithmetic operations on units of weight works the same way as performing arithmetic operations on units of length.

Example 4 Subtract 3 tons 1350 lb from 8 tons 1000 lb.

Solution: To subtract, we line up similar units.

$$\begin{array}{r} 8 \text{ tons } 1000 \text{ lb} \\ - 3 \text{ tons } 1350 \text{ lb} \\ \hline \end{array}$$

Since we cannot subtract 1350 lb from 1000 lb, we borrow 1 ton from the 8 tons. To do so, we write 1 ton as 2000 lb and combine it with the 1000 lb.

$$\begin{array}{r} 7 \text{ tons } + \underbrace{1 \text{ ton } 2000 \text{ lb}} \\ 8 \text{ tons } 1000 \text{ lb} = 7 \text{ tons } 3000 \text{ lb} \\ - 3 \text{ tons } 1350 \text{ lb} = - 3 \text{ tons } 1350 \text{ lb} \\ \hline 4 \text{ tons } 1650 \text{ lb} \end{array}$$

To check, see that the sum of 4 tons 1650 lb and 3 tons 1350 lb is 8 tons 1000 lb.

Work Practice 4

Example 5 Divide 9 lb 6 oz by 2.

Solution: We divide each of the units by 2.

$$\begin{array}{r} 4 \text{ lb } \quad 11 \text{ oz} \\ 2 \overline{) 9 \text{ lb } \quad 6 \text{ oz}} \\ \underline{-8} \\ 1 \text{ lb } = \underline{16 \text{ oz}} \\ 22 \text{ oz} \end{array} \quad \text{Divide 2 into 22 oz to get 11 oz.}$$

To check, multiply 4 pounds 11 ounces by 2. The result is 9 pounds 6 ounces.

Work Practice 5

Example 6 Finding the Weight of a Child

Bryan weighed 8 lb 8 oz at birth. By the time he was 1 year old, he had gained 11 lb 14 oz. Find his weight at age 1 year.

Solution:

$$\begin{array}{rcl} \text{birth weight} & \rightarrow & 8 \text{ lb } 8 \text{ oz} \\ + \text{weight gained} & \rightarrow & + 11 \text{ lb } 14 \text{ oz} \\ \hline \text{total weight} & \rightarrow & 19 \text{ lb } 22 \text{ oz} \end{array}$$

Since 22 oz equals 1 lb 6 oz,

$$\begin{array}{rcl} 19 \text{ lb } 22 \text{ oz} & = & 19 \text{ lb } + \underbrace{1 \text{ lb } 6 \text{ oz}} \\ & = & 20 \text{ lb } 6 \text{ oz} \end{array}$$

Bryan weighed 20 lb 6 oz on his first birthday.

Work Practice 6

Practice 4

Subtract 5 tons 1200 lb from 8 tons 100 lb.

Practice 5

Divide 5 lb 8 oz by 4.

Practice 6

A 5-lb 14-oz batch of cookies is packed into a 6-oz container before it is mailed. Find the total weight.

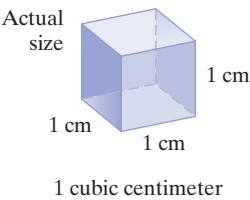
Answers

4. 2 tons 900 lb 5. 1 lb 6 oz
6. 6 lb 4 oz

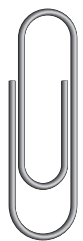
Objective C Defining and Converting Metric System Units of Mass

In scientific and technical areas, a careful distinction is made between **weight** and **mass**. **Weight** is really a measure of the pull of gravity. The farther from Earth an object gets, the less it weighs. However, **mass** is a measure of the amount of substance in the object and does not change. Astronauts orbiting Earth weigh much less than they weigh on Earth, but they have the same mass in orbit as they do on Earth. Here on Earth, weight and mass are the same, so either term may be used.

The basic unit of mass in the metric system is the **gram**. It is defined as the mass of water contained in a cube 1 centimeter (cm) on each side.



The following examples may help you get a feeling for metric masses:

Examples of Metric System – Weight and Mass			
 A tablet contains 200 milligrams of ibuprofen.	 A large paper clip weighs approximately 1 gram.	 NET WEIGHT 16.5 OZ (468g) A box of crackers weighs 468 grams.	 A kilogram is slightly over 2 pounds. An adult woman may weigh 60 kilograms.

The prefixes for units of mass in the metric system are the same as for units of length, as shown in the following table:

Metric Units of Mass
1 kilo gram (kg) = 1000 grams (g)
1 hecto gram (hg) = 100 g
1 deka gram (dag) = 10 g
1 gram (g) = 1 g
1 deci gram (dg) = 1/10 g or 0.1 g
1 centi gram (cg) = 1/100 g or 0.01 g
1 milli gram (mg) = 1/1000 g or 0.001 g

✓ **Concept Check** True or false? A decigram is larger than a dekagram. Explain.

The **milligram**, the **gram**, and the **kilogram** are the three most commonly used units of mass in the metric system.

As with lengths, all units of mass are powers of 10 of the gram, so converting from one unit of mass to another only involves moving the decimal point. To convert from one unit of mass to another in the metric system, list the units of mass in order from largest to smallest.

✓ **Concept Check Answer**
false

Let's convert 4300 milligrams to grams. To convert from milligrams to grams, we move along the list 3 units to the left.

kg hg dag **g** dg cg **mg**

3 units to the left

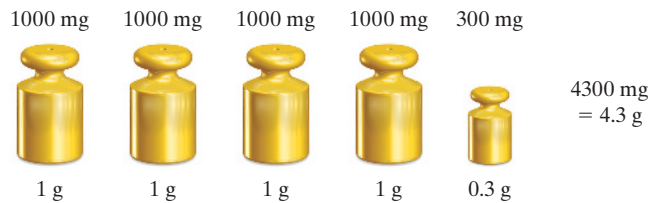
This means that we move the decimal point 3 places to the left to convert from milligrams to grams.

$$4300 \text{ mg} = 4.3 \text{ g}$$

Don't forget, the same conversion can be done with unit fractions.

$$\begin{aligned} 4300 \text{ mg} &= \frac{4300 \text{ mg}}{1} \cdot 1 = \frac{4300 \text{ mg}}{1} \cdot \frac{0.001 \text{ g}}{1 \text{ mg}} \\ &= 4300 \cdot 0.001 \text{ g} \\ &= 4.3 \text{ g} \quad \text{To multiply by 0.001, move the decimal point 3 places to the left.} \end{aligned}$$

To see that this is reasonable, study the diagram:



Thus, $4300 \text{ mg} = 4.3 \text{ g}$

Example 7 Convert 3.2 kg to grams.

Solution: First we convert by using a unit fraction.

$$3.2 \text{ kg} = 3.2 \text{ kg} \cdot 1 = 3.2 \text{ kg} \cdot \frac{1000 \text{ g}}{1 \text{ kg}} = 3200 \text{ g}$$

Unit fraction

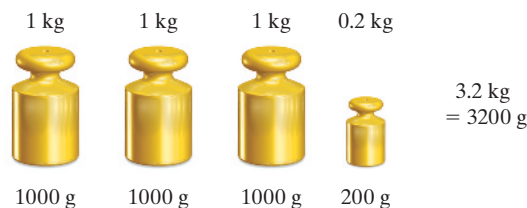
Now let's list the units of mass in order from left to right and move from kilograms to grams.

kg hg dag **g** dg cg mg

3 units to the right

$$3.200 \text{ kg} = 3200. \text{ g}$$

3 places to the right



Work Practice 7

Practice 7

Convert 3.41 g to milligrams.

Answer

7. 3410 mg

Practice 8

Convert 56.2 cg to grams.

Practice 9

Subtract 3.1 dg from 2.5 g.

Practice 10

Twenty-four bags of cement weigh a total of 550 kg. Find the average weight of 1 bag, rounded to the nearest kilogram.

**Answers**

8. 0.562 g 9. 2.19 g or 21.9 dg
10. 23 kg

Example 8 Convert 2.35 cg to grams.**Solution:** We list the units of mass in a chart and move from centigrams to grams.

kg hg dag g dg cg mg

↖ ↗
2 units to the left

$$\begin{array}{r} 02.35 \text{ cg} = 0.0235 \text{ g} \\ \leftarrow \text{2 places to the left} \end{array}$$

Work Practice 8**Objective D** Performing Operations on Metric System Units of Mass

Arithmetic operations can be performed with metric units of mass just as we performed operations with metric units of length. We convert each number to the same unit of mass and add, subtract, multiply, or divide as with decimals.

Example 9 Subtract 5.4 dg from 1.6 g.**Solution:** We convert both numbers to decigrams or to grams before subtracting.

$$\begin{array}{r} 5.4 \text{ dg} = 0.54 \text{ g} \quad \text{or} \quad 1.6 \text{ g} = 16 \text{ dg} \\ \begin{array}{r} 1.60 \text{ g} \\ -0.54 \text{ g} \\ \hline 1.06 \text{ g} \end{array} \quad \begin{array}{r} 16.0 \text{ dg} \\ -5.4 \text{ dg} \\ \hline 10.6 \text{ g} \end{array} \end{array}$$

The difference is 1.06 g or 10.6 dg.

Work Practice 9**Example 10** Calculating Allowable Weight in an Elevator

An elevator has a weight limit of 1400 kg. A sign posted in the elevator indicates that the maximum capacity of the elevator is 17 persons. What is the average allowable weight for each passenger, rounded to the nearest kilogram?

Solution: To solve, notice that the total weight of 1400 kilograms $\div 17 =$ average weight.

$$\begin{array}{r} 82.3 \text{ kg} \approx 82 \text{ kg} \\ 17 \overline{)1400.0 \text{ kg}} \\ \underline{-136} \\ 40 \\ \underline{-34} \\ 60 \\ \underline{-51} \\ 9 \end{array}$$



Each passenger can weigh an average of **82 kg**. (Recall that a kilogram is slightly over 2 pounds, so 82 kilograms is over 164 pounds.)

Work Practice 10

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

mass weight gram

- _____ is a measure of the amount of substance in an object. This measure does not change.
- _____ is the measure of the pull of gravity.
- The basic unit of mass in the metric system is the _____.

Fill in these blanks with the correct number. Choices for these blanks are not shown in the list of terms above.

- One pound equals _____ ounces.
- One ton equals _____ pounds.

Martin-Gay Interactive Videos



See Video 9.5

Watch the section lecture video and answer the following questions.

- Objective A** 6. In Example 2, what units are used in the numerator of the unit fraction and why was this decided?
- Objective B** 7. In Example 4, explain the first step taken to solve the problem.
- Objective C** 8. In Example 5, how many places is the decimal moved and in what direction? What is the final conversion?
- Objective D** 9. What is the answer to Example 7 in decigrams?

9.5 Exercise Set MyLab Math

Objective A Convert as indicated. See Examples 1 through 3.

- | | | |
|-------------------------------------|---|---|
| 1. 2 pounds to ounces | 2. 5 pounds to ounces | 3. 5 tons to pounds |
| 4. 7 tons to pounds | 5. 18,000 pounds to tons | 6. 28,000 pounds to tons |
| 7. 60 ounces to pounds | 8. 90 ounces to pounds | 9. 3500 pounds to tons |
| 10. 11,000 pounds to tons | 11. 12.75 pounds to ounces | 12. 9.5 pounds to ounces |
| 13. 4.9 tons to pounds | 14. 8.3 tons to pounds | 15. $4\frac{3}{4}$ pounds to ounces |
| 16. $9\frac{1}{8}$ pounds to ounces | 17. 2950 pounds to the nearest tenth of a ton | 18. 51 ounces to the nearest tenth of a pound |
| 19. $\frac{4}{5}$ oz to pounds | 20. $\frac{1}{4}$ oz to pounds | 21. $5\frac{3}{4}$ lb to ounces |

22. $2\frac{1}{4}$ lb to ounces

23. 10 lb 1 oz to ounces

24. 7 lb 6 oz to ounces

▶ 25. 89 oz = _____ lb _____ oz

26. 100 oz = _____ lb _____ oz

Objective B Perform each indicated operation. See Examples 4 through 6.

27. 34 lb 12 oz + 18 lb 14 oz

28. 6 lb 10 oz + 10 lb 8 oz

29. 3 tons 1820 lb + 4 tons 930 lb

30. 1 ton 1140 lb + 5 tons 1200 lb

31. 5 tons 1050 lb - 2 tons 875 lb

32. 4 tons 850 lb - 1 ton 260 lb

▶ 33. 12 lb 4 oz - 3 lb 9 oz

34. 45 lb 6 oz - 26 lb 10 oz

35. 5 lb 3 oz $\times$ 6

36. 2 lb 5 oz $\times$ 5

37. 6 tons 1500 lb $\div$ 5

38. 5 tons 400 lb $\div$ 4

Objective C Convert as indicated. See Examples 7 and 8.

39. 500 g to kilograms

40. 820 g to kilograms

▶ 41. 4 g to milligrams

42. 9 g to milligrams

43. 25 kg to grams

44. 18 kg to grams

45. 48 mg to grams

46. 112 mg to grams

▶ 47. 6.3 g to kilograms

48. 4.9 g to kilograms

49. 15.14 g to milligrams

50. 16.23 g to milligrams

51. 6.25 kg to grams

52. 3.16 kg to grams

53. 35 hg to centigrams

54. 4.26 cg to dekagrams

Objective D Perform each indicated operation. Remember to insert units when writing your answers. See Examples 9 and 10.

55. 3.8 mg + 9.7 mg

56. 41.6 g + 9.8 g

57. 205 mg + 5.61 g

58. 2.1 g + 153 mg

▶ 59. 9 g - 7150 mg

60. 6.13 g - 418 mg

61. 1.61 kg - 250 g

62. 4 kg - 2410 g

63. 5.2 kg $\times$ 2.6

64. 4.8 kg $\times$ 9.3

65. 17 kg $\div$ 8

66. 8.25 g $\div$ 6

Objectives A B C D Mixed Practice Solve. Remember to insert units when writing your answers. For Exercises 67 through 74, complete the chart. See Examples 1 through 10.

	Object	Tons	Pounds	Ounces
67.	Statue of Liberty—weight of copper sheeting	100		
68.	Statue of Liberty—weight of steel	125		
69.	 A 12-inch cube of osmium (heaviest metal)		1345	
70.	 A 12-inch cube of lithium (lightest metal)		32	

	Object	Grams	Kilograms	Milligrams	Centigrams
71.	Capsule of amoxicillin (antibiotic)			500	
72.	Tablet of Topamax (epilepsy and migraine uses)			25	
73.	A six-year-old boy		21		
74.	A golf ball	45			

75. A can of 7-Up weighs 336 grams. Find the weight in kilograms of 24 cans.
76. Guy Green normally weighs 73 kg, but he lost 2800 grams after being sick with the flu. Find Guy's new weight.
77. Sudafed is a decongestant that comes in two strengths. Regular strength contains 60 mg of medication. Extra strength contains 0.09 g of medication. How much extra medication is in the extra-strength tablet?
78. A small can of Planters sunflower seeds weighs 177 g. If each can contains 6 servings, find the weight of one serving.



79. Doris Johnson has two open containers of rice. If she combines 1 lb 10 oz from one container with 3 lb 14 oz from the other container, how much total rice does she have?
80. Dru Mizel maintains the records of the amount of coal delivered to his department in the steel mill. In January, 3 tons 1500 lb were delivered. In February, 2 tons 1200 lb were delivered. Find the total amount delivered in these two months.
81. Carla Hamtini was amazed when she grew a 28-lb 10-oz zucchini in her garden, but later she learned that the heaviest zucchini ever grown weighed 64 lb 8 oz in Llanharry, Wales, by B. Lavery in 1990. How far below the record weight was Carla's zucchini? (Source: Guinness World Records)
82. The heaviest baby born in good health weighed an incredible 22 lb 8 oz. He was born in Italy in September 1955. How much heavier is this than a 7-lb 12-oz baby? (Source: Guinness World Records)



83. The smallest surviving premature baby weighed only 8 ounces, less than a can of soda. She was born in Witten, Germany, in 2016. How much lighter was she than an average European baby, who weighs about 7 lb 7.5 oz?
84. A large bottle of Hire's Root Beer weighs 1900 grams. If a carton contains 6 large bottles of root beer, find the weight in kilograms of 5 cartons.

85. Three milligrams of preservatives are added to a 0.5-kg box of dried fruit. How many milligrams of preservatives are in 3 cartons of dried fruit if each carton contains 16 boxes?
86. One box of Swiss Miss Cocoa Mix weighs 0.385 kg, but 39 grams of this weight is the packaging. Find the actual weight of the cocoa in 8 boxes.
87. A carton of 12 boxes of Quaker Oats Oatmeal weighs 6.432 kg. Each box includes 26 grams of packaging material. What is the actual weight of the oatmeal in the carton?
88. The supermarket prepares hamburger in 85-gram market packages. When Leo Gonzalas gets home, he divides the package in half before refrigerating the meat. How much will each package weigh?
89. The Shop 'n Bag supermarket chain ships hamburger meat by placing 10 packages of hamburger in a box, with each package weighing 3 lb 4 oz. How much will 4 boxes of hamburger weigh?
90. The Quaker Oats Company ships its 1-lb 2-oz boxes of oatmeal in cartons containing 12 boxes of oatmeal. How much will 3 such cartons weigh?
91. A carton of Del Monte Pineapple weighs 55 lb 4 oz, but 2 lb 8 oz of this weight is due to packaging. Find the actual weight of the pineapple in 4 cartons.
92. The Hormel Corporation ships cartons of canned ham weighing 43 lb 2 oz each. Of this weight, 3 lb 4 oz is due to packaging. Find the actual weight of the ham found in 3 cartons.

Review

Write each fraction as a decimal. See Section 5.5.

93. $\frac{4}{25}$

94. $\frac{3}{5}$

95. $\frac{7}{8}$

96. $\frac{3}{16}$

Concept Extensions

Determine whether the measurement in each statement is reasonable.

97. The doctor prescribed a pill containing 2 kg of medication.
98. A full-grown cat weighs approximately 15 g.
99. A bag of flour weighs 4.5 kg.
100. A staple weighs 15 mg.
101. A professor weighs less than 150 g.
102. A car weighs 2000 mg.
103. Use a unit other than centigram and write a mass that is equivalent to 25 centigrams. (*Hint: There are many possibilities.*)
104. Use a unit other than pound and write a weight that is equivalent to 4000 pounds. (*Hint: There are many possibilities.*)

True or false? See the second Concept Check in this section.

105. A kilogram is larger than a gram.
106. A decigram is larger than a milligram.
-  107. Why is the decimal point moved to the right when grams are converted to milligrams?
-  108. To change 8 pounds to ounces, multiply by 16. Why is this the correct procedure?

9.6 Capacity

Objective A Defining and Converting U.S. System Units of Capacity

Units of **capacity** are generally used to measure liquids. The number of gallons of gasoline needed to fill a gas tank in a car, the number of cups of water needed in a bread recipe, and the number of quarts of milk sold each day at a supermarket are all examples of using units of capacity. The following summary shows equivalencies between units of capacity:

U.S. Units of Capacity

$$\begin{aligned} 8 \text{ fluid ounces (fl oz)} &= 1 \text{ cup (c)} \\ 2 \text{ cups} &= 1 \text{ pint (pt)} \\ 2 \text{ pints} &= 1 \text{ quart (qt)} \\ 4 \text{ quarts} &= 1 \text{ gallon (gal)} \end{aligned}$$

Just as with units of length and weight, we can form unit fractions to convert between different units of capacity. For instance,

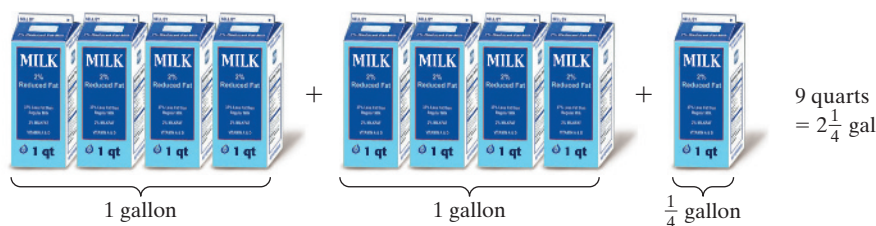
$$\frac{2 \text{ c}}{1 \text{ pt}} = \frac{1 \text{ pt}}{2 \text{ c}} = 1 \quad \text{and} \quad \frac{2 \text{ pt}}{1 \text{ qt}} = \frac{1 \text{ qt}}{2 \text{ pt}} = 1$$

Example 1 Convert 9 quarts to gallons.

Solution: We multiply by the unit fraction $\frac{1 \text{ gal}}{4 \text{ qt}}$.

$$\begin{aligned} 9 \text{ qt} &= \frac{9 \text{ qt}}{1} \cdot 1 \\ &= \frac{9 \text{ qt}}{1} \cdot \frac{1 \text{ gal}}{4 \text{ qt}} \\ &= \frac{9 \text{ gal}}{4} \\ &= 2\frac{1}{4} \text{ gal} \end{aligned}$$

Thus, 9 quarts is the same as $2\frac{1}{4}$ gallons, as shown in the diagram:



Work Practice 1

Objectives

- A** Define U.S. Units of Capacity and Convert from One Unit to Another.
- B** Perform Arithmetic Operations on U.S. Units of Capacity.
- C** Define Metric Units of Capacity and Convert from One Unit to Another.
- D** Perform Arithmetic Operations on Metric Units of Capacity.

Practice 1

Convert 43 pints to quarts.

Answer

1. $21\frac{1}{2} \text{ qt}$

Practice 2

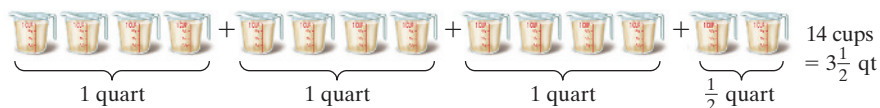
Convert 26 quarts to cups.

Example 2 Convert 14 cups to quarts.**Solution:** Our equivalency table contains no direct conversion from cups to quarts. However, from this table we know that

$$1 \text{ qt} = 2 \text{ pt} = \frac{2 \text{ pt}}{1} \cdot 1 = \frac{2 \text{ pt}}{1} \cdot \frac{2 \text{ c}}{1 \text{ pt}} = 4 \text{ c}$$

so $1 \text{ qt} = 4 \text{ c}$. Now we have the unit fraction $\frac{1 \text{ qt}}{4 \text{ c}}$. Thus,

$$14 \text{ c} = \frac{14 \text{ c}}{1} \cdot 1 = \frac{14 \text{ c}}{1} \cdot \frac{1 \text{ qt}}{4 \text{ c}} = \frac{14 \text{ qt}}{4} = \frac{7}{2} \text{ qt} \text{ or } 3\frac{1}{2} \text{ qt}$$

**Work Practice 2**

✓Concept Check If 50 cups is converted to quarts, will the equivalent number of quarts be less than or greater than 50? Explain.

Objective B Performing Operations on U.S. System Units of Capacity

As is true of units of length and weight, units of capacity can be added, subtracted, multiplied, and divided.

Example 3 Subtract 3 qt from 4 gal 2 qt.**Solution:** To subtract, we line up similar units.

$$\begin{array}{r} 4 \text{ gal } 2 \text{ qt} \\ - \quad 3 \text{ qt} \\ \hline \end{array}$$

We cannot subtract 3 qt from 2 qt. We need to borrow 1 gallon from the 4 gallons, convert it to 4 quarts, and then combine it with the 2 quarts.

$$3 \text{ gal} + \underbrace{1 \text{ gal}}_{4 \text{ qt}} 4 \text{ qt}$$

$$\begin{array}{rcl} 4 \text{ gal } 2 \text{ qt} & = & 3 \text{ gal } 6 \text{ qt} \\ - \quad 3 \text{ qt} & = & - \quad 3 \text{ qt} \\ \hline & & 3 \text{ gal } 3 \text{ qt} \end{array}$$

To check, see that the sum of 3 gal 3 qt and 3 qt is 4 gal 2 qt.

Work Practice 3**Example 4** Finding the Amount of Water in an Aquarium

An aquarium contains 6 gal 3 qt of water. If 2 gal 2 qt of water is added, what is the total amount of water in the aquarium?

$$\begin{array}{rcl} \text{beginning water} & \rightarrow & 6 \text{ gal } 3 \text{ qt} \\ + \text{ water added} & \rightarrow & + 2 \text{ gal } 2 \text{ qt} \\ \hline \text{total water} & \rightarrow & 8 \text{ gal } 5 \text{ qt} \end{array}$$

Practice 3

Subtract 2 qt from 1 gal 1 qt.

Practice 4

A large oil drum contains 15 gal 3 qt of oil. How much will be in the drum if an additional 4 gal 3 qt of oil is poured into it?

Answers

2. 104 c 3. 3 qt 4. 20 gal 2 qt

✓Concept Check Answer

less than 50; answers may vary

Since $5 \text{ qt} = 1 \text{ gal}$, we have

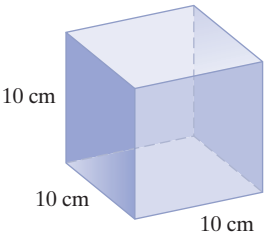
$$\begin{aligned} &= \overbrace{8 \text{ gal}}^{8 \text{ gal}} + \overbrace{1 \text{ gal } 1 \text{ qt}}^{5 \text{ qt}} \\ &= 9 \text{ gal } 1 \text{ qt} \end{aligned}$$

The total amount of water is 9 gal 1 qt.

Work Practice 4

Objective C Defining and Converting Metric System Units of Capacity

Thus far, we know that the basic unit of length in the metric system is the meter and that the basic unit of mass in the metric system is the gram. What is the basic unit of capacity? The **liter**. By definition, a **liter** is the capacity or volume of a cube measuring 10 centimeters on each side.



The following examples may help you get a feeling for metric capacities:

Examples of Metric System – Capacity



- One liter of liquid is slightly more than one quart.



2 liters

- Many soft drinks are packaged in 2-liter bottles.



- Liquid infant pain reliever doses in milliliters (ml or mL)

The metric system was designed to be a consistent system. Once again, the prefixes for metric units of capacity are the same as for metric units of length and mass, as summarized in the following table:

Metric Units of Capacity
1 kilo liter (kl) = 1000 liters (L)
1 hecto liter (hl) = 100 L
1 deka liter (dal) = 10 L
1 liter (L) = 1 L
1 deci liter (dl) = 1/10 L or 0.1 L
1 centi liter (cl) = 1/100 L or 0.01 L
1 milli liter (ml) = 1/1000 L or 0.001 L

Example 6 Convert 0.185 dl to milliliters.

Solution: We list the unit measures in order from left to right and move from deciliters to milliliters.

$$\begin{array}{ccccccc}
 \text{kl} & \text{hl} & \text{dal} & \text{L} & \text{dl} & \text{cl} & \text{ml} \\
 & & & & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \\
 & & & & & \text{2 units to the right} & \\
 0.185 \text{ dl} & = & 18.5 \text{ ml} \\
 & \underbrace{\hspace{1cm}} & \\
 & \text{2 places to the right} &
 \end{array}$$

Work Practice 6**Objective D** Performing Operations on Metric System Units of Capacity

As was true for length and weight, arithmetic operations involving metric units of capacity can also be performed. Make sure that the metric units of capacity are the same before adding, subtracting, multiplying, or dividing.

Example 7 Add 2400 ml to 8.9 L.

Solution: We must convert both to liters or both to milliliters before adding the capacities together.

$$\begin{array}{rcl}
 2400 \text{ ml} = 2.4 \text{ L} & \text{or} & 8.9 \text{ L} = 8900 \text{ ml} \\
 \begin{array}{r} 2.4 \text{ L} \\ + 8.9 \text{ L} \\ \hline 11.3 \text{ L} \end{array} & & \begin{array}{r} 2400 \text{ ml} \\ + 8900 \text{ ml} \\ \hline 11,300 \text{ ml} \end{array}
 \end{array}$$

The total is **11.3 L** or **11,300 ml**. They both represent the same capacity.

Work Practice 7

✓ Concept Check How could you estimate the following operation? Subtract 950 ml from 7.5 L.

Example 8 Finding the Amount of Medication a Person Has Received

A patient hooked up to an IV unit in the hospital is to receive 12.5 ml of medication every hour. How much medication does the patient receive in 3.5 hours?

Solution: We multiply 12.5 ml by 3.5.

$$\begin{array}{rcl}
 \begin{array}{r} \text{medication per hour} \\ \times \hspace{1cm} \text{hours} \\ \hline \text{total medication} \end{array} & \begin{array}{l} \rightarrow \\ \rightarrow \end{array} & \begin{array}{r} 12.5 \text{ ml} \\ \times 3.5 \\ \hline 625 \\ 3750 \\ \hline 43.75 \text{ ml} \end{array}
 \end{array}$$

The patient receives **43.75 ml** of medication.

Work Practice 8**Practice 6**

Convert 2.13 dal to liters.

Practice 7

Add 1250 ml to 2.9 L.

Practice 8

If 28.6 L of water can be pumped every minute, how much water can be pumped in 85 minutes?

Answers

6. 21.3 L **7.** 4150 ml or 4.15 L

8. 2431 L

✓ Concept Check Answer

950 ml = 0.95 L; round 0.95 to 1;
7.5 - 1 = 6.5 L

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

cups	pints	liter
quarts	fluid ounces	capacity

- Units of _____ are generally used to measure liquids.
- The basic unit of capacity in the metric system is the _____.
- One cup equals 8 _____.
- One quart equals 2 _____.
- One pint equals 2 _____.
- One quart equals 4 _____.
- One gallon equals 4 _____.

Martin-Gay Interactive Videos



See Video 9.6

Watch the section lecture video and answer the following questions.

- Objective A** 8. Complete this statement based on Example 1: When using a unit fraction, we are not changing the _____, we are changing the _____. ▶
- Objective B** 9. In Example 4, explain the first step taken to solve the exercise. ▶
- Objective C** 10. In Example 5, how many places is the decimal moved and in what direction? What is the final conversion? ▶
- Objective D** 11. What is the answer to Example 7 in dekaliters? ▶

9.6 Exercise Set MyLab Math

Objective A Convert each measurement as indicated. See Examples 1 and 2.

- | | | | |
|----------------------------------|--------------------------------|----------------------------------|--------------------------------------|
| 1. 32 fluid ounces to cups | 2. 16 quarts to gallons | 3. 8 quarts to pints | 4. 9 pints to quarts |
| ▶ 5. 14 quarts to gallons | 6. 11 cups to pints | 7. 80 fluid ounces to pints | 8. 18 pints to gallons |
| 9. 2 quarts to cups | 10. 3 pints to fluid ounces | 11. 120 fluid ounces to quarts | 12. 20 cups to gallons |
| ▶ 13. 42 cups to quarts | 14. 7 quarts to cups | 15. $4\frac{1}{2}$ pints to cups | 16. $6\frac{1}{2}$ gallons to quarts |
| 17. 5 gal 3 qt to quarts | 18. 4 gal 1 qt to quarts | 19. $\frac{1}{2}$ cup to pints | 20. $\frac{1}{2}$ pint to quarts |
| ▶ 21. 58 qt = _____ gal _____ qt | 22. 70 qt = _____ gal _____ qt | | |

23. $39 \text{ pt} = \underline{\hspace{1cm}} \text{ gal } \underline{\hspace{1cm}} \text{ qt } \underline{\hspace{1cm}} \text{ pt}$

24. $29 \text{ pt} = \underline{\hspace{1cm}} \text{ gal } \underline{\hspace{1cm}} \text{ qt } \underline{\hspace{1cm}} \text{ pt}$

25. $2\frac{3}{4}$ gallons to pints

26. $3\frac{1}{4}$ quarts to cups

Objective B Perform each indicated operation. See Examples 3 and 4.

27. $5 \text{ gal } 3 \text{ qt} + 7 \text{ gal } 3 \text{ qt}$

28. $2 \text{ gal } 2 \text{ qt} + 9 \text{ gal } 3 \text{ qt}$

29. $1 \text{ c } 5 \text{ fl oz} + 2 \text{ c } 7 \text{ fl oz}$

30. $2 \text{ c } 3 \text{ fl oz} + 2 \text{ c } 6 \text{ fl oz}$

▶ 31. $3 \text{ gal} - 1 \text{ gal } 3 \text{ qt}$

32. $2 \text{ pt} - 1 \text{ pt } 1 \text{ c}$

33. $3 \text{ gal } 1 \text{ qt} - 1 \text{ qt } 1 \text{ pt}$

34. $3 \text{ qt } 1 \text{ c} - 1 \text{ c } 4 \text{ fl oz}$

35. $8 \text{ gal } 2 \text{ qt} \times 2$

36. $6 \text{ gal } 1 \text{ pt} \times 2$

37. $9 \text{ gal } 2 \text{ qt} \div 2$

38. $5 \text{ gal } 6 \text{ fl oz} \div 2$

Objective C Convert as indicated. See Examples 5 and 6.

39. 5 L to milliliters

40. 8 L to milliliters

41. 0.16 L to kiloliters

42. 0.127 L to kiloliters

▶ 43. 5600 ml to liters

44. 1500 ml to liters

45. 3.2 L to centiliters

46. 1.7 L to centiliters

47. 410 L to kiloliters

48. 250 L to kiloliters

49. 64 ml to liters

50. 39 ml to liters

▶ 51. 0.16 kl to liters

52. 0.48 kl to liters

53. 3.6 L to milliliters

54. 1.9 L to milliliters

Objective D Perform each indicated operation. Remember to insert units when writing your answers. See Examples 7 and 8.

55. $3.4 \text{ L} + 15.9 \text{ L}$

56. $18.5 \text{ L} + 4.6 \text{ L}$

▶ 57. $2700 \text{ ml} + 1.8 \text{ L}$

58. $4.6 \text{ L} + 1600 \text{ ml}$

59. $8.6 \text{ L} - 190 \text{ ml}$

60. $4.8 \text{ L} - 283 \text{ ml}$

61. $17,500 \text{ ml} - 0.9 \text{ L}$

62. $6850 \text{ ml} - 0.3 \text{ L}$

63. $480 \text{ ml} \times 8$

64. $290 \text{ ml} \times 6$

65. $81.2 \text{ L} \div 0.5$

66. $5.4 \text{ L} \div 3.6$

Objectives A B C D Mixed Practice Solve. Remember to insert units when writing your answers. For Exercises 67 through 70, complete the chart.

	Capacity	Cups	Gallons	Quarts	Pints
67.	An average-size bath of water		21		
68.	A dairy cow's daily milk yield				38
69.	Your kidneys filter about this amount of blood every minute	4			
70.	The amount of water needed in a punch recipe	2			

71. Mike Schaferkotter drank 410 ml of Mountain Dew from a 2-liter bottle. How much Mountain Dew remains in the bottle?
72. The Werners' Volvo has a 54.5-L gas tank. Only 3.8 liters of gasoline still remain in the tank. How much is needed to fill it?
73. Margie Phitts added 354 ml of Prestone dry gas to the 18.6 L of gasoline in her car's tank. Find the total amount of gasoline in the tank.
74. Chris Peckaitis wishes to share a 2-L bottle of Coca-Cola equally with 7 of his friends. How much will each person get?
75. A garden tool engine requires a 30-to-1 gas-to-oil mixture. This means that $\frac{1}{30}$ of a gallon of oil should be mixed with 1 gallon of gas. Convert $\frac{1}{30}$ gallon to fluid ounces. Round to the nearest tenth.
76. Henning's Supermarket sells homemade soup in 1 qt 1 pt containers. How much soup is contained in three such containers?
77. Can 5 pt 1 c of fruit punch and 2 pt 1 c of ginger ale be poured into a 1-gal container without it overflowing?
78. Three cups of prepared Jell-O are poured into 6 dessert dishes. How many fluid ounces of Jell-O are in each dish?
79. Stanley Fisher paid \$14 to fill his car with 44.3 liters of gasoline. Find the price per liter of gasoline to the nearest thousandth of a dollar.
80. A student carelessly misread the scale on a cylinder in the chemistry lab and added 40 cl of water to a mixture instead of 40 ml. Find the excess amount of water.

Review

Write each fraction in simplest form. See Section 4.2.

81. $\frac{20}{25}$

82. $\frac{75}{100}$

83. $\frac{27}{45}$

84. $\frac{56}{60}$

85. $\frac{72}{80}$

86. $\frac{18}{20}$

Concept Extensions

Determine whether the measurement in each statement is reasonable.

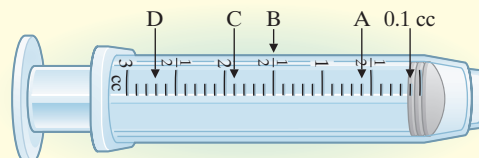
87. Clair took a dose of 2 L of cough medicine to cure her cough.
88. John drank 250 ml of milk for lunch.
89. Jeannie likes to relax in a tub filled with 3000 ml of hot water.
90. Sarah pumped 20 L of gasoline into her car yesterday.

Solve. See the Concept Checks in this section.

91. If 70 pints are converted to gallons, will the equivalent number of gallons be less than or greater than 70? Explain why.
92. If 30 gallons are converted to quarts, will the equivalent number of quarts be less than or greater than 30? Explain why.
93. Explain how to estimate the following operation: Add 986 ml to 6.9 L.
94. Explain how to borrow in order to subtract 1 gal 2 qt from 3 gal 1 qt.
95. Find the number of fluid ounces in 1 gallon.
96. Find the number of fluid ounces in 1.5 gallons.

A cubic centimeter (cc) is the amount of space that a volume of 1 ml occupies. Because of this, we will say that $1 \text{ cc} = 1 \text{ ml}$.

A common syringe is one with a capacity of 3 cc. Use the diagram and give the measurement indicated by each arrow.



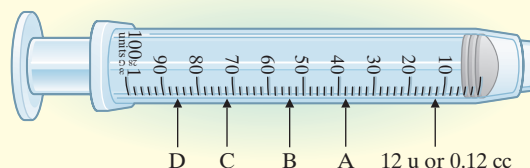
97. B

98. A

99. D

100. C

In order to measure small dosages, such as for insulin, u-100 syringes are used. For these syringes, 1 cc has been divided into 100 equal units (u). Use the diagram and give the measurement indicated by each arrow in units (u) and then in cubic centimeters. Use $100 \text{ u} = 1 \text{ cc}$.



101. B

102. A

103. D

104. C

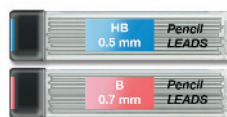
9.7 Temperature and Conversions Between the U.S. and Metric Systems

Objective A Converting Between the U.S. and Metric Systems

The metric system probably had its beginnings in France in the 1600s, but it was the Metric Act of 1866 that made the use of this system legal (although not mandatory) in the United States. Other laws have followed that allow for a slow, but deliberate, transfer to the modernized metric system. In April 2001, for example, the U.S. Stock Exchanges completed their change to decimal trading instead of fractions. By the end of 2009, all products sold in Europe (with some exceptions) were required to have only metric units on their labels. (Source: U.S. Metric Association and National Institute of Standards and Technology)

You may be surprised at the number of everyday items we use that are already manufactured in metric units. We easily recognize 1 L and 2 L soda bottles, but what about the following?

- Pencil leads (0.5 mm or 0.7 mm)
- Camera film (35 mm)
- Sporting events (5-km or 10-km races)



Objectives

- A** Convert Between the U.S. and Metric Systems.
- B** Convert Temperatures from Degrees Celsius to Degrees Fahrenheit.
- C** Convert Temperatures from Degrees Fahrenheit to Degrees Celsius.



Practice 1

The center hole of a standard-sized compact disc is 1.5 centimeters in diameter. Convert this length to inches. Round the result to 2 decimal places.

Practice 2

A full-grown human heart weighs about 8 ounces. Convert this weight to grams. If necessary, round your result to the nearest tenth of a gram.

Answers

1. 0.59 in. 2. 226.8 g

- Medicines (500-mg capsules)



- Labels on retail goods (dual-labeled since 1994)

Since the United States has not completely converted to the metric system, we need to practice converting from one system to the other. Below is a table of mostly approximate conversions.

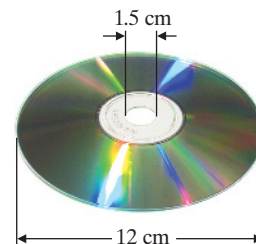
Length:		Capacity:		Weight (mass):	
Metric	U.S. System	Metric	U.S. System	Metric	U.S. System
1 m $\approx$ 1.09 yd		1 L $\approx$ 1.06 qt		1 kg $\approx$ 2.2 lb	
1 m $\approx$ 3.28 ft		1 L $\approx$ 0.26 gal		1 g $\approx$ 0.04 oz	
1 km $\approx$ 0.62 mi		3.79 L $\approx$ 1 gal		0.45 kg $\approx$ 1 lb	
2.54 cm = 1 in.		0.95 L $\approx$ 1 qt		28.35 g $\approx$ 1 oz	
0.30 m $\approx$ 1 ft		29.57 ml $\approx$ 1 fl oz			
1.61 km $\approx$ 1 mi					

There are many ways to perform these metric-to-U.S. conversions. We will do so by using unit fractions.

Example 1 Compact Discs

Standard-sized compact discs are 12 centimeters in diameter. Convert this length to inches. Round the result to two decimal places.

Solution: From our length conversion table, we know that $2.54 \text{ cm} = 1 \text{ in.}$ This fact gives us two unit fractions: $\frac{2.54 \text{ cm}}{1 \text{ in.}}$ and $\frac{1 \text{ in.}}{2.54 \text{ cm}}$. We use the unit fraction with cm in the denominator so that these units divide out.



$$\begin{aligned}
 12 \text{ cm} &= \frac{12 \text{ cm}}{1} \cdot 1 = \frac{12 \text{ cm}}{1} \cdot \frac{1 \text{ in.}}{2.54 \text{ cm}} \quad \begin{array}{l} \text{Unit fraction} \\ \leftarrow \text{Units to convert to} \\ \leftarrow \text{Original units} \end{array} \\
 &= \frac{12}{2.54} \text{ in.} \\
 &\approx 4.72 \text{ in.} \quad \text{Divide.}
 \end{aligned}$$

Thus, the diameter of a standard compact disc is exactly 12 cm or approximately 4.72 inches. For a dimension this size, you can use a ruler to check. Another method is to approximate. Our result, 4.72 in., is close to 5 inches. Since 1 in. is about 2.5 cm, then 5 in. is about $5(2.5 \text{ cm}) = 12.5 \text{ cm}$, which is close to 12 cm.

Work Practice 1

Example 2 Liver

The liver is your largest internal organ. It weighs about 3.5 pounds in a grown man. Convert this weight to kilograms. Round to the nearest tenth. (*Source: Some Body!* by Dr. Pete Rowan)

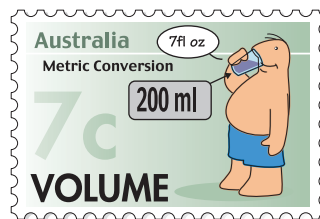
$$\text{Solution: } 3.5 \text{ lb} \approx \frac{3.5 \text{ lb}}{1} \cdot \frac{0.45 \text{ kg}}{1 \text{ lb}} = 3.5(0.45 \text{ kg}) \approx 1.6 \text{ kg}$$

Thus 3.5 pounds is approximately 1.6 kilograms. From the table of conversions, we know that $1 \text{ kg} \approx 2.2 \text{ lb}$. So that means $0.5 \text{ kg} \approx 1.1 \text{ lb}$ and after adding, we have $1.5 \text{ kg} \approx 3.3 \text{ lb}$. Our result is reasonable.

Work Practice 2

Example 3 Postage Stamp

Australia converted to the metric system in 1973. In that year, four postage stamps were issued to publicize this conversion. One such stamp is shown. Let's check the mathematics on the stamp by converting 7 fluid ounces to milliliters. Round to the nearest hundred.



Solution: $7 \text{ fl oz} \approx \frac{7 \text{ fl oz}}{1} \cdot \frac{29.57 \text{ ml}}{1 \text{ fl oz}} = 7(29.57 \text{ ml}) = 206.99 \text{ ml}$

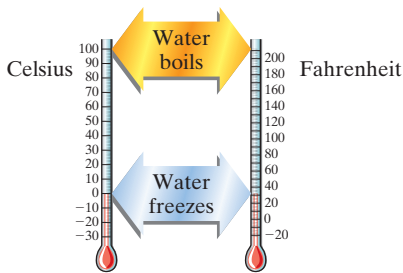
Rounded to the nearest hundred, $7 \text{ fl oz} \approx 200 \text{ ml}$.

Work Practice 3

Now that we have practiced converting between two measurement systems, let's practice converting between two temperature scales.

Temperature When Gabriel Fahrenheit and Anders Celsius independently established units for temperature scales, each based his unit on the heat of water the moment it boils compared to the moment it freezes. One degree Celsius is $\frac{1}{100}$ of the difference in heat. One degree Fahrenheit is $\frac{1}{180}$ of the difference in heat. Celsius arbitrarily labeled the temperature at the freezing point at 0°C , making the boiling point 100°C ; Fahrenheit labeled the freezing point 32°F , making the boiling point 212°F . Water boils at 212°F or 100°C .

By comparing the two scales in the figure, we see that a 20°C day is as warm as a 68°F day. Similarly, a sweltering 104°F day in the Mojave desert corresponds to a 40°C day.



✓Concept Check Which of the following statements is correct? Explain.

- 6°C is below the freezing point of water.
- 6°F is below the freezing point of water.

Objective B Converting Degrees Celsius to Degrees Fahrenheit

To convert from Celsius temperatures to Fahrenheit temperatures, see the box below. In this box, we use the symbol F to represent degrees Fahrenheit and the symbol C to represent degrees Celsius.

Converting Celsius to Fahrenheit

$$F = \frac{9}{5}C + 32 \quad \text{or} \quad F = 1.8C + 32$$

(To convert to Fahrenheit temperature, multiply the Celsius temperature by $\frac{9}{5}$ or 1.8, and then add 32.)

Practice 3

Convert 237 ml to fluid ounces. Round to the nearest whole fluid ounce.

Answer

3. 8 fl oz

✓Concept Check Answer

b

Practice 4

Convert 60°C to degrees Fahrenheit.

Practice 5

Convert 32°C to degrees Fahrenheit.

Practice 6

Convert 68°F to degrees Celsius.

Answers

4. 140°F 5. 89.6°F 6. 20°C

Example 4

Convert 15°C to degrees Fahrenheit.

$$\begin{aligned}\text{Solution: } F &= \frac{9}{5}C + 32 \\ &= \frac{9}{5} \cdot 15 + 32 && \text{Replace } C \text{ with } 15. \\ &= 27 + 32 && \text{Simplify.} \\ &= 59 && \text{Add.}\end{aligned}$$

Thus, 15°C is equivalent to 59°F .

 Work Practice 4

Example 5

Convert 29°C to degrees Fahrenheit.

$$\begin{aligned}\text{Solution: } F &= 1.8C + 32 \\ &= 1.8 \cdot 29 + 32 && \text{Replace } C \text{ with } 29. \\ &= 52.2 + 32 && \text{Multiply } 1.8 \text{ by } 29. \\ &= 84.2 && \text{Add.}\end{aligned}$$

Therefore, 29°C is the same as 84.2°F .

 Work Practice 5

Objective C Converting Degrees Fahrenheit to Degrees Celsius

To convert from Fahrenheit temperatures to Celsius temperatures, see the box below. The symbol C represents degrees Celsius and the symbol F represents degrees Fahrenheit.

Converting Fahrenheit to Celsius

$$C = \frac{5}{9}(F - 32)$$

(To convert to Celsius temperature, subtract 32 from the Fahrenheit temperature, and then multiply by $\frac{5}{9}$.)

Example 6

Convert 59°F to degrees Celsius.

Solution: We evaluate the formula $C = \frac{5}{9}(F - 32)$ when F is 59.

$$\begin{aligned}C &= \frac{5}{9}(F - 32) \\ &= \frac{5}{9} \cdot (59 - 32) && \text{Replace } F \text{ with } 59. \\ &= \frac{5}{9} \cdot (27) && \text{Subtract inside parentheses.} \\ &= 15 && \text{Multiply.}\end{aligned}$$

Therefore, 59°F is the same temperature as 15°C .

 Work Practice 6

Example 7 Convert 114°F to degrees Celsius. If necessary, round to the nearest tenth of a degree.

Solution: $C = \frac{5}{9}(F - 32)$

$$= \frac{5}{9}(114 - 32) \quad \text{Replace F with 114.}$$

$$= \frac{5}{9} \cdot (82) \quad \text{Subtract inside parentheses.}$$

$$\approx 45.6 \quad \text{Multiply.}$$

Therefore, 114°F is approximately 45.6°C .

Work Practice 7

Example 8 Body Temperature

Normal body temperature is 98.6°F . What is this temperature in degrees Celsius?

Solution: We evaluate the formula $C = \frac{5}{9}(F - 32)$ when F is 98.6.

$$C = \frac{5}{9}(F - 32)$$

$$= \frac{5}{9}(98.6 - 32) \quad \text{Replace F with 98.6.}$$

$$= \frac{5}{9} \cdot (66.6) \quad \text{Subtract inside parentheses.}$$

$$= 37 \quad \text{Multiply.}$$

Therefore, normal body temperature is 37°C .

Work Practice 8

✓Concept Check Clarissa must convert 40°F to degrees Celsius. What is wrong with her work shown below?

$$F = 1.8 \cdot C + 32$$

$$F = 1.8 \cdot 40 + 32$$

$$F = 72 + 32$$

$$F = 104$$

Practice 7

Convert 113°F to degrees Celsius. If necessary, round to the nearest tenth of a degree.

Practice 8

During a bout with the flu, Albert's temperature reaches 102.8°F . What is his temperature measured in degrees Celsius? Round to the nearest tenth of a degree.

Answers

7. 45°C 8. 39.3°C

✓Concept Check Answer

She used the conversion for Celsius to Fahrenheit instead of Fahrenheit to Celsius.

Vocabulary, Readiness & Video Check

Martin-Gay Interactive Videos



See Video 9.7

Watch the section lecture video and answer the following questions.

Objective A 1. Write two conversions that may be used to solve

Example 2.

2. Why isn't 0.1125 kg the final answer to Example 3?

Objective B 3. Which version of the formula is used to solve Example 4? What is the replacement value for C ?

Objective C 4. In Example 5, what is the replacement value for F ? What is the final conversion?

9.7 Exercise Set MyLab Math

Note: Because approximations are used, your answers may vary slightly from the answers given in the back of the book.

Objective A Convert as indicated. If necessary, round answers to two decimal places. See Examples 1 through 3.

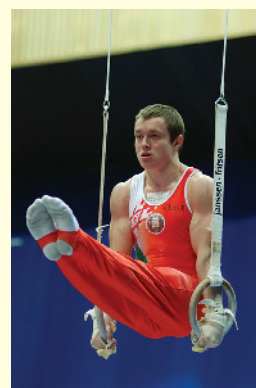
1. 756 milliliters to fluid ounces
2. 18 liters to quarts
3. 86 inches to centimeters
4. 86 miles to kilometers
5. 1000 grams to ounces
6. 100 kilograms to pounds
7. 93 kilometers to miles
8. 9.8 meters to feet
9. 14.5 liters to gallons
10. 150 milliliters to fluid ounces
11. 30 pounds to kilograms
12. 15 ounces to grams

Fill in the chart. Give exact answers or round to one decimal place. See Examples 1 through 3.

	Meters	Yards	Centimeters	Feet	Inches
13. The height of a woman				5	
14. Statue of Liberty length of nose	1.37				
15. Leaning Tower of Pisa		60			
16. Blue whale		36			

Solve. If necessary, round answers to two decimal places. See Examples 1 through 3.

17. The balance beam for female gymnasts is 10 centimeters wide. Convert this width to inches.
18. In men's gymnastics, the rings are 250 centimeters from the floor. Convert this height to inches, then to feet.



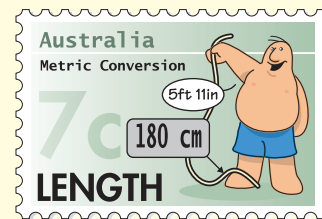
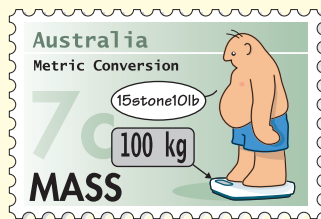
19. In many states, the maximum speed limit for recreational vehicles is 50 miles per hour. Convert this to kilometers per hour.
20. In some states, the speed limit is 70 miles per hour. Convert this to kilometers per hour.
21. Ibuprofen comes in 200-milligram tablets. Convert this to ounces. (Round your answer to this exercise to 3 decimal places.)
22. Vitamin C tablets come in 500-milligram caplets. Convert this to ounces.

The 70-meter-diameter antenna is the largest and most sensitive Deep Space Network antenna. See the Chapter Opener and answer Exercises 23 through 26.



70-Meter Antenna

- 23.** Convert 70 meters to feet.
- 24.** The Deep Space Network sites also have a 26-meter antenna. Convert 26 meters to feet.
- 25.** The 70-meter-diameter antenna can track a spacecraft traveling more than 16 billion kilometers from Earth. Convert this distance to miles.
- 26.** The dish reflector and the mount atop the concrete pedestal of the 70-meter antenna weigh nearly 2.7 million kilograms. Convert this number to tons.
- 27.** A stone is a unit in the British customary system. Use the conversion 14 pounds = 1 stone to check the equivalencies in this 1973 Australian stamp. Is 100 kilograms approximately 15 stone 10 pounds?



- 29.** The Monarch butterfly migrates annually between the northern United States and central Mexico. The trip is about 4500 km long. Convert this to miles.
- 30.** There is a species of African termite that builds nests up to 18 ft high. Convert this to meters.
- 31.** A $3\frac{1}{2}$ -inch diskette is not really $3\frac{1}{2}$ inches. To find its actual width, convert this measurement to centimeters, then to millimeters. Round the result to the nearest ten.
- 32.** The average two-year-old is 84 centimeters tall. Convert this to feet and inches.
- 33.** For an average adult, the weight of the right lung is greater than the weight of the left lung. If the right lung weighs 1.5 pounds and the left lung weighs 1.25 pounds, find the difference in grams. (Source: *Some Body!*)
- 34.** The skin of an average adult weighs 9 pounds and is the heaviest organ. Find the weight in grams. (Source: *Some Body!*)
- 35.** A fast sneeze has been clocked at about 167 kilometers per hour. Convert this to miles per hour. Round to the nearest whole.
- 36.** A Boeing 747 has a cruising speed of about 980 kilometers per hour. Convert this to miles per hour. Round to the nearest whole.
- 37.** The General Sherman giant sequoia tree has a diameter of about 8 meters at its base. Convert this to feet. (Source: *Fantastic Book of Comparisons*)
- 38.** The largest crater on the near side of the moon is Billy Crater. It has a diameter of 303 kilometers. Convert this to miles. (Source: *Fantastic Book of Comparisons*)

39. The total length of the track on a CD is about 4.5 kilometers. Convert this to miles. Round to the nearest whole mile.
40. The distance between Mackinaw City, Michigan, and Cheyenne, Wyoming, is 2079 kilometers. Convert this to miles. Round to the nearest whole mile.
41. A doctor orders a dosage of 5 ml of medicine every 4 hours for 1 week. How many fluid ounces of medicine should be purchased? Round up to the next whole fluid ounce.
42. A doctor orders a dosage of 12 ml of medicine every 6 hours for 10 days. How many fluid ounces of medicine should be purchased? Round up to the next whole fluid ounce.

Without actually converting, choose the most reasonable answer.

43. This math book has a height of about _____.
 a. 28 mm b. 28 cm
 c. 28 m d. 28 km
44. A mile is _____ a kilometer.
 a. shorter than b. longer than
 c. the same length as
45. A liter has _____ capacity than a quart.
 a. less b. greater
 c. the same
46. A foot is _____ a meter.
 a. shorter than b. longer than
 c. the same length as
47. A kilogram weighs _____ a pound.
 a. the same as b. less than
 c. greater than
48. A football field is 100 yards, which is about _____.
 a. 9 m b. 90 m
 c. 900 m d. 9000 m
49. An $8\frac{1}{2}$ -ounce glass of water has a capacity of about _____.
 a. 250 L b. 25 L
 c. 2.5 L d. 250 ml
50. A 5-gallon gasoline can has a capacity of about _____.
 a. 19 L b. 1.9 L
 c. 19 ml d. 1.9 ml
51. The weight of an average man is about _____.
 a. 700 kg b. 7 kg
 c. 0.7 kg d. 70 kg
52. The weight of a pill is about _____.
 a. 200 kg b. 20 kg
 c. 2 kg d. 200 mg

Objectives B C Mixed Practice Convert as indicated. When necessary, round to the nearest tenth of a degree. See Examples 4 through 8.

53. 77°F to degrees Celsius 54. 86°F to degrees Celsius 55. 104°F to degrees Celsius
56. 140°F to degrees Celsius 57. 50°C to degrees Fahrenheit 58. 80°C to degrees Fahrenheit
59. 115°C to degrees Fahrenheit 60. 225°C to degrees Fahrenheit 61. 20°F to degrees Celsius
62. 26°F to degrees Celsius 63. 142.1°F to degrees Celsius 64. 43.4°F to degrees Celsius
65. 92°C to degrees Fahrenheit 66. 75°C to degrees Fahrenheit
67. 12.4°C to degrees Fahrenheit 68. 48.6°C to degrees Fahrenheit

- 69.** The hottest temperature ever recorded in the United States, in Death Valley, was 134°F . Convert this temperature to degrees Celsius. (*Source:* National Climatic Data Center)
- 70.** The hottest temperature ever recorded in the United States in January was 95°F in Los Angeles. Convert this temperature to degrees Celsius. (*Source:* National Climatic Data Center)
- 71.** A weather forecaster in Caracas predicts a high temperature of 27°C . Find this measurement in degrees Fahrenheit.
- 72.** While driving to work, Alan Olda notices a temperature of 18°C flash on the local bank's temperature display. Find the corresponding temperature in degrees Fahrenheit.
- 73.** At Mack Trucks' headquarters, the room temperature is to be set at 70°F , but the thermostat is calibrated in degrees Celsius. Find the temperature to be set.
- 74.** The computer room at Merck, Sharp, and Dohm is normally cooled to 66°F . Find the corresponding temperature in degrees Celsius.
- 75.** In a European cookbook, a recipe requires the ingredients for caramels to be heated to 118°C , but the cook has access only to a Fahrenheit thermometer. Find the temperature in degrees Fahrenheit that should be used to make the caramels.
- 76.** The ingredients for divinity should be heated to 127°C , but the candy thermometer that Myung Kim has is calibrated to degrees Fahrenheit. Find how hot he should heat the ingredients.
- 77.** The temperature of Earth's core is estimated to be 4000°C . Find the corresponding temperature in degrees Fahrenheit.
- 78.** In 2012, the average temperature of Earth's surface was 58.3°F . Convert this temperature to degrees Celsius. (*Source:* NASA)

Review

Perform the indicated operations. See Section 1.7

79. $6 \cdot 4 + 5 \div 1$

80. $10 \div 2 + 9(8)$

81. $3[(1 + 5) \cdot (8 - 6)]$

82. $5[(18 - 8) - 9]$

Concept Extensions

Determine whether the measurement in each statement is reasonable.

- 83.** A 72°F room feels comfortable.
- 84.** Water heated to 110°F will boil.
- 85.** Josiah has a fever if a thermometer shows his temperature to be 40°F .
- 86.** An air temperature of 20°F on a Vermont ski slope can be expected in the winter.
- 87.** When the temperature is 30°C outside, an overcoat is needed.
- 88.** An air-conditioned room at 60°C feels quite chilly.
- 89.** Barbara has a fever when a thermometer records her temperature at 40°C .
- 90.** Water cooled to 32°C will freeze.

Body surface area (BSA) is often used to calculate dosages for some drugs. BSA is calculated in square meters using a person's weight and height.

$$\text{BSA} = \sqrt{\frac{(\text{weight in kg}) \times (\text{height in cm})}{3600}}$$

For Exercises 91 through 96, calculate the BSA for each person. Round to the nearest hundredth. You will need to use the square root key on your calculator.

-  **91.** An adult whose height is 182 cm and weight is 90 kg

-  **92.** An adult whose height is 157 cm and weight is 63 kg

-  **93.** A child whose height is 40 in. and weight is 50 kg (*Hint: Don't forget to first convert inches to centimeters.*)
-  **95.** An adult whose height is 60 in. and weight is 150 lb
- 97.** The most durable storage medium in the world is a nanostructured glass created at Southampton's Optoelectronics Research Center, United Kingdom. This remains stable for 13.8 billion years at a temperature of 371.9°F. Convert this temperature to degrees Celsius. Round your answer to the nearest tenth of a degree. (*Source: Guinness Book of World Records*)
-  **99.** In your own words, describe how to convert from degrees Celsius to degrees Fahrenheit.
-  **100.** In your own words, describe how to convert from degrees Fahrenheit to degrees Celsius.
-  **94.** A child whose height is 26 in. and weight is 13 kg (*Hint: Don't forget to first convert inches to centimeters.*)
-  **96.** An adult whose height is 69 in. and weight is 172 lb
- 98.** The hottest-burning substance known is carbon subnitride. Its flame at one atmospheric pressure reaches 9010°F. Convert this temperature to degrees Celsius. (*Source: Guinness World Records*)

Chapter 9 Group Activity

Map Reading

Sections 9.1, 9.4, and 9.7

Materials:

- ruler
- string
- calculator

This activity may be completed by working in groups or individually.

Investigate the route you would take from Santa Rosa, New Mexico, to San Antonio, New Mexico. Use the map in the figure to answer the following questions. You may find that using string to match the roads on the map is useful when measuring distances.



1. How many miles is it from Santa Rosa to San Antonio via Interstate 40 and Interstate 25? Convert this distance to kilometers.
2. How many miles is it from Santa Rosa to San Antonio via U.S. 54 and U.S. 380? Convert this distance to kilometers.
3. Assume that the speed limit on Interstates 40 and 25 is 65 miles per hour. How long would the trip take if you took this route and traveled 65 miles per hour the entire trip?
4. At what average speed would you have to travel on the U.S. routes to make the trip from Santa Rosa to San Antonio in the same amount of time that it would take on the interstate routes? Do you think this speed is reasonable on this route? Explain your reasoning.
5. Discuss in general the factors that might affect your decision between the different routes.
6. Explain which route you would choose in this case and why.

Chapter 9 Vocabulary Check

Fill in each blank with one of the words or phrases listed below.

transversal	line segment	obtuse	straight	adjacent	right	volume	area
acute	perimeter	vertical	supplementary	ray	angle	line	complementary
vertex	mass	unit fractions	gram	weight	meter	liter	surface area

- _____ is a measure of the pull of gravity.
- _____ is a measure of the amount of substance in an object. This measure does not change.
- The basic unit of length in the metric system is the _____.
- To convert from one unit of length to another, _____ may be used.
- The _____ is the basic unit of mass in the metric system.
- The _____ is the basic unit of capacity in the metric system.
- A(n) _____ is a piece of a line with two endpoints.
- Two angles that have a sum of 90° are called _____ angles.
- A(n) _____ is a set of points extending indefinitely in two directions.
- The _____ of a polygon is the distance around the polygon.
- A(n) _____ is made up of two rays that share the same endpoint. The common endpoint is called the _____.
- _____ measures the amount of surface of a region.
- A(n) _____ is a part of a line with one endpoint. A ray extends indefinitely in one direction.
- A line that intersects two or more lines at different points is called a(n) _____.
- An angle that measures 180° is called a(n) _____ angle.
- The measure of the space of a solid is called its _____.
- When two lines intersect, four angles are formed. Two of these angles that are opposite each other are called _____ angles.
- Two of the angles from Exercise 17 that share a common side are called _____ angles.
- An angle whose measure is between 90° and 180° is called a(n) _____ angle.
- An angle that measures 90° is called a(n) _____ angle.
- An angle whose measure is between 0° and 90° is called a(n) _____ angle.
- Two angles that have a sum of 180° are called _____ angles.
- The _____ of a polyhedron is the sum of the areas of the faces of the polyhedron.

Helpful Hint

▶ Are you preparing for your test?

To help, don't forget to take these:

- Chapter 9 Getting Ready for the Test on page 707
- Chapter 9 Test on page 708

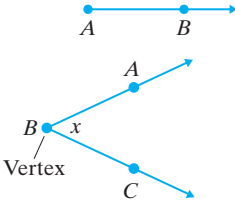
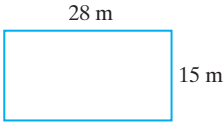
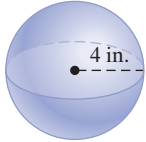
The check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

9

Chapter Highlights

Definitions and Concepts	Examples
Section 9.1 Lines and Angles	
A line is a set of points extending indefinitely in two directions. A line has no width or height, but it does have length. We name a line by any two of its points.	Line AB or $\overleftrightarrow{AB}$ 
A line segment is a piece of a line with two endpoints.	Line segment AB or $\overline{AB}$ 

(Continued on next page)

Definitions and Concepts	Examples
Section 9.1 Lines and Angles (continued)	
A ray is a part of a line with one endpoint. A ray extends indefinitely in one direction. An angle is made up of two rays that share the same endpoint. The common endpoint is called the vertex.	Ray AB or $\overrightarrow{AB}$ $\angle B$ or $\angle x$ or $\angle ABC$ or $\angle CBA$ 
Section 9.2 Perimeter	
Perimeter Formulas Rectangle: $P = 2l + 2w$ Square: $P = 4s$ Triangle: $P = a + b + c$ Circumference of a Circle: $C = 2\pi r$ or $C = \pi d$ where $\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$	Find the perimeter of the rectangle.  $\begin{aligned} P &= 2l + 2w \\ &= 2 \cdot 28 \text{ meters} + 2 \cdot 15 \text{ meters} \\ &= 56 \text{ meters} + 30 \text{ meters} \\ &= 86 \text{ meters} \end{aligned}$ The perimeter is 86 meters.
Section 9.3 Area, Volume, and Surface Area	
Area Formulas Rectangle: $A = lw$ Square: $A = s^2$ Triangle: $A = \frac{1}{2}bh$ Parallelogram: $A = bh$ Trapezoid: $A = \frac{1}{2}(b + B)h$ Circle: $A = \pi r^2$ Volume Formulas Rectangular Solid: $V = lwh$ Cube: $V = s^3$ Sphere: $V = \frac{4}{3}\pi r^3$ Right Circular Cylinder: $V = \pi r^2h$ Cone: $V = \frac{1}{3}\pi r^2h$ Square-Based Pyramid: $V = \frac{1}{3}s^2h$ Surface Area Formulas: See page 644.	Find the area of the square.  $\begin{aligned} A &= s^2 \\ &= (8 \text{ centimeters})^2 \\ &= 64 \text{ square centimeters} \end{aligned}$ The area of the square is 64 square centimeters. Find the volume of the sphere. Use $\frac{22}{7}$ for π.  $\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &\approx \frac{4}{3} \cdot \frac{22}{7} \cdot (4 \text{ inches})^3 \\ &= \frac{4 \cdot 22 \cdot 64}{3 \cdot 7} \text{ cubic inches} \\ &= \frac{5632}{21} \text{ cubic inches or } 268 \frac{4}{21} \text{ cubic inches} \end{aligned}$

Definitions and Concepts

Examples

Section 9.4 Linear Measurement

To convert from one unit of length to another, multiply by a **unit fraction** in the form

$$\frac{\text{units to convert to}}{\text{original units}}$$

Length: U.S. System of Measurement

$$12 \text{ inches (in.)} = 1 \text{ foot (ft)}$$

$$3 \text{ feet} = 1 \text{ yard (yd)}$$

$$5280 \text{ feet} = 1 \text{ mile (mi)}$$

Length: Metric System of Measurement

The basic unit of length in the metric system is the **meter**. A meter is slightly longer than a yard.

Metric Units of Length

$$1 \text{ kilometer (km)} = 1000 \text{ meters (m)}$$

$$1 \text{ hectometer (hm)} = 100 \text{ m}$$

$$1 \text{ dekameter (dam)} = 10 \text{ m}$$

$$1 \text{ meter (m)} = 1 \text{ m}$$

$$1 \text{ decimeter (dm)} = 1/10 \text{ m or } 0.1 \text{ m}$$

$$1 \text{ centimeter (cm)} = 1/100 \text{ m or } 0.01 \text{ m}$$

$$1 \text{ millimeter (mm)} = 1/1000 \text{ m or } 0.001 \text{ m}$$

$$\frac{12 \text{ inches}}{1 \text{ foot}}, \frac{1 \text{ foot}}{12 \text{ inches}}, \frac{3 \text{ feet}}{1 \text{ yard}}$$

Convert 6 feet to inches.

$$\begin{aligned} 6 \text{ ft} &= \frac{6 \text{ ft}}{1} \cdot 1 \\ &= \frac{6 \cancel{\text{ft}}}{1} \cdot \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} \leftarrow \begin{array}{l} \text{units to convert to} \\ \text{original units} \end{array} \\ &= 6 \cdot 12 \text{ in.} \\ &= 72 \text{ in.} \end{aligned}$$

Convert 3650 centimeters to meters.

$$\begin{aligned} 3650 \text{ cm} &= 3650 \text{ cm} \cdot 1 \\ &= \frac{3650 \cancel{\text{cm}}}{1} \cdot \frac{0.01 \text{ m}}{1 \cancel{\text{cm}}} = 36.5 \text{ m} \end{aligned}$$

or

$$\begin{array}{ccccccc} \text{km} & \text{hm} & \text{dam} & \text{m} & \text{dm} & \text{cm} & \text{mm} \\ & & & \underbrace{\hspace{1.5cm}}_{2 \text{ units to the left}} & & & \end{array}$$

$$\begin{array}{c} 3650 \text{ cm} = 36.5 \text{ m} \\ \underbrace{\hspace{1.5cm}}_{2 \text{ places to the left}} \end{array}$$

Section 9.5 Weight and Mass

Weight is really a measure of the pull of gravity. **Mass** is a measure of the amount of substance in an object and does not change.

Weight: U.S. System of Measurement

$$16 \text{ ounces (oz)} = 1 \text{ pound (lb)}$$

$$2000 \text{ pounds} = 1 \text{ ton}$$

Mass: Metric System of Measurement

The **gram** is the basic unit of mass in the metric system. It is the mass of water contained in a cube 1 centimeter on each side. A paper clip weighs about 1 gram.

Metric Units of Mass

$$1 \text{ kilogram (kg)} = 1000 \text{ grams (g)}$$

$$1 \text{ hectogram (hg)} = 100 \text{ g}$$

$$1 \text{ dekagram (dag)} = 10 \text{ g}$$

$$1 \text{ gram (g)} = 1 \text{ g}$$

$$1 \text{ decigram (dg)} = 1/10 \text{ g or } 0.1 \text{ g}$$

$$1 \text{ centigram (cg)} = 1/100 \text{ g or } 0.01 \text{ g}$$

$$1 \text{ milligram (mg)} = 1/1000 \text{ g or } 0.001 \text{ g}$$

Convert 5 pounds to ounces.

$$5 \text{ lb} = 5 \text{ lb} \cdot 1 = \frac{5 \cancel{\text{lb}}}{1} \cdot \frac{16 \text{ oz}}{1 \cancel{\text{lb}}} = 80 \text{ oz}$$

Convert 260 grams to kilograms.

$$260 \text{ g} = \frac{260 \text{ g}}{1} \cdot 1 = \frac{260 \text{ g}}{1} \cdot \frac{1 \text{ kg}}{1000 \text{ g}} = 0.26 \text{ kg}$$

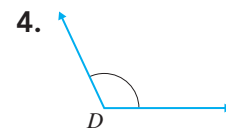
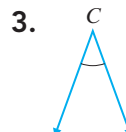
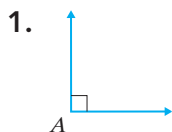
or

$$\begin{array}{ccccccc} \text{kg} & \text{hg} & \text{dag} & \text{g} & \text{dg} & \text{cg} & \text{mg} \\ & \underbrace{\hspace{1.5cm}}_{3 \text{ units to the left}} & & & & & \end{array}$$

$$\begin{array}{c} 260 \text{ g} = 0.26 \text{ kg} \\ \underbrace{\hspace{1.5cm}}_{3 \text{ places to the left}} \end{array}$$

Definitions and Concepts	Examples
Section 9.6 Capacity	
Capacity: U.S. System of Measurement 8 fluid ounces (fl oz) = 1 cup (c) 2 cups = 1 pint (pt) 2 pints = 1 quart (qt) 4 quarts = 1 gallon (gal) Capacity: Metric System of Measurement The liter is the basic unit of capacity in the metric system. It is the capacity or volume of a cube measuring 10 centimeters on each side. A liter of liquid is slightly more than 1 quart. Metric Units of Capacity 1 kiloliter (kl) = 1000 liters (L) 1 hectoliter (hl) = 100 L 1 dekaliter (dal) = 10 L 1 liter (L) = 1 L 1 deciliter (dl) = 1/10 L or 0.1 L 1 centiliter (cl) = 1/100 L or 0.01 L 1 milliliter (ml) = 1/1000 L or 0.001 L	Convert 5 pints to gallons. $1 \text{ gal} = 4 \text{ qt} = 8 \text{ pt}$ $5 \text{ pt} = 5 \text{ pt} \cdot 1 = \frac{5 \cancel{\text{pt}}}{1} \cdot \frac{1 \text{ gal}}{8 \cancel{\text{pt}}} = \frac{5}{8} \text{ gal}$ Convert 1.5 liters to milliliters. $1.5 \text{ L} = \frac{1.5 \text{ L}}{1} \cdot 1 = \frac{1.5 \cancel{\text{L}}}{1} \cdot \frac{1000 \text{ ml}}{1 \cancel{\text{L}}} = 1500 \text{ ml}$ or kl hl dal L dl cl ml 3 units to the left $1.500 \text{ L} = 1500 \text{ ml}$ 3 places to the right
Section 9.7 Temperature and Conversions Between the U.S. and Metric Systems	
To convert between systems, use approximate unit fractions. See page 688. Celsius to Fahrenheit $F = \frac{9}{5}C + 32 \quad \text{or} \quad F = 1.8C + 32$ Fahrenheit to Celsius $C = \frac{5}{9}(F - 32)$	Convert 7 feet to meters. $7 \text{ ft} \approx \frac{7 \cancel{\text{ft}}}{1} \cdot \frac{0.30 \text{ m}}{1 \cancel{\text{ft}}} = 2.1 \text{ m}$ Convert 8 liters to quarts. $8 \text{ L} \approx \frac{8 \cancel{\text{L}}}{1} \cdot \frac{1.06 \text{ qt}}{1 \cancel{\text{L}}} = 8.48 \text{ qt}$ Convert 363 grams to ounces. $363 \text{ g} \approx \frac{363 \cancel{\text{g}}}{1} \cdot \frac{0.04 \text{ oz}}{1 \cancel{\text{g}}} = 14.52 \text{ oz}$ Convert 35°C to degrees Fahrenheit. $F = \frac{9}{5} \cdot 35 + 32 = 63 + 32 = 95$ $35^\circ\text{C} = 95^\circ\text{F}$ Convert 50°F to degrees Celsius. $C = \frac{5}{9} \cdot (50 - 32) = \frac{5}{9} \cdot (18) = 10$ $50^\circ\text{F} = 10^\circ\text{C}$

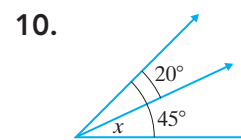
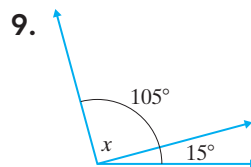
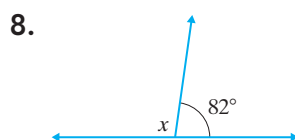
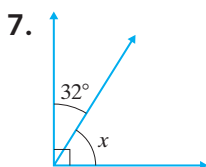
(9.1) Classify each angle as acute, right, obtuse, or straight.



5. Find the complement of a 25° angle.

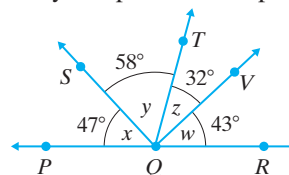
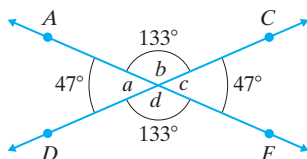
6. Find the supplement of a 105° angle.

Find the measure of angle x in each figure.

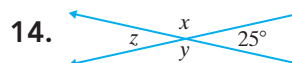
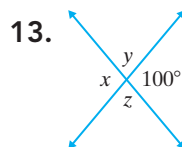


11. Identify the pairs of supplementary angles.

12. Identify the pairs of complementary angles.

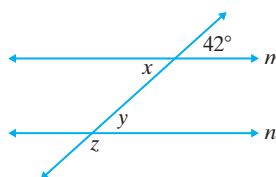
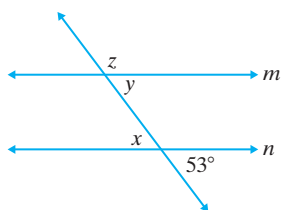


Find the measures of angles x , y , and z in each figure.

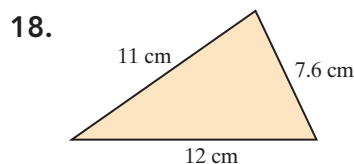
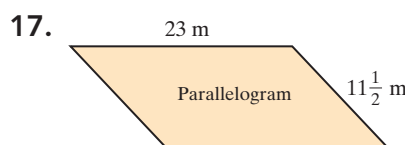


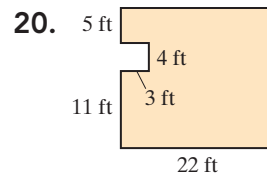
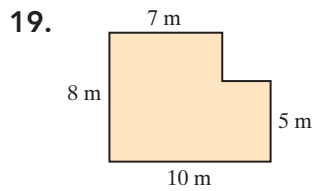
15. Given that $m \parallel n$.

16. Given that $m \parallel n$.



(9.2) Find the perimeter of each figure.



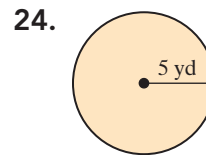
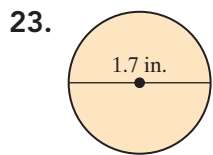


Solve.

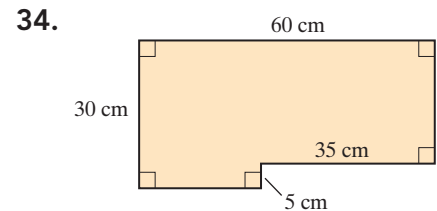
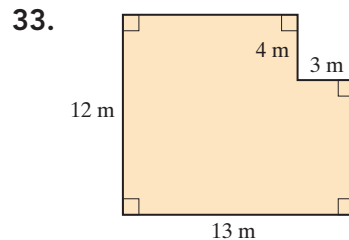
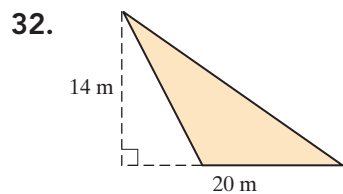
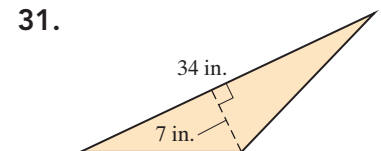
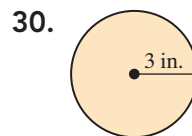
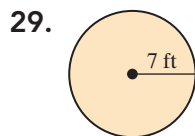
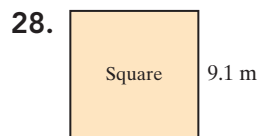
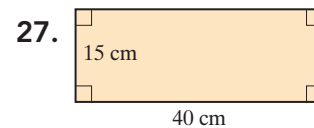
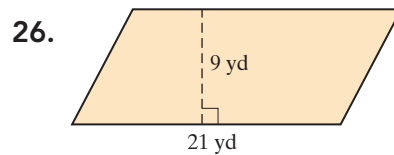
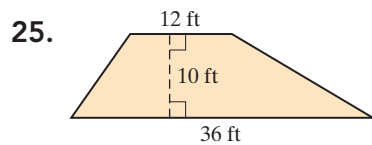
21. Find the perimeter of a rectangular sign that measures 6 feet by 10 feet.

22. Find the perimeter of a town square that measures 110 feet on a side.

Find the circumference of each circle. Use $\pi \approx 3.14$.

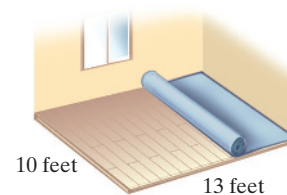


(9.3) Find the area of each figure. For the circles, find the exact area and then use $\pi \approx 3.14$ to approximate the area.



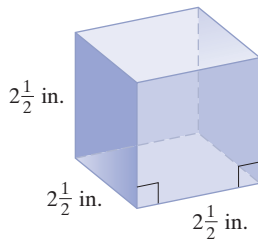
35. The amount of sealer necessary to seal a driveway depends on the area. Find the area of a rectangular driveway 36 feet by 12 feet.

36. Find how much carpet is necessary to cover the floor of the room shown.

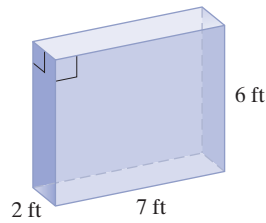
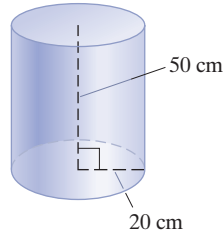
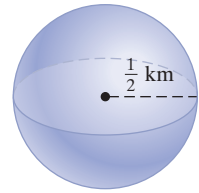


Find the volume and surface area of the solids in Exercises 37 and 38. For Exercises 39 and 40, give the exact volume and an approximation.

37.



38.

39. Use $\pi \approx 3.14$.40. Use $\pi \approx \frac{22}{7}$.

41. Find the volume of a pyramid with a square base 2 feet on a side and a height of 2 feet.

42. Approximate the volume of a tin can 8 inches high and 3.5 inches in radius. Use 3.14 for π .

43. A chest has 3 drawers. If each drawer has inside measurements of $2\frac{1}{2}$ feet by $1\frac{1}{2}$ feet by $\frac{2}{3}$ foot, find the total volume of the 3 drawers.

44. A cylindrical canister for a shop vacuum is 2 feet tall and 1 foot in *diameter*. Find its exact volume.

(9.4) Convert.

45. 108 in. to feet

46. 72 ft to yards

47. 1.5 mi to feet

48. $\frac{1}{2}$ yd to inches

49. 52 ft = _____ yd _____ ft

50. 46 in. = _____ ft _____ in.

51. 42 m to centimeters

52. 82 cm to millimeters

53. 12.18 mm to meters

54. 2.31 m to kilometers

Perform each indicated operation.

55. 4 yd 2 ft + 16 yd 2 ft

56. 7 ft 4 in. $\div$ 2

57. 8 cm + 15 mm

58. 4 m - 126 cm

Solve.

59. A bolt of cloth contains 333 yd 1 ft of cotton ticking. Find the amount of material that remains after 163 yd 2 ft is removed from the bolt.

60. The student activities club is sponsoring a walk for hunger, and all students who participate will receive a sash with the name of the school to wear on the walk. If each sash requires 5 ft 2 in. of material and there are 50 students participating in the walk, how much material will the student activities club need?

61. The trip from El Paso, TX, to Ontario, CA, is about 1235 km each way. Four friends agree to share the driving equally. How far must each drive on this round-trip vacation?
62. The college has ordered that NO SMOKING signs be placed above the doorway of each classroom. Each sign is 0.8 m long and 30 cm wide. Find the area of each sign. (*Hint:* Recall that the area of a rectangle = width $\cdot$ length.)



(9.5) Convert.

63. 66 oz to pounds 64. 2.3 tons to pounds 65. 52 oz = ____ lb ____ oz
66. 10,300 lb = ____ tons ____ lb 67. 27 mg to grams 68. 40 kg to grams
69. 2.1 hg to dekagrams 70. 0.03 mg to decigrams

Perform each indicated operation.

71. 6 lb 5 oz – 2 lb 12 oz 72. 8 lb 6 oz $\times$ 4
73. 4.3 mg $\times$ 5 74. 4.8 kg – 4200 g

Solve.

75. Donshay Berry ordered 1 lb 12 oz of soft-center candies and 2 lb 8 oz of chewy-center candies for his party. Find the total weight of the candy ordered.
76. Four local townships jointly purchase 38 tons 300 lb of cinders to spread on their roads during an ice storm. Determine the weight of the cinders each township receives if they share the purchase equally.

(9.6) Convert.

77. 28 pints to quarts 78. 40 fluid ounces to cups 79. 3 qt 1 pt to pints 80. 18 quarts to cups
81. 9 pt = ____ qt ____ pt 82. 15 qt = ____ gal ____ qt 83. 3.8 L to milliliters
84. 14 hl to kiloliters 85. 30.6 L to centiliters 86. 2.45 ml to liters

Perform each indicated operation.

87. 1 qt 1 pt + 3 qt 1 pt 88. 3 gal 2 qt $\times$ 2 89. 0.946 L – 210 ml 90. 6.1 L + 9400 ml

Solve.

- 91.** Carlos Perez prepared 4 gal 2 qt of iced tea for a block party. During the first 30 minutes of the party, 1 gal 3 qt of the tea is consumed. How much iced tea remains?
- 92.** A recipe for soup stock calls for 1 c 4 fl oz of beef broth. How much should be used if the recipe is cut in half?
- 93.** Each bottle of Kiwi liquid shoe polish holds 85 ml of the polish. Find the number of liters of shoe polish contained in 8 boxes if each box contains 16 bottles.
- 94.** Ivan Miller wants to pour three separate containers of saline solution into a single vat with a capacity of 10 liters. Will 6 liters of solution in the first container combined with 1300 milliliters in the second container and 2.6 liters in the third container fit into the larger vat?

(9.7) *Note: Because approximations are used in this section, your answers may vary slightly from the answers given in the back of the book.*

Convert as indicated. If necessary, round to two decimal places.

- 95.** 7 meters to feet **96.** 11.5 yards to meters **97.** 17.5 liters to gallons
- 98.** 7.8 liters to quarts **99.** 15 ounces to grams **100.** 23 pounds to kilograms
- 101.** A compact disc is 1.2 mm thick. Find the height (in inches) of 50 discs. **102.** If a person weighs 82 kilograms, how many pounds is this?



Convert. Round to the nearest tenth of a degree, if necessary.

- 103.** 42°C to degrees Fahrenheit **104.** 160°C to degrees Fahrenheit
- 105.** 41.3°F to degrees Celsius **106.** 80°F to degrees Celsius

Solve. Round to the nearest tenth of a degree, if necessary.

- 107.** A sharp dip in the jet stream caused the temperature in New Orleans to drop to 35°F. Find the corresponding temperature in degrees Celsius.
- 108.** A recipe for meat loaf calls for a 165°C oven. Find the setting used if the oven has a Fahrenheit thermometer.

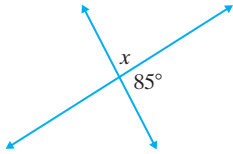
Mixed Review

Find the following.

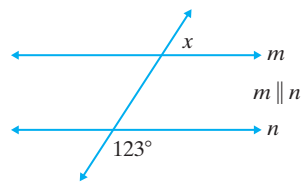
- 109.** Find the supplement of a 72° angle. **110.** Find the complement of a 1° angle.

Find the measure of angle x in each figure.

111.

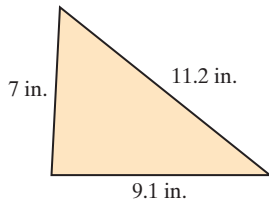


112.

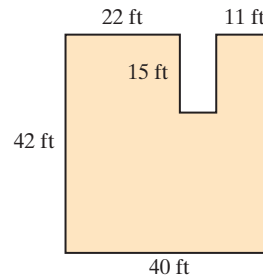


Find the perimeter of each figure.

113.

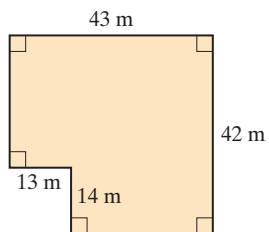


114.

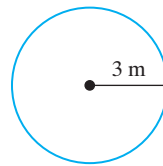


Find the area of each figure. For the circle, find the exact area and then use $\pi \approx 3.14$ to approximate the area.

115.

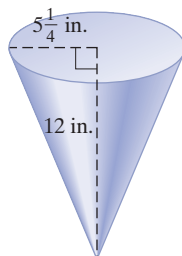


116.

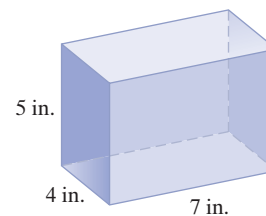


Find the volume of each solid.

117. Give an approximation using $\frac{22}{7}$ for π .



118. Find the surface area also.



Convert the following.

119. 6.25 ft to inches

120. 8200 lb = ____ tons ____ lb

121. 5 m to centimeters

122. 286 mm to kilometers

123. 1400 mg to grams

124. 6.75 gallons to quarts

125. 86°C to degrees Fahrenheit

126. 51.8°F to degrees Celsius

Perform the indicated operations and simplify.

127. 9.3 km - 183 m

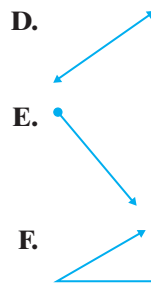
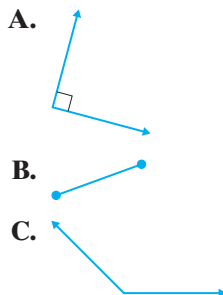
128. 35 L + 700 ml

129. 3 gal 3 qt + 4 gal 2 qt

130. 3.2 kg $\times$ 4

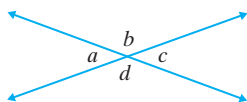
MATCHING Match each word in the first column with its illustration in the second or third column.

- ▶ 1. line
- ▶ 2. line segment
- ▶ 3. ray
- ▶ 4. right angle
- ▶ 5. acute angle
- ▶ 6. obtuse angle



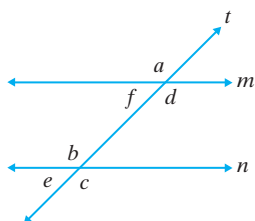
MULTIPLE CHOICE Exercises 7 through 16 are **Multiple Choice**. Choose the correct letter.

Use the given figure for Exercises 7 and 8.



- ▶ 7. Choose two angles that have a sum of 180° .
 A. $\angle a$ and $\angle c$ B. $\angle a$ and $\angle d$ C. $\angle b$ and $\angle d$
- ▶ 8. Choose two angles that have the same measure.
 A. $\angle a$ and $\angle b$ B. $\angle c$ and $\angle d$ C. $\angle b$ and $\angle d$

Use the given figure for Exercises 9 and 10. For this figure, $m \parallel n$.



- ▶ 9. Choose two angles that have a sum of 180° .
 A. $\angle a$ and $\angle e$ B. $\angle a$ and $\angle d$ C. $\angle a$ and $\angle b$
- ▶ 10. Choose two angles that have the same measure.
 A. $\angle a$ and $\angle e$ B. $\angle a$ and $\angle c$ C. $\angle a$ and $\angle f$

For Exercises 11–16, the choices are below. Exercises 13 and 15 have two correct choices.

- A. perimeter B. area C. volume D. circumference

- ▶ 11. Which calculation is measured in square units?
- ▶ 12. Which calculation is measured in cubic units?
- ▶ 13. Which calculation is measured in units.

For Exercises 14 through 16 name the calculation (choices A., B., C., or D. above) to be used to solve each exercise.

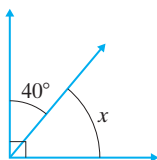
- ▶ 14. The amount of material needed for a rectangular table cloth.
- ▶ 15. The amount of trim needed to go around the edge of a table cloth.
- ▶ 16. The amount of soil needed to fill in a hole in the ground.

Answers

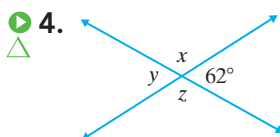
- ▶ 1. Find the complement of a 78° angle.
△

- ▶ 2. Find the supplement of a 124° angle.
△

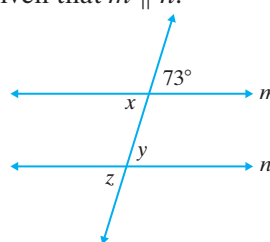
- ▶ 3. Find the measure of $\angle x$.
△



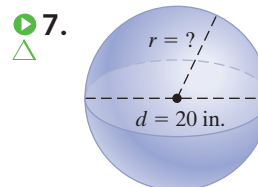
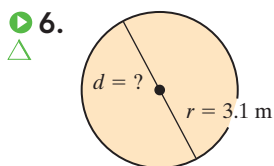
Find the measures of x , y , and z in each figure.



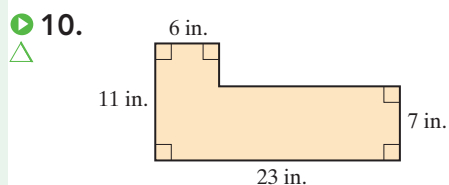
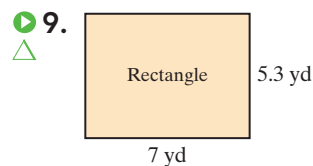
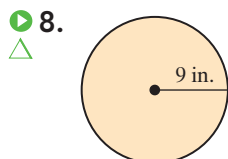
- ▶ 5. Given that $m \parallel n$.
△



Find the unknown diameter or radius as indicated.

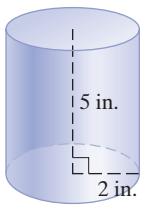


Find the perimeter (or circumference) and area of each figure. For the circle, give the exact value and an approximation using $\pi \approx 3.14$.



Find the volume of each solid. For the cylinder, use $\pi \approx \frac{22}{7}$.

▶ 11.
△



Solve.

▶ 13. Find the perimeter of a square photo with a side length of 4 inches.
△



Convert.

▶ 16. 280 in. = ____ ft ____ in.

▶ 18. 30 oz to pounds

▶ 20. 38 pt to gallons

▶ 22. 2.4 kg to grams

▶ 24. 4.3 dg to grams

Perform each indicated operation.

▶ 26. 3 qt 1 pt + 2 qt 1 pt

▶ 28. 2 ft 9 in. $\times$ 3

▶ 30. 8 cm - 14 mm

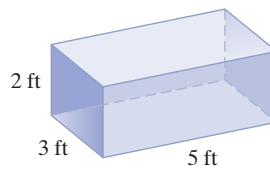
Convert. Round to the nearest tenth of a degree, if necessary.

▶ 32. 84°F to degrees Celsius

▶ 34. The sugar maples in front of Bette MacMillan's house are 8.4 meters tall. Because they interfere with the phone lines, the telephone company plans to remove the top third of the trees. How tall will the maples be after they are shortened?

▶ 36. The engineer in charge of bridge construction said that the span of a certain bridge would be 88 m. But the actual construction required it to be 340 cm longer. Find the span of the bridge, in meters.

▶ 12.
△



▶ 14. How much soil is needed to fill a rectangular hole 3 feet by 3 feet by 2 feet?
△

▶ 15. Find how much baseboard is needed to go around a rectangular room that measures 18 feet by 13 feet. If baseboard costs \$1.87 per foot, also calculate the total cost needed for materials.

▶ 17. $2\frac{1}{2}$ gal to quarts

▶ 19. 2.8 tons to pounds

▶ 21. 40 mg to grams

▶ 23. 3.6 cm to millimeters

▶ 25. 0.83 L to milliliters

▶ 27. 8 lb 6 oz - 4 lb 9 oz

▶ 29. 5 gal 2 qt $\div$ 2

▶ 31. 1.8 km + 456 m

▶ 33. 12.6°C to degrees Fahrenheit

▶ 35. A total of 15 gal 1 qt of oil has been removed from a 20-gallon drum. How much oil still remains in the container?

▶ 37. If 2 ft 9 in. of material is used to manufacture one scarf, how much material is needed for 6 scarves?

11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. _____
24. _____
25. _____
26. _____
27. _____
28. _____
29. _____
30. _____
31. _____
32. _____
33. _____
34. _____
35. _____
36. _____
37. _____

Answers

1. _____
2. _____
3. a. _____
- b. _____
4. a. _____
- b. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____

1. Solve: $3a - 6 = a + 4$

2. Solve: $2x + 1 = 3x - 5$

3. Evaluate:

a. $\left(\frac{2}{5}\right)^4$

b. $\left(-\frac{1}{4}\right)^2$

4. Evaluate:

a. $\left(-\frac{1}{3}\right)^3$

b. $\left(\frac{3}{7}\right)^2$

5. Add: $2\frac{4}{5} + 5 + 1\frac{1}{2}$

6. Add: $2\frac{1}{3} + 4\frac{2}{5} + 3$

7. Simplify by combining like terms:
 $11.1x - 6.3 + 8.9x - 4.6$

8. Simplify by combining like terms:
 $2.5y + 3.7 - 1.3y - 1.9$

9. Simplify: $\frac{5.68 + (0.9)^2 \div 100}{0.2}$

10. Simplify: $\frac{0.12 + 0.96}{0.5}$

11. Insert $<$, $>$, or $=$ to form a true statement.

$0.\overline{7}$ $\frac{7}{9}$

12. Insert $<$, $>$, or $=$ to form a true statement.

0.43 $\frac{2}{5}$

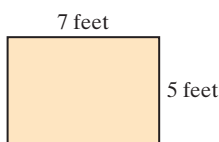
13. Solve: $0.5y + 2.3 = 1.65$

14. Solve: $0.4x - 9.3 = 2.7$

△ 15. An inner-city park is in the shape of a square that measures 300 feet on a side. Find the length of the diagonal of the park, rounded to the nearest whole foot.

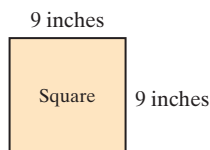
△ 16. A rectangular field is 200 feet by 125 feet. Find the length of the diagonal of the field, rounded to the nearest whole foot.

- △ 17. Given the rectangle shown:



- Find the ratio of its width to its length.
- Find the ratio of its length to its perimeter.

- △ 18. A square is 9 inches by 9 inches.



- Find the ratio of a side to its perimeter.
- Find the ratio of its perimeter to its area.

19. Write the rate as a fraction in simplest form: \$2160 for 12 weeks.

20. Write the rate as a fraction in simplest form: 8 chaperones for 40 students

21. Solve for x : $\frac{1.6}{1.1} = \frac{x}{0.3}$

Round the solution to the nearest hundredth.

22. Solve for x : $\frac{2.4}{3.5} = \frac{0.7}{x}$

Round the solution to the nearest hundredth.

23. The standard dose of an antibiotic is 4 cc (cubic centimeters) for every 25 pounds (lb) of body weight. At this rate, find the standard dose for a 140-lb woman.

24. A recipe that makes 2 pie crusts calls for 3 cups of flour. How much flour is needed to make 5 pie crusts?

25. In a survey of 100 people, 17 people drive blue cars. What percent of people drive blue cars?

26. Of 100 shoppers surveyed at a mall, 38 paid for their purchases using only cash. What percent of shoppers used only cash to pay for their purchases?

27. 13 is $6\frac{1}{2}\%$ of what number?

28. 54 is $4\frac{1}{2}\%$ of what number?

29. What number is 30% of 9?

30. What number is 42% of 30?

31. The number of applications for a mathematics scholarship at one university increased from 34 to 45 in one year. What is the percent of increase? Round to the nearest whole percent.

32. The price of a gallon of paint rose from \$15 to \$19. Find the percent of increase, rounded to the nearest whole percent.

17. a. _____
b. _____
18. a. _____
b. _____
19. _____
20. _____
21. _____
22. _____
23. _____
24. _____
25. _____
26. _____
27. _____
28. _____
29. _____
30. _____
31. _____
32. _____

33. _____

34. _____

35. _____

36. _____

37. _____

38. _____

39. _____

40. _____

41. _____

42. _____

43. _____

44. _____

45. _____

46. _____

47. _____

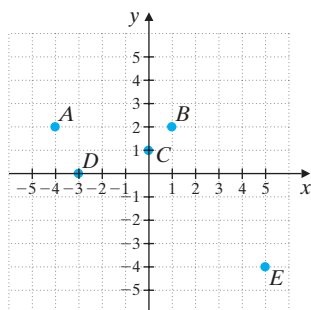
48. _____

49. _____

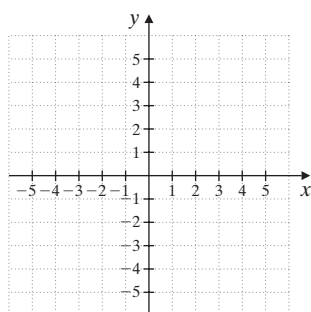
50. _____

33. Find the sales tax and the total price on the purchase of an \$85.50 atlas in a city where the sales tax rate is 7.5%.

35. Find the ordered pair corresponding to each point plotted on the rectangular coordinate system.



37. Graph $y = 4$.



39. Find the median of the list of numbers: 25, 54, 56, 57, 60, 71, 98

41. Find the probability of choosing a red marble from a box containing 1 red, 1 yellow, and 2 blue marbles.

43. Find the complement of a 48° angle.

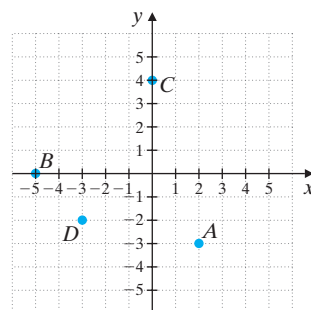
45. Convert 8 feet to inches.

47. Find the area of a circle with a radius of 3 feet. Find the exact area, then an approximation using 3.14 for π .

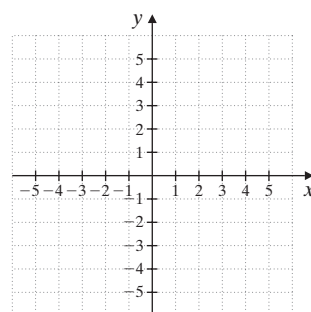
49. Convert 59°F to degrees Celsius.

34. A sofa has a purchase price of \$375. If the sales tax rate is 8%, find the amount of sales tax and the total cost of the sofa.

36. Find the ordered pair corresponding to each point plotted on the rectangular coordinate system.



38. Graph $y = -2$.



40. Find the median of the list of scores: 60, 95, 89, 72, 83

42. Find the probability of choosing a nickel at random in a coin purse that contains 2 pennies, 2 nickels, and 3 quarters.

44. Find the supplement of a 137° angle.

46. Convert 7 yards to feet.

48. Find the area of a circle with a radius of 2 miles. Find the exact area, then an approximation using 3.14 for π .

50. Convert 86°F to degrees Celsius.

Exponents and Polynomials



What Are Wearables, Smartwatches, HMDs, and So On

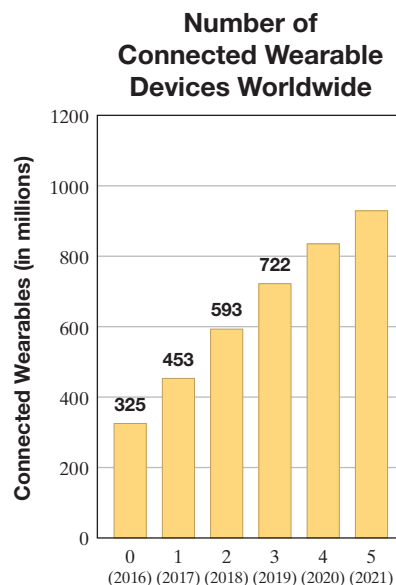
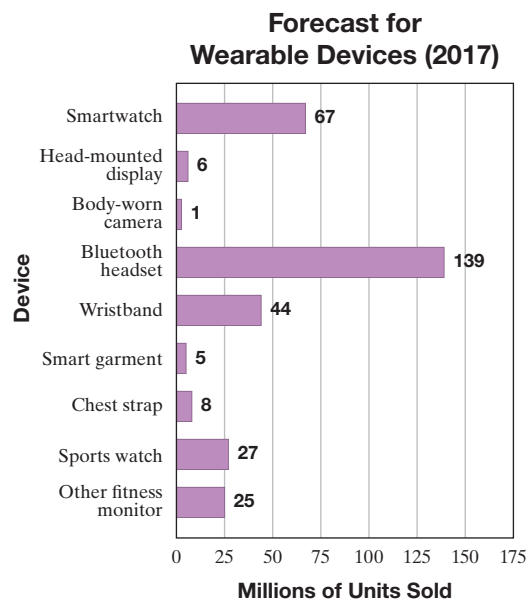
At this writing, definitions of the above are in a state of flux as the technology itself continues to evolve. Basically, a wearable is simply something one can wear and has nothing to do with technology. Nowadays, this word is being used for what is meant as a “smart wearable”; that is, a technological device that can be worn.

With this description of smart wearable (or simply wearable) then, a smartwatch is one of many types of wearable. Oxford Living Dictionary defines a smartwatch as “a mobile device with a touchscreen display, designed to be worn on the wrist.”

An HMD is a head-mounted display. One example is pictured above. These devices can share data and even sensor information or capture video of what a technician is seeing whether the technician is close by or thousands of miles away.

The predicted future of these wearables varies greatly by forecast, and only time will tell which wearables survive. Below we illustrate two forecast for wearables.

In Section 10.1, Exercises 59–62, we study the growth of smart wearables.



10

Recall that an exponent is a shorthand way of representing repeated multiplication. In this chapter, we learn more about exponents and a special type of expression containing exponents, called a *polynomial*. Studying polynomials is a major part of algebra. Polynomials are also useful for modeling many real-world situations. This chapter serves as an introduction to polynomials and some operations that can be performed on them.

Sections

- 10.1** Adding and Subtracting Polynomials
- 10.2** Multiplication Properties of Exponents
- Integrated Review**—Operations on Polynomials
- 10.3** Multiplying Polynomials
- 10.4** Introduction to Factoring Polynomials

Check Your Progress

- Vocabulary Check
- Chapter Highlights
- Chapter Review
- Getting Ready for the Test
- Chapter Test
- Cumulative Review

10.1 Adding and Subtracting Polynomials

Objectives

- A** Add Polynomials. 
- B** Subtract Polynomials. 
- C** Evaluate Polynomials at Given Replacement Values. 

Before we add and subtract polynomials, let's first review some definitions presented in Section 3.1. Recall that the *addends* of an algebraic expression are the *terms* of the expression.

Expressions

$3x + 5$ $\uparrow \quad \uparrow$ 2 terms	$7y^2 + (-6y) + 4$ $\uparrow \quad \uparrow \quad \uparrow$ 3 terms
---	--

Also, recall that *like terms* can be added or subtracted by using the distributive property. For example,

$$7x + 3x = (7 + 3)x = 10x$$

Objective **A** Adding Polynomials

Some terms are also **monomials**. A term is a monomial if the term contains only whole number exponents and no variable in the denominator.

Monomials	Not Monomials
$3x^2$	$\frac{2}{y}$ Variable in denominator
$-\frac{1}{2}a^2bc^3$	$-2x^{-5}$ Not a whole number exponent
7	

A monomial or a sum and/or difference of monomials is called a **polynomial**.

Polynomial

A **polynomial** is a monomial or a sum and/or difference of monomials.

Examples of Polynomials

$$5x^3 - 6x^2 + 2x + 10, \quad -1.2y^3 + 0.7y, \quad z, \quad \frac{1}{3}r - \frac{1}{2}, \quad 0$$

Some polynomials are given special names depending on their number of terms.

Types of Polynomials

A **monomial** is a polynomial with exactly one term.

A **binomial** is a polynomial with exactly two terms.

A **trinomial** is a polynomial with exactly three terms.

The next page contains examples of monomials, binomials, and trinomials. Each of these examples is also a polynomial.

Polynomials			
Monomials	Binomials	Trinomials	More than Three Terms
z	$x + 2$	$x^2 - 2x + 1$	$5x^3 - 6x^2 + 2x - 10$
4	$\frac{1}{3}r - \frac{1}{2}$	$y^5 + 3y^2 - 1.7$	$t^7 - t^5 + t^3 - t + 1$
$0.2x^2$	$-1.2y^3 + 0.7y$	$-a^3 + 2a^2 - 5a$	$z^8 - z^4 + 3z^2 - 2z$
↑ 1 term	↑ 2 terms	↑ 3 terms	

To add polynomials, we use the commutative and associative properties to rearrange and group like terms. Then, we combine like terms.

Adding Polynomials

To add polynomials, combine like terms.

Example 1 Add: $(3x - 1) + (-6x + 2)$

Solution:

$$\begin{aligned}
 (3x - 1) + (-6x + 2) &= (3x - 6x) + (-1 + 2) && \text{Group like terms.} \\
 &= (-3x) + (1) && \text{Combine like terms.} \\
 &= -3x + 1
 \end{aligned}$$

Work Practice 1

Example 2 Add: $(9y^2 - 6y) + (7y^2 + 10y + 2)$

Solution:

$$\begin{aligned}
 (9y^2 - 6y) + (7y^2 + 10y + 2) &= 9y^2 + 7y^2 - 6y + 10y + 2 && \text{Group like terms.} \\
 &= 16y^2 + 4y + 2
 \end{aligned}$$

Work Practice 2

Example 3 Find the sum of $(-y^2 + 2y + 1.7)$ and $(12y^2 - 6y - 3.6)$.

Solution: Recall that “sum” means addition.

$$\begin{aligned}
 (-y^2 + 2y + 1.7) + (12y^2 - 6y - 3.6) \\
 &= \underbrace{-y^2 + 12y^2} + \underbrace{2y - 6y} + \underbrace{1.7 - 3.6} && \text{Group like terms.} \\
 &= 11y^2 - 4y - 1.9 && \text{Combine like terms.}
 \end{aligned}$$

Work Practice 3

Polynomials can also be added vertically. To do this, line up like terms underneath one another. Let's vertically add the polynomials in Example 3.

Example 4 Find the sum of $(-y^2 + 2y + 1.7)$ and $(12y^2 - 6y - 3.6)$. Use a vertical format.

Solution: Line up like terms underneath one another.

$$\begin{array}{r}
 -y^2 + 2y + 1.7 \\
 +12y^2 - 6y - 3.6 \\
 \hline
 11y^2 - 4y - 1.9
 \end{array}$$

Work Practice 4

Helpful Hint

Notice that we are finding the same sum in Example 4 as in Example 3. Of course, the results are the same.

Practice 1

Add:

$$(3y + 7) + (-9y - 14)$$

Practice 2

Add:

$$(x^2 - 4x - 3) + (5x^2 - 6x)$$

Practice 3

Find the sum of

$$(-z^2 - 4.2z + 11) \text{ and } (9z^2 - 1.9z + 6.3).$$

Practice 4

Add vertically:

$$\begin{array}{r}
 (x^2 - x + 1.1) \\
 + (-8x^2 - x - 6.7)
 \end{array}$$

Answers

- $-6y - 7$
- $6x^2 - 10x - 3$
- $8z^2 - 6.1z + 17.3$
- $-7x^2 - 2x - 5.6$

Objective B Subtracting Polynomials

To subtract one polynomial from another, recall how we subtract numbers. Recall from Section 2.3 that to subtract a number, we add its opposite: $a - b = a + (-b)$.

For example,

$$\begin{aligned} 7 - 10 &= 7 + (-10) \\ &= -3 \end{aligned}$$

To subtract a polynomial, we also add its opposite. Just as the opposite of 3 is -3 , the opposite of $(2x^2 - 5x + 1)$ is $-(2x^2 - 5x + 1)$. Let's practice simplifying the opposite of a polynomial.

Practice 5

Simplify: $-(3y^2 + y - 2)$

Example 5 Simplify: $-(2x^2 - 5x + 1)$

Solution: Rewrite $-(2x^2 - 5x + 1)$ as $-1(2x^2 - 5x + 1)$ and use the distributive property.

$$\begin{aligned} -(2x^2 - 5x + 1) &= -1(2x^2 - 5x + 1) \\ &= -1(2x^2) + (-1)(-5x) + (-1)(1) \\ &= -2x^2 + 5x - 1 \end{aligned}$$

Work Practice 5

Notice the result of Example 5.

$$-(2x^2 - 5x + 1) = -2x^2 + 5x - 1$$

This means that **the opposite of a polynomial can be found by changing the signs of the terms of the polynomial**. This leads to the following.

Subtracting Polynomials

To subtract polynomials, change the signs of the terms of the polynomial being subtracted, then add.

Practice 6

Subtract:
 $(9b + 8) - (11b - 20)$

Example 6 Subtract: $(5a + 7) - (2a - 10)$

Solution:

$$\begin{aligned} (5a + 7) - (2a - 10) &= (5a + 7) + (-2a + 10) && \text{Add the opposite of } 2a - 10. \\ &= 5a - 2a + 7 + 10 && \text{Group like terms.} \\ &= 3a + 17 \end{aligned}$$

Work Practice 6**Practice 7**

Subtract:
 $(11x^2 + 7x + 2) - (15x^2 + 4x)$

Example 7 Subtract: $(8x^2 - 4x + 1) - (10x^2 + 4)$

Solution:

$$\begin{aligned} (8x^2 - 4x + 1) - (10x^2 + 4) &= (8x^2 - 4x + 1) + (-10x^2 - 4) && \text{Add the opposite of } 10x^2 + 4. \\ &= 8x^2 - 10x^2 - 4x + 1 - 4 && \text{Group like terms.} \\ &= -2x^2 - 4x - 3 \end{aligned}$$

Work Practice 7**Answers**

5. $-3y^2 - y + 2$ 6. $-2b + 28$
7. $-4x^2 + 3x + 2$

Example 8 Subtract $(-6z^2 - 2z + 13)$ from $(4z^2 - 20z)$.

Solution: Be careful when arranging the polynomials in this example.

$$\begin{aligned}(4z^2 - 20z) - (-6z^2 - 2z + 13) &= (4z^2 - 20z) + (6z^2 + 2z - 13) \\ &= 4z^2 + 6z^2 - 20z + 2z - 13 \\ &= 10z^2 - 18z - 13\end{aligned}$$

Group like terms.

Work Practice 8

✓Concept Check Find and explain the error in the following subtraction.

~~$$\begin{aligned}(3x^2 + 4) - (x^2 - 3x) &= (3x^2 + 4) + (-x^2 - 3x) \\ &= 3x^2 - x^2 - 3x + 4 \\ &= 2x^2 - 3x + 4\end{aligned}$$~~

Just as with adding polynomials, we can subtract polynomials using a vertical format. Let's subtract the polynomials in Example 8 using a vertical format.

Example 9 Subtract $(-6z^2 - 2z + 13)$ from $(4z^2 - 20z)$. Use a vertical format.

Solution: Line up like terms underneath one another.

$$\begin{array}{r} 4z^2 - 20z \\ -(-6z^2 - 2z + 13) \\ \hline \end{array} \quad \text{can be written as} \quad \begin{array}{r} 4z^2 - 20z \\ +6z^2 + 2z - 13 \\ \hline 10z^2 - 18z - 13 \end{array}$$

Work Practice 9

Notice that the answers to Examples 8 and 9 are the same regardless of which format is used.

Objective C Evaluating Polynomials

Polynomials have different values depending on the replacement values for the variables.

Example 10 Find the value of the polynomial $3t^3 - 2t + 5$ when $t = 1$.

Solution: Replace t with 1 and simplify.

$$\begin{aligned}3t^3 - 2t + 5 &= 3(1)^3 - 2(1) + 5 && \text{Let } t = 1. \\ &= 3(1) - 2(1) + 5 && (1)^3 = 1. \\ &= 3 - 2 + 5 \\ &= 6\end{aligned}$$

The value of $3t^3 - 2t + 5$ when $t = 1$ is 6.

Work Practice 10

Many real-world applications can be modeled by polynomials.

Practice 8

Subtract $(-7y^2 + y - 4)$ from $(-3y^2 + 5y)$.

Practice 9

Subtract $(3x^2 - 12x)$ from $(-4x^2 + 20x + 17)$. Use a vertical format.

Practice 10

Find the value of the polynomial $2y^3 + y^2 - 6$ when $y = 3$.

Answers

8. $4y^2 + 4y + 4$ 10. 57
9. $-7x^2 + 32x + 17$

✓Concept Check Answer

$$\begin{aligned}(3x^2 + 4) - (x^2 - 3x) \\ &= (3x^2 + 4) + (-x^2 + 3x) \\ &= 3x^2 - x^2 + 3x + 4 \\ &= 2x^2 + 3x + 4\end{aligned}$$

Practice 11

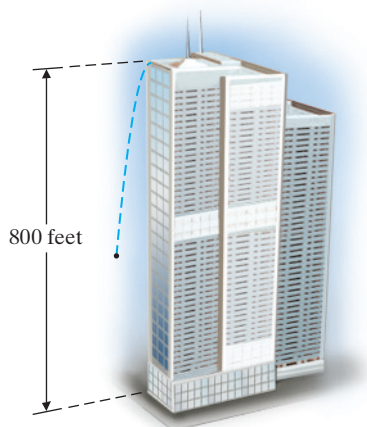
An object is dropped from the top of a 530-foot cliff. Its height in feet at time t seconds is given by the polynomial $-16t^2 + 530$. Find the height of the object when $t = 1$ second and when $t = 4$ seconds.

Helpful Hint

Don't forget to insert units, if appropriate.

Example 11 Finding the Height of an Object

An object is dropped from the top of an 800-foot-tall building. Its height at time t seconds is given by the polynomial $-16t^2 + 800$. Find the height of the object when $t = 1$ second and when $t = 3$ seconds.

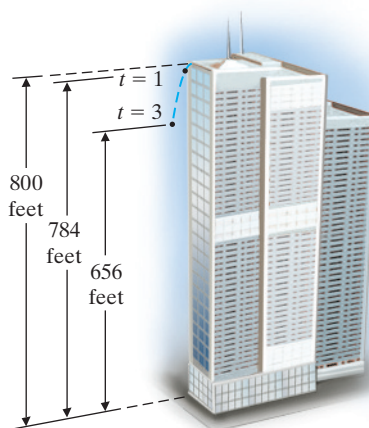


Solution: To find the heights, we evaluate the polynomial when $t = 1$ and when $t = 3$.

$$\begin{aligned} -16t^2 + 800 &= -16(1)^2 + 800 \\ &= -16 + 800 \\ &= 784 \end{aligned}$$

The height of the object at 1 second is 784 feet.

$$\begin{aligned} -16t^2 + 800 &= -16(3)^2 + 800 \\ &= -16(9) + 800 \\ &= -144 + 800 \\ &= 656 \end{aligned}$$



The height of the object at 3 seconds is 656 feet.

Work Practice 11

Answer

11. 514 feet; 274 feet

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

trinomial	binomial	add
terms	monomial	subtract

- The addends of an algebraic expression are the _____ of the expression.
- A polynomial with exactly one term is called a _____.
- A polynomial with exactly two terms is called a _____.
- A polynomial with exactly three terms is called a _____.
- To _____ polynomials, combine like terms.
- To _____ polynomials, change the signs of the terms of the polynomial being subtracted; then add.

Martin-Gay Interactive Videos



See Video 10.1

Watch the section lecture video and answer the following questions.

- Objective A** 7. From Example 2, what's another way of determining when terms are like terms?
- Objective B** 8. Why are we told to be “very, very careful” when translating and subtracting in Example 4?
- Objective C** 9. What value of x is used to evaluate the polynomial in Example 5? What is the final answer?

10.1 Exercise Set MyLab Math



Objective A Add the polynomials. See Examples 1 through 4.

- $(2x + 3) + (-7x - 27)$
- $(9y - 16) + (-43y + 16)$
- $(-3z^2 + 5z - 5) + (-8z^2 - 8z + 4)$
- $(8a^2 + 5a - 9) + (5a^2 - 11a + 6)$
- $(12y - 20) + (9y^2 + 13y - 20)$
- $(5x^2 - 6) + (-3x^2 + 17x - 2)$
- $(4.3a^4 + 5) + (-8.6a^4 - 2a^2 + 4)$
- $(-12.7z^3 - 14z) + (-8.9z^3 + 12z + 2)$

Objective B Simplify. See Example 5.

- $-(9x - 16)$
- $-(4y - 12)$
- $-(-3z^2 + z - 7)$
- $-(-2x^2 - x + 1)$

Subtract the polynomials. See Examples 6 through 9.

- $(8a - 5) - (3a + 8)$
- $(3b + 5) - (-2b + 9)$
- $(3x^2 - 2x + 1) - (5x^2 - 6x)$
- $(-9z^2 + 6z + 2) - (3z^2 + 1)$

17. $(10y^2 - 7) - (20y^3 - 2y^2 - 7)$

18. $(11x^3 + 15x - 9) - (-x^3 + 10x^2 - 9)$

▶ 19. Subtract $(9x^2 + 3x - 4)$ from $(2x + 12)$.

20. Subtract $(4a^2 + 6a + 1)$ from $(-7a + 7)$.

21. Subtract $(5y^2 + 4y - 6)$ from $(13y^2 - 6y - 14)$.

22. Subtract $(16x^2 - x + 1)$ from $(12x^2 - 3x - 12)$.

Objectives A B Mixed Practice Perform each indicated operation. See Examples 1 through 9.

▶ 23. $(25x - 5) + (-20x - 7)$

24. $(14x + 2) + (-7x - 1)$

25. $(4y + 4) - (3y + 8)$

26. $(6z - 3) - (8z + 5)$

27. $(3x^2 + 3x - 4) + (-8x^2 + 9)$

28. $(-2a^2 - 5a) + (6a^2 - 2a + 9)$

29. $(5x + 4.5) + (-x - 8.6)$

30. $(20x - 0.8) + (x + 1.2)$

31. $(a - 5) - (-3a + 2)$

32. $(t + 9) - (-2t + 6)$

▶ 33. $(21y - 4.6) - (36y - 8.2)$

34. $(8.6x + 4) - (9.7x - 93)$

35. $(18t^2 - 4t + 2) - (-t^2 + 7t - 1)$

36. $(35x^2 + x - 5) - (17x^2 - x + 5)$

37. $(2b^3 + 5b^2 - 5b - 8) + (8b^2 + 9b + 6)$

38. $(3z^2 - 8z + 5) + (-3z^3 - 5z^2 - 2z - 4)$

39. Add $(6x^2 - 7)$ and $(-11x^2 - 11x + 20)$.

40. Add $(-2x^2 + 3x)$ and $(9x^2 - x + 14)$.

41. Subtract $\left(3z - \frac{3}{7}\right)$ from $\left(3z + \frac{6}{7}\right)$.

42. Subtract $\left(8y^2 - \frac{7}{10}y\right)$ from $\left(-5y^2 + \frac{3}{10}y\right)$.

Objective C Find the value of each polynomial when $x = 2$. See Examples 10 and 11.

43. $-2x + 9$

44. $-5x - 7$

▶ 45. $x^2 - 6x + 3$

46. $5x^2 + 4x - 100$

47. $\frac{3x^2}{2} - 14$

48. $\frac{7x^3}{14} - x + 5$

Find the value of each polynomial when $x = 5$. See Examples 10 and 11.

49. $2x + 10$

50. $-5x - 6$

51. x^2

52. x^3

53. $2x^2 + 4x - 20$

54. $4x^2 - 5x + 10$

Solve. See Example 11.

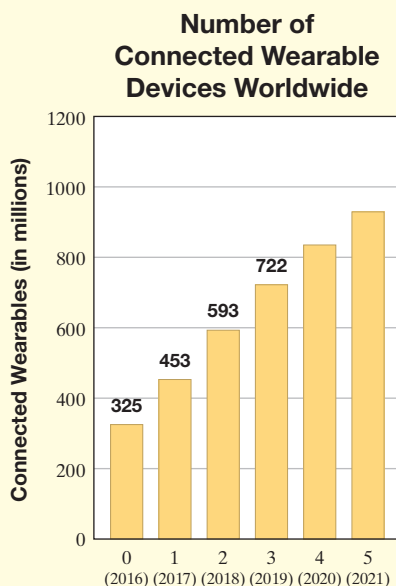
The distance in feet traveled by a free-falling object in t seconds is given by the polynomial $16t^2$. Use this polynomial for Exercises 55 and 56.

- 55.** Find the distance traveled by an object that falls for 6 seconds.
- 56.** It takes 8 seconds for a hard hat to fall from the top of a building. How tall is the building?

Office Supplies, Inc. manufactures office products. The company determines that the total cost for manufacturing x file cabinets is given by the polynomial $3000 + 20x$. Use this polynomial for Exercises 57 and 58.

- 57.** Find the total cost to manufacture 10 file cabinets.
- 58.** Find the total cost to manufacture 100 file cabinets.

The exercises below have to do with smart wearables. For more information about these wearables, see the Chapter Opener.



Source: Statista

- 59.** The Chapter Opener graph is above. The predicted number of wearable devices in millions can be modeled by the polynomial $-5x^2 + 148x + 320$, where x stands for the number of years since 2016.
- a.** Use the graph to predict the number of wearable devices for the year 2020.
- b.** Use the given polynomial and predict the number of wearable devices for the year 2020.
- c.** Compare your answers in parts **a** and **b**. Are they about the same? Should they be? Explain why or why not.
- 60.** Use the information in Exercise 59. Answers parts **a**, **b**, and **c** in Exercise 59 for the year 2021.

The number of smartwatch unit sales worldwide in millions can be estimated by the polynomial $8.7x^2 - 2.1x + 7.4$ where x is the number of years since 2014. Use this polynomial for Exercises 61 and 62.

61. Use the given polynomial to predict the number of smartwatch sales in the year 2022.
62. Use the given polynomial to predict the number of smartwatch sales worldwide in the year 2021.

Devils Tower National Monument in Wyoming became America's first national monument in 1906. This rock formation has a height of 867 feet and is a popular climbing site. It was also used as the alien spacecraft landing site in the 1977 movie *Close Encounters of the Third Kind*.

63. One of the climbers of Devils Tower accidentally drops a piece of climbing chalk when he reaches the summit. The chalk's height above the ground, in feet, can be modeled by the equation $h = 867 - 16t^2$, where t stands for the number of seconds after the chalk is dropped. How far above the ground would the chalk be in 4 seconds?
64. At 7 seconds, the chalk in Exercise 59 passes another climber, who is closer to the base. How far above the ground is this climber?

An object is dropped from the deck of the Royal Gorge Bridge, which stretches across Royal Gorge at a height of 1053 feet above the Arkansas River. The height of the object above the river at t seconds is given by the polynomial $1053 - 16t^2$. Use this polynomial for Exercises 61 and 62. (Source: Royal Gorge Bridge Co.)



65. How far above the river is the object when $t = 3$ seconds?
66. How far above the river is the object when $t = 6$ seconds?

Solve.

67. The percent of households in the United States that only have wireless phone coverage is increasing and can be modeled by the equation $-0.02x^2 + 4.3x + 8.5$, where x stands for the number of years since 2005. If the rate of growth of cell phone-only homes continues, what percent of the U.S. households should be totally cellular by 2025? (Source: CTIA—The Wireless Connection)
68. The continuously growing number of cellular subscribers means that the number of cell phone antennas needs to be constantly increased to carry the additional cellular traffic. The number of cell phone antennas (in thousands) located in the United States can be modeled by $0.3x^2 + 12x + 118$, where x stands for the number of years since 2000. If cellular services continue to grow at this rate, how many cell phone antennas should we expect in the United States in the year 2020? (Source: CTIA—The Wireless Association)

Review

Evaluate. See Sections 1.7 and 2.4.

69. 3^4

70. $(-2)^5$

71. $(-5)^2$

72. 4^3

Write using exponential notation. See Section 1.7

73. $x \cdot x \cdot x$

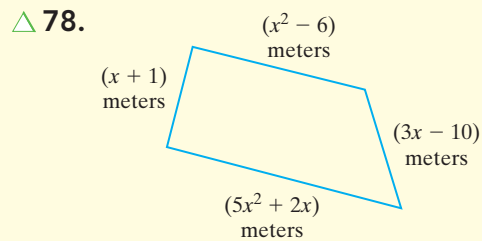
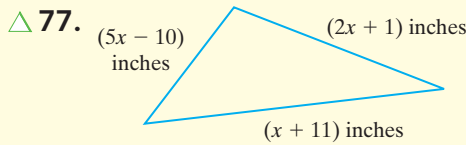
74. $y \cdot y \cdot y \cdot y \cdot y$

75. $2 \cdot 2 \cdot a \cdot a \cdot a \cdot a$

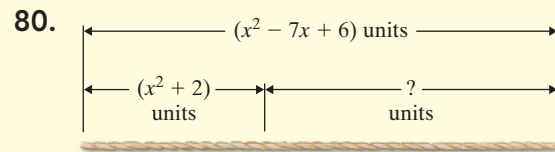
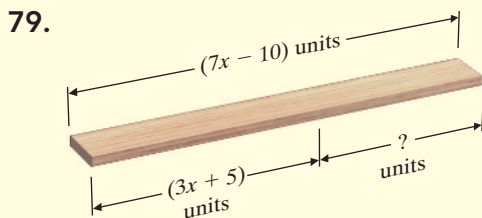
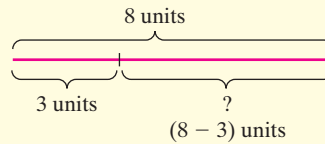
76. $5 \cdot 5 \cdot 5 \cdot b \cdot b$

Concept Extensions

Find the perimeter of each figure.



Given the lengths in the figure below, we find the unknown length by subtracting. Use the information to find the unknown lengths in Exercises 75 and 76.



Fill in the blanks.

81. $(3x^2 + \underline{\hspace{1cm}} x - \underline{\hspace{1cm}}) + (\underline{\hspace{1cm}} x^2 - 6x + 2) = 5x^2 + 14x - 4$

82. $(\underline{\hspace{1cm}} y^2 + 4y - 3) + (8y^2 - \underline{\hspace{1cm}} y + \underline{\hspace{1cm}}) = 9y^2 + 2y + 7$

 83. Find the value of $7a^4 - 6a^2 + 2a - 1$ when $a = 1.2$.

 84. Find the value of $3b^3 + 4b^2 - 100$ when $b = -2.5$.

 85. For Exercises 65 and 66, the polynomial $1053 - 16t^2$ was used to give the height of an object above the river at t seconds. Find the height when $t = 8$ seconds and $t = 9$ seconds. Explain what happened and why.

10.2 Multiplication Properties of Exponents

Objectives

- A** Use the Product Property for Exponents.
- B** Use the Power Property for Exponents.
- C** Use the Power of a Product Property for Exponents.

Objective A Using the Product Property

Recall from Section 1.8 that an exponent has the same meaning whether the base is a number or a variable. For example,

$$5^3 = \underbrace{5 \cdot 5 \cdot 5}_{3 \text{ factors of } 5} \quad \text{and} \quad x^3 = \underbrace{x \cdot x \cdot x}_{3 \text{ factors of } x}$$

We can use this definition of an exponent to discover properties that will help us to simplify products and powers of exponential expressions.

For example, let's use the definition of an exponent to find the product of x^3 and x^4 .

$$\begin{aligned} x^3 \cdot x^4 &= (x \cdot x \cdot x)(x \cdot x \cdot x \cdot x) \\ &= \underbrace{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}_{7 \text{ factors of } x} \\ &= x^7 \end{aligned}$$

Notice that the result is the same if we add the exponents.

$$x^3 \cdot x^4 = x^{3+4} = x^7$$

This suggests the following product rule or property for exponents.

Product Property for Exponents

If m and n are positive integers and a is a real number, then

$$a^m \cdot a^n = a^{m+n}$$

In other words, to multiply two exponential expressions with the same base, keep the base and add the exponents.

Example 1 Multiply: $y^7 \cdot y^2$

Solution: $y^7 \cdot y^2 = y^{7+2}$ Use the product property for exponents.
 $= y^9$ Simplify.

Work Practice 1

Example 2 Multiply: $3x^5 \cdot 6x^3$

Solution: $3x^5 \cdot 6x^3 = (3 \cdot 6)(x^5 \cdot x^3)$ Apply the commutative and associative properties.
 $= 18x^{5+3}$ Use the product property for exponents.
 $= 18x^8$ Simplify.

Work Practice 2

Example 3 Multiply: $(-2a^4b^{10})(9a^5b^3)$

Solution: Use properties of multiplication to group numbers and like variables together.

$$\begin{aligned} (-2a^4b^{10})(9a^5b^3) &= (-2 \cdot 9)(a^4 \cdot a^5)(b^{10} \cdot b^3) \\ &= -18a^{4+5}b^{10+3} \\ &= -18a^9b^{13} \end{aligned}$$

Work Practice 3

Practice 1

Multiply: $z^5 \cdot z^6$

Practice 2

Multiply: $8y^5 \cdot 4y^9$

Practice 3

Multiply: $(-4r^6s^2)(-3r^2s^5)$

Answers

1. z^{11} 2. $32y^{14}$ 3. $12r^8s^7$

Example 4 Multiply: $2x^3 \cdot 3x \cdot 5x^6$

Solution: First notice the factor $3x$. Since there is one factor of x in $3x$, it can also be written as $3x^1$.

$$\begin{aligned} 2x^3 \cdot 3x^1 \cdot 5x^6 &= (2 \cdot 3 \cdot 5)(x^3 \cdot x^1 \cdot x^6) \\ &= 30x^{10} \end{aligned}$$

Helpful Hint

Don't forget that if an exponent is not written, it is assumed to be 1.

Work Practice 4

Helpful Hint

These examples will remind you of the difference between adding and multiplying terms.

Addition

$$\begin{aligned} 5x^3 + 3x^3 &= (5 + 3)x^3 = 8x^3 \\ 7x + 4x^2 &= 7x + 4x^2 \end{aligned}$$

Multiplication

$$\begin{aligned} (5x^3)(3x^3) &= 5 \cdot 3 \cdot x^3 \cdot x^3 = 15x^{3+3} = 15x^6 \\ (7x)(4x^2) &= 7 \cdot 4 \cdot x \cdot x^2 = 28x^{1+2} = 28x^3 \end{aligned}$$

Objective B Using the Power Property

Next suppose that we want to simplify an exponential expression raised to a power. To see how we simplify $(x^2)^3$, we again use the definition of an exponent.

$$\begin{aligned} (x^2)^3 &= \underbrace{(x^2) \cdot (x^2) \cdot (x^2)}_{\text{3 factors of } x^2} && \text{Apply the definition of an exponent.} \\ &= x^{2+2+2} && \text{Use the product property for exponents.} \\ &= x^6 && \text{Simplify.} \end{aligned}$$

Notice the result is exactly the same if we multiply the exponents.

$$(x^2)^3 = x^{2 \cdot 3} = x^6$$

This suggests the following power rule or property for exponents.

Power Property for Exponents

If m and n are positive integers and a is a real number, then

$$(a^m)^n = a^{m \cdot n}$$

In other words, to raise a power to a power, keep the base and multiply the exponents.

Helpful Hint

Take a moment to make sure that you understand when to apply the product property and when to apply the power property.

Product Property → Add Exponents

$$x^5 \cdot x^7 = x^{5+7} = x^{12}$$

$$y^6 \cdot y^2 = y^{6+2} = y^8$$

Power Property → Multiply Exponents

$$(x^5)^7 = x^{5 \cdot 7} = x^{35}$$

$$(y^6)^2 = y^{6 \cdot 2} = y^{12}$$

Practice 4

Multiply: $11y^5 \cdot 3y^2 \cdot y$. (Recall that $y = y^1$.)

Answer

4. $33y^8$

Practice 5Simplify: $(z^3)^6$ **Practice 6**Simplify: $(z^4)^5 \cdot (z^3)^7$ **Practice 7**Simplify: $(3b)^4$ **Practice 8**Simplify: $(4x^2y^6)^3$ **Answers**5. z^{18} 6. z^{41} 7. $81b^4$ 8. $64x^6y^{18}$ **Concept Check Answer**

b

Example 5 Simplify: $(y^8)^2$ **Solution:** $(y^8)^2 = y^{8 \cdot 2}$ Use the power property.
 $= y^{16}$ **Work Practice 5****Example 6** Simplify: $(a^3)^4 \cdot (a^2)^9$ **Solution:** $(a^3)^4 \cdot (a^2)^9 = a^{12} \cdot a^{18}$ Use the power property.
 $= a^{12+18}$ Use the product property.
 $= a^{30}$ Simplify. **Work Practice 6****Objective C** Using the Power of a Product Property

Next, let's simplify the power of a product.

$$\begin{aligned}
 (xy)^3 &= xy \cdot xy \cdot xy && \text{Apply the definition of an exponent.} \\
 &= (x \cdot x \cdot x)(y \cdot y \cdot y) && \text{Group like bases.} \\
 &= x^3y^3 && \text{Simplify.}
 \end{aligned}$$

Notice that the power of a product can be written as the product of powers. This leads to the following power of a product rule or property.

Power of a Product Property for ExponentsIf n is a positive integer and a and b are real numbers, then

$$(ab)^n = a^n b^n$$

In other words, to raise a product to a power, raise each factor to the power.

Concept Check Which property is needed to simplify $(x^6)^3$? Explain.

- Product property for exponents
- Power property for exponents
- Power of a product property for exponents

Example 7 Simplify: $(5t)^3$ **Solution:** $(5t)^3 = 5^3 t^3$ Apply the power of a product property.
 $= 125t^3$ Write 5^3 as 125. **Work Practice 7****Example 8** Simplify: $(2a^5b^3)^3$ **Solution:** $(2a^5b^3)^3 = 2^3(a^5)^3(b^3)^3$ Apply the power of a product property.
 $= 8a^{15}b^9$ Apply the power property. **Work Practice 8**

Example 9 Simplify: $(3y^4z^2)^4(2y^3z^5)^5$ **Solution:**

$$\begin{aligned}
 (3y^4z^2)^4(2y^3z^5)^5 &= 3^4(y^4)^4(z^2)^4 \cdot 2^5(y^3)^5(z^5)^5 && \text{Apply the power of a product property.} \\
 &= 81y^{16}z^8 \cdot 32y^{15}z^{25} && \text{Apply the power property.} \\
 &= (81 \cdot 32)(y^{16} \cdot y^{15})(z^8 \cdot z^{25}) && \text{Group like bases.} \\
 &= 2592y^{31}z^{33} && \text{Apply the product property.}
 \end{aligned}$$

Work Practice 9**Practice 9**Simplify: $(2x^2y^4)^4(3x^6y^9)^2$ **Answer**9. $144x^{20}y^{34}$ **Vocabulary, Readiness & Video Check**

Use the choices below to fill in each blank. Not all choices will be used.

add	multiply	exponent	$6x^2$
subtract	divide	$36x$	$36x^2$

- In $7x^2$, the 2 is called the _____.
- To simplify $x^4 \cdot x^3$, we _____ the exponents.
- To simplify $(x^4)^3$, we _____ the exponents.
- The expression $(6x)^2$ simplifies to _____.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 10.2

- Objective A** 5. In the lecture before Example 1, what example is used to illustrate the product rule for exponents?
- Objective B** 6. In Example 4, how are we told to see the difference between applying the product rule and applying the power rule?
- Objective C** 7. What are the factors in the product being raised to a power in Example 6?

10.2 Exercise Set**Objective A** Multiply. See Examples 1 through 4.

- | | | | |
|---------------------------|----------------------------|----------------------------------|----------------------------------|
| 1. $x^5 \cdot x^9$ | 2. $y^4 \cdot y^7$ | 3. $a^3 \cdot a$ | 4. $b \cdot b^4$ |
| 5. $3z^3 \cdot 5z^2$ | 6. $8r^2 \cdot 2r^{15}$ | 7. $-4x \cdot 10x$ | 8. $-9y \cdot 3y$ |
| 9. $2x \cdot 3x \cdot 7x$ | 10. $4y \cdot 3y \cdot 5y$ | 11. $a \cdot 4a^{11} \cdot 3a^5$ | 12. $b \cdot 7b^{10} \cdot 5b^8$ |
| 13. $(-5x^2y^3)(-5x^4y)$ | 14. $(-2xy^4)(-6x^3y^7)$ | 15. $(7ab)(4a^4b^5)$ | 16. $(3a^3b^6)(12a^2b^9)$ |

Objectives B C Mixed Practice Simplify. See Examples 5 through 9.

17. $(x^5)^3$ 18. $(y^4)^7$ 19. $(z^3)^{10}$ 20. $(a^9)^3$
 21. $(b^7)^6 \cdot (b^2)^{10}$ 22. $(x^2)^9 \cdot (x^5)^3$ 23. $(3a)^4$ 24. $(2y)^5$
 25. $(a^{11}b^8)^3$ 26. $(x^7y^4)^8$ 27. $(10x^5y^3)^3$ 28. $(8a^5b^7)^2$
 29. $(-3y)(2y^7)^3$ 30. $(-2x)(5x^2)^4$ 31. $(4xy)^3(2x^3y^5)^2$ 32. $(2xy)^4(3x^4y^3)^3$

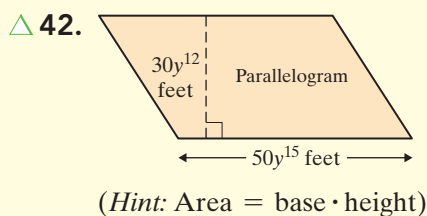
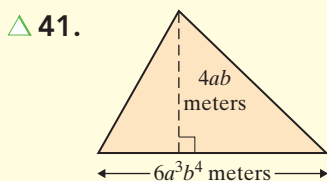
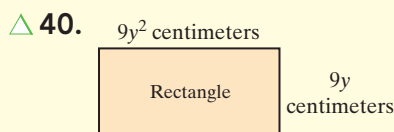
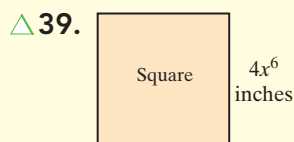
Review

Multiply. See Section 3.1.

33. $7(x - 3)$ 34. $4(y + 2)$ 35. $-2(3a + 2b)$
 36. $-3(8r + 3s)$ 37. $9(x + 2y - 3)$ 38. $5(a + 7b - 3)$

Concept Extensions

Find the area of each figure.



Multiply and simplify.

43. $(14a^7b^6)^3(9a^6b^3)^4$ 44. $(5x^{14}y^6)^7(3x^{20}y^{19})^5$
 45. $(8.1x^{10})^5$ 46. $(4.6a^{14})^4$
 47. $(x^{90}y^{72})^3$ 48. $(a^{20}b^{10}c^5)^5(a^9b^{12})^3$
 49. In your own words, explain why $x^2 \cdot x^3 = x^5$ and $(x^2)^3 = x^6$.

Operations on Polynomials

Add or subtract the polynomials as indicated.

1. $(3x + 5) + (-x - 8)$

2. $(15y - 7) + (5y - 4)$

3. $(7x + 1) - (-3x - 2)$

4. $(14y - 6) - (19y - 2)$

5. $(a^4 + 5a) - (3a^4 - 3a^2 - 4a)$

6. $(2a^3 - 6a^2 + 11) - (6a^3 + 6a^2 + 11)$

7. $(4.5x^2 + 8.1x) + (2.8x^2 - 12.3x - 5.3)$

8. $(1.2y^2 - 3.6y) + (0.6y^2 + 1.2y - 5.6)$

9. Subtract $(2x - 6)$ from $(8x + 1)$.

10. Subtract $(3x^2 - x + 2)$ from $(5x^2 + 2x - 10)$.

Find the value of each polynomial when $x = 3$.

11. $2x - 7$

12. $x^2 + 5x + 2$

Simplify.

13. $x^9 \cdot x^{11}$

14. $x^5 \cdot x^5$

15. $y^3 \cdot y$

16. $a \cdot a^{10}$

17. $(x^7)^{11}$

18. $(x^6)^6$

19. $(x^3)^4 \cdot (x^5)^6$

20. $(y^2)^9 \cdot (y^3)^3$

21. $(5x)^3$

22. $(2y)^5$

23. $(-6xy^2)(2xy^5)$

24. $(-4a^2b^3)(-3ab)$

25. $(y^{11}z^{13})^3$

26. $(a^5b^{12})^4$

27. $(10x^2y)^2(3y)$

28. $(8y^3z)^2(2z^5)$

29. $(2a^5b)^4(3a^9b^4)^2$

30. $(5x^4y^6)^3(x^2y^2)^5$

31. There is an ever-increasing amount of money spent on mobile advertising. The amount of money (in billions) being spent on mobile advertising can be modeled by $1.4x^2 - 1.06x + 0.9$, where x is the number of years since 2010. If the growth continues as it has, how much money (in billions) should we expect to be spent in 2020? (Source: Pew Research Center)

Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. _____
24. _____
25. _____
26. _____
27. _____
28. _____
29. _____
30. _____
31. _____

10.3 Multiplying Polynomials

Objectives

- A** Multiply a Monomial and Any Polynomial.
- B** Multiply Two Binomials.
- C** Square a Binomial.
- D** Use the FOIL Order to Multiply Binomials.
- E** Multiply Any Two Polynomials.

Practice 1

Multiply: $4y(8y^2 + 5)$

Practice 2

Multiply: $3r(8r^2 - r + 11)$

Objective A Multiplying a Monomial and a Polynomial

Recall from Section 10.1 that a polynomial that consists of one term is called a **monomial**. For example, $5x$ is a monomial. To multiply a monomial and any polynomial, we use the distributive property

$$a(b + c) = a \cdot b + a \cdot c$$

and apply properties of exponents.

Example 1 Multiply: $5x(3x^2 + 2)$

Solution:

$$\begin{aligned} 5x(3x^2 + 2) &= 5x \cdot 3x^2 + 5x \cdot 2 && \text{Apply the distributive property.} \\ &= 15x^3 + 10x \end{aligned}$$

Work Practice 1

Example 2 Multiply: $2z(4z^2 + 6z - 9)$

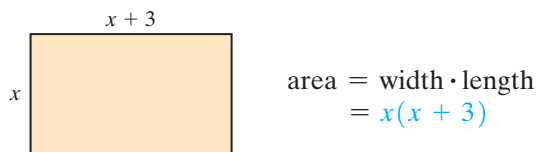
Solution:

$$\begin{aligned} 2z(4z^2 + 6z - 9) &= 2z \cdot 4z^2 + 2z \cdot 6z + 2z(-9) \\ &= 8z^3 + 12z^2 - 18z \end{aligned}$$

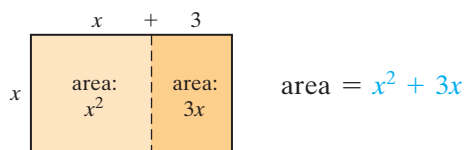
Work Practice 2

To visualize multiplication by a monomial, let's look at two ways we can represent the area of the same rectangle.

Method 1: The width of the rectangle is x and its length is $x + 3$. One way to calculate the area of the rectangle is



Method 2: Another way to calculate the area of the rectangle is to find the sum of the areas of the smaller figures.



Since the areas must be equal, we have that

$$x(x + 3) = x^2 + 3x \quad \text{As expected from the distributive property.}$$

Answers

1. $32y^3 + 20y$ 2. $24r^3 - 3r^2 + 33r$

Objective B Multiplying Binomials

Recall also from Section 10.1 that a polynomial that consists of exactly two terms is called a **binomial**. To multiply two binomials, we use a version of the distributive property:

$$(b + c)a = b \cdot a + c \cdot a$$

Example 3 Multiply: $(x + 2)(x + 3)$ **Solution:**

$$\begin{aligned}(x + 2)(x + 3) &= \overbrace{x(x + 3)} + \overbrace{2(x + 3)} && \text{Apply the distributive property.} \\ &= x \cdot x + x \cdot 3 + 2 \cdot x + 2 \cdot 3 && \text{Apply the distributive property.} \\ &= x^2 + 3x + 2x + 6 && \text{Multiply.} \\ &= x^2 + 5x + 6 && \text{Combine like terms.}\end{aligned}$$

 Work Practice 3**Example 4** Multiply: $(4y + 9)(3y - 2)$ **Solution:**

$$\begin{aligned}(4y + 9)(3y - 2) &= \overbrace{4y(3y - 2)} + \overbrace{9(3y - 2)} && \text{Apply the distributive property.} \\ &= 4y \cdot 3y + 4y(-2) + 9 \cdot 3y + 9(-2) && \text{Apply the distributive property.} \\ &= 12y^2 - 8y + 27y - 18 && \text{Multiply.} \\ &= 12y^2 + 19y - 18 && \text{Combine like terms.}\end{aligned}$$

 Work Practice 4**Objective C** Squaring a Binomial

Raising a binomial to the power of 2 is also called squaring a binomial. To square a binomial, we use the definition of an exponent, and then multiply.

Example 5 Multiply: $(2x + 1)^2$ **Solution:**

$$\begin{aligned}(2x + 1)^2 &= (2x + 1)(2x + 1) && \text{Apply the definition of an exponent.} \\ &= \overbrace{2x(2x + 1)} + \overbrace{1(2x + 1)} && \text{Apply the distributive property.} \\ &= 2x \cdot 2x + 2x \cdot 1 + 1 \cdot 2x + 1 \cdot 1 && \text{Apply the distributive property.} \\ &= 4x^2 + 2x + 2x + 1 && \text{Multiply.} \\ &= 4x^2 + 4x + 1 && \text{Combine like terms.}\end{aligned}$$

 Work Practice 5

 **Concept Check** Correct and explain the error:

~~$$(x + 5)^2 = x^2 + 25$$~~

Practice 3Multiply: $(b + 3)(b + 5)$ **Practice 4**Multiply: $(7x - 1)(5x + 4)$ **Practice 5**Multiply: $(6y - 1)^2$ **Answers**3. $b^2 + 8b + 15$ 4. $35x^2 + 23x - 4$ 5. $36y^2 - 12y + 1$ **Concept Check Answer**

$$\begin{aligned}(x + 5)^2 &= (x + 5)(x + 5) \\ &= x^2 + 10x + 25\end{aligned}$$

Objective D Using the FOIL Order to Multiply Binomials

Recall from Example 3 that

$$\begin{aligned}(x + 2)(x + 3) &= x \cdot x + x \cdot 3 + 2 \cdot x + 2 \cdot 3 \\ &= x^2 + 5x + 6\end{aligned}$$

One way to remember the products $x \cdot x$, $x \cdot 3$, $2 \cdot x$, and $2 \cdot 3$ is to use a special order for multiplying binomials, called the FOIL order. Of course, the product is the same no matter what order or method you choose to use.

FOIL stands for the products of the **F**irst terms, **O**uter terms, **I**nnner terms, and then **L**ast terms. For example,

$$\begin{aligned}(x + 2)(x + 3) &= \overset{\text{F}}{x} \cdot \overset{\text{F}}{x} + \overset{\text{O}}{x} \cdot \overset{\text{O}}{3} + \overset{\text{I}}{2} \cdot \overset{\text{I}}{x} + \overset{\text{L}}{2} \cdot \overset{\text{L}}{3} = x^2 + 3x + 2x + 6 \\ &= x^2 + 5x + 6\end{aligned}$$

Helpful Hint

The product is the same no matter what order or method you choose to use.

Practice 6–7

Use the FOIL order to multiply.

6. $(10x - 7)(2x + 3)$

7. $(3x + 2)^2$

Helpful Hint

Remember that the FOIL order can only be used to multiply **two binomials**.

Examples

Use the FOIL order to multiply.

$$\begin{aligned}6. \quad (3x - 6)(2x + 1) &= \overset{\text{F}}{3x} \cdot \overset{\text{F}}{2x} + \overset{\text{O}}{3x} \cdot \overset{\text{O}}{1} + \overset{\text{I}}{-6} \cdot \overset{\text{I}}{2x} + \overset{\text{L}}{-6} \cdot \overset{\text{L}}{1} \\ &= 6x^2 + 3x - 12x - 6 \\ &= 6x^2 - 9x - 6\end{aligned}$$

Multiply.

Combine like terms.

$$\begin{aligned}7. \quad (3x - 5)^2 &= (3x - 5)(3x - 5) \\ &= \overset{\text{F}}{3x} \cdot \overset{\text{F}}{3x} + \overset{\text{O}}{3x} \cdot \overset{\text{O}}{-5} + \overset{\text{I}}{-5} \cdot \overset{\text{I}}{3x} + \overset{\text{L}}{-5} \cdot \overset{\text{L}}{-5} \\ &= 9x^2 - 15x - 15x + 25 \\ &= 9x^2 - 30x + 25\end{aligned}$$

Multiply.

Combine like terms.

Work Practice 6–7

Objective E Multiplying Polynomials

In Section 10.1, we learned that a polynomial that consists of exactly three terms is called a **trinomial**. Next, we multiply a binomial by a trinomial.

Example 8

Multiply: $(3a + 2)(a^2 - 6a + 3)$

Solution: Use the distributive property to multiply $3a$ by the trinomial $(a^2 - 6a + 3)$ and then 2 by the trinomial.

$$\begin{aligned}(3a + 2)(a^2 - 6a + 3) &= 3a(a^2 - 6a + 3) + 2(a^2 - 6a + 3) \\ &= 3a \cdot a^2 + 3a(-6a) + 3a \cdot 3 + 2 \cdot a^2 + 2(-6a) + 2 \cdot 3 \\ &= 3a^3 - 18a^2 + 9a + 2a^2 - 12a + 6 \\ &= 3a^3 - 16a^2 - 3a + 6\end{aligned}$$

Apply the distributive property.

Apply the distributive property.

Multiply.

Combine like terms.

Work Practice 8

Practice 8

Multiply:

$(2x + 5)(x^2 + 4x - 1)$

Answers

6. $20x^2 + 16x - 21$

7. $9x^2 + 12x + 4$

8. $2x^3 + 13x^2 + 18x - 5$

In general, we have the following.

To Multiply Two Polynomials

Multiply each term of the first polynomial by each term of the second polynomial, and then combine like terms.

A convenient method of multiplying polynomials is to use a vertical format similar to multiplying real numbers.

✓ **Concept Check** True or false? When a trinomial is multiplied by a trinomial, the result will have at most nine terms. Explain.

Example 9 Find the product of $(a^2 - 6a + 3)$ and $(3a + 2)$ vertically.

Solution:

$$\begin{array}{r}
 a^2 - 6a + 3 \\
 \times \quad 3a + 2 \\
 \hline
 2a^2 - 12a + 6 \quad \text{Multiply } a^2 - 6a + 3 \text{ by } 2. \\
 3a^3 - 18a^2 + 9a \quad \text{Multiply } a^2 - 6a + 3 \text{ by } 3a. \text{ Line up like terms.} \\
 \hline
 3a^3 - 16a^2 - 3a + 6 \quad \text{Combine like terms.}
 \end{array}$$

Notice that this example is the same as Example 8, and that of course the products are the same.

Work Practice 9

Practice 9

Multiply $(x^2 + 3x - 2)$ and $(3x + 4)$ vertically.

Answer

9. $3x^3 + 13x^2 + 6x - 8$

✓ **Concept Check Answer**
true

Vocabulary, Readiness & Video Check

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.



See Video 10.3

- Objective A** 1. How is the distributive property used to solve Example 1? ▶
- Objective B** 2. In Example 2, how many times is the distributive property used? List each distribution using specific steps from the example. ▶
- Objective C** 3. In Example 3, why is the power rule for exponents mentioned? ▶
- Objective D** 4. From Examples 4 and 5, what's the only type of multiplication for which we can apply the FOIL order of multiplying? ▶
- Objective E** 5. Can the FOIL order of multiplying be used to solve Example 6? Why or why not? ▶

10.3 Exercise Set MyLab Math

Objective A Multiply. See Examples 1 and 2.

1. $3x(9x^2 - 3)$

2. $4y(10y^3 + 2y)$

3. $-3a(2a^2 - 3a - 5)$

4. $-4b(-2b^2 - 5b + 8)$

 5. $7x^2(6x^2 - 5x + 7)$

6. $6z^2(-3z^2 - z + 4)$

Objectives B C D Mixed Practice Multiply. See Examples 3 through 7

7. $(x + 3)(x + 10)$

8. $(y + 5)(y + 9)$

 9. $(2x - 6)(x + 4)$

10. $(7z + 1)(z - 6)$

11. $(6a + 4)^2$

12. $(8b - 3)^2$

Objective E Multiply. See Examples 8 and 9.

13. $(a + 6)(a^2 - 6a + 3)$

14. $(y + 4)(y^2 + 8y - 2)$

15. $(4x - 5)(2x^2 + 3x - 10)$

16. $(9z - 2)(2z^2 + z + 1)$

17. $(x^3 + 2x + x^2)(3x + 1 + x^2)$

18. $(y^2 - 2y + 5)(y^3 + 2 + y)$

Objectives A B C D E Mixed Practice Multiply. See Examples 1 through 9.

19. $10r(-3r + 2)$

20. $5x(4x^2 + 5)$

21. $-2y^2(3y + y^2 - 6)$

22. $3z^3(4z^4 - 2z + z^3)$

23. $(x + 2)(x + 12)$

24. $(y + 7)(y - 7)$

 25. $(2a + 3)(2a - 3)$

26. $(6s + 1)(3s - 1)$

27. $(x + 5)^2$

28. $(x + 3)^2$

 29. $\left(b + \frac{3}{5}\right)\left(b + \frac{4}{5}\right)$

30. $\left(a - \frac{7}{10}\right)\left(a + \frac{3}{10}\right)$

31. $(6x + 1)(x^2 + 4x + 1)$

32. $(9y - 1)(y^2 + 3y - 5)$

 33. $(7x + 5)^2$

34. $(5x + 9)^2$

35. $(2x - 1)^2$

36. $(4a - 3)^2$

37. $(2x^2 - 3)(4x^3 + 2x - 3)$

38. $(3y^2 + 2)(5y^2 - y + 2)$

39. $(x^3 + x^2 + x)(x^2 + x + 1)$

40. $(a^4 + a^2 + 1)(a^4 + a^2 - 1)$

41. $(2z^2 - z + 1)(5z^2 + z - 2)$

42. $(2b^2 - 4b + 3)(b^2 - b + 2)$

Review

Write each number as a product of prime numbers. See Section 4.2.

43. 50

44. 48

45. 72

46. 36

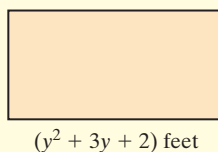
47. 200

48. 300

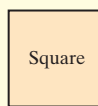
Concept Extensions

Find the area of each figure.

△ 49.

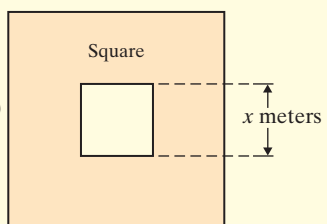
 $(y - 6)$ feet $(y^2 + 3y + 2)$ feet

△ 50.

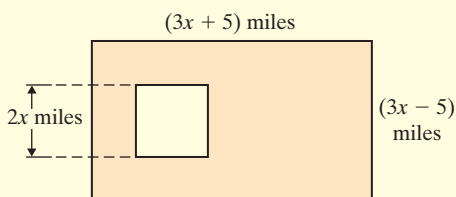
 $(2x + 11)$
centimeters

Find the area of the shaded figure. To do so, subtract the area of the smaller square from the area of the larger geometric figure.

△ 51.

 $(x^2 - 1)$
meters

△ 52.



53. Suppose that a classmate asked you why $(2x + 1)^2$ is **not** $4x^2 + 1$. Write down your response to this classmate.

10.4 Introduction to Factoring Polynomials

Objectives

- A** Find the Greatest Common Factor of a List of Integers. ▶
- B** Find the Greatest Common Factor of a List of Terms. ▶
- C** Factor the Greatest Common Factor from the Terms of a Polynomial. ▶

Recall from Section 2.4 that when an integer is written as the product of two or more integers, each of these integers is called a *factor* of the product. This is true of polynomials also. When a polynomial is written as the product of two or more other polynomials, each of these polynomials is called a factor of the product.

factor · factor = product
$-2 \cdot 4 = -8$
$x^3 \cdot x^7 = x^{10}$
$5(x + 2) = 5x + 10$

The process of writing a polynomial as a product is called **factoring**. Notice that factoring is the reverse process of multiplying.

$$\underbrace{5x + 10}_{\text{multiplying}} = \underbrace{5(x + 2)}_{\text{factoring}}$$

Objective A Finding the GCF of a List of Integers ▶

Before we factor polynomials, let's practice finding the greatest common factor of a list of integers. The **greatest common factor (GCF)** of a list of integers is the largest integer that is a factor of all the integers in the list. For example,

the GCF of 30 and 18 is 6

because 6 is the largest integer that is a factor of both 30 and 18.

If the GCF cannot be found by inspection, the following steps can be used.

To Find the GCF of a List of Integers

Step 1: Write each number as a product of prime numbers.

Step 2: Identify the common prime factors.

Step 3: The product of all common prime factors found in Step 2 is the greatest common factor. If there are no common prime factors, the greatest common factor is 1.

✓ **Concept Check** Which of the following is the prime factorization of 36?

- a. $4 \cdot 9$ b. $2 \cdot 2 \cdot 3 \cdot 3$ c. $6 \cdot 6$

Recall from Section 4.2 that a prime number is a whole number other than 1 whose only factors are 1 and itself.

Example 1 Find the GCF of 12 and 20.

Solution:

Step 1: Write each number as a product of primes.

$$12 = 2 \cdot 2 \cdot 3$$

$$20 = 2 \cdot 2 \cdot 5$$

Practice 1

Find the GCF of 42 and 28.

Answer

1. 14

✓ **Concept Check Answer**

b

$$\begin{array}{l} \text{Step 2: } 12 = \underbrace{(2 \cdot 2)}_{\downarrow} \cdot 3 \\ \quad \quad 20 = \underbrace{(2 \cdot 2)}_{\downarrow} \cdot 5 \\ \quad \quad \quad 2 \cdot 2 \end{array}$$

Identify the common factors.

Step 3: The GCF is $2 \cdot 2 = 4$.**Work Practice 1****Objective B** Finding the GCF of a List of Terms

How do we find the GCF of a list of variables raised to powers? For example, what is the GCF of y^3 , y^5 , and y^{10} ? Notice that each variable term contains a factor of y^3 and no higher power of y is a factor of each term.

$$y^3 = y^3$$

$$y^5 = y^3 \cdot y^2 \quad \text{Recall the product property for exponents.}$$

$$y^{10} = y^3 \cdot y^7$$

The GCF of y^3 , y^5 , and y^{10} is y^3 . From this example, we can see that **the GCF of a list of variables raised to powers is the variable raised to the smallest exponent in the list.**

Example 2 Find the GCF of x^{11} , x^4 , and x^6 .**Solution:** The GCF is x^4 since 4 is the smallest exponent to which x is raised.**Work Practice 2**In general, **the GCF of a list of terms is the product of all common factors.****Example 3** Find the GCF of $4x^3$, $12x$, and $10x^5$.**Solution:** The GCF of 4, 12, and 10 is 2.The GCF of x^3 , x^1 , and x^5 is x^1 .Thus, the GCF of $4x^3$, $12x$, and $10x^5$ is $2x^1$ or $2x$.**Work Practice 3****Helpful Hint**

If you ever have trouble finding the GCF, remember that you can always use the method below.

Example 3:

$$\begin{array}{l} 4x^3 = \underbrace{2 \cdot 2 \cdot x}_{\text{common}} \cdot x \cdot x \\ 12x = \underbrace{2 \cdot 2 \cdot 3 \cdot x}_{\text{common}} \\ 10x^5 = \underbrace{2 \cdot 5 \cdot x}_{\text{common}} \cdot x \cdot x \cdot x \cdot x \\ \text{GCF} = 2 \cdot x \text{ or } 2x \end{array}$$

Objective C Factoring Out the GCF

Next, we practice factoring a polynomial by factoring the GCF from its terms. To do so, we write each term of the polynomial as a product of the GCF and another factor, and then apply the distributive property.

Practice 2Find the GCF of z^7 , z^8 , and z .**Practice 3**Find the GCF of $6a^4$, $3a^5$, and $15a^2$.**Answers**2. z 3. $3a^2$

Practice 4Factor: $10y^7 + 5y^9$ **Practice 5**Factor: $4z^2 - 12z + 2$ **Helpful Hint**

Don't forget to include the term 1.

Practice 6Factor: $-3y^2 - 9y + 15x^2$ **Answers**

4. $5y^7(2 + y^2)$ 5. $2(2z^2 - 6z + 1)$
 6. $-3(y^2 + 3y - 5x^2)$ or $3(-y^2 - 3y + 5x^2)$

✓ **Concept Check Answer**
 answers may vary

Example 4 Factor: $7x^3 + 14x^2$ **Solution:** The GCF of $7x^3$ and $14x^2$ is $7x^2$.

$$\begin{aligned} 7x^3 + 14x^2 &= 7x^2 \cdot x^1 + 7x^2 \cdot 2 \\ &= 7x^2(x + 2) \end{aligned}$$

Apply the distributive property.

Work Practice 4

Notice in Example 4 that we factored $7x^3 + 14x^2$ by writing it as the product $7x^2(x + 2)$. Also notice that to check factoring, we multiply

$$\begin{aligned} 7x^2(x + 2) &= 7x^2 \cdot x + 7x^2 \cdot 2 \\ &= 7x^3 + 14x^2 \end{aligned}$$

which is the original binomial.

Example 5 Factor: $6x^2 - 24x + 6$ **Solution:** The GCF of the terms is 6.

$$\begin{aligned} 6x^2 - 24x + 6 &= 6 \cdot x^2 - 6 \cdot 4x + 6 \cdot 1 \\ &= 6(x^2 - 4x + 1) \end{aligned}$$

Work Practice 5**Helpful Hint**

A common mistake in the example above is to forget to write down the term of 1. Remember to mentally check by multiplying.

$$\begin{aligned} 6(x^2 - 4x) &= 6x^2 - 24x && \text{Not the original trinomial} \\ 6(x^2 - 4x + 1) &= 6x^2 - 24x + 6 && \text{The original trinomial} \end{aligned}$$

Example 6 Factor: $-2a + 20b - 4b^2$ **Solution:**

$$\begin{aligned} -2a + 20b - 4b^2 &= 2 \cdot -a + 2 \cdot 10b - 2 \cdot 2b^2 \\ &= 2(-a + 10b - 2b^2) \end{aligned}$$

When the coefficient of the first term is a negative number, we often factor out a negative common factor.

$$\begin{aligned} -2a + 20b - 4b^2 &= (-2)(a) + (-2)(-10b) + (-2)(2b^2) \\ &= -2(a - 10b + 2b^2) \end{aligned}$$

Both $2(-a + 10b - 2b^2)$ and $-2(a - 10b + 2b^2)$ are factorizations of $-2a + 20b - 4b^2$.

Work Practice 6

✓ **Concept Check** Check both factorizations given in Example 6.

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

factoring smallest factor
product largest greatest common factor (GCF)

- In $-3 \cdot x^4 = -3x^4$, the -3 and the x^4 are each called a _____ and $-3x^4$ is called a _____.
- The _____ of a list of integers is the largest integer that is a factor of all integers in the list.
- The GCF of a list of variables raised to powers is the variable raised to the _____ exponent in the list.
- _____ is the process of writing an expression as a product.

Martin-Gay Interactive Videos



See Video 10.4

Watch the section lecture video and answer the following question.

- Objective A** 5. In the lecture before Example 1, what are we reminded that the word “factor” means?
- Objective B** 6. In Example 3, what is the GCF of the numerical coefficients? Of the variable parts? Of the terms?
- Objective C** 7. When starting to solve Example 4, what is the plan to factor the binomial?

10.4 Exercise Set MyLab Math



Objective A Find the greatest common factor of each list of numbers. See Example 1.

- | | | |
|-------------------|--------------------|-------------------|
| 1. 48 and 15 | 2. 36 and 20 | 3. 60 and 72 |
| 4. 96 and 45 | 5. 12, 20, and 36 | 6. 18, 24, and 60 |
| 7. 8, 32, and 100 | 8. 30, 50, and 200 | |

Objective B Find the greatest common factor of each list of terms. See Examples 2 and 3.

- | | | | |
|----------------------------|----------------------------|-----------------------|------------------------|
| 9. y^7, y^2, y^{10} | 10. x^3, x, x^5 | 11. a^5, a^5, a^5 | 12. b^6, b^6, b^4 |
| 13. x^3y^2, xy^2, x^4y^2 | 14. a^5b^3, a^5b^2, a^5b | 15. $3x^4, 5x^7, 10x$ | 16. $9z^6, 4z^5, 2z^3$ |
| 17. $2z^3, 14z^5, 18z^3$ | 18. $6y^7, 9y^6, 15y^5$ | | |

Objective C Factor. Check by multiplying. See Examples 4 through 6.

- | | | |
|----------------------|--------------------------|-------------------------|
| 19. $3y^2 + 18y$ | 20. $2x^2 + 18x$ | 21. $10a^6 - 5a^8$ |
| 22. $21y^5 + y^{10}$ | 23. $4x^3 + 12x^2 + 20x$ | 24. $9b^3 - 54b^2 + 9b$ |

25. $z^7 - 6z^5$

26. $y^{10} + 4y^5$

27. $-35 + 14y - 7y^2$

28. $-20x + 4x^2 - 2$

29. $12a^5 - 36a^6$

30. $25z^3 - 20z^2$

Review

Solve. See Sections 71 through 73.

31. Find 30% of 120.

32. Find 45% of 265.

33. Write 80% as a fraction in simplified form.

34. Write 65% as a fraction in simplified form.

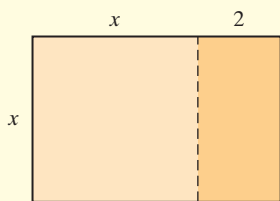
35. Write $\frac{3}{8}$ as a percent.36. Write $\frac{3}{4}$ as a percent.

Concept Extensions

△ 37. The area of the largest rectangle below is $x(x + 2)$.

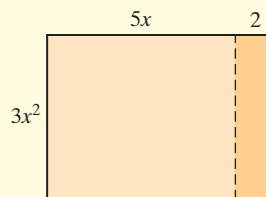
a. Find another expression for the area by writing the sum of the areas of the smaller rectangles.

b. Explain how $x(x + 2)$ and the answer to part a are related.



△ 38. a. Write an expression for the area of the largest rectangle in two different ways.

b. Explain how the two answers to part a are related.



39. In your own words, define the greatest common factor of a list of numbers.

40. Suppose that a classmate asks you why $4x^2 + 6x + 2$ does not factor as $2(2x^2 + 3x)$. Write down your response to this classmate.

41. For the expression $(xy + z)^x$, let $x = 2$ and $z = 7$. Then multiply and simplify.

42. For the expression $(xy + z)^x$, let $x = 2$ and $z = -7$. Then multiply and simplify.

43. Explain two ways in which $(2y + 7)^2$ and your answer to Exercise 41 are related. Use the words *multiply* and *factor* in your explanations.

44. Explain two ways in which $(2y - 7)^2$ and your answer to Exercise 42 are related. Use the words *multiply* and *factor* in your explanations.

Chapter 10 Group Activity

Business Analysis

This activity may be completed by working in groups or individually.

Suppose you own a small business that manufactures specialized iPod covers. You need to decide how many covers to make. The more covers you make, the lower the price you will have to charge to sell them all. Naturally, each cover you make costs you money because you must buy the materials to make each cover. The following table summarizes some factors you must consider in deciding how many covers to make, along with algebraic representations of those factors.

Number of Covers	Description	Algebraic Expression
	Unknown	x
Total manufacturing expenses	This is the total amount that it will cost to manufacture all the iPod covers. It will cost \$100 to buy special equipment to manufacture the covers in addition to materials costing \$0.50 per cover.	$100 + 0.50x$
Price charged per cover	For each additional cover produced, the price that must be charged per cover decreases from \$40 by an additional \$0.05.	$40 - 0.05x$

- Revenue* is the amount of money collected from selling the iPod covers. Revenue can be found by multiplying the price charged per cover by the number of covers sold. Use the algebraic expressions given in the table above to find a polynomial that represents the revenue from sales of covers. Then write this polynomial in the Polynomial column next to “Revenue” in the table to the right.
- Profit* is the amount of money you make from selling the iPod covers after deducting the expenses for making the covers. Profit can be found by subtracting total manufacturing expenses from revenue. Find a polynomial that represents the profit from the sales of covers. Then write this polynomial in the Polynomial column next to “Profit” in the table to the right.
- Complete the following table by evaluating each polynomial for each of the numbers of covers given in the table.
- Study the table. Which number of covers will give you the largest profit from making and selling iPod covers?

		Number of Covers, x				
	Polynomial	200	300	400	500	600
Revenue						
Total manufacturing expenses	$100 + 0.50x$					
Profit						

Chapter 10 Vocabulary Check

Fill in each blank with one of the words or phrases listed below.

trinomial monomial greatest common factor binomial
exponent factoring polynomials FOIL

- _____ is the process of writing an expression as a product.
- The _____ of a list of terms is the product of all common factors.
- The _____ method may be used when multiplying two binomials.
- A polynomial with exactly 3 terms is called a(n) _____.

5. A polynomial with exactly 2 terms is called a(n) _____.
6. A polynomial with exactly 1 term is called a(n) _____.
7. Monomials, binomials, and trinomials are all examples of _____.
8. In $5x^3$, the 3 is called a(n) _____.

Helpful Hint

▶ Are you preparing for your test?

To help, don't forget to take these:

- Chapter 10 Getting Ready for the Test on page 746
- Chapter 10 Test on page 747

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

10 Chapter Highlights

Definitions and Concepts	Examples
Section 10.1 Adding and Subtracting Polynomials	
A monomial is a term that contains whole number exponents and no variable in the denominator. A polynomial is a monomial or a sum or difference of monomials. A binomial is a polynomial with two terms. A trinomial is a polynomial with three terms. To add polynomials, combine like terms. To subtract polynomials, change the signs of the terms being subtracted, then add.	Monomial: $-2x^2y^3$ Polynomials: $5x^2 - 6x + 2, \quad -\frac{9}{10}y, \quad 7$ Binomial: $5x - y$ Trinomial: $7z^3 + 0.5z + 1$ Add: $(7z^2 - 6z + 2) + (5z^2 - 4z + 5)$ $\begin{aligned} &(7z^2 - 6z + 2) + (5z^2 - 4z + 5) \\ &= \underbrace{7z^2 + 5z^2}_{12z^2} - \underbrace{6z - 4z}_{-2z} + \underbrace{2 + 5}_{7} \quad \text{Group like terms.} \\ &= 12z^2 - 2z + 7 \quad \text{Combine like terms.} \end{aligned}$ Subtract: $(20x - 6) - (30x - 6)$ $\begin{aligned} &(20x - 6) - (30x - 6) \\ &= (20x - 6) + (-30x + 6) \\ &= \underbrace{20x - 30x}_{-10x} + \underbrace{-6 + 6}_0 \quad \text{Group like terms.} \\ &= -10x \quad \text{Combine like terms.} \end{aligned}$
Section 10.2 Multiplication Properties of Exponents	
Product property for exponents $a^m \cdot a^n = a^{m+n}$ Power property for exponents $(a^m)^n = a^{m \cdot n}$ Power of a product property for exponents $(ab)^n = a^n b^n$	$x^3 \cdot x^{11} = x^{3+11} = x^{14}$ $(y^5)^3 = y^{5 \cdot 3} = y^{15}$ $(2z^5)^4 = 2^4 (z^5)^4 = 16z^{20}$

Definitions and Concepts	Examples
Section 10.3 Multiplying Polynomials	
To multiply two polynomials, multiply each term of the first polynomial by each term of the second polynomial, and then combine like terms.	$ \begin{aligned} &(x + 2)(x^2 + 5x - 1) \\ &= x(x^2 + 5x - 1) + 2(x^2 + 5x - 1) \\ &= x \cdot x^2 + x \cdot 5x + x(-1) + 2 \cdot x^2 + 2 \cdot 5x + 2(-1) \\ &= x^3 + 5x^2 - x + 2x^2 + 10x - 2 \\ &= x^3 + 7x^2 + 9x - 2 \end{aligned} $
Section 10.4 Introduction to Factoring Polynomials	
To Find the Greatest Common Factor of a List of Integers Step 1: Write each number as a product of prime numbers. Step 2: Identify the common prime factors. Step 3: The product of all common prime factors found in Step 2 is the greatest common factor. If there are no common prime factors, the greatest common factor is 1. The GCF of a list of variables raised to powers is the variable raised to the smallest exponent in the list. The GCF of a list of terms is the product of all common factors. To factor the GCF from the terms of a polynomial, write each term as a product of the GCF and another factor, then apply the distributive property.	Find the GCF of 18 and 30. $ \begin{array}{rcl} 18 & = & \underbrace{(2)}_{\downarrow} \cdot \underbrace{(3)}_{\downarrow} \cdot 3 \\ 30 & = & \underbrace{(2)}_{\downarrow} \cdot \underbrace{(3)}_{\downarrow} \cdot 5 \end{array} $ The GCF is $2 \cdot 3$ or 6. The GCF of x^6, x^8, and x^3 is x^3. Find the GCF of $6y^3$, $12y$, and $4y^7$. The GCF of 6, 12, and 4 is 2. The GCF of y^3, y, and y^7 is y. The GCF of $6y^3$, $12y$, and $4y^7$ is $2y$. Factor $4y^6 + 6y^5$. The GCF of $4y^6$ and $6y^5$ is $2y^5$. $ \begin{aligned} 4y^6 + 6y^5 &= 2y^5 \cdot 2y + 2y^5 \cdot 3 \\ &= 2y^5(2y + 3) \end{aligned} $

Chapter 10

Review

(10.1) Perform each indicated operation.

1. $(2b + 7) + (8b - 10)$

2. $(7s - 6) + (14s - 9)$

3. $(3x + 0.2) - (4x - 2.6)$

4. $(10y - 6) - (11y + 6)$

5. $(4z^2 + 6z - 1) + (5z - 5)$

6. $(17a^3 + 11a^2 + a) + (14a^2 - a)$

7. $\left(9y^2 - y + \frac{1}{2}\right) - \left(20y^2 - \frac{1}{4}\right)$

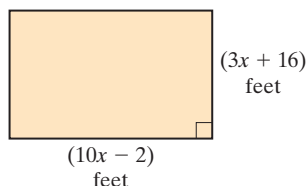
8. Subtract $(x - 2)$ from $(x^2 - 6x + 1)$.

Find the value of each polynomial when $x = 3$.

9. $5x^2$

10. $2 - 7x$

△11. Find the perimeter of the given rectangle.



12. Find the perimeter of a square whose side length is $(4x^2 + 1)$ meters.

(10.2) Multiply and simplify.

13. $x^{10} \cdot x^{14}$

14. $y \cdot y^6$

15. $4z^2 \cdot 6z^5$

16. $(-3x^2y)(5xy^4)$

17. $(a^5)^7$

18. $(x^2)^4 \cdot (x^{10})^2$

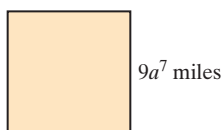
19. $(9b)^2$

20. $(a^4b^2c)^5$

21. $(7x)(2x^5)^3$

22. $(3x^6y^5)^3(2x^6y^5)^2$

△23. Find the area of the square.



24. Find the area of a rectangle whose length is $3x^4$ inches and whose width is $9x$ inches.

(10.3) Multiply.

25. $2a(5a^2 - 6)$

26. $-3y^2(y^2 - 2y + 1)$

27. $(x + 2)(x + 6)$

28. $(3x - 1)(5x - 9)$

29. $(y - 5)^2$

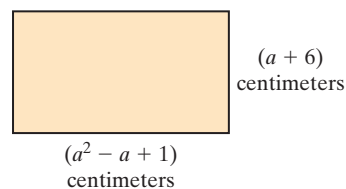
30. $(7a + 1)^2$

31. $(x + 1)(x^2 - 2x + 3)$

32. $(4y^2 - 3)(2y^2 + y + 1)$

33. $(3z^2 + 2z + 1)(z^2 + z + 1)$

△34. Find the area of the given rectangle.



(10.4) Find the greatest common factor (GCF) of each list.

35. 20 and 35

36. 12 and 32

37. 24, 30, and 60

38. 10, 20, and 25

39. x^3, x^2, x^{10}

40. y^{10}, y^7, y^7

41. xy^2, xy, x^3y^3

42. a^5b^4, a^6b^3, a^7b^2

43. $5a^3, 10a, 20a^4$

44. $12y^2z, 20y^2z, 24y^5z$

Factor out the GCF.

45. $2x^2 + 12x$

46. $6a^2 - 12a$

47. $6y^4 - y^6$

48. $7x^2 + 14x + 7$

49. $5a^7 - a^4 + a^3$

50. $10y^6 - 10y$

Mixed Review

Perform the indicated operations.

51. $(z^2 - 5z + 8) + (6z - 4)$

52. $(8y - 5) - (12y - 3)$

53. $x^5 \cdot x^{16}$

54. $y^8 \cdot y$

55. $(a^3b^5c)^6$

56. $(9x^2) \cdot (3x^2)^2$

57. $3a(4a^3 - 5)$

58. $(x + 4)(x + 5)$

59. $(3x + 4)^2$

60. $(6z + 5)(z - 2)$

Find the greatest common factor (GCF) of each list.

61. 28, 32, and 40

62. $5z^5, 12z^8, 3z^4$

Factor out the GCF.

63. $z^9 + 4z^7$

64. $x^{12} - 6x^5$

65. $15a^4 + 45a^5$

66. $16z^5 - 24z^8$

MULTIPLE CHOICE The exercises below are **Multiple Choice**. Choose the correct letter.

For Exercises 1 through 5, the choices are:

A. addition B. subtraction C. multiplication D. division

- ▶1. To simplify $(x^6)^2$, what operation is performed on the exponents?
▶2. To simplify $x^6 \cdot x^2$ what operation is performed on the exponents?

For the polynomials $(3x + 1)$ and $(x - 2)$. Choose the operation that leads to each answer in Exercises 3 through 5.

- ▶3. Answer: $4x - 1$
▶4. Answer: $2x + 3$
▶5. Answer: $3x^2 - 5x - 2$

For Exercises 6 through 9, choose the correct letter.

- ▶6. Choose the value of $-3x - 4$ when $x = 2$.
A. 2 B. -10 C. 10 D. -2
- ▶7. Choose the value of $300 - 16x^2$ when $x = 3$.
A. 252 B. 204 C. 144 D. 156
- ▶8. $(2x + 3) - (x - 2) = \underline{\hspace{2cm}}$
A. $x + 1$ B. 1 C. $x + 5$ D. $2x^2 - x - 6$
- ▶9. $-(4x^2 + 7x - 9) = \underline{\hspace{2cm}}$
A. $-4x^2 + 7x - 9$ B. $-4x^2 - 7x - 9$ C. $4x^2 + 7x + 9$ D. $-4x^2 - 7x + 9$

Add or subtract as indicated.

► 1. $(11x - 3) + (4x - 1)$

► 2. $(11x - 3) - (4x - 1)$

► 3. $(1.3y^2 + 5y) + (2.1y^2 - 3y - 3)$

► 4. Subtract $(8a^2 + a)$ from $(6a^2 + 2a + 1)$.

► 5. Find the value of $x^2 - 6x + 1$ when $x = 8$.

Multiply and simplify.

► 6. $y^3 \cdot y^{11}$

► 7. $(y^3)^{11}$

► 8. $(2x^2)^4$

► 9. $(6a^3)(-2a^7)$

► 10. $(p^6)^7(p^2)^6$

► 11. $(3a^4b)^2(2ba^4)^3$

► 12. $5x(2x^2 + 1.3)$

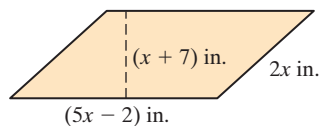
► 13. $-2y(y^3 + 6y^2 - 4)$

► 14. $(x - 3)(x + 2)$

► 15. $(5x + 2)^2$

► 16. $(a + 2)(a^2 - 2a + 4)$

► 17. Find the area and the perimeter of the parallelogram. (Hint: $A = b \cdot h$.)



Find the greatest common factor of each list.

► 18. 45 and 60

► 19. $6y^3, 9y^5, 18y^4$

Factor out the GCF.

► 20. $3y^2 - 15y$

► 21. $10a^2 + 12a$

► 22. $6x^2 - 12x - 30$

► 23. $7x^6 - 6x^4 + x^3$

Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. _____

Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____

△ 1. The state of Colorado is in the shape of a rectangle whose length is 380 miles and whose width is 280 miles. Find its area.

2. In a pecan orchard, there are 21 trees in each row and 7 rows of trees. How many pecan trees are there?

3. Add: $1 + (-10) + (-8) + 9$

4. Add: $-2 + (-7) + 3 + (-4)$

Subtract.

5. $8 - 15$

6. $4 - 7$

7. $-4 - (-5)$

8. $3 - (-2)$

9. Solve: $x = -60 + 4 + 10$

10. Solve: $x = -12 + 3 + 7$

11. Solve: $17 - 7x + 3 = -3x + 21 - 3x$

12. $20 - 6x + 4 = -2x + 18 + 2x$

13. Add: $\frac{2x}{15} + \frac{3x}{10}$

14. Subtract: $\frac{5}{7y} - \frac{9}{14y}$

15. Round 736.2359 to the nearest tenth.

16. Round 328.174 to the nearest tenth.

17. Add: $23.85 + 1.604$

18. Add: $12.762 + 4.29$

19. Is -9 a solution of the equation $3.7y = -3.33$?

20. Is 6 a solution of the equation $2.8x = 16.8$?

Divide.

21. $\frac{786.1}{1000}$

22. $\frac{818}{1000}$

23. $\frac{0.12}{10}$

24. $\frac{5.03}{100}$

25. Evaluate $-2x + 5$ for $x = 3.8$.

26. Evaluate $6x - 1$ for $x = -2.1$.

27. Write $\frac{22}{7}$ as a decimal. Round to the nearest hundredth.

28. Write $\frac{37}{19}$ as a decimal. Round to the nearest thousandth.

29. Find: $\sqrt{\frac{1}{36}}$

30. Find: $\sqrt{\frac{4}{25}}$

31. Mel Wagstaff is a 6-foot-tall park ranger who needs to know the height of a particular tree. He measures the shadow of the tree to be 69 feet long when his own shadow is 9 feet long. Find the height of the tree.

32. Phoebe, a very intelligent dog, wants to estimate the height of a fire hydrant. She notices that when her shadow is 2 feet long, the shadow of the hydrant is 6 feet long. Find the height of the hydrant if Phoebe is 1 foot tall.

33. Translate to an equation: 1.2 is 30% of what number?

34. Translate to an equation: 9 is 45% of what number?

35. What percent of 50 is 8?

36. What percent of 16 is 4?

37. Mr. Percy, the principal at Slidell High School, counted 31 freshmen absent during a particular day. If this is 4% of the total number of freshmen, how many freshmen are there at Slidell High School?

38. Two percent of the apples in a shipment are rotten. If there are 29 rotten apples, how many apples are in the shipment?

21. _____

22. _____

23. _____

24. _____

25. _____

26. _____

27. _____

28. _____

29. _____

30. _____

31. _____

32. _____

33. _____

34. _____

35. _____

36. _____

37. _____

38. _____

39.

40.

41.

42.

43.

44.

45.

46.

47.

48.

49.

50.

51.

52.

53.

54.

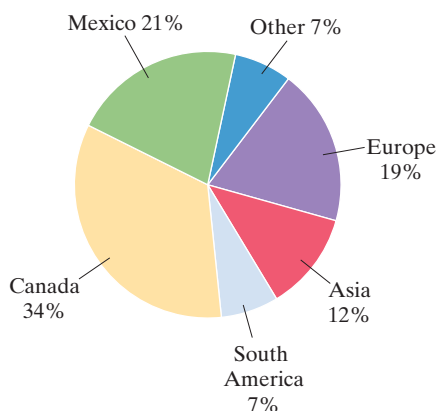
39. A recent college graduate borrowed \$2400 at 10% simple interest for 8 months to buy a used Toyota Corolla. Find the simple interest he paid.

40. Find the amount of simple interest earned on a \$1000 CD for 10 months at an interest rate of 3%.

41. Using the circle graph shown, determine the percent of visitors who came to the United States from Mexico or Canada.

42. Using the circle graph for Exercise 41, find the percent of visitors who came to the United States from Europe or Asia.

Visitors to U.S. by Region

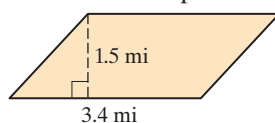


Source: Office of Travel and Tourism Industries, 2012

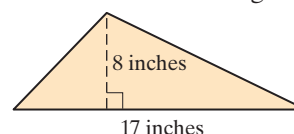
- △ 43. Find the perimeter of a rectangle with a length of 11 inches and a width of 3 inches.

- △ 44. Find the perimeter of a triangular yard whose sides are 6 feet, 8 feet, and 11 feet.

- △ 45. Find the area of the parallelogram.



- △ 46. Find the area of the triangle.



47. Subtract 3 tons 1350 lb from 8 tons 1000 lb.

48. Multiply 5 tons 700 lb by 3.

49. Convert 3210 ml to liters.

50. Convert 4321 cl to liters.

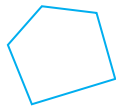
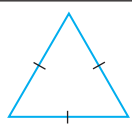
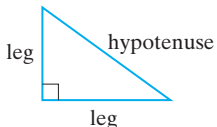
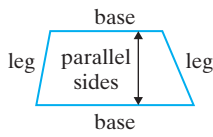
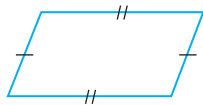
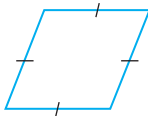
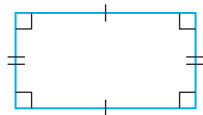
51. Add: $(3x - 1) + (-6x + 2)$

52. Subtract: $(7a + 4) - (3a - 8)$

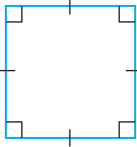
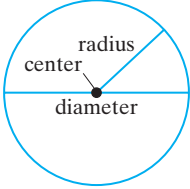
53. Multiply: $(x + 2)(x + 3)$

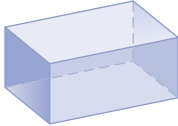
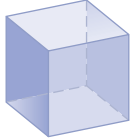
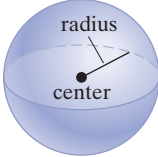
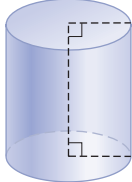
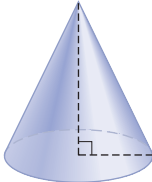
54. Multiply: $(2x + 5)(x + 7)$

A.1 Tables of Geometric Figures

Plane Figures Have Length and Width but No Thickness or Depth		
Name	Description	Figure
Polygon	Union of three or more coplanar line segments that intersect with each other only at each endpoint, with each endpoint shared by two segments.	
Triangle	Polygon with three sides (sum of measures of three angles is 180°).	
Scalene Triangle	Triangle with no sides of equal length.	
Isosceles Triangle	Triangle with two sides of equal length.	
Equilateral Triangle	Triangle with all sides of equal length.	
Right Triangle	Triangle that contains a right angle.	
Quadrilateral	Polygon with four sides (sum of measures of four angles is 360°).	
Trapezoid	Quadrilateral with exactly one pair of opposite sides parallel.	
Isosceles Trapezoid	Trapezoid with legs of equal length.	
Parallelogram	Quadrilateral with both pairs of opposite sides parallel.	
Rhombus	Parallelogram with all sides of equal length.	
Rectangle	Parallelogram with four right angles.	

(Continued)

Plane Figures Have Length and Width but No Thickness or Depth (continued)		
Name	Description	Figure
Square	Rectangle with all sides of equal length.	
Circle	All points in a plane the same distance from a fixed point called the center .	

Solid Figures Have Length, Width, and Height or Depth		
Name	Description	Figure
Rectangular Solid	A solid with six sides, all of which are rectangles.	
Cube	A rectangular solid whose six sides are squares.	
Sphere	All points the same distance from a fixed point called the center .	
Right Circular Cylinder	A cylinder having two circular bases that are perpendicular to its altitude.	
Right Circular Cone	A cone with a circular base that is perpendicular to its altitude.	

A.2 Table of Percents, Decimals, and Fraction Equivalents

Percent	Decimal	Fraction
1%	0.01	$\frac{1}{100}$
5%	0.05	$\frac{1}{20}$
10%	0.1	$\frac{1}{10}$
12.5% or $12\frac{1}{2}\%$	0.125	$\frac{1}{8}$
$16.\overline{6}\%$ or $16\frac{2}{3}\%$	$0.1\overline{6}$	$\frac{1}{6}$
20%	0.2	$\frac{1}{5}$
25%	0.25	$\frac{1}{4}$
30%	0.3	$\frac{3}{10}$
$33.\overline{3}\%$ or $33\frac{1}{3}\%$	$0.\overline{3}$	$\frac{1}{3}$
37.5% or $37\frac{1}{2}\%$	0.375	$\frac{3}{8}$
40%	0.4	$\frac{2}{5}$
50%	0.5	$\frac{1}{2}$
60%	0.6	$\frac{3}{5}$
62.5% or $62\frac{1}{2}\%$	0.625	$\frac{5}{8}$
$66.\overline{6}\%$ or $66\frac{2}{3}\%$	$0.\overline{6}$	$\frac{2}{3}$
70%	0.7	$\frac{7}{10}$
75%	0.75	$\frac{3}{4}$
80%	0.8	$\frac{4}{5}$
$83.\overline{3}\%$ or $83\frac{1}{3}\%$	$0.8\overline{3}$	$\frac{5}{6}$
87.5% or $87\frac{1}{2}\%$	0.875	$\frac{7}{8}$
90%	0.9	$\frac{9}{10}$
100%	1.0	1
110%	1.1	$1\frac{1}{10}$
125%	1.25	$1\frac{1}{4}$
$133.\overline{3}\%$ or $133\frac{1}{3}\%$	$1.\overline{3}$	$1\frac{1}{3}$
150%	1.5	$1\frac{1}{2}$
$166.\overline{6}\%$ or $166\frac{2}{3}\%$	$1.\overline{6}$	$1\frac{2}{3}$
175%	1.75	$1\frac{3}{4}$
200%	2.0	2

A.3 Table on Finding Common Percents of a Number

Common Percent Equivalences*	Shortcut Method for Finding Percent	Examples
$1\% = 0.01$ (or $\frac{1}{100}$)	To find 1% of a number, multiply by 0.01. To do so, move the decimal point 2 places to the left.	1% of 210 is 2.10 or 2.1. 1% of 1500 is 15. 1% of 8.6 is 0.086.
$10\% = 0.1$ (or $\frac{1}{10}$)	To find 10% of a number, multiply by 0.1, or move the decimal point of the number 1 place to the left.	10% of 140 is 14. 10% of 30 is 3. 10% of 17.6 is 1.76.
$25\% = \frac{1}{4}$	To find 25% of a number, find $\frac{1}{4}$ of the number, or divide the number by 4.	25% of 20 is $\frac{20}{4}$ or 5. 25% of 8 is 2. 25% of 10 is $\frac{10}{4}$ or $2\frac{1}{2}$.
$50\% = \frac{1}{2}$	To find 50% of a number, find $\frac{1}{2}$ of the number, or divide the number by 2.	50% of 64 is $\frac{64}{2}$ or 32. 50% of 1000 is 500. 50% of 9 is $\frac{9}{2}$ or $4\frac{1}{2}$.
$100\% = 1$	To find 100% of a number, multiply the number by 1. In other words, 100% of a number is the number.	100% of 98 is 98. 100% of 1407 is 1407. 100% of 18.4 is 18.4.
$200\% = 2$	To find 200% of a number, multiply the number by 2.	200% of 31 is $31 \cdot 2$ or 62. 200% of 750 is 1500. 200% of 6.5 is 13.

*See Appendix A.2.

A.4 Table of Squares and Square Roots

n	n^2	$\sqrt{n}$	n	n^2	$\sqrt{n}$
1	1	1.000	51	2601	7.141
2	4	1.414	52	2704	7.211
3	9	1.732	53	2809	7.280
4	16	2.000	54	2916	7.348
5	25	2.236	55	3025	7.416
6	36	2.449	56	3136	7.483
7	49	2.646	57	3249	7.550
8	64	2.828	58	3364	7.616
9	81	3.000	59	3481	7.681
10	100	3.162	60	3600	7.746
11	121	3.317	61	3721	7.810
12	144	3.464	62	3844	7.874
13	169	3.606	63	3969	7.937
14	196	3.742	64	4096	8.000
15	225	3.873	65	4225	8.062
16	256	4.000	66	4356	8.124
17	289	4.123	67	4489	8.185
18	324	4.243	68	4624	8.246
19	361	4.359	69	4761	8.307
20	400	4.472	70	4900	8.367
21	441	4.583	71	5041	8.426
22	484	4.690	72	5184	8.485
23	529	4.796	73	5329	8.544
24	576	4.899	74	5476	8.602
25	625	5.000	75	5625	8.660
26	676	5.099	76	5776	8.718
27	729	5.196	77	5929	8.775
28	784	5.292	78	6084	8.832
29	841	5.385	79	6241	8.888
30	900	5.477	80	6400	8.944
31	961	5.568	81	6561	9.000
32	1024	5.657	82	6724	9.055
33	1089	5.745	83	6889	9.110
34	1156	5.831	84	7056	9.165
35	1225	5.916	85	7225	9.220
36	1296	6.000	86	7396	9.274
37	1369	6.083	87	7569	9.327
38	1444	6.164	88	7744	9.381
39	1521	6.245	89	7921	9.434
40	1600	6.325	90	8100	9.487
41	1681	6.403	91	8281	9.539
42	1764	6.481	92	8464	9.592
43	1849	6.557	93	8649	9.644
44	1936	6.633	94	8836	9.695
45	2025	6.708	95	9025	9.747
46	2116	6.782	96	9216	9.798
47	2209	6.856	97	9409	9.849
48	2304	6.928	98	9604	9.899
49	2401	7.000	99	9801	9.950
50	2500	7.071	100	10,000	10.000

Quotient Rule and Negative Exponents

Objectives

- A** Use the Quotient Rule for Exponents, and Define a Number Raised to the 0 Power. ▶
- B** Simplify Expressions Containing Negative Exponents. ▶
- C** Use the Rules and Definitions for Exponents to Simplify Exponential Expressions. ▶

Objective A Using the Quotient Rule and Defining the Zero Exponent ▶

Let's study a pattern for simplifying exponential expressions that involves quotients.

$$\begin{aligned}\frac{x^5}{x^3} &= \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} \\ &= \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} \\ &= 1 \cdot 1 \cdot 1 \cdot x \cdot x \\ &= x \cdot x \\ &= x^2\end{aligned}$$

Notice that the result is exactly the same if we subtract exponents of the common bases.

$$\frac{x^5}{x^3} = x^{5-3} = x^2$$

The following rule states this result in a general way.

Quotient Rule for Exponents

If m and n are positive integers and a is a real number, then

$$\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$$

For example,

$$\frac{x^6}{x^2} = x^{6-2} = x^4, \quad x \neq 0$$

In other words, to divide one exponential expression by another with a common base, we keep the base and subtract the exponents.

Practice 1–3

Simplify each quotient.

1. $\frac{y^{10}}{y^6}$ 2. $\frac{5^{11}}{5^8}$ 3. $\frac{12a^4b^{11}}{ab}$

Answers

1. y^4 2. 125 3. $12a^3b^{10}$

Examples

Simplify each quotient.

1. $\frac{x^5}{x^2} = x^{5-2} = x^3$ Use the quotient rule.

2. $\frac{4^7}{4^3} = 4^{7-3} = 4^4 = 256$ Use the quotient rule.

3. $\frac{2x^5y^2}{xy} = 2 \cdot \frac{x^5}{x^1} \cdot \frac{y^2}{y^1}$
 $= 2 \cdot (x^{5-1}) \cdot (y^{2-1})$ Use the quotient rule.
 $= 2x^4y^1$ or $2x^4y$

Work Practice 1–3

Let's now give meaning to an expression such as x^0 . To do so, we will simplify $\frac{x^3}{x^3}$ in two ways and compare the results.

$$\frac{x^3}{x^3} = x^{3-3} = x^0 \quad \text{Apply the quotient rule.}$$

$$\frac{x^3}{x^3} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = 1 \quad \text{Apply the fundamental property for fractions.}$$

Since $\frac{x^3}{x^3} = x^0$ and $\frac{x^3}{x^3} = 1$, we define that $x^0 = 1$ as long as x is not 0.

Zero Exponent

$a^0 = 1$, as long as a is not 0.

For example, $5^0 = 1$.

In other words, a base raised to the 0 power is 1, as long as the base is not 0.

Examples

Simplify each expression.

4. $3^0 = 1$

5. $(-4)^0 = 1$

6. $-4^0 = -1 \cdot 4^0 = -1 \cdot 1 = -1$

7. $5x^0 = 5 \cdot x^0 = 5 \cdot 1 = 5$

Work Practice 4–7

Practice 4–7

Simplify each expression.

4. 6^0 5. $(-8)^0$ 6. -8^0

7. $7y^0$

Objective B Simplifying Expressions Containing Negative Exponents

Our work with exponential expressions so far has been limited to exponents that are positive integers or 0. Here we will also give meaning to an expression like x^{-3} .

Suppose that we wish to simplify the expression $\frac{x^2}{x^5}$. If we use the quotient rule for exponents, we subtract exponents:

$$\frac{x^2}{x^5} = x^{2-5} = x^{-3}, \quad x \neq 0$$

But what does x^{-3} mean? Let's simplify $\frac{x^2}{x^5}$ using the definition of a^n .

$$\begin{aligned} \frac{x^2}{x^5} &= \frac{x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} \\ &= \frac{\cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x} \quad \text{Divide numerator and denominator by common factors by applying the fundamental principle for fractions.} \\ &= \frac{1}{x^3} \end{aligned}$$

If the quotient rule is to hold true for negative exponents, then x^{-3} must equal $\frac{1}{x^3}$.

Answers

4. 1 5. 1 6. -1 7. 7

From this example, we state the definition for negative exponents.

Negative Exponents

If a is a real number other than 0 and n is an integer, then

$$a^{-n} = \frac{1}{a^n}$$

For example,

$$x^{-3} = \frac{1}{x^3}$$

In other words, another way to write a^{-n} is to take the reciprocal of a and change the sign of its exponent.

Practice 8–10

Simplify by writing each expression with positive exponents only.

8. 5^{-2} 9. $5x^{-2}$ 10. $4^{-1} + 3^{-1}$

Helpful Hint

Don't forget that since there are no parentheses, only x is the base for the exponent -3 .

Examples

Simplify by writing each expression with positive exponents only.

8. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ Use the definition of negative exponents.

9. $2x^{-3} = 2^1 \cdot \frac{1}{x^3} = \frac{2^1}{x^3}$ or $\frac{2}{x^3}$ Use the definition of negative exponents.

10. $2^{-1} + 4^{-1} = \frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

Work Practice 8–10

Helpful Hint

A negative exponent *does not* affect the sign of its base.

Remember: Another way to write a^{-n} is to take the reciprocal of a and change the sign of its exponent: $a^{-n} = \frac{1}{a^n}$. For example,

$$\begin{aligned} x^{-2} &= \frac{1}{x^2}, & 2^{-3} &= \frac{1}{2^3} \text{ or } \frac{1}{8} \\ \frac{1}{y^{-4}} &= \frac{1}{\frac{1}{y^4}} = y^4, & \frac{1}{5^{-2}} &= 5^2 \text{ or } 25 \end{aligned}$$

From the preceding Helpful Hint, we know that $x^{-2} = \frac{1}{x^2}$ and $\frac{1}{y^{-4}} = y^4$. We can use this to include another statement in our definition of negative exponents.

Negative Exponents

If a is a real number other than 0 and n is an integer, then

$$a^{-n} = \frac{1}{a^n} \text{ and } \frac{1}{a^{-n}} = a^n$$

Answers

8. $\frac{1}{25}$ 9. $\frac{5}{x^2}$ 10. $\frac{7}{12}$

Examples

Simplify each expression. Write each result using positive exponents only.

$$11. \left(\frac{2}{x}\right)^{-3} = \frac{2^{-3}}{x^{-3}} = \frac{2^{-3}}{1} \cdot \frac{1}{x^{-3}} = \frac{1}{2^3} \cdot \frac{x^3}{1} = \frac{x^3}{2^3} = \frac{x^3}{8} \quad \text{Use the negative exponents rule.}$$

$$12. \frac{y}{y^{-2}} = \frac{y^1}{y^{-2}} = y^{1-(-2)} = y^3 \quad \text{Use the quotient rule.}$$

$$13. \frac{x^{-5}}{x^7} = x^{-5-7} = x^{-12} = \frac{1}{x^{12}}$$

Work Practice 11–13**Objective C** Simplifying Exponential Expressions

All the previously stated rules for exponents apply for negative exponents also. Here is a summary of the rules and definitions for exponents. Notice that there is a power of a quotient rule.

Summary of Exponent Rules

If m and n are integers and a , b , and c are real numbers, then

Product rule for exponents: $a^m \cdot a^n = a^{m+n}$

Power rule for exponents: $(a^m)^n = a^{m \cdot n}$

Power of a product: $(ab)^n = a^n b^n$

Power of a quotient: $\left(\frac{a}{c}\right)^n = \frac{a^n}{c^n}, \quad c \neq 0$

Quotient rule for exponents: $\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$

Zero exponent: $a^0 = 1, \quad a \neq 0$

Negative exponent: $a^{-n} = \frac{1}{a^n}, \quad a \neq 0$

Examples

Simplify each expression. Write each result using positive exponents only.

$$14. \overbrace{x^{-3} \cdot x^2} \cdot x^{-7} = x^{-3+2} \cdot x^{-7} \quad \text{Use the product rule.}$$

$$= x^{-1} \cdot x^{-7}$$

$$= x^{-1+(-7)} \quad \text{Use the product rule.}$$

$$= x^{-8}$$

$$= \frac{1}{x^8} \quad \text{Use the definition of negative exponents.}$$

$$15. (x^4 y^{-9})(x^{-1} y^{11}) = x^{4+(-1)} \cdot y^{-9+11} \quad \text{Use the product rule.}$$

$$= x^3 y^2 \quad \text{Simplify.}$$

$$16. (4m^2 n^5)(5m^6 n^{-8}) = 4 \cdot 5 \cdot m^{2+6} \cdot n^{5+(-8)} \quad \text{Use the product rule.}$$

$$= 20 \cdot m^8 \cdot n^{-3} \quad \text{Simplify.}$$

$$= \frac{20m^8}{n^3} \quad \text{Use the definition of a negative exponent.}$$

Work Practice 14–16**Practice 11–13**

Simplify each expression. Write each result using positive exponents only.

$$11. \left(\frac{6}{7}\right)^{-2} \quad 12. \frac{x}{x^{-4}} \quad 13. \frac{y^{-4}}{y^6}$$

Practice 14–16

Simplify each expression. Write each result using positive exponents only.

$$14. y^{-6} \cdot y^3 \cdot y^{-4}$$

$$15. (a^6 b^{-4})(a^{-3} b^8)$$

$$16. (3y^9 z^{10})(2y^3 z^{-12})$$

Answers

$$11. \frac{49}{36} \quad 12. x^5 \quad 13. \frac{1}{y^{10}}$$

$$14. \frac{1}{y^7} \quad 15. a^3 b^4 \quad 16. \frac{6y^{12}}{z^2}$$

B Exercise Set MyLab Math **Objective A** Use the quotient rule and simplify each expression. See Examples 1 through 3.

1. $\frac{x^3}{x}$

2. $\frac{y^{10}}{y^9}$

3. $\frac{9^8}{9^6}$

4. $\frac{5^7}{5^4}$

5. $\frac{p^7 q^{20}}{pq^{15}}$

6. $\frac{x^8 y^6}{xy^5}$

7. $\frac{7x^3 y^6}{14x^2 y^3}$

8. $\frac{9a^4 b^7}{27ab^2}$

Simplify each expression. See Examples 4 through 7.

9. 7^0

10. 23^0

11. $2x^0$

12. $4y^0$

13. -7^0

14. -2^0

15. $(-7)^0$

16. $(-2)^0$

Objective B Simplify each expression. Write each result using positive exponents only. See Examples 8 through 13.

17. 4^{-3}

18. 6^{-2}

19. $7x^{-3}$

20. $5y^{-4}$

21. $3^{-1} + 2^{-1}$

22. $4^{-1} + 4^{-2}$

23. $\frac{1}{p^{-3}}$

24. $\frac{1}{q^{-5}}$

25. $\frac{x^{-2}}{x}$

26. $\frac{y}{y^{-3}}$

27. $\frac{z^{-4}}{z^{-7}}$

28. $\frac{x^{-4}}{x^{-1}}$

29. $3^{-2} + 3^{-1}$

30. $4^{-2} - 4^{-3}$

31. $\left(\frac{5}{y}\right)^{-2}$

32. $\left(\frac{3}{x}\right)^{-3}$

33. $\frac{1}{p^{-4}}$

34. $\frac{1}{y^{-6}}$

Objective C Simplify each expression. Write each result using positive exponents only. See Examples 14 through 16.

35. $a^2 \cdot a^{-9} \cdot a^{13}$

36. $z^4 \cdot z^{-5} \cdot z^3$

37. $(x^8 y^{-6})(x^{-2} y^{12})$

38. $(a^{-20} b^8)(a^{22} b^{-4})$

39. $x^{-7} \cdot x^{-8} \cdot x^4$

40. $y^{-6} \cdot y^{-3} \cdot y^2$

41. $(5x^{-7})(3x^4)$

42. $(4x^9)(6x^{-13})$

43. $y^5 \cdot y^{-7} \cdot y^{-10}$

44. $x^8 \cdot x^{-11} \cdot x^{-2}$

45. $(8m^5 n^{-1})(7m^2 n^{-4})$

46. $(2x^{10} y^{-3})(9x^4 y^{-7})$

Objectives A B C Mixed Practice Simplify each expression. Write each result using positive exponents only. See Examples 1 through 16.

47. $\frac{x^{15}}{x^8}$

48. $\frac{y^{19}}{y^{10}}$

49. $\frac{a^9 b^{14}}{ab}$

50. $\frac{x^{11} y^7}{xy}$

51. $\frac{x^3}{x^9}$

52. $\frac{z^4}{z^{12}}$

53. $3z^0$

54. $5y^0$

55. 5^{-3}

56. 7^{-2}

57. $8x^{-9}$

58. $9y^{-7}$

59. $5^{-1} + 10^{-1}$

60. $7^{-1} + 14^{-1}$

61. $\frac{z^{-8}}{z^{-1}}$

62. $\frac{r^{-15}}{r^{-4}}$

63. $x^{-7} \cdot x^5 \cdot x^{-7}$

64. $y^{-9} \cdot y^6 \cdot y^{-9}$

65. $(a^{-2} b^3)(a^{10} b^{-11})$

66. $(x^{-4} y^5)(x^{13} y^{-14})$

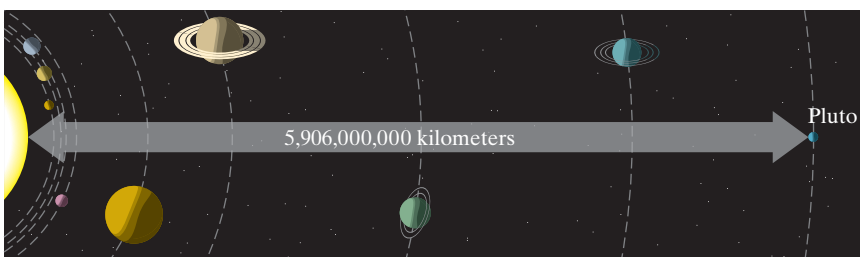
67. $(3x^{20} y^{-1})(10x^{-11} y^{-5})$

68. $(4m^{16} n^{-3})(11m^{-6} n^{-3})$

Scientific Notation

Objective A Writing Numbers in Scientific Notation

Both very large and very small numbers frequently occur in many fields of science. For example, the distance between the Sun and the dwarf planet Pluto is approximately 5,906,000,000 kilometers, and the mass of a proton is approximately 0.000000000000000000000000165 gram. It can be tedious to write these numbers in this standard decimal notation, so **scientific notation** is used as a convenient shorthand for expressing very large and very small numbers.



Scientific Notation

A positive number is written in scientific notation if it is written as the product of a number a , where $1 \leq a < 10$, and an integer power r of 10: $a \times 10^r$.

The following numbers are written in scientific notation. The $\times$ sign for multiplication is used as part of the notation.

$$\begin{array}{c} \text{(Mass of a proton)} \quad 1.65 \times 10^{-24} \quad 5.906 \times 10^9 \quad \text{(Distance between the Sun and Pluto)} \\ \text{-----} \quad \text{-----} \quad \text{-----} \end{array}$$

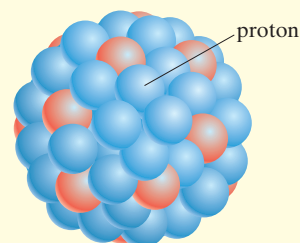
The following steps are useful when writing numbers in scientific notation.

To Write a Number in Scientific Notation

- Step 1:** Move the decimal point in the original number so that the new number has a value between 1 and 10.
- Step 2:** Count the number of decimal places the decimal point is moved in Step 1. If the original number is 10 or greater, the count is positive. If the original number is less than 1, the count is negative.
- Step 3:** Multiply the new number in Step 1 by 10 raised to an exponent equal to the count found in Step 2.

Objectives

- A** Write Numbers in Scientific Notation.
- B** Convert Numbers in Scientific Notation to Standard Form.
- C** Perform Operations on Numbers Written in Scientific Notation.



Mass of proton is approximately 0.000000000000000000000000165 gram

Practice 1

Write each number in scientific notation.

- a. 760,000
b. 0.00035

Example 1

Write each number in scientific notation.

a. 367,000,000

b. 0.000003

Solution: Move each decimal point until the number is between 1 and 10.

a. $367,000,000. = 3.67 \times 10^8$ The original number is 10 or greater, so the exponent is positive.

b. $0.000003 = 3.0 \times 10^{-6}$ The original number is less than 1, so the exponent is negative.

 Work Practice 1

Objective B Converting Numbers to Standard Form

A number written in scientific notation can be rewritten in standard form. For example, to write 8.63×10^3 in standard form, recall that $10^3 = 1000$.

$$8.63 \times 10^3 = 8.63(1000) = 8630$$

Notice that the exponent on the 10 is positive 3, and we moved the decimal point 3 places to the right.

$$\text{To write } 7.29 \times 10^{-3} \text{ in standard form, recall that } 10^{-3} = \frac{1}{10^3} = \frac{1}{1000}.$$

$$7.29 \times 10^{-3} = 7.29 \left(\frac{1}{1000} \right) = \frac{7.29}{1000} = 0.00729$$

The exponent on the 10 is negative 3, and we moved the decimal to the left 3 places.

In general, **to write a scientific notation number in standard form**, move the decimal point the same number of places as the exponent on 10. If the exponent is positive, move the decimal point to the right; if the exponent is negative, move the decimal point to the left.

Practice 2

Write the numbers in standard form, without exponents.

- a. 9.062×10^{-4}
b. 8.002×10^6

Example 2

Write each number in standard notation, without exponents.

a. 1.02×10^5

b. 7.358×10^{-3}

Solution:

a. Move the decimal point 5 places to the right.

$$1.02 \times 10^5 = 102,000.$$

b. Move the decimal point 3 places to the left.

$$7.358 \times 10^{-3} = 0.007358$$

 Work Practice 2

Objective C Performing Operations on Numbers
Written in Scientific Notation

Performing operations on numbers written in scientific notation makes use of the rules and definitions for exponents.

Answers

1. a. 7.6×10^5 b. 3.5×10^{-4}
2. a. 0.0009062 b. 8,002,000

Example 3

Perform each indicated operation. Write each result in standard decimal form.

a. $(8 \times 10^{-6})(7 \times 10^3)$

b. $\frac{12 \times 10^2}{6 \times 10^{-3}}$

Solution:

$$\begin{aligned} \text{a. } (8 \times 10^{-6})(7 \times 10^3) &= 8 \cdot 7 \cdot 10^{-6} \cdot 10^3 \\ &= 56 \times 10^{-3} \\ &= 0.056 \end{aligned}$$

$$\text{b. } \frac{12 \times 10^2}{6 \times 10^{-3}} = \frac{12}{6} \times 10^{2-(-3)} = 2 \times 10^5 = 200,000$$

Work Practice 3**Practice 3**

Perform each indicated operation. Write each result in standard decimal form.

a. $(8 \times 10^7)(3 \times 10^{-9})$

b. $\frac{8 \times 10^4}{2 \times 10^{-3}}$

Answers

3. a. 0.24 b. 40,000,000

C**Exercise Set MyLab Math**

Objective A Write each number in scientific notation. See Example 1.

1. 78,000

2. 9,300,000,000

3. 0.00000167

4. 0.00000017

5. 0.00635

6. 0.00194

7. 1,160,000

8. 700,000

9. When it is completed in 2022, the Thirty Meter Telescope is expected to be the world's largest optical telescope. Located in an observatory complex at the summit of Mauna Kea in Hawaii, the elevation of the Thirty Meter Telescope will be roughly 4200 meters above sea level. Write 4200 in scientific notation.

10. The Thirty Meter Telescope (see Exercise 9) will have the ability to view objects 13,000,000,000 light-years away. Write 13,000,000,000 in scientific notation.

Objective B Write each number in standard form. See Example 2.

11. 8.673×10^{-10}

12. 9.056×10^{-4}

13. 3.3×10^{-2}

14. 4.8×10^{-6}

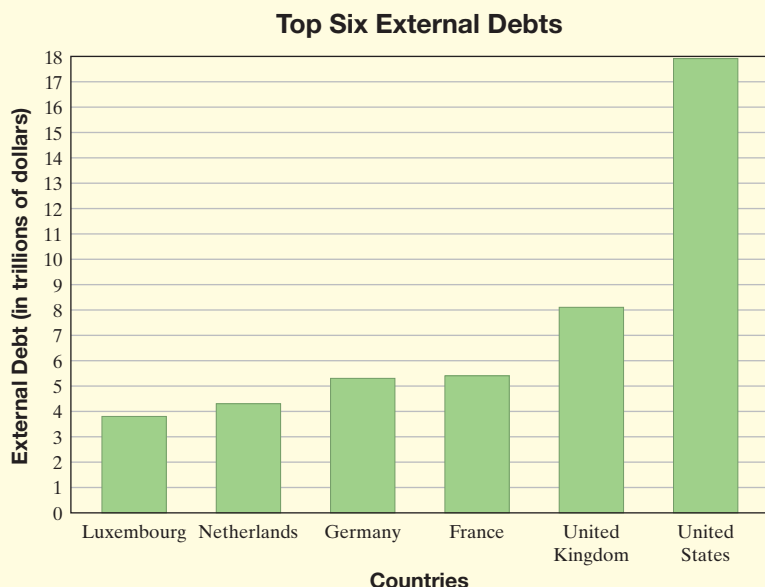
15. 2.032×10^4

16. 9.07×10^{10}

17. Each second, the Sun converts 7.0×10^8 tons of hydrogen into helium and energy in the form of gamma rays. Write this number in standard form. (Source: Students for the Exploration and Development of Space)

18. In chemistry, Avogadro's number is the number of atoms in one mole of an element. Avogadro's number is $6.02214199 \times 10^{23}$. Write this number in standard form. (Source: National Institute of Standards and Technology)

Objectives A B Mixed Practice See Examples 1 and 2. The bar graph below shows estimates of the top six external debts of countries as of March 2016. If a number is written in standard form, write it in scientific notation. If a number is written in scientific notation, write it in standard form.



Source: CIA World Factbook

19. Germany's external debt as of March 2016 was \$5,300,000,000,000.
20. Luxembourg's external debt as of March 2016 was \$3,800,000,000,000.
21. The United Kingdom's external debt as of March 2016 was 8.1×10^{12} .
22. France's external debt as of March 2016 was 5.4×10^{12} .
23. Use the bar graph to estimate the external debt of the Netherlands and then express it in both standard and scientific notation.
24. Use the bar graph to estimate the external debt of the United States and then express it in both standard and scientific notation.

Objectives C Evaluate each expression using exponential rules. Write each result in standard form. See Example 3.

25. $(1.2 \times 10^{-3})(3 \times 10^{-2})$

26. $(2.5 \times 10^6)(2 \times 10^{-6})$

27. $(4 \times 10^{-10})(7 \times 10^{-9})$

28. $(5 \times 10^6)(4 \times 10^{-8})$

29. $\frac{8 \times 10^{-1}}{16 \times 10^5}$

30. $\frac{25 \times 10^{-4}}{5 \times 10^{-9}}$

31. $\frac{1.4 \times 10^{-2}}{7 \times 10^{-8}}$

32. $\frac{0.4 \times 10^5}{0.2 \times 10^{11}}$

33. Although the actual amount varies by season and time of day, the average volume of water that flows over Niagara Falls (the American and Canadian falls combined) each second is 7.5×10^5 gallons. How much water flows over Niagara Falls in an hour? Write the result in scientific notation. (Hint: 1 hour equals 3600 seconds.) (Source: niagarafallslive.com)

34. A beam of light travels 9.460×10^{12} kilometers per year. How far does light travel in 10,000 years? Write the result in scientific notation.

Geometric Formulas

Appendix

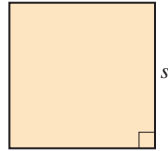
D

Rectangle



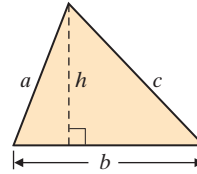
Perimeter: $P = 2l + 2w$
Area: $A = lw$

Square



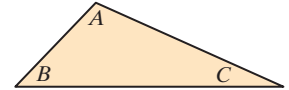
Perimeter: $P = 4s$
Area: $A = s^2$

Triangle



Perimeter: $P = a + b + c$
Area: $A = \frac{1}{2}bh$

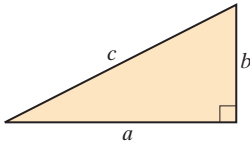
Sum of Angles of Triangle



$A + B + C = 180^\circ$

The sum of the measures of the three angles is 180° .

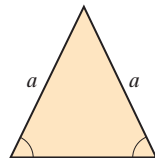
**Pythagorean Theorem
(for right triangles)**



Perimeter: $P = a + b + c$
Area: $A = \frac{1}{2}ab$

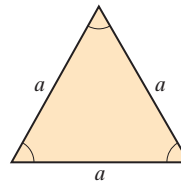
One 90° (right) angle

Isosceles Triangle



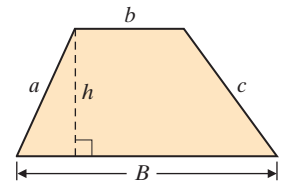
Triangle has:
two equal sides and
two equal angles.

Equilateral Triangle



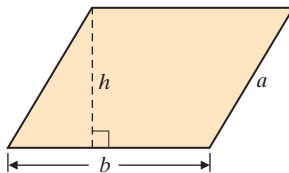
Triangle has:
three equal sides and
three equal angles.
Measure of each angle is 60° .

Trapezoid



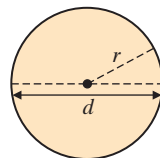
Perimeter: $P = a + b + c + B$
Area: $A = \frac{1}{2}h(B + b)$

Parallelogram



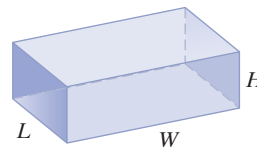
Perimeter: $P = 2a + 2b$
Area: $A = bh$

Circle



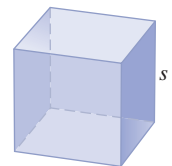
Circumference: $C = \pi d$
 $C = 2\pi r$
Area: $A = \pi r^2$

Rectangular Solid



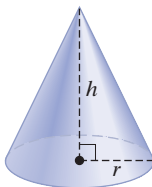
Volume: $V = LWH$
Surface Area:
 $S = 2LW + 2HL + 2HW$

Cube



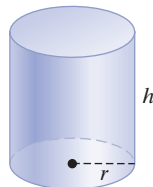
Volume: $V = s^3$
Surface Area: $S = 6s^2$

Cone



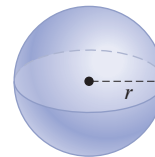
Volume: $V = \frac{1}{3}\pi r^2 h$
Surface Area:
 $S = \pi r\sqrt{r^2 + h^2} + \pi r^2$

Right Circular Cylinder



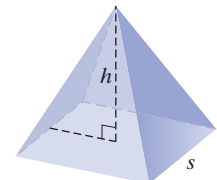
Volume: $V = \pi r^2 h$
Surface Area: $S = 2\pi r^2 + 2\pi rh$

Sphere



Volume: $V = \frac{4}{3}\pi r^3$
Surface Area: $S = 4\pi r^2$

Square-Based Pyramid



Volume: $V = \frac{1}{3}s^2 h$

Contents of Student Resources

Study Skills Builders

Attitude and Study Tips:

1. Have You Decided to Complete This Course Successfully?
2. Tips for Studying for an Exam
3. What to Do the Day of an Exam
4. Are You Satisfied with Your Performance on a Particular Quiz or Exam?
5. How Are You Doing?
6. Are You Preparing for Your Final Exam?

Organizing Your Work:

7. Learning New Terms
8. Are You Organized?
9. Organizing a Notebook
10. How Are Your Homework Assignments Going?

MyLab Math and MathXL:

11. Tips for Turning in Your Homework on Time
12. Tips for Doing Your Homework Online
13. Organizing Your Work
14. Getting Help with Your Homework Assignments
15. Tips for Preparing for an Exam
16. How Well Do You Know the Resources Available to You in MyLab Math?

Additional Help Inside and Outside Your Textbook:

17. How Well Do You Know Your Textbook?
18. Are You Familiar with Your Textbook Supplements?
19. Are You Getting All the Mathematics Help That You Need?

Bigger Picture—Study Guide Outline

Practice Final Exam

Answers to Selected Exercises

Solutions to Selected Exercises

Student Resources

Study Skills Builders

Attitude and Study Tips

Study Skills Builder 1

Have You Decided to Complete This Course Successfully?

Ask yourself if one of your current goals is to complete this course successfully.

If it is not a goal of yours, ask yourself why. One common reason is fear of failure. Amazingly enough, fear of failure alone can be strong enough to keep many of us from doing our best in any endeavor.

Another common reason is that you simply haven't taken the time to think about or write down your goals for this course. To help accomplish this, answer the questions below.

Exercises

1. Write down your goal(s) for this course.

- Now list steps you will take to make sure your goal(s) in Exercise 1 are accomplished.
- Rate your commitment to this course with a number between 1 and 5. Use the diagram below to help.

High Commitment		Average Commitment		Not Committed at All	
5	4	3	2	1	

- If you have rated your personal commitment level (from the exercise above) as a 1, 2, or 3, list the reasons why this is so. Then determine whether it is possible to increase your commitment level to a 4 or 5.

Good luck, and don't forget that a positive attitude will make a big difference.

Study Skills Builder 2

Tips for Studying for an Exam

To prepare for an exam, try the following study techniques:

- Start the study process days before your exam.
- Make sure that you are up-to-date on your assignments.
- If there is a topic that you are unsure of, use one of the many resources that are available to you. For example,
 - See your instructor.
 - View a lecture video on the topic.
 - Visit a learning resource center on campus.
 - Read the textbook material and examples on the topic.
- Reread your notes and carefully review the Chapter Highlights at the end of any chapter.
- Work the review exercises at the end of the chapter.
- Find a quiet place to take the Chapter Test found at the end of the chapter. Do not use any resources when taking this sample test. This way, you will have a clear indication of how prepared you are for your exam. Check your answers and use the Chapter Test Prep Videos to make sure that you correct any missed exercises.

Good luck, and keep a positive attitude.

Exercises

Let's see how you did on your last exam.

1. How many days before your last exam did you start studying for that exam?
2. Were you up-to-date on your assignments at that time or did you need to catch up on assignments?
3. List the most helpful text supplement (if you used one).
4. List the most helpful campus supplement (if you used one).
5. List your process for preparing for a mathematics test.
6. Was this process helpful? In other words, were you satisfied with your performance on your exam?
7. If not, what changes can you make in your process that will make it more helpful to you?

Study Skills Builder 3

What to Do the Day of an Exam

Your first exam may be soon. On the day of an exam, don't forget to try the following:

- Allow yourself plenty of time to arrive.
- Read the directions on the test carefully.
- Read each problem carefully as you take your test. Make sure that you answer the question asked.
- Watch your time and pace yourself so that you may attempt each problem on your test.
- Check your work and answers.
- **Do not turn your test in early.** If you have extra time, spend it double-checking your work.

Good luck!

Exercises

Answer the following questions based on your most recent mathematics exam, whenever that was.

1. How soon before class did you arrive?
2. Did you read the directions on the test carefully?
3. Did you make sure you answered the question asked for each problem on the exam?
4. Were you able to attempt each problem on your exam?
5. If your answer to Exercise 4 is no, list reasons why.
6. Did you have extra time on your exam?
7. If your answer to Exercise 6 is yes, describe how you spent that extra time.

Study Skills Builder 4

Are You Satisfied with Your Performance on a Particular Quiz or Exam?

If not, don't forget to analyze your quiz or exam and look for common errors. Were most of your errors a result of:

- *Carelessness?* Did you turn in your quiz or exam before the allotted time expired? If so, resolve to use any extra time to check your work.
- *Running out of time?* Try completing any questions that you are unsure of last and delay checking your work until all questions have been answered.
- *Not understanding a concept?* If so, review that concept and correct your work so that you make sure you understand the concept before the next quiz or the final exam.
- *Test conditions?* When studying for a quiz or exam, make sure you place yourself in conditions similar to test conditions. For example, before your next quiz or exam, take a sample test without the aid of your notes or text.

(For a sample test, see your instructor or use the Chapter Test at the end of each chapter.)

Exercises

1. Have you corrected all your previous quizzes and exams?
2. List any errors you have found common to two or more of your graded papers.
3. Is one of your common errors not understanding a concept? If so, are you making sure you understand all the concepts for the next quiz or exam?
4. Is one of your common errors making careless mistakes? If so, are you now taking all the time allotted to check over your work so that you can minimize the number of careless mistakes?
5. Are you satisfied with your grades thus far on quizzes and tests?
6. If your answer to Exercise 5 is no, are there any more suggestions you can make to your instructor or yourself to help? If so, list them here and share them with your instructor.

Study Skills Builder 5

How Are You Doing?

If you haven't done so yet, take a few moments and think about how you are doing in this course. Are you working toward your goal of successfully completing this course? Is your performance on homework, quizzes, and tests satisfactory? If not, you might want to see your instructor to see if he/she has any suggestions on how you can improve your performance. Reread Section 1.1 for ideas on places to get help with your mathematics course.

Exercises

Answer the following.

1. List any textbook supplements you are using to help you through this course.
2. List any campus resources you are using to help you through this course.
3. Write a short paragraph describing how you are doing in your mathematics course.
4. If improvement is needed, list ways that you can work toward improving your situation as described in Exercise 3.

Study Skills Builder 6

Are You Preparing for Your Final Exam?

To prepare for your final exam, try the following study techniques:

- Review the material that you will be responsible for on your exam. This includes material from your textbook, your notebook, and any handouts from your instructor.
- Review any formulas that you may need to memorize.
- Check to see if your instructor or mathematics department will be conducting a final exam review.
- Check with your instructor to see whether final exams from previous semesters/quarters are available to students for review.

- Use your previously taken exams as a practice final exam. To do so, rewrite the test questions in mixed order on blank sheets of paper. This will help you prepare for exam conditions.
- If you are unsure of a few concepts, see your instructor or visit a learning lab for assistance. Also, view the video segment of any troublesome sections.
- If you need further exercises to work, try the Cumulative Reviews at the end of the chapters.

Once again, good luck! I hope you are enjoying this textbook and your mathematics course.

Organizing Your Work

Study Skills Builder 7

Learning New Terms

Many of the terms used in this text may be new to you. It will be helpful to make a list of new mathematical terms and symbols as you encounter them and to review them frequently. Placing these new terms (including page references) on 3×5 index cards might help you later when you're preparing for a quiz.

Exercises

1. Name one way you might place a word and its definition on a 3×5 card.
2. How do new terms stand out in this text so that they can be found?

Study Skills Builder 8

Are You Organized?

Have you ever had trouble finding a completed assignment? When it's time to study for a test, are your notes neat and organized? Have you ever had trouble reading your own mathematics handwriting? (Be honest—I have.)

When any of these things happen, it's time to get organized. Here are a few suggestions:

- Write your notes and complete your homework assignments in a notebook with pockets (spiral or ring binder).
- Take class notes in this notebook, and then follow the notes with your completed homework assignment.
- When you receive graded papers or handouts, place them in the notebook pocket so that you will not lose them.
- Mark (possibly with an exclamation point) any note(s) that seem extra important to you.
- Mark (possibly with a question mark) any notes or homework that you are having trouble with.
- See your instructor or a math tutor for help with the concepts or exercises that you are having trouble understanding.

- If you are having trouble reading your own handwriting, *slow down* and write your mathematics work clearly!

Exercises

1. Have you been completing your assignments on time?
2. Have you been correcting any exercises you may be having difficulty with?
3. If you are having trouble understanding a mathematical concept or correcting any homework exercises, have you visited your instructor, a tutor, or your campus math lab?
4. Are you taking lecture notes in your mathematics course? (By the way, these notes should include worked-out examples solved by your instructor.)
5. Is your mathematics course material (handouts, graded papers, lecture notes) organized?
6. If your answer to Exercise 5 is no, take a moment and review your course material. List at least two ways that you might better organize it.

Study Skills Builder 9

Organizing a Notebook

It's never too late to get organized. If you need ideas about organizing a notebook for your mathematics course, try some of these:

- Use a spiral or ring binder notebook with pockets and use it for mathematics only.
- Start each page by writing the book's section number you are working on at the top.
- When your instructor is lecturing, take notes. *Always* include any examples your instructor works for you.
- Place your worked-out homework exercises in your notebook immediately after the lecture notes from that section. This way, a section's worth of material is together.
- Homework exercises: Attempt and check all assigned homework.
- Place graded quizzes in the pockets of your notebook or a special section of your binder.

Exercises

Check your notebook organization by answering the following questions.

1. Do you have a spiral or ring binder notebook for your mathematics course only?
2. Have you ever had to flip through several sheets of notes and work in your mathematics notebook to determine what section's work you are in?
3. Are you now writing the textbook's section number at the top of each notebook page?
4. Have you ever lost or had trouble finding a graded quiz or test?
5. Are you now placing all your graded work in a dedicated place in your notebook?
6. Are you attempting all of your homework and placing all of your work in your notebook?
7. Are you checking and correcting your homework in your notebook? If not, why not?
8. Are you writing in your notebook the examples your instructor works for you in class?

Study Skills Builder 10

How Are Your Homework Assignments Going?

It is very important in mathematics to keep up with homework. Why? Many concepts build on each other. Often your understanding of a day's concepts depends on an understanding of the previous day's material.

Remember that completing your homework assignment involves a lot more than attempting a few of the problems assigned.

To complete a homework assignment, remember these four things:

- Attempt all of it.
- Check it.
- Correct it.
- If needed, ask questions about it.

Exercises

Take a moment and review your completed homework assignments. Answer the questions below based on this review.

1. Approximate the fraction of your homework you have attempted.
2. Approximate the fraction of your homework you have checked (if possible).
3. If you are able to check your homework, have you corrected it when errors have been found?
4. When working homework, if you do not understand a concept, what do you do?

MyLab Math and MathXL

Study Skills Builder 11

Tips for Turning in Your Homework on Time

It is very important to keep up with your mathematics homework assignments. Why? Many concepts in mathematics build upon each other.

Remember these 4 tips to help ensure your work is completed on time:

- Know the assignments and due dates set by your instructor.
- Do not wait until the last minute to submit your homework.
- Set a goal to submit your homework 6–8 hours before the scheduled due date in case you have unexpected technology trouble.
- Schedule enough time to complete each assignment.

Following the tips above will also help you avoid potentially losing points for late or missed assignments.

Exercises

Take a moment to consider your work on your homework assignments to date and answer the following questions:

1. What percentage of your assignments have you turned in on time?
2. Why might it be a good idea to submit your homework 6–8 hours before the scheduled deadline?
3. If you have missed submitting any homework by the due date, list some of the reasons why this occurred.
4. What steps do you plan to take in the future to ensure your homework is submitted on time?

Study Skills Builder 12

Tips for Doing Your Homework Online

Practice is one of the main keys to success in any mathematics course. Did you know that MyLab Math/MathXL provides you with **immediate feedback** for each exercise? If you are incorrect, you are given hints to work the exercise correctly. You have **unlimited practice opportunities** and can rework any exercises you have trouble with until you master them, and submit homework assignments unlimited times before the deadline.

Remember these success tips when doing your homework online:

- Attempt all assigned exercises.
- Write down (neatly) your step-by-step work for each exercise before entering your answer.
- Use the immediate feedback provided by the program to help you check and correct your work for each exercise.
- Rework any exercises you have trouble with until you master them.
- Work through your homework assignment as many times as necessary until you are satisfied.

Exercises

Take a moment to think about your homework assignments to date and answer the following:

1. Have you attempted all assigned exercises?
2. Of the exercises attempted, have you also written out your work before entering your answer—so that you can check it?
3. Are you familiar with how to enter answers using the MathXL player so that you avoid answer-entry type errors?
4. List some ways the immediate feedback and practice supports have helped you with your homework. If you have not used these supports, how do you plan to use them with the success tips above on your next assignment?

Study Skills Builder 13

Organizing Your Work

Have you ever used any readily available paper (such as the back of a flyer, another course assignment, Post-its, etc.) to work out your homework exercises before entering the answer in MathXL? To save time, have you ever entered answers directly into MathXL without working the exercises on paper? When it's time to study, have you ever been unable to find your completed work or read and follow your own mathematics handwriting?

When any of these things happen, it's time to get organized. Here are some suggestions:

- Write your step-by-step work for each homework exercise (neatly) on lined, loose-leaf paper and keep this in a 3-ring binder.
- Refer to your step-by-step work when you receive feedback that your answer is incorrect in MathXL. Double-check using the steps and hints provided by the program and correct your work accordingly.
- Keep your written homework with your class notes for that section.

- Identify any exercises you are having trouble with and ask questions about them.
- Keep all graded quizzes and tests in this binder as well to study later.

If you follow the suggestions above, you and your instructor or tutor will be able to follow your steps and correct any mistakes. You will also have a written copy of your work to refer to later to ask questions and study for tests.

Exercises

1. Why is it important that you write out your step-by-step work for homework exercises and keep a hard copy of all work submitted online?
2. If you have gotten an incorrect answer, are you able to follow your steps and find your error?
3. If you were asked today to review your previous homework assignments and 1st test, could you find them? If not, list some ways you might better organize your work.

Study Skills Builder 14

Getting Help with Your Homework Assignments

There are many resources available to you through MathXL to help you work through any homework exercises you may have trouble with. It is important that you know what these resources are and know when and how to use them.

Let's review the features found on the right side of the screen in the homework exercises:

- **Help Me Solve This**—provides step-by-step help for the exercise you are working. You must work an additional exercise of the same type (without this help) before you can get credit for having worked it correctly.
- **View an Example**—allows you to view a correctly worked exercise similar to the one you are having trouble with. You can then go back to your original exercise and work it on your own.
- **E-Book**—allows you to read examples from your text and find similar exercises.

- **Video**—your text author, Elayn Martin-Gay, works an exercise similar to the one you need help with. **Not all exercises have an accompanying video clip.
- **Ask My Instructor**—allows you to e-mail your instructor for help with an exercise.

Exercises

1. How does the "Help Me Solve This" feature work?
2. If the "View an Example" feature is used, is it necessary to work an additional problem before continuing the assignment?
3. When might be a good time to use the "Video" feature? Do all exercises have an accompanying video clip?
4. Which of the features above have you used? List those you found the most helpful to you.
5. If you haven't used the features discussed, list those you plan to try on your next homework assignment.

Study Skills Builder 15

Tips for Preparing for an Exam

Did you know that you can rework your previous homework assignments in MyLab Math and MathXL? This is a great way to prepare for tests. To do this, open a previous homework assignment and click “similar exercise.” This will generate new exercises similar to the homework you have submitted. You can then rework the exercises and assignments until you feel confident that you understand them.

To prepare for an exam, follow these tips:

- Review your written work for your previous homework assignments along with your class notes.
- Identify any exercises or topics that you have questions on or have difficulty understanding.
- Rework your previous assignments in MyLab Math and MathXL until you fully understand them and can do them without help.
- Get help for any topics you feel unsure of or for which you have questions.

Exercises

1. Are your current homework assignments up to date and is your written work for them organized in a binder or notebook? If the answer is no, it's time to get organized. For tips on this, see Study Skills Builder 13—Organizing Your Work.
2. How many days in advance of an exam do you usually start studying?
3. List some ways you think that working previous homework assignments can help you prepare for your test.
4. List 2–3 resources you can use to get help for any topics you are unsure of or have questions on.

Good luck!

Study Skills Builder 16

How Well Do You Know the Resources Available to You in MyLab Math?

There are many helpful resources available to you in MyLab Math. Let's take a moment to locate and explore a few of them now. Go into your MyLab Math course, and visit the Multimedia Library, Tools for Success, and E-Book.

Let's see what you found.

Exercises

1. List the resources available to you in the Multimedia Library.
2. List the resources available to you in the Tools for Success folder.
3. Where did you find the English/Spanish Audio Glossary?
4. Can you view videos from the E-Book?
5. Did you find any resources you did not know about? If so, which ones?
6. Which resources have you used most often or find most helpful?

Additional Help Inside and Outside Your Textbook

Study Skills Builder 17

How Well Do You Know Your Textbook?

The questions below will help determine whether you are familiar with your textbook. For additional information, see Section 1.1 in this text.

1. What does the  icon mean?
2. What does the  icon mean?
3. What does the  icon mean?
4. Where can you find a review for each chapter? What answers to this review can be found in the back of your text?
5. The end of each chapter contains an overview of the chapter along with examples. What is this feature called?
6. The end of each chapter contains a review of vocabulary. What is this feature called?
7. There are practice exercises that are contained in this text. What are they and how can they be used?
8. This text contains a student section in the back entitled Student Resources. List the contents of this section and how they might be helpful.
9. What exercise answers are available in this text? Where are they located?

Study Skills Builder 18

Are You Familiar with Your Textbook Supplements?

Below is a review of some of the student supplements available for additional study. Check to see if you are using the ones most helpful to you.

- Getting Ready for the Test Videos and Chapter Test Prep Videos. These videos provide video clip solutions to the Getting Ready for the Test and Chapter Test exercises in this text. You will find them extremely useful when studying for tests or exams.
- Interactive DVD Lecture Series. These are keyed to each section of the text. The material is presented by me, Elayn Martin-Gay, and I have placed a  by the exercises in the text that I have worked on the video.
- The *Student Solutions Manual*. This contains worked-out solutions to odd-numbered exercises as well as every exercise in the Integrated Reviews, Chapter Reviews, Getting Ready for the Tests, Chapter Tests, and Cumulative Reviews.
- Pearson Tutor Center. Mathematics questions may be phoned, faxed, or e-mailed to this center.
- MyLab Math is a text-specific online course. MathXL is an online homework, tutorial, and assessment system. Take a moment and determine whether these are available to you.

As usual, your instructor is your best source of information.

Exercises

Let's see how you are doing with textbook supplements.

1. Name one way the Lecture Videos can be helpful to you.
2. Name one way the Getting Ready for the Test Videos and the Chapter Test Prep Videos can help you prepare for a chapter test.
3. List any textbook supplements that you have found useful.
4. Have you located and visited a learning resource lab located on your campus?
5. List the textbook supplements that are currently housed in your campus' learning resource lab.

Study Skills Builder 19

Are You Getting All the Mathematics Help That You Need?

Remember that, in addition to your instructor, there are many places to get help with your mathematics course. For example:

- This text has an accompanying video lesson for every section.
- The back of the book contains answers to odd-numbered exercises and selected solutions.
- A *Student Solutions Manual* is available that contains worked-out solutions to odd-numbered exercises as well as solutions to every exercise in the Integrated Reviews, Chapter Reviews, Getting Ready for the Tests, Chapter Tests, and Cumulative Reviews.
- Don't forget to check with your instructor for other local resources available to you, such as a tutor center.

Exercises

1. List items you find helpful in the text and all student supplements to this text.
2. List all the campus help that is available to you for this course.
3. List any help (besides the textbook) from Exercises **1** and **2** above that you are using.
4. List any help (besides the textbook) that you feel you should try.
5. Write a goal for yourself that includes trying anything you listed in Exercise **4** during the next week.

Bigger Picture— Study Guide Outline

I. Operations on Sets of Numbers

A. Whole Numbers

1. **Add or Subtract:** (Sec. 1.3)

$$\begin{array}{r} 14 \\ + 39 \\ \hline 53 \end{array} \qquad \begin{array}{r} 300 \\ - 27 \\ \hline 273 \end{array}$$

2. **Multiply or Divide:** (Sec. 1.5 and Sec. 1.6)

$$\begin{array}{r} 238 \\ \times 47 \\ \hline 1666 \\ 9520 \\ \hline 11,186 \end{array} \qquad \begin{array}{r} 127 \text{ R}2 \\ 7 \overline{)891} \\ \underline{-7} \\ 19 \\ \underline{-14} \\ 51 \\ \underline{-49} \\ 2 \end{array}$$

3. **Exponent:** (Sec. 1.7) 4 factors of 3

$$3^4 = \underbrace{3 \cdot 3 \cdot 3 \cdot 3}_{4 \text{ factors of } 3} = 81$$

4. **Order of Operations:** (Sec. 1.7)

$$\begin{aligned} 24 \div 3 \cdot 2 - (2 + 8) &= 24 \div 3 \cdot 2 - (10) && \text{Simplify within parentheses.} \\ &= 8 \cdot 2 - 10 && \text{Multiply or divide from left to right.} \\ &= 16 - 10 && \text{Multiply or divide from left to right.} \\ &= 6 && \text{Add or subtract from left to right.} \end{aligned}$$

5. **Square Root:** (Sec. 6.4)

$$\sqrt{25} = 5 \text{ because } 5 \cdot 5 = 25 \text{ and } 5 \text{ is a positive number.}$$

B. Integers

1. **Add:** (Sec. 2.2) $-5 + (-2) = -7$ Adding like signs
Add absolute values. Attach the common sign.

$$-5 + 2 = -3$$
Adding unlike signs
Subtract absolute values. Attach the sign of the number with the larger absolute value.

2. **Subtract:** Add the first number to the opposite of the second number. (Sec. 2.3)

$$7 - 10 = 7 + (-10) = -3$$

3. **Multiply or Divide:** Multiply or divide as usual. If the signs of the two numbers are the same, the answer is positive. If the signs of the two numbers are different, the answer is negative. (Sec. 2.4)

$$-5 \cdot 5 = -25, \quad \frac{-32}{-8} = 4$$

C. Fractions

1. **Simplify:** Factor the numerator and denominator. Then divide out factors of 1 by dividing out common factors in the numerator and denominator. (Sec. 4.2)

$$\text{Simplify: } \frac{20}{28} = \frac{4 \cdot 5}{4 \cdot 7} = \frac{5}{7}$$

- 2. Multiply:** Numerator times numerator over denominator times denominator. (Sec. 4.3)

$$\frac{5}{9} \cdot \frac{2}{7} = \frac{10}{63}$$

- 3. Divide:** First fraction times the reciprocal of the second fraction. (Sec. 4.3)

$$\frac{2}{11} \div \frac{3}{4} = \frac{2}{11} \cdot \frac{4}{3} = \frac{8}{33}$$

- 4. Add or Subtract:** Must have same denominators. If not, find the LCD, and write each fraction as an equivalent fraction with the LCD as denominator. (Sec. 4.4 and Sec. 4.5)

$$\frac{2}{5} + \frac{1}{15} = \frac{2}{5} \cdot \frac{3}{3} + \frac{1}{15} = \frac{6}{15} + \frac{1}{15} = \frac{7}{15}$$

D. Decimals

- 1. Add or Subtract:** Line up decimal points. (Sec. 5.2)

$$\begin{array}{r} 1.27 \\ + 0.6 \\ \hline 1.87 \end{array}$$

- 2. Multiply:** (Sec. 5.3)

$$\begin{array}{r} 2.56 \\ \times 3.2 \\ \hline 512 \\ 7680 \\ \hline 8.192 \end{array}$$

2 decimal places
↓ 1 decimal place
2 + 1 = 3
↓ 3 decimal places

- 3. Divide:** (Sec. 5.4)

$$\begin{array}{r} 0.7 \\ 8 \overline{)5.6} \end{array} \qquad \begin{array}{r} 1.31 \\ 0.6 \overline{)0.786} \end{array}$$

II. Solving Equations

- A. Equations in General:** Simplify both sides of the equation by removing parentheses and combining any like terms. Then use the addition property to write variable terms on one side and constants (numbers) on the other side. Then use the multiplication property to solve for the variable by dividing both sides of the equation by the coefficient of the variable. (Sec. 3.3)

$$\text{Solve: } 2(x - 5) = 80$$

$$2x - 10 = 80 \quad \text{Use the distributive property.}$$

$$2x - 10 + 10 = 80 + 10 \quad \text{Add 10 to both sides.}$$

$$2x = 90 \quad \text{Simplify.}$$

$$\frac{2x}{2} = \frac{90}{2} \quad \text{Divide both sides by 2.}$$

$$x = 45 \quad \text{Simplify.}$$

- B. Proportions:** Set cross products equal to each other. Then solve. (Sec. 6.2)

$$\frac{14}{3} = \frac{2}{n} \text{ or } 14 \cdot n = 3 \cdot 2 \text{ or } 14 \cdot n = 6 \text{ or } n = \frac{6}{14} = \frac{3}{7}$$

C. Percent Problems

- 1. Solved by Equations:** Remember that “of” means multiplication and “is” means equals. (Sec. 7.2)

“12% of some number is 6” translates to

$$12\% \cdot n = 6 \text{ or } 0.12 \cdot n = 6 \text{ or } n = \frac{6}{0.12} \text{ or } n = 50$$

- 2. Solved by Proportions:** Remember that percent, p , is identified by % or “percent”; base, b , usually appears after “of”; and amount, a , is the part compared to the whole. (Sec. 7.3)

“12% of some number is 6” translates to

$$\frac{6}{b} = \frac{12}{100} \text{ or } 6 \cdot 100 = b \cdot 12 \text{ or } \frac{600}{12} = b \text{ or } 50 = b$$

Practice Final Exam

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. _____
24. _____
25. _____
26. _____

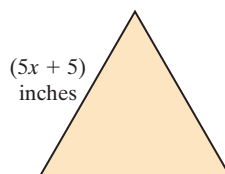
Simplify by performing the indicated operations.

1. $2^3 \cdot 5^2$
2. $16 + 9 \div 3 \cdot 4 - 7$
3. $18 - 24$
4. $5 \cdot (-20)$
5. $\sqrt{49}$
6. $(-5)^3 - 24 \div (-3)$
7. $0 \div 49$
8. $62 \div 0$
9. $-\frac{8}{15y} - \frac{2}{15y}$
10. $\frac{11}{12} - \frac{3}{8} + \frac{5}{24}$
11. $\frac{3a}{8} \cdot \frac{16}{6a^3}$
12. $-\frac{16}{3} \div -\frac{3}{12}$
13. $\frac{19}{-2\frac{3}{11}}$
14. $\frac{0.23 + 1.63}{-0.3}$
15. 10.2×4.01
16. Write 0.6% as a decimal.
17. Write 6.1 as a percent.
18. Write $\frac{3}{8}$ as a percent.
19. Write 0.345 as a fraction.
20. Write $-\frac{13}{26}$ as a decimal.
21. Round 34.8923 to the nearest tenth.

Evaluate each expression for the given replacement values.

22. $5(x^3 - 2)$ for $x = 2$
23. $10 - y^2$ for $y = -3$
24. $x \div y$ for $x = \frac{1}{2}$ and $y = 3\frac{7}{8}$
25. Simplify: $-(3z + 2) - 5z - 18$

- △ 26. Write an expression that represents the perimeter of the equilateral triangle. Then simplify the expression.



Solve each equation.

27. $\frac{n}{-7} = 4$

28. $-4x + 7 = 15$

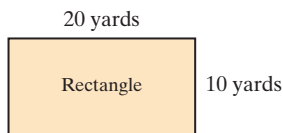
29. $-4(x - 11) - 34 = 10 - 12$

30. $\frac{x}{5} + x = -\frac{24}{5}$

31. $2(x + 5.7) = 6x - 3.4$

32. $\frac{8}{x} = \frac{11}{6}$

- △ 33. Find the perimeter and area.



Solve.

35. The difference of three times a number and five times the same number is 4. Find the number.

36. During a 258-mile trip, a car used
- $10\frac{3}{4}$
- gallons of gas. How many miles would we expect the car to travel on 1 gallon of gas?

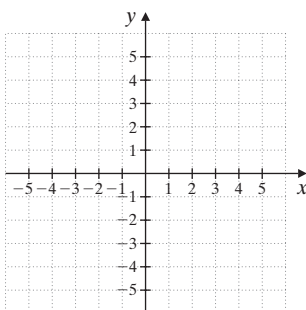
37. In a 10-kilometer race, there are 112 more men entered than women. Find the number of female runners if the total number of runners in the race is 600.

38. The standard dose of medicine for a dog is 10 grams for every 15 pounds of body weight. What is the standard dose for a dog that weighs 80 pounds?

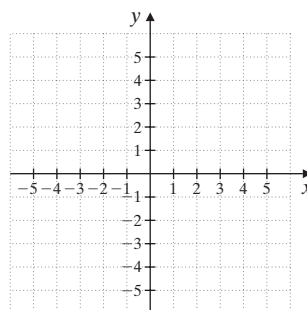
39. A \$120 framed picture is on sale for 15% off. Find the discount and the sale price.

Graph each linear equation.

40. $y + x = -4$



41. $y = 3x - 5$



27. _____

28. _____

29. _____

30. _____

31. _____

32. _____

33. _____

34. _____

35. _____

36. _____

37. _____

38. _____

39. _____

40. _____

41. _____

42. _____

43. _____

44. _____

45. _____

46. _____

47. _____

48. _____

49. _____

50. _____

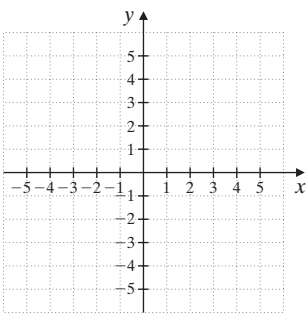
51. _____

52. _____

53. _____

54. _____

42. $y = -4$



44. Subtract $(8a^2 + a)$ from $(6a^2 + 2a + 1)$.

Multiply and simplify.

45. $(6a^3)(-2a^7)$

47. $(x - 3)(x + 2)$

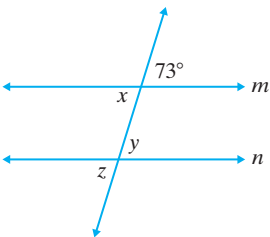
49. Find the complement of a 78° angle.

43. Add: $(11x - 3) + (4x - 1)$

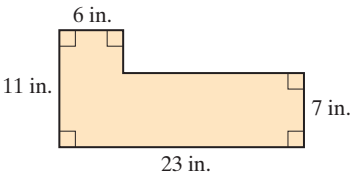
46. $(3a^4b)^2(2ba^4)^3$

48. Factor out the GCF: $3y^2 - 15y$

50. Given that $m \parallel n$, find the measures of x , y , and z .



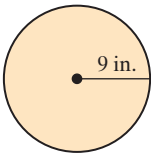
51. Find the perimeter and area.



Convert.

53. $2\frac{1}{2}$ gallons to quarts

52. Find the circumference and area. Give the exact values and then approximations using $\pi \approx 3.14$.



54. 2.4 kilograms to grams

Answers to Selected Exercises

Chapter 1 The Whole Numbers

Section 1.2

Vocabulary, Readiness & Video Check 1. whole 3. words 5. period 7. hundreds 9. 80,000

Exercise Set 1.2 1. tens 3. thousands 5. hundred-thousands 7. millions 9. three hundred fifty-four 11. eight thousand, two hundred seventy-nine 13. twenty-six thousand, nine hundred ninety 15. two million, three hundred eighty-eight thousand 17. twenty-four million, three hundred fifty thousand, one hundred eighty-five 19. thirty-seven thousand, nine hundred seventeen 21. two thousand, seven hundred twenty 23. eighteen million, three hundred thousand 25. fourteen thousand, four hundred thirty-three 27. seventeen thousand, four hundred ten 29. 6587 31. 59,800 33. 13,601,011 35. 7,000,017 37. 260,997 39. 250 41. 1024 43. \$247,966,675 45. 565 47. $200 + 9$ 49. $3000 + 400 + 70$ 51. $80,000 + 700 + 70 + 4$ 53. $60,000 + 6000 + 40 + 9$ 55. $30,000,000 + 9,000,000 + 600,000 + 80,000$ 57. 1786 59. Mount Baker 61. Glacier Peak 63. German shepherd 65. Labrador retriever; seventy-five 67. 70 pounds 69. 9861 71. no; one hundred five 73. answers may vary 75. 1,000,000,000,000

Section 1.3

Calculator Explorations 1. 134 3. 340 5. 2834 7. 770 9. 109 11. 8978

Vocabulary, Readiness & Video Check 1. number 3. 0 5. minuend; subtrahend; difference 7. order; commutative 9. place; right; left 11. triangle; 3

Exercise Set 1.3 1. 36 3. 292 5. 49 7. 5399 9. 209,078 11. 25 13. 212 15. 11,926 17. 16,717 19. 35,901 21. 632,389 23. 600 25. 25 27. 288 29. 168 31. 5723 33. 504 35. 79 37. 32,711 39. 5041 41. 31,213 43. 1034 45. 9 47. 8518 49. 22,876 51. 25 ft 53. 24 in. 55. 29 in. 57. 44 m 59. 2093 61. 266 63. 20 65. 544 67. 72 69. 88 71. 3170 thousand 73. \$619 75. 264,000 sq mi 77. 283,000 sq mi 79. 340 ft 81. 264 pages 83. 27,441,402 85. 100 dB 87. 58 dB 89. 3444 91. 124 ft 93. California 95. 2282 97. Florida and Georgia 99. 5894 mi 101. minuend: 48; subtrahend: 1 103. minuend: 70; subtrahend: 7 105. answers may vary 107. correct 109. incorrect; 530 111. incorrect: 685 113. correct 115. $\begin{array}{r} 5269 \\ -2385 \\ \hline 2884 \end{array}$ 117. answers may vary 119. no; 1089 more pages

Section 1.4

Vocabulary, Readiness & Video Check 1. graph 3. 70; 60 5. 3 is the place we're rounding to (tens), and the digit to the right of this place is 5 or greater, so we need to add 1 to the 3. 7. Each circled digit is to the right of the place value being rounded to and is used to determine whether or not we add 1 to the digit in the place value being rounded to.

Exercise Set 1.4 1. 420 3. 640 5. 2800 7. 500 9. 21,000 11. 34,000 13. 328,500 15. 36,000 17. 39,990 19. 30,000,000 21. 5280; 5300; 5000 23. 9440; 9400; 9000 25. 14,880; 14,900; 15,000 27. 311,000 miles 29. 60,100 days 31. \$150,000,000,000 33. \$4,200,000 35. USA: 220,000,000; India: 292,000,000 37. 130 39. 80 41. 5700 43. 300 45. 11,400 47. incorrect 49. correct 51. correct 53. \$3400 55. 900 mi 57. 6000 ft 59. Springfield was larger by approximately 40,000. 61. The decrease was 182,000 63. 74,900,000; 75,000,000; 70,000,000 65. 64,100,000; 64,000,000; 60,000,000 67. 5723, for example 69. a. 8550 b. 8649 71. answers may vary 73. 140 m

Section 1.5

Calculator Explorations 1. 3456 3. 15,322 5. 272,291

Vocabulary, Readiness & Video Check 1. 0 3. product; factor 5. grouping; associative 7. length 9. distributive 11. Area is measured in square units, and here we have meters times meters, or square meters; the correct answer is 63 *square* meters, or the correct units are square meters.

Exercise Set 1.5 1. 24 3. 0 5. 0 7. 87 9. $6 \cdot 3 + 6 \cdot 8$ 11. $4 \cdot 3 + 4 \cdot 9$ 13. $20 \cdot 14 + 20 \cdot 6$ 15. 512 17. 3678 19. 1662 21. 6444 23. 1157 25. 24,418 27. 24,786 29. 15,600 31. 0 33. 6400 35. 48,126 37. 142,506 39. 2,369,826 41. 64,790 43. 3,949,935 45. area: 63 sq m; perimeter: 32 m 47. area: 680 sq ft; perimeter: 114 ft 49. 240,000 51. 300,000 53. c 55. c 57. 880 59. 4200 61. 4480 63. 375 cal 65. \$3290 67. a. 20 b. 100 c. 2000 lb 69. 8800 sq ft 71. 56,000 sq ft 73. 5828 pixels 75. 2100 characters 77. 1280 cal 79. \$10, \$60; \$10, \$200; \$12, \$36; \$12, \$36: total cost \$372 81. 1,440,000 tea bags 83. 135 85. 2144 87. 23 89. 15 91. $5 \cdot 6$ or $6 \cdot 5$ 93. a. $5 + 5 + 5$ or $3 + 3 + 3 + 3 + 3$ b. answers may vary 95. $\begin{array}{r} 203 \\ \times 14 \\ \hline 812 \\ 2030 \\ \hline 2842 \end{array}$ 97. $\begin{array}{r} 42 \\ \times 93 \\ \hline 126 \\ 378 \\ \hline 3906 \end{array}$ 99. answers may vary 101. 506 windows

A2

Answers to Selected Exercises

Section 1.6

Calculator Explorations 1. 53 3. 62 5. 261 7. 0

Vocabulary, Readiness & Video Check 1. quotient; dividend; divisor 3. 1 5. undefined 7. 0 9. $202 \cdot 102 + 15 = 20,619$
11. addition and division

Exercise Set 1.6 1. 6 3. 12 5. 0 7. 31 9. 1 11. 8 13. undefined 15. 1 17. 0 19. 9 21. 29 23. 74 25. 338
27. undefined 29. 9 31. 25 33. 68 R 3 35. 236 R 5 37. 38 R 1 39. 326 R 4 41. 13 43. 49 45. 97 R 8 47. 209 R 11
49. 506 51. 202 R 7 53. 54 55. 99 R 100 57. 202 R 15 59. 579 R 72 61. 17 63. 511 R 3 65. 2132 R 32 67. 6080
69. 23 R 2 71. 5 R 25 73. 20 R 2 75. 33 students 77. 165 lb 79. 310 yd 81. 89 bridges 83. 11 light poles 85. 5 mi
87. 1760 yd 89. 20 91. 387 93. 79 95. 74° 97. 9278 99. 15,288 101. 679 103. undefined 105. 9 R 12 107. c
109. b 111. \$3,090,000,000 113. increase; answers may vary 115. no; answers may vary 117. 12 ft 119. answers may vary
121. 5 R 1

Integrated Review 1. 194 2. 6555 3. 4524 4. 562 5. 67 6. undefined 7. 1 8. 5 9. 0 10. 0 11. 0 12. 3 13. 63
14. 9 15. 138 16. 276 17. 1169 18. 9826 19. 182 R 4 20. 79,317 21. 1099 R 2 22. 111 R 1 23. 663 R 24 24. 1076 R 60
25. 1037 26. 9899 27. 30,603 28. 47,500 29. 71 30. 558 31. 6 R 8 32. 53 33. 183 34. 231 35. 9740; 9700; 10,000
36. 1430; 1400; 1000 37. 20,800; 20,800; 21,000 38. 432,200; 432,200; 432,000 39. perimeter: 24 ft; area: 36 sq ft 40. perimeter:
42 in.; area: 98 sq in. 41. 28 mi 42. 26 m 43. 24 44. 124 45. Lake Pontchartrain Bridge; 2175 ft 46. \$5562

Section 1.7

Calculator Explorations 1. 4096 3. 3125 5. 2048 7. 2526 9. 4295 11. 8

Vocabulary, Readiness & Video Check 1. base; exponent 3. addition 5. division 7. 1 9. The area of a rectangle is
length $\cdot$ width. A square is a special rectangle where length = width. Thus, the area of a square is side $\cdot$ side or $(\text{side})^2$.

Exercise Set 1.7 1. 4^3 3. 7^6 5. 12^3 7. $6^2 \cdot 5^3$ 9. $9 \cdot 8^2$ 11. $3 \cdot 2^4$ 13. $3 \cdot 2^4 \cdot 5^5$ 15. 64 17. 125 19. 32 21. 1 23. 7
25. 128 27. 256 29. 256 31. 729 33. 144 35. 100 37. 20 39. 729 41. 192 43. 162 45. 21 47. 7 49. 5 51. 16
53. 46 55. 8 57. 64 59. 83 61. 2 63. 48 65. 4 67. undefined 69. 59 71. 52 73. 44 75. 12 77. 21 79. 3 81. 43
83. 8 85. 16 87. area: 49 sq m; perimeter: 28 m 89. area: 529 sq mi; perimeter: 92 mi 91. true 93. false 95. $(2 + 3) \cdot 6 - 2$
97. $24 \div (3 \cdot 2) + 2 \cdot 5$ 99. 1260 ft 101. 6,384,814 103. answers may vary

Section 1.8

Vocabulary, Readiness & Video Check 1. expression 3. expression; variables 5. equation 7. multiplication 9. decreased by

Exercise Set 1.8 1. 28; 14; 147; 3 3. 152; 152; 0; undefined 5. 57; 55; 56; 56 7. 9 9. 8 11. 6 13. 5 15. 117 17. 94 19. 5
21. 34 23. 20 25. 4 27. 4 29. 0 31. 33 33. 125 35. 121 37. 100 39. 60 41. 4 43. 16; 64; 144; 256 45. yes 47. no
49. no 51. yes 53. no 55. yes 57. 12 59. 6 61. 4 63. none 65. 11 67. $x + 8$ 69. $x + 8$ 71. $20 - x$ 73. $512x$
75. $8 \div x$ or $\frac{8}{x}$ 77. $5x + (17 + x)$ 79. $5x$ 81. $11 - x$ 83. $x - 5$ 85. $6 \div x$ or $\frac{6}{x}$ 87. $50 - 8x$ 89. correct answer: 21
91. correct answer: 46 93. 274,657 95. 777 97. $5x$; answers may vary 99. As t gets larger $16t^2$ gets larger.

Chapter 1 Vocabulary Check 1. whole numbers 2. perimeter 3. place value 4. exponent 5. area 6. digits 7. variable
8. equation 9. expression 10. solution 11. set 12. sum 13. divisor 14. dividend 15. quotient 16. factor 17. product
18. minuend 19. subtrahend 20. difference 21. addend

Chapter 1 Review 1. tens 2. ten-millions 3. seven thousand, six hundred forty 4. forty-six million, two hundred thousand,
one hundred twenty 5. $3000 + 100 + 50 + 8$ 6. $400,000,000 + 3,000,000 + 200,000 + 20,000 + 5000$ 7. 81,900 8. 6,304,000,000
9. 615,000,000 10. 28,000,000 11. Oceania/Australia 12. Asia 13. 67 14. 67 15. 65 16. 304 17. 449 18. 840 19. 3914
20. 7908 21. 4211 22. 1967 23. 1334 24. 886 25. 17,897 26. 34,658 27. 7523 mi 28. \$197,699 29. 216 ft 30. 66 km
31. 82 million or 82,000,000 32. 113 million or 113,000,000 33. May 34. August 35. \$110 36. \$240 37. 40 38. 50 39. 880
40. 500 41. 3800 42. 58,000 43. 40,000,000 44. 800,000 45. 7300 46. 4100 47. 2700 mi 48. Europe: 832,000,000; Latin
America/Caribbean: 626,000,000; difference: 206,000,000 49. 2208 50. 1396 51. 2280 52. 2898 53. 560 54. 900 55. 0
56. 0 57. 16,994 58. 8954 59. 113,634 60. 44,763 61. 411,426 62. 636,314 63. 1500 64. 4920 65. \$898 66. \$403,980
67. 91 sq mi 68. 500 sq cm 69. 7 70. 4 71. 5 R 2 72. 4 R 2 73. undefined 74. 0 75. 33 R 2 76. 19 R 7 77. 24 R 2
78. 35 R 15 79. 506 R 10 80. 907 R 40 81. 2793 R 140 82. 2012 R 60 83. 18 R 2 84. 21 R 2 85. 27 boxes 86. 13 miles
87. 51 88. 59 89. 64 90. 125 91. 405 92. 400 93. 16 94. 10 95. 15 96. 7 97. 12 98. 9 99. 42 100. 33 101. 9
102. 2 103. 6 104. 29 105. 40 106. 72 107. 5 108. 7 109. 49 sq m 110. 9 sq in. 111. 5 112. 17 113. undefined
114. 0 115. 121 116. 2 117. 4 118. 20 119. $x - 5$ 120. $x + 7$ 121. $10 \div x$ or $\frac{10}{x}$ 122. $5x$ 123. yes 124. no 125. no
126. yes 127. 11 128. 175 129. 14 130. none 131. 417 132. 682 133. 2196 134. 2516 135. 1101 136. 1411
137. 458 R 8 138. 237 R 1 139. 70,848 140. 95,832 141. 1644 142. 8481 143. 840 144. 300,000 145. 12 146. 6
147. no 148. yes 149. 53 full boxes with 18 left over 150. \$86

Chapter 1 Getting Ready for the Test 1. D 2. E 3. F 4. B 5. C 6. B 7. D 8. A 9. C 10. B 11. D 12. A 13. C

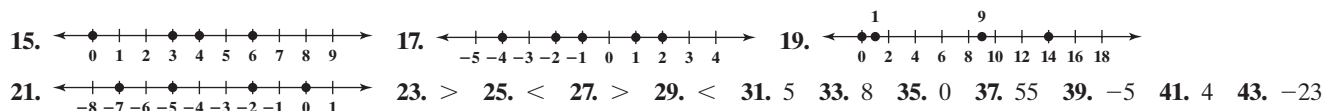
Chapter 1 Test 1. eighty-two thousand, four hundred twenty-six 2. 402,550 3. 141 4. 113 5. 14,880 6. 766 R 42 7. 200 8. 98 9. 0 10. undefined 11. 33 12. 21 13. 48 14. 36 15. 5,698,000 16. 82 17. 52,000 18. 13,700 19. 1600 20. 92 21. 122 22. 1605 23. 7 R 2 24. \$17 25. \$126 26. 360 cal 27. \$7905 28. 20 cm; 25 sq cm 29. 60 yd; 200 sq yd 30. 30 31. 1 32. a. $x \div 17$ or $\frac{x}{17}$ b. $2x - 20$ 33. yes 34. 10

Chapter 2 Integers and Introduction to Solving Equations

Section 2.1

Vocabulary, Readiness & Video Check 1. integers 3. inequality symbols 5. is less than; is greater than 7. absolute value 9. number of feet a miner works underground 11. negative 13. opposite of

Exercise Set 2.1 1. -1235 3. -35,814 5. +120 7. -11,810 9. -1270 million 11. -160, -147; Guillermo 13. -60° or -60°F



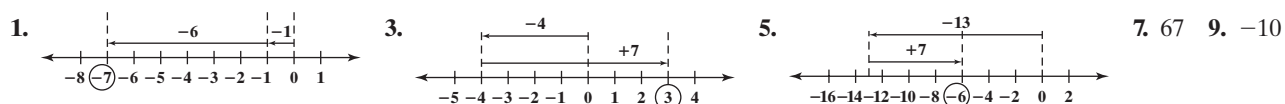
45. 85 47. 7 49. -20 51. -3 53. 43 55. 15 57. 33 59. 6 61. -2 63. 32 65. -7 67. < 69. < 71. = 73. < 75. > 77. < 79. > 81. < 83. 31; -31 85. 28; 28 87. Caspian Sea 89. Lake Superior 91. iodine 93. oxygen 95. 13 97. 35 99. 360 101. $-|-8|$, $-|3|$, 2^2 , $-(-5)$ 103. $-|-6|$, $-|1|$, $|1|$, $-(-6)$ 105. -10, $-|-9|$, $-(-2)$, $|12|$, 5^2 107. a, d 109. 8 111. false 113. true 115. false 117. answers may vary 119. no; answers may vary

Section 2.2

Calculator Explorations 1. -159 3. 44 5. -894,855

Vocabulary, Readiness & Video Check 1. 0 3. a 5. Negative; the numbers have different signs and the sign of the sum is the same as the sign of the number with the larger absolute value, -6. 7. The diver's current depth is 231 feet below the surface.

Exercise Set 2.2



11. 0 13. 4 15. -6 17. -2 19. -9 21. -24 23. -840 25. 7 27. -3 29. -30 31. 40 33. -20 35. -125 37. -7 39. -246 41. 16 43. 13 45. -28 47. -11 49. 20 51. -34 53. -1 55. 0 57. -42 59. -70 61. 3 63. -21 65. $-6 + 25$; 19 67. $-31 + (-9) + 30$; -10 69. $0 + (-215) + (-16) = -231$; 231 ft below the surface 71. Walker: -3; Matsuyama: -2 73. \$45,687,000,000 75. $-\$7,707,000,000$ 77. 2°C 79. \$13,609 81. -31°F 83. -7535 m 85. 44 87. 141 89. answers may vary 91. -3 93. -22 95. true 97. false 99. answers may vary

Section 2.3

Vocabulary, Readiness & Video Check 1. b 3. d 5. additive inverse 7. to follow the order of operations

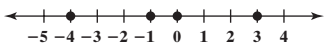
Exercise Set 2.3 1. 0 3. 3 5. -5 7. 22 9. 3 11. -20 13. -12 15. -13 17. 508 19. -14 21. -4 23. -12 25. $-25 - 17$; -42 27. $-22 - (-3)$; -19 29. $2 - (-12)$; 14 31. -56 33. -5 35. -145 37. -37 39. 3 41. 1 43. -1 45. -31 47. 44 49. -32 51. -9 53. 14 55. -11 57. 31 59. 12 61. 20 63. 111°F 65. 180°F 67. 263°F 69. 26 strokes 71. -10°C 73. 154 ft 75. 69 ft 77. 652 ft 79. 144 ft 81. 1197°F 83. $-\$45$ billion 85. $-5 + x$ 87. $-20 - x$ 89. 5 91. 1058 93. answers may vary 95. 16 97. -20 99. -4 101. 0 103. -14 105. false 107. answers may vary

Section 2.4

Vocabulary, Readiness & Video Check 1. negative 3. positive 5. 0 7. undefined 9. multiplication 11. The phrase "lost four yards" in the example translates to the negative number -4.

Exercise Set 2.4 1. 12 3. -36 5. -81 7. 0 9. 48 11. -12 13. 80 15. 0 17. -15 19. -9 21. -27 23. -36 25. -64 27. -8 29. -5 31. 7 33. 0 35. undefined 37. -14 39. 0 41. -15 43. -63 45. 42 47. -24 49. 49 51. -5 53. -9 55. -6 57. 120 59. -1080 61. undefined 63. -6 65. -7 67. 3 69. -1 71. -32 73. 180 75. 1 77. -30 79. -1104 81. -2870 83. -56 85. -18 87. 35 89. -1 91. undefined 93. 6 95. 16; 4 97. 0; 0 99. $-54 \div 9$; -6 101. $-42(-6)$; 252 103. $-71 \cdot x$ or $-71x$ 105. $-16 - x$ 107. $-29 + x$ 109. $\frac{x}{-33}$ or $x \div (-33)$ 111. $3 \cdot (-4) = -12$; a loss of 12 yd

113. $5 \cdot (-20) = -100$; a depth of 100 feet **115.** -210°C **117.** -189°C **119.** $-\$132$ million per month **121. a.** $-23,705$ screens **b.** -4741 screens per year **123.** 109 **125.** 8 **127.** -19 **129.** -28 **131.** -8 **133.** negative **135.** $(-5)^{17}$, $(-2)^{17}$, $(-2)^{12}$, $(-5)^{12}$
137. answers may vary

Integrated Review **1.** -50 ; $+122$ or 122 **2.**  **3.** $>$ **4.** $<$ **5.** $<$ **6.** $>$ **7.** 3 **8.** 9 **9.** -4
10. 5 **11.** -11 **12.** 3 **13.** -64 **14.** 0 **15.** 12 **16.** -20 **17.** -48 **18.** -9 **19.** 10 **20.** -2 **21.** 106 **22.** -3 **23.** 0
24. 4 **25.** 42 **26.** 6 **27.** 19 **28.** -900 **29.** -12 **30.** -19 **31.** undefined **32.** 0 **33.** $-12 - (-8)$; -4 **34.** $-17 + (-27)$; -44
35. $-5(-25)$; 125 **36.** $-100 \div (-5)$; 20 **37.** $\frac{x}{-17}$ or $x \div (-17)$ **38.** $-3 + x$ **39.** $x - (-18)$ **40.** $-7 \cdot x$ or $-7x$ **41.** 9
42. -15 **43.** 27 **44.** 33 **45.** -15 **46.** -4

Section 2.5

Calculator Explorations **1.** 48 **3.** -258

Vocabulary, Readiness & Video Check **1.** division **3.** average **5.** subtraction **7.** A fraction bar means divided by and it is a grouping symbol. **9.** Finding the average is a good application of both order of operations and adding and dividing integers.

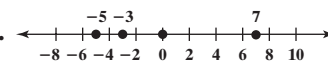
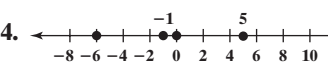
Exercise Set 2.5 **1.** -125 **3.** -64 **5.** 32 **7.** -8 **9.** -11 **11.** -43 **13.** -8 **15.** 17 **17.** -1 **19.** 4 **21.** -3 **23.** 16
25. 13 **27.** -77 **29.** 80 **31.** 256 **33.** 53 **35.** 4 **37.** -64 **39.** 4 **41.** 16 **43.** -27 **45.** 34 **47.** 65 **49.** -7 **51.** 36 **53.** -117
55. 30 **57.** -3 **59.** -30 **61.** 1 **63.** -12 **65.** 0 **67.** -20 **69.** 9 **71.** -16 **73.** -128 **75.** 1 **77.** -50 **79.** -2 **81.** -19
83. 18 **85.** 0 **87.** no; answers may vary **89.** 4050 **91.** 45 **93.** 32 in. **95.** 30 ft **97.** $2 \cdot (7 - 5) \cdot 3$ **99.** $-6 \cdot (10 - 4)$
101. answers may vary **103.** answers may vary **105.** 20,736 **107.** 8900 **109.** 9

Section 2.6

Vocabulary, Readiness & Video Check **1.** expression **3.** equation; expression **5.** solution **7.** addition
9. an equal sign **11.** original; true

Exercise Set 2.6 **1.** yes **3.** no **5.** yes **7.** yes **9.** 18 **11.** -12 **13.** 9 **15.** -17 **17.** 4 **19.** -4 **21.** -14 **23.** -17 **25.** 0
27. 1 **29.** -7 **31.** -50 **33.** -25 **35.** 36 **37.** 21 **39.** 12 **41.** -80 **43.** -2 **45.** $x - (-2)$ **47.** $-6 \cdot x$ or $-6x$
49. $-15 + x$ **51.** $-8 \div x$ or $-\frac{8}{x}$ **53.** 41,574 **55.** -409 **57.** answers may vary **59.** answers may vary

Chapter 2 Vocabulary Check **1.** opposites **2.** absolute value **3.** integers **4.** negative **5.** positive **6.** inequality symbols **7.** solution **8.** average **9.** expression **10.** equation **11.** is less than; is greater than **12.** addition **13.** multiplication

Chapter 2 Review **1.** -1572 **2.** $+11,239$ **3.**  **4.** 
5. 11 **6.** 0 **7.** -8 **8.** 9 **9.** -16 **10.** 2 **11.** $>$ **12.** $<$ **13.** $>$ **14.** $>$ **15.** 18 **16.** -42 **17.** false **18.** true **19.** true
20. true **21.** 2 **22.** 3 **23.** -5 **24.** -10 **25.** Elevator D **26.** Elevator B **27.** 2 **28.** 14 **29.** 4 **30.** 17 **31.** -23 **32.** -22
33. -21 **34.** -70 **35.** 0 **36.** 0 **37.** -151 **38.** -606 **39.** -20°C **40.** -150 ft **41.** -19 **42.** 11 **43.** 8 **44.** -16 **45.** -24
46. -10 **47.** 20 **48.** 8 **49.** 0 **50.** -32 **51.** 0 **52.** 7 **53.** -10 **54.** -27 **55.** 692 ft **56.** -25 **57.** -14 or 14 ft below ground
58. 82 ft **59.** true **60.** false **61.** 21 **62.** -18 **63.** -64 **64.** 60 **65.** 25 **66.** -1 **67.** 0 **68.** 24 **69.** -5 **70.** 3 **71.** 0
72. undefined **73.** -20 **74.** -9 **75.** 38 **76.** -5 **77.** $(-5)(2) = -10$ **78.** $(-50)(4) = -200$ **79.** $-1024 \div 4 = -256$
80. $-45 \div 9 = -5$ **81.** 49 **82.** -49 **83.** 0 **84.** -8 **85.** -16 **86.** 35 **87.** -32 **88.** -8 **89.** 7 **90.** -14 **91.** 39 **92.** -117
93. -2 **94.** -12 **95.** -3 **96.** -35 **97.** -5 **98.** 5 **99.** -1 **100.** -7 **101.** no **102.** yes **103.** -13 **104.** -20 **105.** -3
106. -9 **107.** -13 **108.** -31 **109.** 44 **110.** -26 **111.** -19 **112.** 38 **113.** 6 **114.** -5 **115.** -15 **116.** -19 **117.** 48
118. -21 **119.** 21 **120.** -5 **121.** -27°C **122.** 6°C **123.** 13,118 ft **124.** -9 **125.** 2 **126.** 3 **127.** -5 **128.** -25 **129.** -20
130. 17 **131.** -21 **132.** -17 **133.** 12 **134.** -9 **135.** -200 **136.** 3

Chapter 2 Getting Ready for the Test **1.** A **2.** A **3.** A **4.** B **5.** C **6.** C **7.** A **8.** D **9.** C **10.** D **11.** B **12.** B
13. D **14.** A

Chapter 2 Test **1.** 3 **2.** -6 **3.** -100 **4.** 4 **5.** -30 **6.** 12 **7.** 65 **8.** 5 **9.** 12 **10.** -6 **11.** 50 **12.** -2 **13.** -11
14. -46 **15.** -117 **16.** 3456 **17.** 28 **18.** -213 **19.** -2 **20.** 2 **21.** -5 **22.** -32 **23.** -17 **24.** 1 **25.** -1
26. 88 ft below sea level **27.** 45 **28.** 31,642 **29.** 3820 ft below sea level **30.** -4 **31. a.** $17 \cdot x$ or $17x$ **b.** $20 - x$ **32.** 5
33. -28 **34.** -20 **35.** -4

Cumulative Review **1.** hundred-thousands; Sec. 1.2, Ex. 1 **2.** hundreds; Sec. 1.2 **3.** thousands; Sec. 1.2, Ex. 2 **4.** thousands; Sec. 1.2
5. ten-millions; Sec. 1.2, Ex. 3 **6.** hundred-thousands; Sec. 1.2 **7. a.** $<$ **b.** $>$ **c.** $>$; Sec. 2.1, Ex. 3 **8. a.** $>$ **b.** $>$ **c.** $<$; Sec. 2.1
9. 39; Sec. 1.3, Ex. 3 **10.** 39; Sec. 1.3 **11.** 7321; Sec. 1.3, Ex. 6 **12.** 3013; Sec. 1.3 **13.** 36,184 mi; Sec. 1.3, Ex. 11 **14.** $\$525$; Sec. 1.3
15. 570; Sec. 1.4, Ex. 1 **16.** 600; Sec. 1.4 **17.** 1800; Sec. 1.4, Ex. 5 **18.** 5000; Sec. 1.4 **19. a.** $5 \cdot 6 + 5 \cdot 5$ **b.** $20 \cdot 4 + 20 \cdot 7$
c. $2 \cdot 7 + 2 \cdot 9$; Sec. 1.5, Ex. 2 **20. a.** $5 \cdot 2 + 5 \cdot 12$ **b.** $9 \cdot 3 + 9 \cdot 6$ **c.** $4 \cdot 8 + 4 \cdot 1$; Sec. 1.5 **21.** 78,875; Sec. 1.5, Ex. 5 **22.** 31,096; Sec. 1.5

23. a. 6 b. 8 c. 7; Sec. 1.6, Ex. 1 24. a. 7 b. 8 c. 12; Sec. 1.6 25. 741; Sec. 1.6, Ex. 4 26. 456; Sec. 1.6 27. 12 cards each; 10 cards left over; Sec. 1.6, Ex. 11 28. \$9; Sec. 1.6 29. 81; Sec. 1.7, Ex. 5 30. 125; Sec. 1.7 31. 6; Sec. 1.7, Ex. 6 32. 4; Sec. 1.7 33. 180; Sec. 1.7, Ex. 8 34. 56; Sec. 1.7 35. 2; Sec. 1.7, Ex. 13 36. 5; Sec. 1.7 37. 14; Sec. 1.8, Ex. 1 38. 14; Sec. 1.8 39. a. 9 b. 8 c. 0; Sec. 2.1, Ex. 4 40. a. 4 b. 7; Sec. 2.1 41. 23; Sec. 2.2, Ex. 7 42. 5; Sec. 2.2 43. 22; Sec. 2.3, Ex. 12 44. 5; Sec. 2.3 45. -21; Sec. 2.4, Ex. 1 46. -10; Sec. 2.4 47. 0; Sec. 2.4, Ex. 3 48. -54; Sec. 2.4 49. -16; Sec. 2.5, Ex. 8 50. -27; Sec. 2.5

Chapter 3 Solving Equations and Problem Solving

Section 3.1

Vocabulary, Readiness & Video Check 1. expression; term 3. combine like terms 5. variable; constant 7. associative 9. numerical coefficient 11. distributive property 13. addition; multiplication; P = perimeter, A = area

Exercise Set 3.1 1. $8x$ 3. $-n$ 5. $-2c$ 7. $-6x$ 9. $12a - 5$ 11. $42x$ 13. $-33y$ 15. $72a$ 17. $2y + 6$ 19. $3a - 18$
21. $-12x - 28$ 23. $2x + 1$ 25. $15c + 3$ 27. $-21n + 20$ 29. $7w + 15$ 31. $11x - 8$ 33. $-2y + 16$ 35. $-2y$ 37. $-7z$
39. $8d - 3c$ 41. $6y - 14$ 43. $-q$ 45. $2x + 22$ 47. $-3x - 35$ 49. $-3z - 15$ 51. $-6x + 6$ 53. $3x - 30$ 55. $-r + 8$
57. $-7n + 3$ 59. $9z - 14$ 61. -6 63. $-4x + 10$ 65. $2x + 20$ 67. $7a + 12$ 69. $3y + 5$ 71. $(14y + 22)$ m
73. $(11a + 12)$ ft 75. $(125x + 55)$ in. 77. $36y$ sq in. 79. $(32x - 64)$ sq km 81. $(60y + 20)$ sq mi 83. 4700 sq ft
85. 64 ft 87. $(3x + 6)$ ft 89. -3 91. 8 93. 0 95. incorrect; $15x - 10$ 97. incorrect; $2xy$ 99. correct
101. incorrect; $4y - 12 + 11$ or $4y - 1$ 103. distributive 105. associative 107. $(20x + 16)$ sq mi 109. $4824q + 12,274$
111. answers may vary 113. answers may vary

Section 3.2

Vocabulary, Readiness & Video Check 1. equivalent 3. simplifying 5. addition 7. Simplify the left side of the equation by combining like terms. 9. addition property of equality.

Exercise Set 3.2 1. 6 3. 8 5. -4 7. 6 9. -1 11. 18 13. -8 15. -50 17. 3 19. -22 21. -6 23. 24 25. -30
27. 12 29. 4 31. 3 33. -3 35. 1 37. 5 39. -11 41. -4 43. -3 45. -1 47. 0 49. 3 51. -6 53. -35 55. 10
57. -2 59. 0 61. 28 63. -5 65. -28 67. -28 69. 5 71. -4 73. $-7 + x$ 75. $x - 11$ 77. $-13x$ 79. $\frac{x}{-12}$ or $-\frac{x}{12}$
81. $-11x + 5$ 83. $-10 - 7x$ 85. $4x + 7$ 87. $2x - 17$ 89. $-6(x + 15)$ 91. $\frac{45}{-5x}$ or $-\frac{45}{5x}$ 93. $\frac{17}{x} + (-15)$ or $\frac{17}{x} - 15$
95. Caracas 97. \$1535 99. answers may vary 101. no; answers may vary 103. answers may vary 105. 67,896 107. -48 109. 42

Integrated Review 1. expression 2. equation 3. equation 4. expression 5. simplify 6. solve 7. $8x$ 8. $-4y$ 9. $-2a - 2$
10. $5a - 26$ 11. $-8x - 14$ 12. $-6x + 30$ 13. $5y - 10$ 14. $15x - 31$ 15. $(12x - 6)$ sq m 16. $(2x + 9)$ ft 17. -4 18. -3
19. -10 20. 6 21. -15 22. -120 23. -5 24. -13 25. -24 26. -54 27. 12 28. -42 29. 2 30. 2 31. -3 32. 5
33. -5 34. 6 35. $x - 10$ 36. $-20 + x$ 37. $10x$ 38. $\frac{10}{x}$ 39. $-2x + 5$ 40. $-4(x - 1)$

Section 3.3

Calculator Explorations 1. yes 3. no 5. yes

Vocabulary, Readiness & Video Check 1. $3x - 9 + x - 16$; $5(2x + 6) - 1 = 39$ 3. addition 5. distributive 7. the addition property of equality; to make sure we get an equivalent equation 9. gives; amounts to

Exercise Set 3.3 1. -12 3. -3 5. 1 7. -45 9. -9 11. 6 13. 8 15. 5 17. 0 19. -5 21. -22 23. 6 25. -11
27. -7 29. -5 31. 270 33. 5 35. 3 37. 9 39. -6 41. 11 43. 3 45. 4 47. -4 49. 3 51. -1 53. -4 55. -5
57. 0 59. 4 61. 1 63. -30 65. $-42 + 16 = -26$ 67. $-5(-29) = 145$ 69. $3(-14 - 2) = -48$ 71. $\frac{100}{2(50)} = 1$
73. 134,000,000 returns 75. 39,000,000 returns 77. 33 79. -37 81. b 83. a 85. $6x - 10 = 5x - 7$; $6x = 5x + 3$; $x = 3$
87. 0 89. -4 91. no; answers may vary

Section 3.4

Vocabulary, Readiness & Video Check 1. The phrase is "a number subtracted from -20" so -20 goes first and we subtract the number from that. 3. The original application asks for the fastest speeds of a pheasant and a falcon. The value of x is the speed in mph for a pheasant, so the falcon's speed still needs to be found.

Exercise Set 3.4 1. $-5 + x = -7$ 3. $3x = 27$ 5. $-20 - x = 104$ 7. $2x = 108$ 9. $5(-3 + x) = -20$ 11. $9 + 3x = 33$; 8
13. $3 + 4 + x = 16$; 9 15. $x - 3 = \frac{10}{5}$; 5 17. $30 - x = 3(x + 6)$; 3 19. $5x - 40 = x + 8$; 12 21. $3(x - 5) = \frac{108}{12}$; 8

23. $4x = 30 - 2x$; 5 25. California: 55 votes; Florida: 29 votes 27. falcon: 185 mph; pheasant: 37 mph 29. *USA Today*: 2283 thousand; *The Wall Street Journal*: 2057 thousand 31. Apple: \$154 billion; Google: \$83 billion 33. Xbox: \$330; games \$110
35. 5470 mi 37. Beaver Stadium: 107,282; Michigan Stadium: 106,201 39. China: 140 million; Spain: 70 million 41. United States: 12,200,000; Germany: 6,100,000 43. \$225 45. Olympic: 120 yards; Under-10 field: 70 yards 47. 77 points 49. TV: \$74.7 billion; Internet: \$75.3 billion 51. 590 53. 1000 55. 3000 57. yes; answers may vary 59. \$216,200 61. \$549

Chapter 3 Vocabulary Check 1. simplified; combined. 2. like 3. variable 4. algebraic expression 5. terms 6. numerical coefficient 7. evaluating the expression 8. constant 9. equation 10. solution 11. distributive 12. multiplication
13. addition

Chapter 3 Review 1. $10y - 15$ 2. $-6y - 10$ 3. $-6a - 7$ 4. $-8y + 2$ 5. $2x + 10$ 6. $-3y - 24$ 7. $11x - 12$ 8. $-4m - 12$
9. $-5a + 4$ 10. $12y - 9$ 11. $16y - 5$ 12. $x - 2$ 13. $(4x + 6)$ yd 14. 20y m 15. $(6x - 3)$ sq yd 16. $(45x + 8)$ sq cm
17. -2 18. 10 19. -7 20. 5 21. -12 22. 45 23. -6 24. -1 25. -25 26. -8 27. -2 28. -2 29. -8 30. -45
31. 5 32. -5 33. -63 34. -15 35. 5 36. 12 37. -6 38. 4 39. $-5x$ 40. $x - 3$ 41. $-5 + x$ 42. $-\frac{2}{x}$ or $-\frac{2}{x}$

43. $-5x - 50$ 44. $2x + 11$ 45. $\frac{70}{x + 6}$ 46. $2(x - 13)$ 47. 21 48. -10 49. 2 50. 2 51. 11 52. -5 53. -15 54. 10

55. -2 56. -6 57. -1 58. 1 59. 0 60. 20 61. $20 - (-8) = 28$ 62. $-2 - 19 = -21$ 63. $\frac{-75}{5 + 20} = -3$

64. $5[2 + (-6)] = -20$ 65. $2x - 8 = 40$ 66. $6x = x + 20$ 67. $\frac{x}{2} - 12 = 10$ 68. $x - 3 = \frac{x}{4}$ 69. 5 70. -16 71. 2386 votes

72. 84 DVDs 73. $-11x$ 74. $-35x$ 75. $22x - 19$ 76. $-9x - 32$ 77. -1 78. -25 79. 13 80. -6 81. -22 82. -6
83. -15 84. 18 85. -5 86. 11 87. 2 88. -1 89. 0 90. -6 91. 5 92. 1 93. Vermont: 108.1 trillion BTU; District of Columbia: 132.4 trillion BTU 94. North Dakota 86,843 mi; South Dakota: 82,354 mi

Chapter 3 Getting Ready for the Test 1. A 2. B 3. A 4. C 5. C 6. B 7. B 8. A 9. C 10. A 11. D 12. E
13. B 14. C 15. E 16. F

Chapter 3 Test 1. $-5x + 5$ 2. $-6y - 14$ 3. $-8z - 20$ 4. $(15x + 15)$ in. 5. $(12x - 4)$ sq m 6. -6 7. -6 8. 24
9. -2 10. 6 11. 3 12. -2 13. 0 14. 4 15. $-23 + x$ 16. $-2 - 3x$ 17. $2 \cdot 5 + (-15) = -5$ 18. $3x + 6 = -30$
19. -2 20. 8 free throws 21. 244 women

Cumulative Review Chapters 1–3 1. three hundred eight million, sixty-three thousand, five hundred fifty-seven; Sec. 1.2, Ex. 7
2. two hundred seventy-six thousand, four; Sec. 1.2 3. 13 in.; Sec. 1.3, Ex. 9 4. 18 in.; Sec. 1.3 5. 726; Sec. 1.3, Ex. 8
6. 9585; Sec. 1.3 7. 249,000; Sec. 1.4, Ex. 3 8. 844,000; Sec. 1.4 9. 200; Sec. 1.5, Ex. 3a 10. 29,230; Sec. 1.5
11. 208; Sec. 1.6, Ex. 5 12. 86; Sec. 1.6 13. 7; Sec. 1.7, Ex. 9 14. 35; Sec. 1.7 15. 26; Sec. 1.8, Ex. 4 16. 10; Sec. 1.8
17. 20 is a solution; 26 and 40 are not solutions; Sec. 1.8, Ex. 7 18. a. < b. >; Sec. 2.1 19. 3; Sec. 2.2, Ex. 1
20. -7; Sec. 2.2 21. -25; Sec. 2.2, Ex. 5 22. -4; Sec. 2.2 23. 23; Sec. 2.2, Ex. 7 24. 17; Sec. 2.2 25. -14; Sec. 2.3, Ex. 2
26. -5; Sec. 2.3 27. 11; Sec. 2.3, Ex. 3 28. 29; Sec. 2.3 29. -4; Sec. 2.3, Ex. 4 30. -3; Sec. 2.3 31. -2; Sec. 2.4, Ex. 10
32. 6; Sec. 2.4 33. 5; Sec. 2.4, Ex. 11 34. -13; Sec. 2.4 35. -16; Sec. 2.4, Ex. 12 36. -10; Sec. 2.4 37. 9; Sec. 2.5, Ex. 1
38. -32; Sec. 2.5 39. -9; Sec. 2.5, Ex. 2 40. 25; Sec. 2.5 41. $6y + 2$; Sec. 3.1, Ex. 2 42. $3x + 9$; Sec. 3.1 43. not a solution;
Sec. 2.6, Ex. 1 44. solution; Sec. 2.6 45. 3; Sec. 2.6, Ex. 7 46. -5; Sec. 2.6 47. 12; Sec. 3.2, Ex. 7 48. -2; Sec. 3.2
49. software: \$420; computer system: \$1680; Sec. 3.4, Ex. 4 50. 11; Sec. 3.4

Chapter 4 Fractions and Mixed Numbers

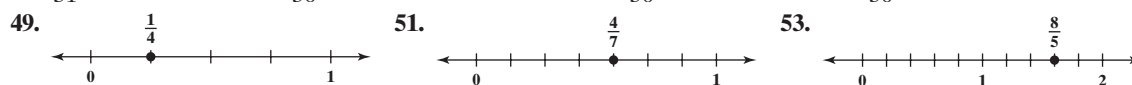
Section 4.1

Vocabulary, Readiness & Video Check 1. fraction; denominator; numerator 3. improper; proper; mixed number
5. equal; improper 7. how many equal parts to divide each whole number into 9. addition; +

Exercise Set 4.1

1. numerator: 1; denominator: 2; proper 3. numerator: 10; denominator: 3; improper 5. numerator: 15; denominator: 15;
improper 7. $\frac{1}{3}$ 9. a. $\frac{11}{4}$ b. $2\frac{3}{4}$ 11. a. $\frac{23}{6}$ b. $3\frac{5}{6}$ 13. $\frac{7}{12}$ 15. $\frac{3}{7}$ 17. $\frac{4}{9}$ 19. a. $\frac{4}{3}$ b. $1\frac{1}{3}$ 21. a. $\frac{11}{2}$ b. $5\frac{1}{2}$ 23. $\frac{1}{6}$
25. $\frac{5}{8}$ 27.  29.  31.  33. $\frac{42}{131}$ of the students
35. a. 89 b. $\frac{89}{131}$ of the students 37. $\frac{7}{44}$ of the presidents 39. $\frac{10}{15}$ of the tropical storms 41. $\frac{11}{31}$ of the month

43. $\frac{10}{31}$ of the class 45. a. $\frac{33}{50}$ of the states b. 17 states c. $\frac{17}{50}$ of the states 47. a. $\frac{21}{50}$ of the marbles b. 29 c. $\frac{29}{50}$ of the marbles



55.  57. 1 59. -5 61. 0 63. 1 65. undefined 67. 3 69. $\frac{7}{3}$ 71. $\frac{18}{5}$ 73. $\frac{53}{8}$ 75. $\frac{83}{7}$

77. $\frac{187}{20}$ 79. $\frac{500}{3}$ 81. $3\frac{2}{5}$ 83. $4\frac{5}{8}$ 85. $3\frac{2}{15}$ 87. 15 89. $1\frac{7}{175}$ 91. $6\frac{65}{112}$ 93. 9 95. 125 97. $-\frac{11}{2}, \frac{11}{-2}$ 99. $\frac{13}{-15}, -\frac{13}{15}$

101. answers may vary 103.  105. 7 107. $\frac{947}{2247}$ of the stores 109. $\frac{1500}{1580}$ of the affiliates

Section 4.2

- Calculator Explorations 1. $\frac{4}{7}$ 3. $\frac{20}{27}$ 5. $\frac{8}{15}$ 7. $\frac{2}{9}$

Vocabulary, Readiness & Video Check 1. prime factorization 3. prime 5. equivalent 7. Check that every factor is a prime number and check that the product of the factors is the original number 9. You can simplify the two fractions and then compare them. $\frac{3}{9}$ and $\frac{6}{18}$ both simplify to $\frac{1}{3}$ so the original fractions are equivalent.

- Exercise Set 4.2** 1. $2^2 \cdot 5$ 3. $2^4 \cdot 3$ 5. 3^4 7. $2 \cdot 3^4$ 9. $2 \cdot 5 \cdot 11$ 11. $5 \cdot 17$ 13. $2^4 \cdot 3 \cdot 5$ 15. $2^2 \cdot 3^2 \cdot 23$ 17. $\frac{1}{4}$ 19. $\frac{2x}{21}$

21. $\frac{7}{8}$ 23. $\frac{2}{3}$ 25. $\frac{7}{10}$ 27. $-\frac{7}{9}$ 29. $\frac{5x}{6}$ 31. $\frac{27}{64}$ 33. $\frac{5x}{8}$ 35. $-\frac{5}{8}$ 37. $\frac{3x^2y}{2}$ 39. $\frac{3}{4}$ 41. $\frac{5}{8z}$ 43. $\frac{3}{14}$ 45. $-\frac{11}{17y}$ 47. $\frac{7}{8}$

49. $14a^2$ 51. equivalent 53. not equivalent 55. equivalent 57. equivalent 59. not equivalent 61. not equivalent

63. $\frac{1}{4}$ of a shift 65. $\frac{1}{2}$ mi 67. a. $\frac{8}{25}$ b. 34 states c. $\frac{17}{25}$ 69. $\frac{5}{12}$ of the wall 71. a. 12 states b. $\frac{1}{4}$ 73. $\frac{15}{122}$ of U.S. astronauts

75. -3 77. -14 79. answers may vary 81. $\frac{3}{5}$ 83. $\frac{9}{25}$ 85. $\frac{1}{25}$ 87. $2^2 \cdot 3^5 \cdot 5 \cdot 7$ 89. answers may vary 91. no; answers

- may vary 93. $\frac{3}{50}$ 95. answers may vary 97. $\frac{6}{25}$ 99. answers may vary 101. 786, 222, 900, 1470 103. 6; answers may vary

Section 4.3

Vocabulary, Readiness & Video Check 1. $\frac{a \cdot c}{b \cdot d}$ 3. $\frac{2 \cdot 2 \cdot 2}{7} \cdot \frac{2}{7} \cdot \frac{2}{7} \cdot \frac{2}{7}$ 5. $\frac{a \cdot d}{b \cdot c}$ 7. We have a negative fraction times a positive fraction and a negative number times a positive number is a negative number. 9. numerator; denominator 11. radius = $\frac{1}{2} \cdot$ diameter

- Exercise Set 4.3** 1. $\frac{18}{77}$ 3. $-\frac{5}{28}$ 5. $\frac{1}{15}$ 7. $\frac{18x}{55}$ 9. $\frac{3a^2}{4}$ 11. $\frac{x^2}{y}$ 13. 0 15. $-\frac{17}{25}$ 17. $\frac{1}{56}$ 19. $\frac{1}{125}$ 21. $\frac{4}{9}$ 23. $-\frac{4}{27}$

25. $\frac{4}{5}$ 27. $-\frac{1}{6}$ 29. $-\frac{16}{9x}$ 31. $\frac{121y}{60}$ 33. $-\frac{1}{6}$ 35. $\frac{x}{25}$ 37. $\frac{10}{27}$ 39. $\frac{18x^2}{35}$ 41. $\frac{10}{9}$ 43. $-\frac{1}{4}$ 45. $\frac{9}{16}$ 47. xy^2 49. $\frac{77}{2}$

51. $-\frac{36}{x}$ 53. $\frac{3}{49}$ 55. $-\frac{19y}{7}$ 57. $\frac{4}{11}$ 59. $\frac{8}{3}$ 61. $\frac{15x}{4}$ 63. $\frac{8}{9}$ 65. $\frac{1}{60}$ 67. b 69. $\frac{2}{5}$ 71. $-\frac{5}{3}$ 73. a. $\frac{1}{3}$ b. $\frac{12}{25}$

75. a. $-\frac{36}{55}$ b. $-\frac{44}{45}$ 77. yes 79. no 81. 50 83. 20 85. 112 87. 84 million astronauts 89. 868 mi 91. $\frac{3}{16}$ in.

93. \$1838 95. 50 libraries 97. $\frac{1}{14}$ sq ft 99. 3840 mi 101. 2400 mi 103. 201 105. 196 107. answers may vary 109. 5

111. 40,630,000 people 113. 8505

Section 4.4

Vocabulary, Readiness & Video Check 1. like; unlike 3. $-\frac{a}{b}$ 5. least common denominator (LCD) 7. numerators; denominator 9. $P = \frac{5}{12} + \frac{7}{12} + \frac{5}{12} + \frac{7}{12}$; 2 meters 11. Multiplying by 1 does not change the value of a fraction.

- Exercise Set 4.4** 1. $\frac{7}{11}$ 3. $\frac{2}{3}$ 5. $-\frac{1}{4}$ 7. $-\frac{1}{2}$ 9. $\frac{2}{3x}$ 11. $-\frac{x}{9}$ 13. $\frac{6}{11}$ 15. $\frac{3}{4}$ 17. $-\frac{3}{y}$ 19. $-\frac{19}{33}$ 21. $-\frac{1}{3}$ 23. $\frac{7a-3}{4}$
 25. $\frac{9}{10}$ 27. $-\frac{13x}{14}$ 29. $\frac{9x+1}{15}$ 31. $-\frac{x}{2}$ 33. $-\frac{2}{3}$ 35. $\frac{3x}{4}$ 37. $\frac{5}{4}$ 39. $\frac{2}{5}$ 41. 1 in. 43. 2 m 45. $\frac{7}{10}$ mi 47. $\frac{8}{25}$ 49. $\frac{7}{25}$
 51. $\frac{1}{50}$ 53. 45 55. 72 57. 150 59. $24x$ 61. 126 63. 168 65. 14 67. 20 69. 25 71. $56x$ 73. $8y$ 75. $20a$
 77. $\frac{54}{100}, \frac{50}{100}, \frac{46}{100}, \frac{50}{100}, \frac{15}{100}, \frac{65}{100}, \frac{45}{100}, \frac{52}{100}, \frac{60}{100}, \frac{61}{100}, \frac{48}{100}, \frac{50}{100}$ 79. drugs, health and beauty aids 81. 9 83. 125 85. 49 87. 24
 89. $\frac{2}{7} + \frac{9}{7} = \frac{11}{7}$ 91. answers may vary 93. 1; answers may vary 95. $814x$ 97. answers may vary 99. a, b, and d

Section 4.5

- Calculator Explorations** 1. $\frac{37}{80}$ 3. $\frac{95}{72}$ 5. $\frac{394}{323}$

- Vocabulary, Readiness & Video Check** 1. equivalent; least common denominator 3. $\frac{4}{24}, \frac{15}{24}, \frac{19}{24}$ 5. expression; equation
 7. They are unlike terms and so cannot be combined. 9. $\frac{3}{2} + \frac{1}{3}; 6$

- Exercise Set 4.5** 1. $\frac{5}{6}$ 3. $\frac{1}{6}$ 5. $-\frac{4}{33}$ 7. $-\frac{3}{14}$ 9. $\frac{3x}{5}$ 11. $\frac{24-y}{12}$ 13. $\frac{11}{36}$ 15. $-\frac{44}{7}$ 17. $\frac{89a}{99}$ 19. $\frac{4y-1}{6}$ 21. $\frac{x+6}{2x}$
 23. $-\frac{8}{33}$ 25. $\frac{3}{14}$ 27. $\frac{11y-10}{35}$ 29. $-\frac{11}{36}$ 31. $\frac{1}{20}$ 33. $\frac{33}{56}$ 35. $\frac{17}{16}$ 37. $-\frac{2}{9}$ 39. $-\frac{11}{30}$ 41. $\frac{15+11y}{33}$ 43. $-\frac{53}{42}$ 45. $\frac{7x}{8}$
 47. $\frac{11}{18}$ 49. $\frac{44a}{39}$ 51. $-\frac{11}{60}$ 53. $\frac{5y+9}{9y}$ 55. $\frac{19}{20}$ 57. $\frac{56}{45}$ 59. $\frac{40+9x}{72x}$ 61. $-\frac{5}{24}$ 63. $\frac{37x-20}{56}$ 65. $<$ 67. $>$ 69. $>$
 71. $\frac{13}{12}$ 73. $\frac{1}{4}$ 75. $\frac{11}{6}$ 77. $\frac{34}{15}$ cm or $2\frac{4}{15}$ cm 79. $\frac{17}{10}$ m or $1\frac{7}{10}$ m 81. $x + \frac{1}{2}$ 83. $-\frac{3}{8} - x$ 85. $\frac{7}{100}$ mph 87. $\frac{5}{8}$ in. 89. $\frac{49}{100}$ of
 students 91. $\frac{47}{32}$ in. 93. $\frac{19}{25}$ 95. $\frac{3}{40}$ 97. $\frac{63}{100}$ 99. -20 101. -6 103. $\frac{3}{5} + \frac{4}{5} = \frac{7}{5}$ 105. $\frac{223}{540}$ 107. $\frac{49}{44}$
 109. answers may vary 111. standard mail

- Integrated Review** 1. $\frac{3}{7}$ 2. $\frac{5}{4}$ or $1\frac{1}{4}$ 3. $\frac{73}{85}$ 4.

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

 5. -1 6. 17 7. 0
 8. undefined 9. $5 \cdot 13$ 10. $2 \cdot 5 \cdot 7$ 11. $3^2 \cdot 5 \cdot 7$ 12. $3^2 \cdot 7^2$ 13. $\frac{1}{7}$ 14. $\frac{6}{5}$ 15. $-\frac{14}{15}$ 16. $-\frac{9}{10}$ 17. $\frac{2x}{5}$ 18. $\frac{3}{8y}$ 19. $\frac{11z^2}{14}$
 20. $\frac{7}{11ab^2}$ 21. not equivalent 22. equivalent 23. a. $\frac{1}{25}$ b. 48 c. $\frac{24}{25}$ 24. a. $\frac{11}{47}$ b. 108,000 c. $\frac{36}{47}$ 25. 30 26. 14 27. 90
 28. $\frac{28}{36}$ 29. $\frac{55}{75}$ 30. $\frac{40}{48}$ 31. $\frac{4}{5}$ 32. $-\frac{2}{5}$ 33. $\frac{3}{25}$ 34. $\frac{1}{3}$ 35. $\frac{4}{5}$ 36. $\frac{5}{9}$ 37. $-\frac{1}{6y}$ 38. $\frac{3x}{2}$ 39. $\frac{1}{18}$ 40. $\frac{4}{21}$ 41. $-\frac{7}{48z}$
 42. $-\frac{9}{50}$ 43. $\frac{37}{40}$ 44. $\frac{11}{36}$ 45. $\frac{11}{18}$ 46. $\frac{25y+12}{50}$ 47. $\frac{2}{3} \cdot x$ or $\frac{2}{3}x$ 48. $x \div \left(-\frac{1}{5}\right)$ 49. $-\frac{8}{9} - x$ 50. $\frac{6}{11} + x$ 51. 1020
 52. 24 lots 53. $\frac{3}{4}$ ft

Section 4.6

- Vocabulary, Readiness & Video Check** 1. complex 3. division 5. addition 7. distributive property 9. Since x is squared and the replacement value is negative, we use parentheses to make sure the whole value of x is squared. Without parentheses, the exponent would not apply to the negative sign.

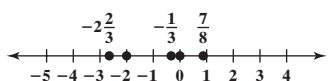
- Exercise Set 4.6** 1. $\frac{1}{6}$ 3. $\frac{7}{3}$ 5. $\frac{x}{6}$ 7. $\frac{23}{22}$ 9. $\frac{2x}{13}$ 11. $\frac{17}{60}$ 13. $\frac{5}{8}$ 15. $\frac{35}{9}$ 17. $-\frac{17}{45}$ 19. $\frac{11}{8}$ 21. $\frac{29}{10}$ 23. $\frac{27}{32}$ 25. $\frac{1}{100}$
 27. $\frac{9}{64}$ 29. $\frac{7}{6}$ 31. $-\frac{2}{5}$ 33. $-\frac{2}{9}$ 35. $\frac{11}{9}$ 37. $\frac{5a}{2}$ 39. $\frac{7}{2}$ 41. $\frac{9}{20}$ 43. $-\frac{13}{2}$ 45. $\frac{9}{25}$ 47. $-\frac{5}{32}$ 49. 1 51. $-\frac{2}{5}$ 53. $-\frac{11}{40}$
 55. $\frac{x+6}{16}$ 57. $\frac{7}{2}$ or $3\frac{1}{2}$ 59. $\frac{49}{6}$ or $8\frac{1}{6}$ 61. no; answers may vary 63. $\frac{5}{8}$ 65. $\frac{11}{56}$ 67. halfway between a and b 69. false
 71. true 73. true 75. addition; answers may vary 77. subtraction, multiplication, addition, division 79. division, multiplication, subtraction, addition 81. $-\frac{77}{16}$ 83. $-\frac{55}{16}$

Section 4.7

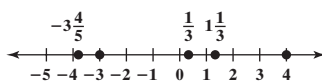
Calculator Explorations 1. $\frac{280}{11}$ 3. $\frac{3776}{35}$ 5. $26\frac{1}{14}$ 7. $92\frac{3}{10}$

Vocabulary, Readiness & Video Check 1. mixed number 3. round 5. The denominator of the mixed number we're graphing, $-3\frac{4}{5}$, is 5. 7. The fractional part of a mixed number should always be a proper fraction. 9. We're adding two mixed numbers with unlike signs, so the answer has the sign of the mixed number with the larger absolute value, which in this case is negative.

Exercise Set 4.7 1.



3.



5. b 7. a 9. $\frac{8}{21}$ 11. $4\frac{3}{8}$ 13. $7\frac{7}{10}$; 8

15. $23\frac{31}{35}$; 24 17. $12\frac{1}{2}$ 19. $5\frac{1}{2}$ 21. $18\frac{2}{3}$ 23. a 25. b 27. $6\frac{2}{3}$; 7 29. $13\frac{11}{14}$; 14 31. $17\frac{7}{25}$ 33. $47\frac{53}{84}$ 35. $25\frac{5}{14}$ 37. $13\frac{13}{24}$
 39. $2\frac{3}{5}$; 3 41. $7\frac{5}{14}$; 7 43. $\frac{24}{25}$ 45. $3\frac{5}{9}$ 47. $15\frac{3}{4}$ 49. 4 51. $5\frac{11}{14}$ 53. $6\frac{2}{9}$ 55. $\frac{25}{33}$ 57. $35\frac{13}{18}$ 59. $2\frac{1}{2}$ 61. $72\frac{19}{30}$ 63. $\frac{11}{14}$
 65. $5\frac{4}{7}$ 67. $13\frac{16}{33}$ 69. $-5\frac{2}{7} - x$ 71. $1\frac{9}{10} \cdot x$ 73. $3\frac{3}{16}$ mi 75. $9\frac{2}{5}$ in. 77. $7\frac{13}{20}$ in. 79. $3\frac{1}{2}$ sq yd 81. $\frac{15}{16}$ sq in. 83. $21\frac{5}{24}$ m
 85. no; she will be short $\frac{1}{2}$ ft 87. $4\frac{2}{3}$ m 89. $9\frac{3}{4}$ min 91. $1\frac{4}{5}$ min 93. $-10\frac{3}{25}$ 95. $-24\frac{7}{8}$ 97. $-13\frac{59}{60}$ 99. $4\frac{2}{7}$ 101. $-1\frac{23}{24}$
 103. $\frac{73}{1000}$ 105. $1x$ or x 107. $1a$ or a 109. a, b, c 111. Incorrect; to divide mixed numbers, first write each mixed number as an improper fraction. 113. answers may vary 115. answers may vary 117. answers may vary

Section 4.8

Vocabulary, Readiness & Video Check 1. 6 3. 15 5. addition property of equality 7. We multiply by 12 because it is the LCD of all fractions in the equation. The equation no longer contains fractions.

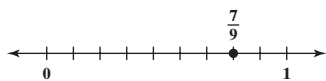
Exercise Set 4.8 1. $-\frac{2}{3}$ 3. $\frac{1}{13}$ 5. $\frac{4}{5}$ 7. $\frac{11}{12}$ 9. $-\frac{7}{10}$ 11. $\frac{11}{16}$ 13. $\frac{11}{18}$ 15. $\frac{2}{7}$ 17. 12 19. -27 21. $\frac{27}{8}$ 23. $\frac{1}{21}$ 25. $\frac{2}{11}$
 27. $-\frac{3}{10}$ 29. 1 31. 10 33. -1 35. -15 37. $\frac{3x-28}{21}$ 39. $\frac{y+10}{2}$ 41. $\frac{7x}{15}$ 43. $\frac{4}{3}$ 45. 2 47. $\frac{21}{10}$ 49. $-\frac{1}{14}$ 51. -3
 53. $-\frac{1}{9}$ 55. 50 57. $-\frac{3}{7}$ 59. 4 61. $\frac{3}{5}$ 63. $-\frac{1}{24}$ 65. $-\frac{5}{14}$ 67. 4 69. -36 71. 57,200 73. 330 75. answers may vary
 77. $\frac{112}{11}$ 79. area: $\frac{3}{16}$ sq in.; perimeter: 2 in.

Chapter 4 Vocabulary Check 1. reciprocals 2. composite number 3. equivalent 4. improper fraction

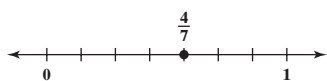
5. prime number 6. simplest form 7. proper fraction 8. mixed number 9. numerator; denominator 10. prime factorization 11. undefined 12. 0 13. like 14. least common denominator 15. complex fraction 16. cross products

Chapter 4 Review 1. $\frac{2}{6}$ 2. $\frac{4}{7}$ 3. $\frac{7}{3}$ or $2\frac{1}{3}$ 4. $\frac{13}{4}$ or $3\frac{1}{4}$ 5. $\frac{11}{12}$ 6. a. 108 b. $\frac{108}{131}$ 7. -1 8. 1 9. 0 10. undefined

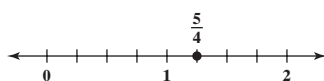
11.



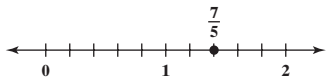
12.



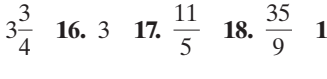
13.



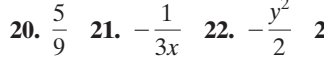
14.



15.



16.



17. $\frac{11}{5}$ 18. $\frac{35}{9}$ 19. $\frac{3}{7}$ 20. $\frac{5}{9}$ 21. $-\frac{1}{3x}$ 22. $-\frac{y^2}{2}$ 23. $\frac{29}{32c}$ 24. $\frac{18z}{23}$
 25. $\frac{5x}{3y^2}$ 26. $\frac{7b}{5c^2}$ 27. $\frac{2}{3}$ of a foot 28. $\frac{3}{5}$ of the cars 29. no 30. yes 31. $\frac{3}{10}$ 32. $-\frac{5}{14}$ 33. $\frac{9}{x^2}$ 34. $\frac{y}{2}$ 35. $-\frac{1}{27}$ 36. $-\frac{25}{144}$
 37. -2 38. $\frac{15}{4}$ 39. $\frac{27}{2}$ 40. $-\frac{5}{6y}$ 41. $\frac{12}{7}$ 42. $-\frac{63}{10}$ 43. $\frac{77}{48}$ sq ft 44. $\frac{4}{9}$ sq m 45. $\frac{10}{11}$ 46. $\frac{2}{3}$ 47. $-\frac{1}{3}$ 48. $\frac{4x}{5}$ 49. $\frac{4y-3}{21}$
 50. $-\frac{1}{15}$ 51. $3x$ 52. 24 53. 20 54. 35 55. $49a$ 56. $45b$ 57. 40 58. 10 59. $\frac{3}{4}$ of his homework 60. $\frac{3}{2}$ mi 61. $\frac{11}{18}$
 62. $\frac{7}{26}$ 63. $-\frac{1}{12}$ 64. $-\frac{5}{12}$ 65. $\frac{25x+2}{55}$ 66. $\frac{4+3b}{15}$ 67. $\frac{7y}{36}$ 68. $\frac{11x}{18}$ 69. $\frac{4y+45}{9y}$ 70. $\frac{9y-33}{11y}$ 71. $\frac{91}{150}$ 72. $\frac{5}{18}$ 73. $\frac{19}{9}$ m
 74. $\frac{3}{2}$ ft 75. $\frac{21}{50}$ of the donors 76. $\frac{1}{4}$ yd 77. $\frac{4x}{7}$ 78. $\frac{3y}{11}$ 79. -2 80. $-7y$ 81. $\frac{15}{4}$ 82. $-\frac{5}{24}$ 83. $\frac{8}{13}$ 84. $-\frac{1}{27}$ 85. $\frac{29}{110}$
 86. $-\frac{1}{7}$ 87. $20\frac{7}{24}$ 88. $2\frac{51}{55}$; 3 89. $5\frac{1}{5}$; 6 90. $5\frac{1}{4}$ 91. 22 mi 92. $36\frac{2}{3}$ g 93. each measurement is $4\frac{1}{4}$ in. 94. $\frac{7}{10}$ yd 95. $-27\frac{5}{14}$

96. $-\frac{33}{40}$ 97. $1\frac{5}{27}$ 98. $-3\frac{15}{16}$ 99. $\frac{5}{6}$ 100. $-\frac{9}{10}$ 101. -10 102. -6 103. 15 104. 1 105. 5 106. -4 107. $\frac{1}{4}$ 108. $\frac{y^2}{2x}$
 109. $\frac{1}{5}$ 110. $-\frac{7}{12x}$ 111. $\frac{11x}{12}$ 112. $-\frac{23}{55}$ 113. $-6\frac{2}{5}$ 114. $12\frac{3}{8}; 13$ 115. $2\frac{19}{35}; 2$ 116. $\frac{22}{7}$ or $3\frac{1}{7}$ 117. $-\frac{1}{12}$ 118. $-\frac{9}{14}$
 119. $-\frac{4}{9}$ 120. -19 121. $44\frac{1}{2}$ yd 122. $40\frac{1}{2}$ sq ft

Chapter 4 Getting Ready for the Test 1. A 2. B 3. D 4. C 5. C 6. B 7. C 8. D 9. B 10. A 11. F
 12. H 13. B 14. D 15. C 16. A 17. D 18. C 19. D 20. A 21. B

- Chapter 4 Test** 1. $\frac{7}{16}$ 2. $\frac{23}{3}$ 3. $18\frac{3}{4}$ 4. $\frac{4}{35}$ 5. $-\frac{3x}{5}$ 6. not equivalent 7. equivalent 8. $2^2 \cdot 3 \cdot 7$ 9. $3^2 \cdot 5 \cdot 11$
 10. $\frac{4}{3}$ 11. $-\frac{4}{3}$ 12. $\frac{8x}{9}$ 13. $\frac{x-21}{7x}$ 14. y^2 15. $\frac{16}{45}$ 16. $\frac{9a+4}{10}$ 17. $-\frac{2}{3y}$ 18. $\frac{1}{a^2}$ 19. $\frac{3}{4}$ 20. $14\frac{1}{40}$ 21. $16\frac{8}{11}$
 22. $\frac{64}{3}$ or $21\frac{1}{3}$ 23. $22\frac{1}{2}$ 24. $-\frac{5}{3}$ or $-1\frac{2}{3}$ 25. $\frac{9}{16}$ 26. $\frac{3}{8}$ 27. $\frac{11}{12}$ 28. $\frac{3}{4x}$ 29. $\frac{76}{21}$ or $3\frac{13}{21}$ 30. -2 31. -4 32. 1 33. $\frac{5}{2}$ or $2\frac{1}{2}$
 34. $\frac{4}{31}$ 35. $3\frac{3}{4}$ ft 36. $\frac{23}{50}$ 37. $\frac{13}{50}$ 38. \$2820 39. perimeter: $3\frac{1}{3}$ ft; area: $\frac{2}{3}$ sq ft 40. 24 mi

Cumulative Review Chapters 1–4 1. five hundred forty-six; Sec. 1.2, Ex. 5 2. one hundred fifteen; Sec. 1.2 3. twenty-seven thousand, thirty-four; Sec. 1.2, Ex. 6 4. six thousand, five hundred seventy-three; Sec. 1.2 5. 759; Sec. 1.3, Ex. 1 6. 631; Sec. 1.3 7. 514; Sec. 1.3, Ex. 7 8. 933; Sec. 1.3 9. 278,000; Sec. 1.4, Ex. 2 10. 1440; Sec. 1.4 11. 57,600 megabytes; Sec. 1.5, Ex. 7 12. 1305 mi; Sec. 1.5 13. 7089 R 5; Sec. 1.6, Ex. 7 14. 379 R 10; Sec. 1.6 15. 7^3 ; Sec. 1.7, Ex. 1 16. 7^2 ; Sec. 1.7 17. $3^4 \cdot 9^3$; Sec. 1.7, Ex. 4 18. $9^4 \cdot 5^2$; Sec. 1.7 19. 6; Sec. 1.8, Ex. 2 20. 52; Sec. 1.8 21. -7188 ; Sec. 2.1, Ex. 1 22. -21 ; Sec. 2.1 23. -4 ; Sec. 2.2, Ex. 3 24. 5; Sec. 2.2 25. 3; Sec. 2.3, Ex. 9 26. 10; Sec. 2.3 27. 25; Sec. 2.4, Ex. 8 28. -16 ; Sec. 2.4 29. -16 ; Sec. 2.5, Ex. 8 30. 25; Sec. 2.5 31. $6y + 2$; Sec. 3.1, Ex. 2 32. $6x + 9$; Sec. 3.1 33. -14 ; Sec. 3.2, Ex. 4 34. -18 ; Sec. 3.2 35. -1 ; Sec. 3.3, Ex. 2 36. -11 ; Sec. 3.3 37. $\frac{2}{5}$; Sec. 4.1, Ex. 3 38. $2^2 \cdot 3 \cdot 13$; Sec. 4.2 39. a. $\frac{38}{9}$ b. $\frac{19}{11}$; Sec. 4.1, Ex. 20 40. $7\frac{4}{5}$; Sec. 4.1 41. $\frac{7x}{11}$; Sec. 4.2, Ex. 5 42. $\frac{2}{3y}$; Sec. 4.2 43. $2\frac{11}{12}$; Sec. 4.7, Ex. 2 44. $2\frac{2}{3}$; Sec. 4.7 45. $\frac{5}{12}$; Sec. 4.3, Ex. 11 46. $\frac{11}{56}$; Sec. 4.7

Chapter 5 Decimals

Section 5.1

Vocabulary, Readiness & Video Check 1. words; standard form 3. decimals 5. tenths; tens 7. as “and” 9. Reading a decimal correctly gives you the correct place value, which tells you the denominator of your equivalent fraction. 11. When rounding, we look to the digit to the right of the place value we’re rounding to. In this case we look to the hundredths-place digit, which is 7.

Exercise Set 5.1 1. five and sixty-two hundredths 3. sixteen and twenty-three hundredths 5. negative two hundred five thousandths 7. one hundred sixty-seven and nine thousandths 9. three thousand and four hundredths 11. one hundred five and six tenths 13. two and forty-three hundredths

15.

<i>Preprinted Name</i> <i>Preprinted Address</i>	<i>Current date</i> DATE
PAY TO THE ORDER OF <u>R.W. Financial</u> \$ 321.42 <u>Three hundred twenty-one and 42/100</u> DOLLARS	
FOR _____	Signature _____

17.

<i>Preprinted Name</i> <i>Preprinted Address</i>	<i>Current date</i> DATE
PAY TO THE ORDER OF <u>Verizon</u> \$ 91.68 <u>Ninety-one and 68/100</u> DOLLARS	
FOR _____	Signature _____

19. 2.8 21. 9.08 23. -705.625 25. 0.0046 27. $\frac{7}{10}$ 29. $\frac{27}{100}$ 31. $\frac{2}{5}$ 33. $5\frac{2}{5}$ 35. $-\frac{29}{500}$ 37. $7\frac{1}{125}$ 39. $15\frac{401}{500}$ 41. $\frac{601}{2000}$
 43. $0.8; \frac{8}{10}$ or $\frac{4}{5}$ 45. seventy-seven thousandths; $\frac{77}{1000}$ 47. $<$ 49. $<$ 51. $<$ 53. $=$ 55. $<$ 57. $>$ 59. $<$ 61. $>$ 63. 0.6
 65. 98,210 67. -0.23 69. 0.594 71. 3.1 73. 3.142 75. \$27 77. \$0.20 79. 0.7 in. 81. 1.92 min 83. \$68 85. 225 days 87. 5766
 89. 35 91. b 93. a 95. answers may vary 97. 712 99. $\frac{26,849,577}{100,000,000,000}$ 101. answers may vary 103. answers may vary
 105. 0.26499, 0.25786 107. 0.10299, 0.1037, 0.1038, 0.9 109. \$4200 million

Section 5.2

Calculator Explorations 1. 328.742 3. 5.2414 5. 865.392

Vocabulary, Readiness & Video Check 1. last 3. like 5. false 7. Lining up the decimal points also lines up place values, so we only add or subtract digits in the same place values. 9. So the subtraction can be written vertically with decimal points lined up. 11. perimeter

Exercise Set 5.2 1. 77 3. 10.35 5. 27.0578 7. -8.57 9. 10.33 11. 465.56; $\begin{array}{r} 230 \\ + 230 \\ \hline 460 \end{array}$ 13. 115.123; $\begin{array}{r} 100 \\ + 9 \\ \hline 109 \end{array}$ 15. 50.409 17. 4.4

 19. 15.3 21. 598.23 23. 1.83; $6 - 4 = 2$ 25. 876.6; $\begin{array}{r} 1000 \\ - 100 \\ \hline 900 \end{array}$ 27. 194.4 29. -6.32 31. -6.15 33. 3.1 35. 2.9988 37. 16.3

 39. 3.1 41. -5.62 43. 363.36 45. -549.8 47. 861.6 49. 216.114 51. 0.088 53. -180.44 55. -1.1 57. 3.81 59. 3.39
 61. 1.61 63. no 65. yes 67. no 69. $6.9x + 6.9$ 71. $3.47y - 10.97$ 73. \$7.52 75. $-\$0.42$ 77. 28.56 m 79. 16.16 in.
 81. 195.8 mph 83. 42.7 texts per day 85. \$2356 million 87. 326.3 in. 89. 67.44 ft 91. 4.234 mph 93. Switzerland

 95. 7.9 lb 97.

Country	Pounds of Chocolate per Person
Switzerland	19.8
Germany	17.4
UK	16.3
Ireland	16.2
Norway	14.6
Sweden	11.9

 99. 138 101. $\frac{4}{9}$ 103. incorrect; $\begin{array}{r} 9.200 \\ 8.630 \\ + 4.005 \\ \hline 21.835 \end{array}$ 105. 6.08 in. 107. \$1.20

109. 1 nickel, 1 dime, and 2 pennies; 3 nickels and 2 pennies; 1 dime and 7 pennies; 2 nickels and 7 pennies 111. answers may vary

 113. answers may vary 115. $22.181x - 22.984$

Section 5.3

Vocabulary, Readiness & Video Check 1. sum 3. right; zeros 5. circumference 7. Knowing whether we placed the decimal point correctly in our product 9. $3(5.7) - (-0.2)$ 11. This is an application problem and needs units attached. The complete answer is 24.8 grams.

Exercise Set 5.3 1. 1.36 3. 0.6 5. -17.595 7. 55.008 9. $28.56; 7 \times 4 = 28$ 11. 0.1041 13. 8.23854; $\begin{array}{r} 1 \\ \times 8 \\ \hline 8 \end{array}$ 15. 11.2746 17. 65

 19. 0.83 21. -7093 23. 70 25. 0.0983 27. 0.02523 29. 0.0492 31. 14,790 33. 1.29 35. -9.3762 37. 0.5623 39. 36.024
 41. 1,500,000,000 43. 49,800,000 45. -0.6 47. 173 49. no 51. yes 53. $10\pi \text{ cm} \approx 31.4 \text{ cm}$ 55. $18.2\pi \text{ yd} \approx 57.148 \text{ yd}$
 57. \$715.20 59. 24.8 g 61. 14.144 sq in. 63. $250\pi \text{ ft} \approx 785 \text{ ft}$ 65. $135\pi \text{ m} \approx 423.9 \text{ m}$ 67. 64.9605 in. 69. a. 62.8 m and 125.6 m
 b. yes 71. \$404.25 73. 9774 Canadian dollars 75. 16,406.16 Mexican pesos 77. 486 79. -9 81. 3.64 83. 3.56 85. -0.1105
 87. 3,831,600 mi 89. answers may vary 91. answers may vary

Section 5.4

Calculator Explorations 1. not reasonable 3. reasonable

Vocabulary, Readiness & Video Check 1. quotient; divisor; dividend 3. left; zeros 5. a whole number 7. We just need to know how to move the decimal point. 1000 has three zeros, so we move the decimal point in the decimal number three places to the left. 9. We want the answer rounded to the nearest tenth, so we go to one extra place value, to the hundredths place, in order to round.

Exercise Set 5.4 1. 4.6 3. 0.094 5. 300 7. 7.3 9. $6.6; 6\overline{)36}$ 11. 0.413 13. -600 15. 7 17. 4.8 19. 2100 21. 5.8 23. 5.5
 25. $9.8; 7\overline{)70}$ 27. 9.6 29. 45 31. 54.592 33. 0.0055 35. 23.87 37. 114.0 39. 0.83397 41. 2.687 43. -0.0129 45. 12.6
 47. 1.31 49. 0.045625 51. 0.413 53. -8 55. -7.2 57. 1400 59. 30 61. $-58,000$ 63. -0.69 65. 0.024 67. 65
 69. -5.65 71. -7.0625 73. yes 75. no 77. 11 qt 79. 5.1 m 81. 11.4 boxes 83. 24 tsp 85. 8 days 87. 146.6 mi per week
 89. 345.5 thousand books per hr 91. $\frac{21}{50}$ 93. $-\frac{1}{10}$ 95. 4.26 97. 1.578 99. -26.66 101. 904.29 103. c 105. b 107. 85.5
 109. 8.6 ft 111. answers may vary 113. $65.2-82.6$ knots 115. 273 m

Integrated Review 1. 2.57 2. 4.05 3. 8.9 4. 3.5 5. 0.16 6. 0.24 7. 0.27 8. 0.52 9. -4.8 10. 6.09 11. 75.56 12. 289.12
 13. -24.974 14. -43.875 15. -8.6 16. 5.4 17. -280 18. 1600 19. 224.938 20. 145.079 21. 0.56 22. -0.63 23. 27.6092
 24. 145.6312 25. 5.4 26. -17.74 27. -414.44 28. -1295.03 29. -34 30. -28 31. 116.81 32. 18.79 33. 156.2 34. 1.562
 35. 25.62 36. 5.62 37. Exact: 204.1 mi; Estimate: 200 mi 38. \$1.89 39. \$8.8 billion or \$8,800,000,000

Section 5.5

Vocabulary, Readiness & Video Check 1. false 3. false 5. We place a bar over just the repeating digits and only 6 repeats in our decimal answer. 7. The fraction bar serves as a grouping symbol 9. $4(0.3) - (-2.4)$

Exercise Set 5.5 1. 0.2 3. 0.68 5. 0.75 7. -0.08 9. 2.25 11. $0.91\overline{6}$ 13. 0.425 15. 0.45 17. $-0.\overline{3}$ 19. 0.4375 21. $0.\overline{63}$
 23. 5.85 25. 0.624 27. -0.33 29. 0.44 31. 0.6 33. 0.62 35. 0.86 37. 0.02 39. $<$ 41. $=$ 43. $<$ 45. $<$ 47. $<$ 49. $>$
 51. $<$ 53. $<$ 55. 0.32, 0.34, 0.35 57. 0.49, 0.491, 0.498 59. $5.23, \frac{42}{8}, 5.34$ 61. $0.612, \frac{5}{8}, 0.649$ 63. 0.59 65. -3 67. 5.29
 69. 9.24 71. 0.2025 73. -1.29 75. -15.4 77. -3.7 79. 25.65 sq in. 81. 0.248 sq yd 83. 5.76 85. 5.7 87. 3.6 89. $\frac{77}{50}$
 91. $\frac{5}{2}$ 93. $= 1$ 95. > 1 97. < 1 99. 0.057 101. 8200 stations 103. answers may vary

Section 5.6

Vocabulary, Readiness & Video Check 1. So that we are no longer working with decimals.

Exercise Set 5.6 1. 5.9 3. -0.43 5. 10.2 7. -4.5 9. 4 11. 0.45 13. 4.2 15. -4 17. 1.8 19. 10 21. 7.6 23. 60
 25. -0.07 27. 20 29. 0.0148 31. -8.13 33. 1.5 35. -1 37. -7 39. 7 41. 53.2 43. $3x - 16$ 45. $\frac{3}{5x}$ 47. $\frac{13x}{21}$ 49. 3.7
 51. $6x - 0.61$ 53. $-2y + 6.8$ 55. 9.1 57. -3 59. $-4z + 16.67$ 61. 15.7 63. 5.85 65. $-2.1z - 10.1$ 67. answers may vary
 69. answers may vary 71. 7.683 73. 4.683

Section 5.7

Vocabulary, Readiness & Video Check 1. average 3. mean (or average) 5. grade point average 7. Place the data numbers in numerical order (or verify that they already are)

Exercise Set 5.7 1. mean: 21; median: 23; no mode 3. mean: 8.1; median: 8.2; mode: 8.2 5. mean: 0.5; median: 0.5; mode: 0.2 and 0.5
 7. mean: 370.9; median: 313.5; no mode 9. 2109.2 ft 11. 1968.5 ft 13. answers may vary 15. 2.79 17. 3.64 19. 6.8 21. 6.9
 23. 88.5 25. 73 27. 70 and 71 29. 9 rates 31. mean: 3773 mi; median: 3812 mi 33. $\frac{3}{5y}$ 35. $\frac{11}{15}$ 37. 35, 35, 37, 43
 39. yes; answers may vary

Chapter 5 Vocabulary Check 1. decimal 2. numerator; denominator 3. vertically 4. and 5. sum 6. mode 7. circumference
 8. median; mean 9. mean 10. standard form

Chapter 5 Review 1. tenths 2. hundred-thousandths 3. negative twenty-three and forty-five hundredths 4. three hundred forty-five hundred-thousandths 5. one hundred nine and twenty-three hundredths 6. two hundred and thirty-two millionths

7. 8.06 8. -503.102 9. 16,025.0014 10. 14.011 11. $\frac{4}{25}$ 12. $-12\frac{23}{1000}$ 13. 0.00231 14. 25.25 15. $>$ 16. $=$ 17. $<$ 18. $>$
 19. 0.6 20. 0.94 21. -42.90 22. 16.349 23. 890,500,000 24. 600,000 25. 18.1 26. 5.1 27. -7.28 28. -12.04 29. 320.312
 30. 148.74236 31. 1.7 32. 2.49 33. -1324.5 34. -10.136 35. 65.02 36. 199.99802 37. 52.6 mi 38. -5.7 39. 22.2 in.
 40. 38.9 ft 41. 72 42. 9345 43. -78.246 44. 73,246.446 45. $14\pi \text{ m} \approx 43.96 \text{ m}$ 46. $20\pi \text{ in.} \approx 62.8 \text{ in.}$ 47. 0.0877 48. 15.825
 49. 70 50. -0.21 51. 8.059 52. 30.4 53. 0.02365 54. -9.3 55. 7.3 m 56. 45 months 57. 0.8 58. -0.923 59. $2.\overline{3}$ or 2.333
 60. $0.21\overline{6}$ or 0.217 61. $=$ 62. $<$ 63. $<$ 64. $<$ 65. 0.832, 0.837, 0.839 66. $\frac{5}{8}, 0.626, 0.685$ 67. $0.42, \frac{3}{7}, 0.43$ 68. $\frac{19}{12}, 1.63, \frac{18}{11}$
 69. -11.94 70. 3.89 71. 7.26 72. 0.81 73. 55 74. -129 75. 6.9 sq ft 76. 5.46 sq in. 77. 0.3 78. 92.81 79. 8.6 80. -80
 81. 1.98 82. -1.5 83. -20 84. 1 85. mean: 178; median: 14; no mode 86. mean: 58.1; median: 60; mode: 45 and 86 87. mean:
 24,500; median: 20,000; mode: 20,000 88. mean: 447.3; median: 420; mode: 400 89. 3.25 90. 2.57 91. two hundred and thirty-
 two ten-thousandths 92. -16.09 93. $\frac{847}{10,000}$ 94. $0.75, \frac{6}{7}, \frac{8}{9}$ 95. -0.07 96. 0.1125 97. 51.057 98. $>$ 99. $<$ 100. 86.91
 101. 3.115 102. \$123.00 103. \$3646.00 104. -1.7 105. 5.26 106. -12.76 107. -14.907 108. 8.128 109. -7.245 110. 4900
 111. 23.904 112. 9600 sq ft 113. yes 114. 0.1024 115. 3.6 116. mean: 74.4; median: 73; mode: none 117. mean: 619.17;
 median: 647.5; mode: 327

Chapter 5 Getting Ready for the Test 1. C 2. A 3. B 4. E 5. C 6. A 7. B 8. D 9. B 10. B 11. C 12. B
 13. C 14. A

Chapter 5 Test 1. forty-five and ninety-two thousandths 2. 3000.059 3. 17.595 4. -51.20 or -51.2 5. -20.42 6. 40.902
 7. 0.037 8. 34.9 9. 0.862 10. $<$ 11. $<$ 12. $\frac{69}{200}$ 13. $-24\frac{73}{100}$ 14. -0.5 15. 0.941 16. 1.93 17. -6.2 18. $0.5x - 13.4$

19. -3 20. 3.7 21. mean: 38.4; median: 42; no mode 22. mean: 12.625; median: 12.5; mode: 12 and 16 23. 3.07 24. 4,583,000,000
25. 2.31 sq mi 26. 18π mi $\approx$ 56.52 mi 27. a. 9904 sq ft b. 198.08 oz 28. 54 mi

Chapters 1–5 Cumulative Review 1. seventy-two; Sec. 1.2, Ex. 4 2. one hundred seven; Sec. 1.2 3. five hundred forty-six;
Sec. 1.2, Ex. 5 4. five thousand, twenty-six; Sec. 1.2 5. 759; Sec. 1.3, Ex. 1 6. 19 in.; Sec. 1.3 7. 514; Sec. 1.3, Ex. 7 8. 121 R 1;
Sec. 1.6 9. 278,000; Sec. 1.4, Ex. 2 10. $2 \cdot 3 \cdot 5$; Sec. 4.2 11. 20,296; Sec. 1.5, Ex. 4 12. 0; Sec. 1.5 13. a. 7 b. 12 c. 1 d. 1
e. 20 f. 1; Sec. 1.6, Ex. 2 14. 25; Sec. 1.6 15. 7; Sec. 1.7, Ex. 9 16. 49; Sec. 1.7 17. 81; Sec. 1.7, Ex. 5 18. 125; Sec. 1.7
19. 81; Sec. 1.7, Ex. 7 20. 1000; Sec. 1.7 21. 2; Sec. 1.8, Ex. 3 22. 6; Sec. 1.8 23. a. -13 b. 2 c. 0; Sec. 2.1, Ex. 5 24. a. 7
b. -4 c. 1; Sec. 2.1 25. -23; Sec. 2.2, Ex. 4 26. -22; Sec. 2.2 27. 180; Sec. 1.7, Ex. 8 28. 32; Sec. 1.7 29. -49; Sec. 2.4, Ex. 9
30. -32; Sec. 2.4 31. 25; Sec. 2.4, Ex. 8 32. -9; Sec. 2.4 33. $\frac{4}{3}; 1\frac{1}{3}$; Sec. 4.1, Ex. 10 34. $\frac{7}{4}; 1\frac{3}{4}$; Sec. 4.1 35. $\frac{11}{4}; 2\frac{3}{4}$; Sec. 4.1, Ex. 11
36. $\frac{14}{3}; 4\frac{2}{3}$; Sec. 4.1 37. $2^2 \cdot 3^2 \cdot 7$; Sec. 4.2, Ex. 3 38. 62; Sec. 1.3 39. $-\frac{36}{13}$; Sec. 4.2, Ex. 8 40. $\frac{79}{8}$; Sec. 4.1 41. equivalent;
Sec. 4.2, Ex. 10 42. $>$; Sec. 4.5 43. $\frac{10}{33}$; Sec. 4.3, Ex. 1 44. $1\frac{1}{2}$; Sec. 4.7 45. $\frac{1}{8}$; Sec. 4.3, Ex. 2 46. 37; Sec. 4.7 47. -24;
Sec. 3.2, Ex. 3 48. -8; Sec. 3.2 49. 829.6561; Sec. 5.2, Ex. 2 50. 230.8628; Sec. 5.2 51. 18.408; Sec. 5.3, Ex. 1 52. 28.251; Sec. 5.3

Chapter 6 Ratio, Proportion, and Triangle Applications

Section 6.1

Vocabulary, Readiness & Video Check 1. unit 3. division 5. numerator; denominator 7. false 9. The units are different in
Example 5 (shrubs and feet); they were the same in Example 4 (days). 11. When shopping for the best buy

Exercise Set 6.1 1. $\frac{2}{3}$ 3. $\frac{77}{100}$ 5. $\frac{463}{821}$ 7. $\frac{3}{8}$ 9. $\frac{8}{25}$ 11. $\frac{12}{7}$ 13. $\frac{2}{7}$ 15. $\frac{4}{1}$ 17. $\frac{10}{29}$ 19. $\frac{25}{144}$ 21. $\frac{5}{4}$ 23. $\frac{15}{1}$ 25. $\frac{2}{3}$ 27. $\frac{10}{21}$
29. $\frac{17}{40}$ 31. $\frac{124}{67}$ 33. $\frac{1 \text{ shrub}}{3 \text{ ft}}$ 35. $\frac{3 \text{ returns}}{20 \text{ sales}}$ 37. $\frac{3 \text{ laser printers}}{14 \text{ computers}}$ 39. $\frac{9 \text{ gal}}{2 \text{ acres}}$ 41. 110 cal/oz 43. 75 riders/car
45. 90 wingbeats/sec 47. \$50,000/yr 49. 1,655,975 voters/senator 51. 300 good/defective 53. \$4,517,559/player 55. a. 31.25
computer boards/hr b. 33.5 computer boards/hr c. Suellen 57. a. $\approx$ 27.6 miles/gal b. $\approx$ 29.2 miles/gal c. the truck
59. \$11.50 per compact disc 61. \$0.17 per banana 63. 8 oz: \$0.411 per oz; 12 oz: \$0.399 per oz; 12 oz 65. 16 oz: \$0.118 per oz;
6 oz: \$0.115 per oz; 6 oz 67. 12 oz: \$0.191 per oz; 8 oz: \$0.186 per oz; 8 oz 69. 100: \$0.006 per napkin; 180: \$0.005 per napkin;
180 napkins 71. 2.3 73. 0.15 75. no; answers may vary 77. no; $\frac{2}{5}$ 79. yes, the machine should be repaired 81. 257; 19.2
83. 347; 21.6 85. 1.5 steps/ft 87. a. $\frac{19}{50}$ b. $\frac{19}{31}$ c. no; answers may vary 89. answers may vary 91. no; answers may vary

Section 6.2

Vocabulary, Readiness & Video Check 1. proportion; ratio 3. true 5. equals or = 7. It is a ratio equal to a ratio

Exercise Set 6.2 1. $\frac{10 \text{ diamonds}}{6 \text{ opals}} = \frac{5 \text{ diamonds}}{3 \text{ opals}}$ 3. $\frac{20 \text{ students}}{5 \text{ microscopes}} = \frac{4 \text{ students}}{1 \text{ microscope}}$ 5. $\frac{6 \text{ eagles}}{58 \text{ sparrows}} = \frac{3 \text{ eagles}}{29 \text{ sparrows}}$
7. $\frac{2\frac{1}{4} \text{ cups flour}}{24 \text{ cookies}} = \frac{6\frac{3}{4} \text{ cups flour}}{72 \text{ cookies}}$ 9. $\frac{22 \text{ vanilla wafers}}{1 \text{ cup cookie crumbs}} = \frac{55 \text{ vanilla wafers}}{2.5 \text{ cups cookie crumbs}}$ 11. true 13. false 15. true 17. true
19. false 21. true 23. true 25. false 27. true 29. $\frac{10}{15} = \frac{4}{6}$; true 31. $\frac{11}{4} = \frac{5}{2}$; false 33. $\frac{0.15}{3} = \frac{0.35}{7}$; true 35. $\frac{\frac{2}{3}}{\frac{1}{5}} = \frac{\frac{2}{5}}{\frac{1}{9}}$; false
37. 3 39. -9 41. 5 43. 3.2 45. 38.4 47. 25 49. 0.0025 51. 1 53. $\frac{9}{20}$ 55. 12 57. $\frac{3}{4}$ 59. $\frac{35}{18}$ 61. 14.9 63. 0.07 65. 1.9
67. 3.163 69. $<$ 71. $>$ 73. $\frac{3}{5}$ 75. $\frac{2x}{7}$ 77. $\frac{9}{3} = \frac{15}{5}$; $\frac{5}{15} = \frac{3}{9}$; $\frac{15}{9} = \frac{5}{3}$ 79. $\frac{6}{1} = \frac{18}{3}$; $\frac{3}{18} = \frac{1}{6}$; $\frac{18}{6} = \frac{3}{1}$ 81. possible
answers: $\frac{d}{b} = \frac{c}{a}$; $\frac{a}{c} = \frac{b}{d}$; $\frac{b}{d} = \frac{a}{c}$ 83. answers may vary 85. 0 87. 1400 89. 252.5

Integrated Review 1. $\frac{9}{10}$ 2. $\frac{9}{25}$ 3. $\frac{47}{50}$ 4. $\frac{8}{23}$ 5. $\frac{173}{139}$ 6. $\frac{6}{7}$ 7. $\frac{7}{26}$ 8. $\frac{20}{33}$ 9. $\frac{2}{3}$ 10. $\frac{1}{8}$ 11. $\frac{2}{3}$ 12. a. 185
b. $\frac{37}{227}$ 13. $\frac{1 \text{ professor}}{5 \text{ graduate assistants}}$ 14. $\frac{3 \text{ lights}}{10 \text{ ft}}$ 15. $\frac{2 \text{ Senators}}{1 \text{ state}}$ 16. $\frac{1 \text{ teacher}}{28 \text{ students}}$ 17. $\frac{3 \text{ inches}}{1 \text{ second}}$ 18. $\frac{\$8}{1 \text{ hour}}$

19. $\frac{19 \text{ households with computers}}{25 \text{ households}}$ 20. $\frac{269 \text{ electoral votes}}{25 \text{ states}}$ 21. 140 ft/sec 22. 65 mi/hr 23. 21 employees/fax line
 24. 17 phone calls/teenager 25. 26 mi/gal 26. 16 teachers/computer 27. 6.5 books/student 28. 165 lb/adult 29. 8 lb: \$0.27 per lb;
 18 lb: \$0.277 per lb; 8 lb 30. 100: \$0.020 per plate; 500: \$0.018 per plate; 500 paper plates 31. 3 packs: \$0.797 per pack; 8 packs:
 \$0.749 per pack; 8 packs 32. 4: \$1.173 per battery; 10: \$1.489 per battery; 4 batteries 33. no 34. yes 35. 24 36. 32.5
 37. $2.\overline{72}$ or $2\frac{8}{11}$ 38. 18

Section 6.3

Vocabulary, Readiness & Video Check 1. ones

- Exercise Set 6.3** 1. 360 baskets 3. 165 min 5. 630 applications 7. 23 ft 9. 270 sq ft 11. 25 gal 13. 450 km 15. 16 bags
 17. 15 hits 19. 27 people 21. 18 applications 23. 5 weeks 25. $10\frac{2}{3}$ servings 27. 37.5 seconds 29. a. 18 tsp
 b. 6 tbsp 31. 6 people 33. 112 ft; 11-in. difference 35. 102.9 mg 37. 1257 ft; The actual height of the Empire State Building is
 1248 ft. 39. 434 emergency room visits 41. 28 workers 43. 2.4 c 45. a. 0.1 gal b. 13 fl oz 47. a. 2062.5 mg b. no
 49. $2^3 \cdot 5^2$ 51. 2^5 53. 0.8 ml 55. 1.25 ml 57. $11 \approx 12$ or 1 dozen; $1.5 \times 8 = 12$; 12 cups of milk 59. $4\frac{2}{3}$ ft 61. answers may vary

Section 6.4

Calculator Explorations 1. 32 3. 3.873 5. 9.849

Vocabulary, Readiness & Video Check 1. 10; -10 3. radical 5. perfect squares 7. $c^2; b^2$ 9. The square roots of 49 are 7 and
 -7 since $7^2 = 49$ and $(-7)^2 = 49$. The radical sign means the positive square root only, so $\sqrt{49} = 7$. 11. The hypotenuse is the side
 across from the right angle.

- Exercise Set 6.4** 1. 2 3. 11 5. $\frac{1}{9}$ 7. $\frac{4}{8} = \frac{1}{2}$ 9. 1.732 11. 3.873 13. 5.568 15. 5.099 17. 6, 7 19. 10, 11 21. 16 23. 9.592
 25. $\frac{7}{12}$ 27. 8.426 29. 13 in. 31. 6.633 cm 33. 52.802 m 35. 117 mm 37. 5 39. 12 41. 17205 43. 44.822 45. 42.426
 47. 1.732 49. 8.5 51. 141.42 yd 53. 25.0 ft 55. 340 ft 57. $\frac{5}{6}$ 59. $\frac{x}{30}$ 61. $\frac{21}{13y}$ 63. $\frac{9x}{64}$ 65. 6 67. 10 69. answers may vary
 71. yes 73. $(\sqrt{80} - 6)$ in. ≈ 2.94 in.

Section 6.5

Vocabulary, Readiness & Video Check 1. false 3. true 5. false 7. The ratios of corresponding sides are the same.

- Exercise Set 6.5** 1. congruent; SSS 3. not congruent 5. congruent; ASA 7. congruent; SAS 9. $\frac{2}{1}$ 11. $\frac{3}{2}$ 13. 4.5 15. 6
 17. 5 19. 13.5 21. 17.5 23. 10 25. 28.125 27. 10 29. 520 ft 31. 500 ft 33. 14.4 ft 35. 52 neon tetras 37. 381 ft 39. 4.01
 41. -1.23 43. $3\frac{8}{9}$ in.; no 45. 8.4 47. answers may vary 49. 200 ft, 300 ft, 425 ft

Chapter 6 Vocabulary Check 1. ratio 2. proportion 3. unit rate 4. unit price 5. rate 6. cross products 7. equal
 8. not equal 9. Congruent 10. Similar 11. leg 12. leg 13. hypotenuse 14. right 15. Pythagorean

- Chapter 6 Review** 1. $\frac{23}{37}$ 2. $\frac{11}{13}$ 3. $\frac{17}{35}$ 4. $\frac{18}{35}$ 5. $\frac{9}{4}$ 6. $\frac{2}{13}$ 7. $\frac{5 \text{ people}}{2 \text{ pets}}$ 8. $\frac{5 \text{ pages}}{2 \text{ min}}$ 9. 52 mi/hr 10. 15 ft/sec
 11. \$1.74/diskette 12. 13 bushels/tree 13. 8 oz: \$0.124 per oz; 12 oz: \$0.141 per oz; 8-oz size 14. 18 oz: \$0.083; 28 oz: \$0.085; 18-oz size
 15. $\frac{24 \text{ uniforms}}{8 \text{ players}} = \frac{3 \text{ uniforms}}{1 \text{ player}}$ 16. $\frac{12 \text{ tires}}{3 \text{ cars}} = \frac{4 \text{ tires}}{1 \text{ car}}$ 17. no 18. yes 19. 5 20. 21 21. -5.625 22. 60 23. 0.94 24. $1\frac{1}{8}$
 25. 0.63 26. 30.9 27. 14 28. 35 29. 8 bags 30. 16 bags 31. $213\frac{1}{3}$ mi 32. 9.6 in. 33. 8 34. 12 35. 3.464 36. 3.873
 37. 0 38. 1 39. 7.071 40. 8.062 41. $\frac{2}{5}$ 42. $\frac{1}{10}$ 43. 13 44. 29 45. 10.7 46. 55.1 47. 28.28 cm 48. 88.2 ft 49. congruent; ASA
 50. not congruent 51. $13\frac{1}{3}$ 52. 17.4 53. 33 ft 54. $x = \frac{5}{6}$ in.; $y = 2\frac{1}{6}$ in. 55. $\frac{3}{5}$ 56. $\frac{1}{27}$ 57. $\frac{1 \text{ teacher}}{9 \text{ students}}$ 58. $\frac{1 \text{ nurse}}{4 \text{ patients}}$
 59. 34 mi/hr 60. 2 gal/cow 61. $\frac{14}{1051}$ 62. 166 steps/min 63. 4 oz: \$1.235 per oz; 8 oz: \$1.248 per oz; 4-oz size 64. 12 oz: \$0.054
 per oz; 64 oz: \$0.047 per oz; 64-oz size 65. $\frac{2 \text{ cups cookie dough}}{30 \text{ cookies}} = \frac{4 \text{ cups cookie dough}}{60 \text{ cookies}}$ 66. $\frac{5 \text{ nickels}}{3 \text{ dollars}} = \frac{20 \text{ nickels}}{12 \text{ dollars}}$ 67. 1.6

68. 25 69. 3.75 70. $\frac{2}{5}$ 71. 6 72. $\frac{4}{9}$ 73. 10.247 74. 5.657 75. 86.6 76. 20.8 77. 12 78. $6\frac{1}{2}$

Chapter 6 Getting Ready for the Test 1. A 2. A 3. D 4. D 5. A 6. B 7. B 8. A

Chapter 6 Test 1. $\frac{9}{13}$ 2. $\frac{3 \text{ in.}}{10 \text{ days}}$ 3. $\frac{43}{50}$ 4. $\frac{47}{78}$ 5. $\frac{293}{93}$ 6. 81.25 km/hr 7. 28 students/teacher 8. 16 in./ozmin 9. 8 oz;
\$0.149 per oz; 12 oz: \$0.158 per oz; 8-oz size 10. 16 oz: \$0.093 per oz; 24 oz: \$0.100 per oz; 16-oz size 11. true 12. false 13. 5
14. $4\frac{4}{11}$ 15. $\frac{7}{3}$ 16. 8 17. $49\frac{1}{2}$ ft 18. $3\frac{3}{4}$ hr 19. $53\frac{1}{3}$ g 20. 7 21. 12.530 22. $\frac{8}{10} = \frac{4}{5}$ 23. 5.66 cm 24. 7.5 25. 69 ft

Cumulative Review Chapters 1–6 1. a. 3 b. 15 c. 0 d. 70; Sec. 1.3, Ex. 5 2. a. 0 b. 20 c. 0 d. 20; Sec. 1.5
3. 249,000; Sec. 1.4, Ex. 3 4. 249,000; Sec. 1.4 5. a. 200 b. 1230; Sec. 1.5, Ex. 3 6. 373 R 24; Sec. 1.6 7. -8; Sec. 2.2, Ex. 15
8. -84; Sec. 2.4 9. $2^4 \cdot 5$; Sec. 4.2, Ex. 2 10. 8; Sec. 1.7 11. $\frac{3}{5}$; Sec. 4.2, Ex. 4 12. 243; Sec. 1.7 13. $\frac{2}{5}$; Sec. 4.3, Ex. 6 14. $15\frac{3}{8}$; Sec. 4.7
15. 2; Sec. 4.4, Ex. 3 16. $\frac{4}{5}$; Sec. 4.4 17. 14; Sec. 4.4, Ex. 12 18. $\frac{49}{50}$; Sec. 4.5 19. $\frac{15}{20}$; Sec. 4.4, Ex. 17 20. yes; Sec. 4.2 21. $-\frac{8}{33}$;
Sec. 4.5, Ex. 4 22. $7\frac{47}{72}$; Sec. 4.7 23. $\frac{1}{6}$ hr; Sec. 4.5, Ex. 11 24. 27; Sec. 1.7 25. $7\frac{17}{24}$; Sec. 4.7, Ex. 9 26. $\frac{16}{27}$; Sec. 4.6 27. <; Sec. 4.5,
Ex. 7 28. -8; Sec. 3.2 29. -3; Sec. 3.2, Ex. 1 30. 0; Sec. 3.3 31. 5; Sec. 3.3, Ex. 1 32. 5; Sec. 3.3 33. 2; Sec. 3.3, Ex. 4 34. 0.075;
Sec. 5.1 35. 736.2; Sec. 5.1, Ex. 15 36. 736.236; Sec. 5.1 37. 25.454; Sec. 5.2, Ex. 1 38. 681.24; Sec. 5.2 39. 0.8496; Sec. 5.3, Ex. 2
40. 0.375; Sec. 5.5 41. -0.052; Sec. 5.4, Ex. 3 42. $\frac{79}{10}$; Sec. 5.5 43. -3.7; Sec. 5.5, Ex. 12 44. 3; Sec. 6.2 45. $\frac{4}{9}, \frac{9}{20}, 0.456$; Sec. 5.5,
Ex. 10 46. 140 m/sec; Sec. 6.1 47. $\frac{3}{2}$; Sec. 6.1, Ex. 2 48. $\frac{1}{3}$; Sec. 6.1 49. $\frac{50}{63}$; Sec. 6.1, Ex. 3 50. $\frac{1}{10}$; Sec. 6.1

Chapter 7 Percent

Section 7.1

Vocabulary, Readiness & Video Check 1. Percent 3. percent 5. 0.01 7. Percent means “per 100.” 9. 1

Exercise Set 7.1 1. 96% 3. football; 37% 5. 50% 7. 0.41 9. 0.06 11. 1.00 or 1 13. 0.736 15. 0.028 17. 0.006
19. 3.00 or 3 21. 0.3258 23. $\frac{2}{25}$ 25. $\frac{1}{25}$ 27. $\frac{9}{200}$ 29. $\frac{7}{4}$ or $1\frac{3}{4}$ 31. $\frac{1}{16}$ 33. $\frac{31}{300}$ 35. $\frac{179}{800}$ 37. 22% 39. 0.6% 41. 530%
43. 5.6% 45. 22.28% 47. 300% 49. 70% 51. 70% 53. 80% 55. 68% 57. $37\frac{1}{2}\%$ 59. $33\frac{1}{3}\%$ 61. 450% 63. 190%
65. 81.82% 67. 26.67% 69. $0.6; \frac{3}{5}; 23\frac{1}{2}\%; \frac{47}{200}; 80\%; 0.8; 0.333\bar{3}; \frac{1}{3}; 87.5\%; 0.875; 0.075; \frac{3}{40}$ 71. 2; 2; 280%; $2\frac{4}{5}; 705; 7\frac{1}{20}; 454\%; 4.54$
73. $0.38; \frac{19}{50}$ 75. $0.99; \frac{99}{100}$ 77. $0.492; \frac{123}{250}$ 79. $0.005; \frac{1}{200}$ 81. $0.162; \frac{81}{500}$ 83. 78.1% 85. 0.7% 87. 0.47 89. $\frac{11}{36}$ 91. $1\frac{5}{6}$
93. a. 52.9% b. 52.86% 95. b, d 97. 4% 99. 75% 101. greater 103. 0.266; 26.6% 105. occupational therapy assistant
107. 0.30 or 0.3 109. answers may vary

Section 7.2

Vocabulary, Readiness & Video Check 1. is 3. amount; base; percent 5. greater 7. “of” translates to multiplication; “is” (or something equivalent) translates to an equal sign; “what” or “unknown” translates to our variable

Exercise Set 7.2 1. $18\% \cdot 81 = x$ 3. $20\% \cdot x = 105$ 5. $0.6 = 40\% \cdot x$ 7. $x \cdot 80 = 3.8$ 9. $x = 9\% \cdot 43$ 11. $x \cdot 250 = 150$
13. 3.5 15. 28.7 17. 10 19. 600 21. 110% 23. 34% 25. 1 27. 645 29. 500 31. 5.16% 33. 25.2 35. 35% 37. 35
39. 0.624 41. 0.5% 43. 145 45. 63% 47. 4% 49. 30 51. $3\frac{7}{11}$ 53. $\frac{17}{12} = \frac{x}{20}$ 55. $\frac{8}{9} = \frac{14}{x}$ 57. c 59. b
61. answers may vary 63. b 65. c 67. c 69. a 71. a 73. answers may vary 75. 686.625 77. 12,285

Section 7.3

Vocabulary, Readiness & Video Check 1. amount; base; percent 3. amount 5. 45 follows the word “of” so it is the base

Exercise Set 7.3 1. $\frac{a}{45} = \frac{98}{100}$ 3. $\frac{a}{150} = \frac{4}{100}$ 5. $\frac{14.3}{b} = \frac{26}{100}$ 7. $\frac{84}{b} = \frac{35}{100}$ 9. $\frac{70}{400} = \frac{p}{100}$ 11. $\frac{8.2}{82} = \frac{p}{100}$ 13. 26
15. 18.9 17. 600 19. 10 21. 120% 23. 28% 25. 37 27. 1.68 29. 1000 31. 210% 33. 55.18 35. 45% 37. 75
39. 0.864 41. 0.5% 43. 140 45. 9.6 47. 113% 49. $-\frac{7}{8}$ 51. $3\frac{2}{15}$ 53. 0.7 55. 2.19 57. answers may vary
59. no; $a = 16$ 61. yes 63. answers may vary 65. 12,011.2 67. 7270.6

Integrated Review 1. 94% 2. 17% 3. 37.5% 4. 350% 5. 470% 6. 800% 7. 45% 8. 106% 9. 675% 10. 325%
 11. 2% 12. 6% 13. 0.71 14. 0.31 15. 0.03 16. 0.04 17. 2.24 18. 7 19. 0.029 20. 0.066 21. $0.07; \frac{7}{100}$ 22. $0.05; \frac{1}{20}$
 23. $0.068; \frac{17}{250}$ 24. $0.1125; \frac{9}{80}$ 25. $0.74; \frac{37}{50}$ 26. $0.45; \frac{9}{20}$ 27. $0.163; \frac{49}{300}$ 28. $0.127; \frac{19}{150}$ 29. 13.5 30. 100 31. 350 32. 120%
 33. 28% 34. 76 35. 34 36. 130% 37. 46% 38. 378 39. 150 40. 62

Section 7.4

Vocabulary, Readiness & Video Check 1. The price of the home is \$175,000.

Exercise Set 7.4 1. 1600 bolts 3. 8.8 lb 5. 14% 7. 16,748 screens 9. 38% 11. 496 chairs; 5704 chairs
 13. 59,917 occupational therapy assistants 15. 1,843,920 17. 30% 19. 50% 21. 12.5% 23. 29.2% 25. \$175,000 27. 31.2 hr
 29. \$867.87; \$20,153.87 31. 35 ft 33. increase: \$1043; tuition in 2016–2017: \$10,037 35. increase: 1,494,100 enrolled in associate
 degrees programs; 8,194,100 projected enrollment in 2024–2025 37. 30; 60% 39. 52; 80% 41. 2; 25% 43. 120; 75%
 45. 44% 47. 137.5% 49. 374.6% 51. 9.8% 53. 14.8% 55. 1.9% 57. 141.4% 59. 51.4% 61. 4.56 63. 11.18 65. $\frac{1}{24}$
 67. $\frac{28}{39}$ 69. The increased number is double the original number. 71. 331,778 73. 93.0% 75. 324,228,700 77. answers may vary
 79. percent of increase = $\frac{30}{150} = 20\%$ 81. False; the percents are different.

Section 7.5

Vocabulary, Readiness & Video Check 1. sales tax 3. commission 5. sale price 7. We write the commission rate as a percent.

Exercise Set 7.5 1. \$750 3. \$858.93 5. 7% 7. a. \$120 b. \$130.20 9. \$117; \$1917 11. \$485 13. 6% 15. \$16.10; \$246.10
 17. \$53,176.04 19. 14% 21. \$4888.50 23. \$185,500 25. \$8.90; \$80.10 27. \$98.25; \$98.25 29. \$143.50; \$266.50 31. \$3255;
 \$18,445 33. \$45; \$255 35. \$2745; \$332.45 37. \$3.08; \$59.08 39. \$7074 41. 8% 43. 1200 45. 132 47. 16 49. d
 51. \$4.00; \$6.00; \$8.00 53. \$720; \$10.80; \$14.40 55. a discount of 60% is better; answers may vary 57. \$26,838.45

Section 7.6

Calculator Explorations 1. \$936.31 3. \$9674.77 5. \$634.49

Vocabulary, Readiness & Video Check 1. simple 3. Compound 5. Total amount 7. principal

Exercise Set 7.6 1. \$32 3. \$73.60 5. \$750 7. \$33.75 9. \$700 11. \$101,562.50; \$264,062.50 13. \$5562.50 15. \$14,280
 17. \$46,815.37 19. \$2327.14 21. \$58,163.65 23. \$2915.75 25. \$2938.66 27. \$2971.89 29. 32 yd 31. 35 m 33. $\frac{9x}{20}$ 35. $-\frac{131}{225}$
 37. answers may vary 39. answers may vary

Chapter 7 Vocabulary Check 1. of 2. is 3. Percent 4. Compound interest 5. $\frac{\text{amount}}{\text{base}}$ 6. 100% 7. 0.01

8. $\frac{1}{100}$ 9. base; amount 10. percent of decrease 11. percent of increase 12. sales tax 13. total price 14. commission
 15. amount of discount 16. sale price

Chapter 7 Review 1. 37% 2. 77% 3. 0.26 4. 0.75 5. 0.035 6. 0.015 7. 2.75 8. 4.00 or 4 9. 0.4785 10. 0.8534

11. 160% 12. 5.5% 13. 76% 14. 8.5% 15. 71% 16. 65% 17. 600% 18. 900% 19. $\frac{7}{100}$ 20. $\frac{3}{20}$ 21. $\frac{1}{4}$ 22. $\frac{17}{200}$
 23. $\frac{51}{500}$ 24. $\frac{1}{6}$ 25. $\frac{1}{3}$ 26. $1\frac{1}{10}$ 27. 40% 28. 70% 29. $58\frac{1}{3}\%$ 30. $166\frac{2}{3}\%$ 31. 125% 32. 60% 33. 6.25% 34. 62.5%
 35. 100,000 36. 8000 37. 23% 38. 114.5 39. 108.8 40. 150% 41. 418 42. 300 43. 159.6 44. 180% 45. 110%
 46. 165 47. 66% 48. 16% 49. 20.9% 50. 106.25% 51. \$13.23 52. \$206,400 53. \$273.75 54. \$2.17 55. \$5000
 56. \$300.38 57. discount: \$900; sale price: \$2100 58. discount: \$9; sale price: \$81 59. \$160 60. \$325 61. \$30,104.61
 62. \$17,506.54 63. \$180.61 64. \$33,830.10 65. 0.038 66. 1.245 67. 54% 68. 9520% 69. $\frac{47}{100}$ 70. $\frac{7}{125}$ 71. 12.5%
 72. 120% 73. 268.75 74. 110% 75. 708.48 76. 134% 77. 300% 78. 38.4 79. 560 80. 325% 81. 26% 82. \$6786.50
 83. \$61770 84. \$3.45 85. 12.5% 86. \$1491 87. \$11,687.50

Chapter 7 Getting Ready for the Test 1. D 2. A 3. B 4. C 5. B 6. D 7. A 8. C 9. C 10. A 11. D 12. C

Chapter 7 Test 1. 0.85 2. 5 3. 0.006 4. 5.6% 5. 610% 6. 35% 7. $1\frac{1}{5}$ 8. $\frac{77}{200}$ 9. $\frac{1}{500}$ 10. 55% 11. 37.5%

12. 175% 13. 25% 14. $\frac{9}{20}$ 15. 33.6 16. 1250 17. 75% 18. 38.4 lb 19. \$56,750 20. \$383.21 21. 5% 22. discount: \$18;
 sale price: \$102 23. \$395 24. 9% 25. \$647.50 26. \$2005.63 27. \$427 28. 2.3%

- Cumulative Review Chapters 1–7**
- 20,296; Sec. 1.5, Ex. 4
 - 31,084; Sec. 1.5
 - 10; Sec. 2.3, Ex. 8
 - 10; Sec. 2.3
 - 1; Sec. 2.6, Ex. 2
 - 1; Sec. 2.6
 - 2; Sec. 3.3, Ex. 4
 - 5; Sec. 3.3
 - $\frac{21}{7}$; Sec. 4.4, Ex. 20
 - $\frac{40}{5}$; Sec. 4.4
 - $-\frac{10}{27}$; Sec. 4.2, Ex. 6
 - $\frac{5y}{16}$; Sec. 4.2
 - $\frac{7}{10}$; Sec. 4.3, Ex. 13
 - $-\frac{4}{7}$; Sec. 4.3
 - $-\frac{1}{2}$; Sec. 4.4, Ex. 9
 - $\frac{1}{5}$; Sec. 4.4
 - $\frac{1}{28}$; Sec. 4.5, Ex. 5
 - $\frac{16}{45}$; Sec. 4.5
 - $\frac{3}{2}$; Sec. 4.6, Ex. 2
 - $\frac{50}{9}$; Sec. 4.6
 - 3; Sec. 4.8, Ex. 9
 - 4; Sec. 4.8
 - a. $\frac{38}{9}$ b. $\frac{19}{11}$; Sec. 4.1, Ex. 20
 - a. $\frac{17}{5}$ b. $\frac{44}{7}$; Sec. 4.1
 - $\frac{1}{8}$; Sec. 5.1, Ex. 9
 - $\frac{17}{20}$; Sec. 5.1
 - 105 $\frac{83}{1000}$; Sec. 5.1, Ex. 11
 - $17\frac{3}{200}$; Sec. 5.1
 - 67.69; Sec. 5.2, Ex. 6
 - 2794; Sec. 5.2
 - 76.8; Sec. 5.3, Ex. 5
 - 1248.3; Sec. 5.3
 - 76,300; Sec. 5.3, Ex. 7
 - 8537.5; Sec. 5.3
 - 50; Sec. 5.4, Ex. 10
 - no; Sec. 5.4
 - 80.5; Sec. 5.7, Ex. 5
 - 48; Sec. 5.7
 - $\frac{50}{63}$; Sec. 6.1, Ex. 3
 - $\frac{29}{38}$; Sec. 6.1
 - \$0.21/oz; Sec. 6.1, Ex. 11
 - \$2.25 per sq ft; Sec. 6.1
 - no; Sec. 6.2, Ex. 3
 - yes; Sec. 6.2
 - 17.5 mi; Sec. 6.3, Ex. 1
 - 35; Sec. 6.3
 - $\frac{19}{1000}$; Sec. 7.1, Ex. 9
 - $\frac{23}{1000}$; Sec. 7.1
 - $\frac{1}{3}$; Sec. 7.1, Ex. 11
 - $1\frac{2}{25}$; Sec. 7.1

Chapter 8 Graphing and Introduction to Statistics and probability

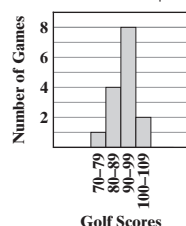
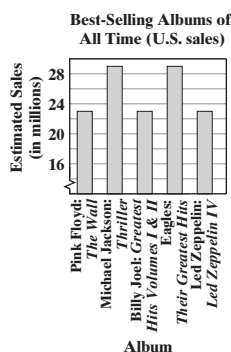
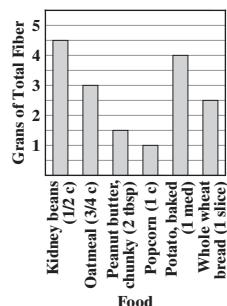
Section 8.1

Vocabulary, Readiness & Video Check

- bar
- line
- range
- Count the number of symbols and multiply this number by how much each symbol stands for (from the key).
- bar graph
- answers may vary; for example: 6, 6, 6, 6.

- Exercise Set 8.1**
- Kansas
 - 5.5 million or 5,500,000 acres
 - South Dakota, Colorado, and Washington
 - Montana
 - 48,000
 - 2016
 - 18,000
 - 60,000 wildfires/year
 - September
 - 78
 - $\frac{1}{39}$
 - Tokyo, Japan; about 38 million or 38,000,000
 - New York; 20.6 million or 20,600,000
 - approximately 3 million

29. **Fiber Content of Selected Foods**
31. **Best-Selling Albums of All Time (U.S. sales)**
33. 15 adults
35. 61 adults
37. 24 adults
39. 12 adults
41. $\frac{9}{100}$
43. 55–64
45. 24 million householders
47. 47 million householders
49. 2 million householders
51. |; 1
53. ||||; 8
55. ||||; 6
57. ||||; 6
59. ||; 2
61. **Golf Scores**
63. 8 goals/game
65. 2003
67. increase
69. 2001, 2007, 2013
71. 12
73. 350
75. 2
77. 1.7
79. a. 8.2 b. 8 c. 9
81. a. 6.1 b. 6 c. 6, 7, 8
83. a. 15 b. 15 c. 15
85. a. 6.1 b. 6 c. 4, 6
87. 3.6
89. 6.2
91. 25%
93. 34%
95. 83°F
97. Sunday; 68°F
99. Tuesday; 13°F
101. answers may vary

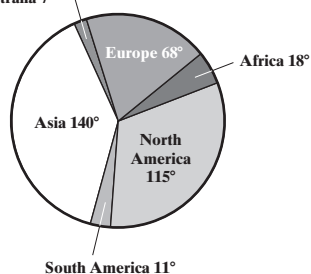
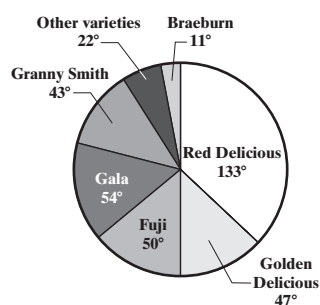


Section 8.2

Vocabulary, Readiness & Video Check

- circle
- 360
- 100%

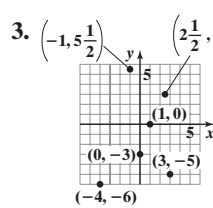
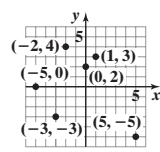
- Exercise Set 8.2**
- parent or guardian's home
 - $\frac{9}{35}$
 - $\frac{9}{16}$
 - Asia
 - 37%
 - 17,100,000 sq mi
 - 2,850,000 sq mi
 - 55%
 - nonfiction
 - 31,400 books
 - 27,632 books
 - 25,120 books
 25. Other varieties 22° Braeburn 11°
 27. Australia 7°
 29. $2^2 \times 5$
 31. $2^3 \times 5$
 33. 5×17
 35. answers may vary
 - 129,600,002 sq km
 39. 55,542,858 sq km
 41. 672 respondents
 43. 2408 respondents
 45. $\frac{12}{31}$
 47. no; answers may vary



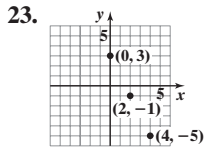
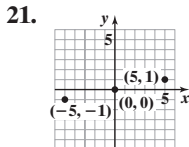
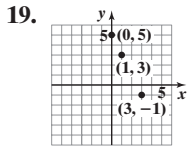
Section 8.3

Vocabulary, Readiness & Video Check 1. x, y 3. four 5. plotting 7. plane 9. right; coordinate 11. -7

Exercise Set 8.3



5. $A(0, 0); B(3\frac{1}{2}, 0); C(3, 2); D(-1, 3); E(-2, -2); F(0, -1); G(2, -1)$
 7. yes 9. no 11. yes 13. yes 15. yes 17. no



25. $(1, -9), (0, 0), (2, -18)$ 27. $(2, -12), (6, -8), (0, -14)$

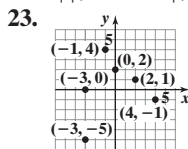
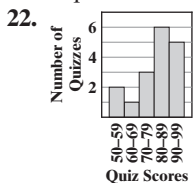
29. $(-2, 0), (1, -3), (-7, 5)$ 31. $(-16, -12), (-1, 3), (100, 104)$

33. $(1, -2), (2, 1), (3, 4)$ 35. $(0, 0), (3, -3), (9, -9)$

37. $(4, -6), (-2, -3), (0, -4)$ 39. 1.7 41. 21.84 43. -23.6 45. true

47. true 49. false 51. false 53. right 55. rectangle 57. 20 units

Integrated Review 1. 320,000 2. 440,000 3. personal care aides 4. nursing assistants 5. Oroville Dam; 755 ft 6. New Bullards Bar Dam; 635 ft 7. 15 ft 8. 4 dams 9. Thursday and Saturday; 100°F 10. Monday; 82°F 11. Sunday, Monday, and Tuesday 12. Wednesday, Thursday, Friday, and Saturday 13. 70 qt containers 14. 52 qt containers 15. 2 qt containers 16. 6 qt containers 17. $||; 2$ 18. $|; 1$ 19. $|||; 3$ 20. $||||; 6$ 21. $||||; 5$

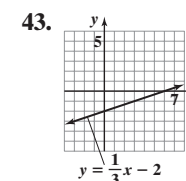
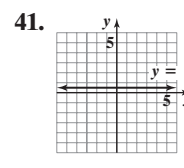
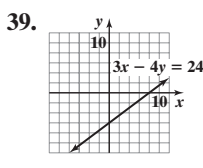
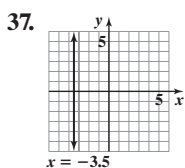
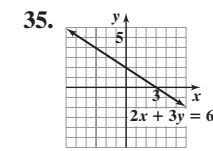
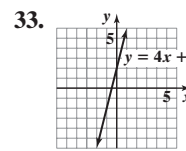
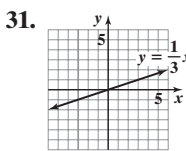
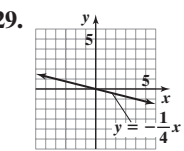
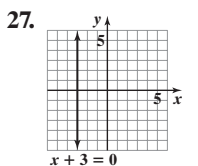
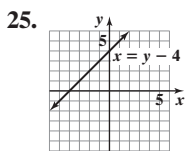
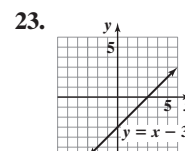
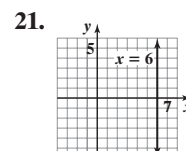
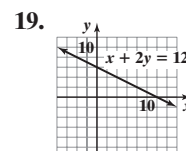
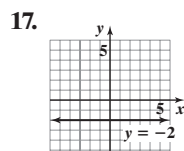
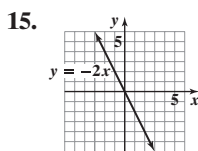
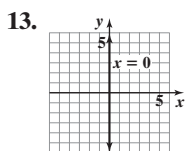
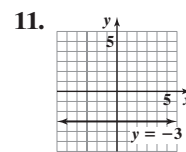
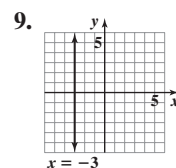
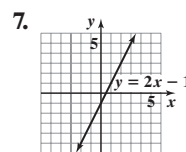
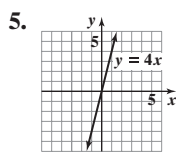
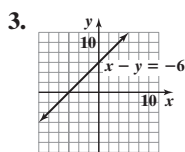
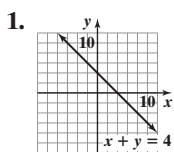


24. no 25. yes 26. $(0, -6), (6, 0), (2, -4)$

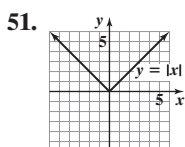
Section 8.4

Vocabulary, Readiness & Video Check 1. linear 3. horizontal 5. infinite 7. Because there's no solving to do, only evaluating, since the equation is solved for y .

Exercise Set 8.4



45. $-\frac{1}{16}$ 47. $-\frac{15}{19}$ 49. $\frac{2-x}{11}$

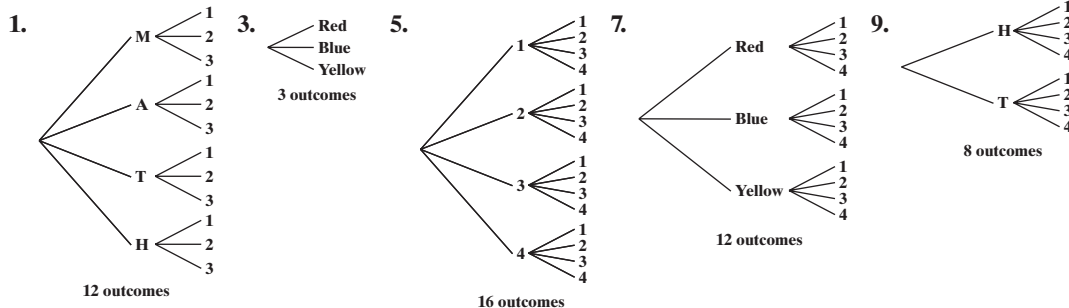


53. answers may vary 55. increasing 57. 30,000 59. answers may vary 61. decreasing
 63. 7.9 million or 7,900,000 65. answers may vary

Section 8.5

Vocabulary, Readiness & Video Check 1. outcome 3. probability 5. 0 7. The number of outcomes equal the ending number of branches drawn.

Exercise Set 8.5



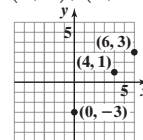
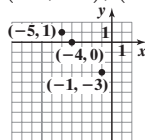
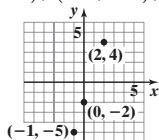
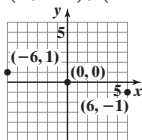
11. $\frac{1}{6}$ 13. $\frac{1}{3}$ 15. $\frac{1}{2}$ 17. $\frac{2}{3}$ 19. $\frac{1}{3}$ 21. 1 23. $\frac{2}{3}$ 25. $\frac{1}{7}$ 27. $\frac{2}{7}$ 29. $\frac{4}{7}$ 31. $\frac{19}{100}$ 33. $\frac{1}{20}$ 35. $\frac{5}{6}$ 37. $\frac{1}{6}$ 39. $6\frac{2}{3}$ 41. $\frac{1}{52}$
43. $\frac{1}{13}$ 45. $\frac{1}{4}$ 47. $\frac{1}{2}$ 49. $\frac{5}{36}$ 51. 0 53. answers may vary

Chapter 8 Vocabulary Check 1. bar 2. outcomes 3. pictograph 4. line 5. x ; y 6. tree diagram 7. experiment 8. circle 9. probability 10. histogram; class interval; class frequency 11. origin 12. quadrants 13. plotting 14. linear

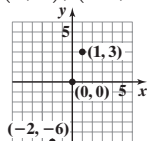
Chapter 8 Review 1. 2,500,000 2. 7,500,000 3. South 4. Northeast 5. South, West 6. Northeast, Midwest 7. 30% 8. 2015 9. 1990, 2000, 2010, 2015 10. answers may vary 11. 962 (exact number) 12. 927 (exact number) 13. 971 (exact number) 14. 842 (exact number) 15. 31 16. answers may vary 17. 1 employee 18. 4 employees 19. 18 employees 20. 9 employees 21. 22. 23.

24. 25. 3 26. 7 27. 35 28. 19 29. 5 30. 2.84 31. a. 62.1 b. 62 c. 63 32. a. 62.5 b. 63 c. 64 33. a. 12.9 b. 12 c. 11 and 12 34. a. 18.1 b. 18 c. 18 and 20 35. a. 59.8 b. 60 c. 60 36. a. 13.9 b. 14 c. 10 and 18 37. mortgage payment 38. utilities 39. \$1225 40. \$700 41. $\frac{39}{160}$ 42. $\frac{7}{40}$ 43. 78 44. 21 45. 2 46. 7

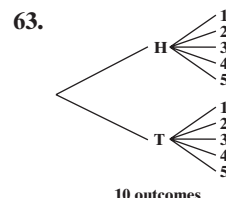
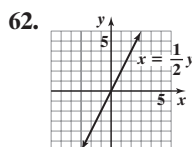
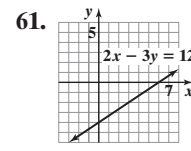
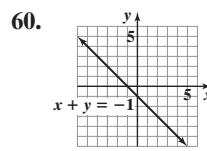
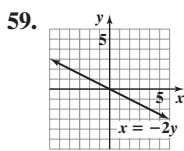
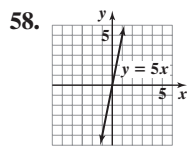
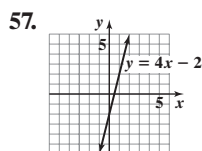
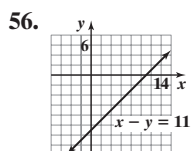
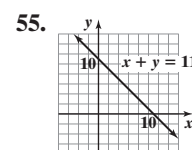
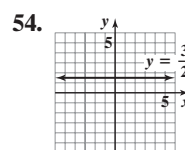
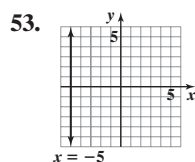
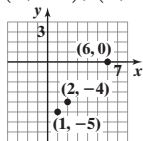
47. $(0, 0)$, $(6, -1)$, $(-6, 1)$ 48. $(0, -2)$, $(-1, -5)$, $(2, 4)$ 49. $(-1, -3)$, $(-4, 0)$, $(-5, 1)$ 50. $(4, 1)$, $(0, -3)$, $(6, 3)$



51. $(1, 3)$, $(-2, -6)$, $(0, 0)$



52. $(1, -5)$, $(6, 0)$, $(2, -4)$



30. \$0.53 per oz; Sec. 6.1 31. -2 ; Sec. 6.2, Ex. 6 32. 16; Sec. 6.2 33. $\frac{12}{19}$; Sec. 6.5, Ex. 2 34. $\frac{4}{9}$; Sec. 6.5 35. 0.046; Sec. 7.1, Ex. 4
 36. 0.32; Sec. 7.1 37. 0.0074; Sec. 7.1, Ex. 6 38. 0.027; Sec. 7.1 39. 21; Sec. 7.2, Ex. 7 40. 14.4; Sec. 7.2 41. 52; Sec. 7.3, Ex. 9
 42. 38; Sec. 7.3 43. 8.5%; Sec. 7.5, Ex. 2 44. 6.5%; Sec. 7.5 45. \$6772.12; Sec. 7.6, Ex. 5 46. \$144.05; Sec. 7.6 47. 57; Sec. 5.7, Ex. 4
 48. 48.5; Sec. 5.7 49. $\frac{1}{3}$; Sec. 8.5, Ex. 4 50. $\frac{1}{2}$; Sec. 8.5

Chapter 9 Geometry and Measurement

Section 9.1

Vocabulary, Readiness & Video Check 1. plane 3. Space 5. ray 7. straight 9. acute 11. Parallel; intersecting
 13. degrees 15. vertical 17. $\angle WUV, \angle VUW, \angle U, \angle x$ 19. $180^\circ - 17^\circ = 163^\circ$

Exercise Set 9.1 1. line; line CD or line l or $\overleftrightarrow{CD}$ 3. line segment; line segment MN or $\overline{MN}$ 5. angle; $\angle GHI$ or $\angle IHG$ or $\angle H$
 7. ray; ray UW or $\overrightarrow{UW}$ 9. $\angle CPR, \angle RPC$ 11. $\angle TPM, \angle MPT$ 13. straight 15. right 17. obtuse 19. acute 21. 67°
 23. 163° 25. 32° 27. 30° 29. $\angle MNP$ and $\angle RNO$; $\angle PNQ$ and $\angle QNR$ 31. $\angle SPT$ and $\angle TPQ$; $\angle SPR$ and $\angle RPQ$; $\angle SPT$
 and $\angle SPR$; $\angle TPQ$ and $\angle QPR$ 33. 27° 35. 132° 37. $m\angle x = 30^\circ$; $m\angle y = 150^\circ$; $m\angle z = 30^\circ$ 39. $m\angle x = 77^\circ$; $m\angle y = 103^\circ$;
 $m\angle z = 77^\circ$ 41. $m\angle x = 100^\circ$; $m\angle y = 80^\circ$; $m\angle z = 100^\circ$ 43. $m\angle x = 134^\circ$; $m\angle y = 46^\circ$; $m\angle z = 134^\circ$ 45. $\angle ABC$ or $\angle CBA$
 47. $\angle DBE$ or $\angle EBD$ 49. 15° 51. 50° 53. 65° 55. 95° 57. $\frac{9}{8}$ or $1\frac{1}{8}$ 59. $\frac{7}{32}$ 61. $\frac{5}{6}$ 63. $1\frac{1}{3}$ 65. 360° 67. 45°
 69. 54.8° 71. false; answers may vary 73. true 75. $m\angle a = 60^\circ$; $m\angle b = 50^\circ$; $m\angle c = 110^\circ$; $m\angle d = 70^\circ$; $m\angle e = 120^\circ$
 77. no; answers may vary 79. 45° ; 45°

Section 9.2

Vocabulary, Readiness & Video Check 1. perimeter 3. π 5. $\frac{22}{7}$ (or 3.14); 3.14 (or $\frac{22}{7}$) 7. Opposite sides of a rectangle have
 the same measure, so we can just find the sum of the measures of all four sides.

Exercise Set 9.2 1. 64 ft 3. 120 cm 5. 21 in. 7. 48 ft 9. 42 in. 11. 155 cm 13. 21 ft 15. 624 ft 17. 346 yd 19. 22 ft
 21. \$55 23. a. 8 b. 72 in. 25. 28 in. 27. \$36.12 29. 96 m 31. 66 ft 33. 74 cm 35. 17π cm; 53.38 cm 37. 16π mi; 50.24 mi
 39. 26π m; 81.64 m 41. 15π ft; 47.1 ft 43. 12,560 ft 45. 30.7 mi 47. 14π cm $\approx$ 43.96 cm 49. 40 mm 51. 84 ft 53. 23
 55. 1 57. 6 59. 10 61. a. width: 30 yd; length: 40 yd b. 140 yd 63. b 65. a. 62.8 m; 125.6 m b. yes 67. answers may vary
 69. 27.4 m 71. 75.4 m

Section 9.3

Vocabulary, Readiness & Video Check 1. surface area 3. Area 5. square 7. We don't have a formula for an L-shaped figure,
 so we divide it into two rectangles, use the formula to find the area of each, and then add these two areas.

Exercise Set 9.3 1. 7 sq m 3. $9\frac{3}{4}$ sq yd 5. 15 sq yd 7. 2.25π sq in. $\approx$ 7.065 sq in. 9. 36.75 sq ft 11. 28 sq m 13. 22 sq yd
 15. $13\frac{3}{5}$ sq ft 17. $22\frac{1}{2}$ sq in. 19. 25 sq cm 21. 86 sq mi 23. 24 sq cm 25. 36π sq in. $\approx$ $113\frac{1}{7}$ sq in. 27. $V = 72$ cu in.;
 $SA = 108$ sq in. 29. $V = 512$ cu cm; $SA = 384$ sq cm 31. $V = 4\pi$ cu yd $\approx$ $12\frac{4}{7}$ cu yd; $SA = (2\pi\sqrt{13} + 4\pi)$ sq yd $\approx$ 35.20 sq yd
 33. $V = \frac{500}{3}\pi$ cu in. $\approx$ $523\frac{17}{21}$ cu in.; $SA = 100\pi$ sq in. $\approx$ $314\frac{2}{7}$ sq in. 35. $V = 9\pi$ cu in. $\approx$ $28\frac{2}{7}$ cu in. 37. $V = 75$ cu cm
 39. $2\frac{10}{27}$ cu in. 41. $V = 8.4$ cu ft; $SA = 26$ sq ft 43. 113,625 sq ft 45. 168 sq ft 47. 960 cu cm 49. 9200 sq ft
 51. $V = \frac{1372}{3}\pi$ cu in. or $457\frac{1}{3}\pi$ cu in.; $SA = 196\pi$ sq in. 53. a. 381 sq ft b. 4 squares 55. $V = 5.25\pi$ cu in. 57. 4π sq ft $\approx$ 12.56 sq ft
 59. $V = 4.5\pi$ cu m; 14.13 cu m 61. 168 sq ft 63. $10\frac{5}{6}$ cu in. 65. 8.8 cu in. 67. 25 69. 9 71. 5 73. 20 75. perimeter
 77. area 79. area 81. perimeter 83. 12-in. pizza 85. 2093.33 cu m 87. no; answers may vary 89. 7.74 sq in. 91. 298.5 sq m
 93. no; answers may vary

Integrated Review 1. 153° ; 63° 2. $m\angle x = 75^\circ$; $m\angle y = 105^\circ$; $m\angle z = 75^\circ$ 3. $m\angle x = 128^\circ$; $m\angle y = 52^\circ$; $m\angle z = 128^\circ$

4. $m\angle x = 52^\circ$ 5. 4.6 in. 6. $4\frac{1}{4}$ in. 7. 20 m; 25 sq m 8. 12 ft; 6 sq ft 9. 10π cm $\approx$ 31.4 cm; 25π sq cm $\approx$ 78.5 sq cm
 10. 32 mi; 44 sq mi 11. 54 cm; 143 sq cm 12. 62 ft; 238 sq ft 13. 9800 ft; 4,900,000 sq ft 14. $V = 64$ cu in.; $SA = 96$ sq in.
 15. $V = 30.6$ cu ft; $SA = 63$ sq ft 16. $V = 400$ cu cm 17. $V = 4\frac{1}{2}\pi$ cu mi $\approx$ $14\frac{1}{7}$ cu mi

Section 9.4

Vocabulary, Readiness & Video Check 1. meter 3. yard 5. feet 7. feet 9. Both mean addition; $5\frac{2}{5} = 5 + \frac{2}{5}$ and

5 ft 2 in. = 5 ft + 2 in. 11. Since the metric system is based on base 10, we just need to move the decimal point to convert from one unit to another.

Exercise Set 9.4 1. 5 ft 3. 36 ft 5. 8 mi 7. 102 in. 9. $3\frac{1}{3}$ yd 11. 33,792 ft 13. 4.5 yd 15. 0.25 ft 17. 13 yd 1 ft 19. 7 ft 1 in.

21. 1 mi 4720 ft 23. 62 in. 25. 26 ft 27. 84 in. 29. 11 ft 2 in. 31. 22 yd 1 ft 33. 6 ft 5 in. 35. 7 ft 6 in. 37. 14 ft 4 in.
39. 83 yd 1 ft 41. 6000 cm 43. 4 cm 45. 0.5 km 47. 1.7 m 49. 15 m 51. 42,000 cm 53. 7000 m 55. 83 mm 57. 0.201 dm
59. 40 mm 61. 8.94 m 63. 2.94 m or 2940 mm 65. 1.29 cm or 12.9 mm 67. 12.64 km or 12,640 m 69. 54.9 m 71. 1.55 km
73. $348\frac{2}{3}$; 12,552 75. $11\frac{2}{3}$; 420 77. 5000; 0.005; 500 79. 0.065; 65; 0.000065 81. 342,000; 342,000,000; 34,200,000 83. 10 ft 6 in.
85. 5100 ft 87. 5.0 times 89. 26.7 mm 91. 15 ft 9 in. 93. 3.35 m 95. $72\frac{2}{3}$ yd 97. $\frac{21}{100}$ 99. 0.13 101. 0.25 103. no 105. yes

107. no 109. Estimate: 13 yd 111. answers may vary; for example, $1\frac{1}{3}$ yd or 48 in. 113. answers may vary 115. 334.89 sq m

Section 9.5

Vocabulary, Readiness & Video Check 1. Mass 3. gram 5. 2000 7. We can't subtract 9 oz from 4 oz, so we borrow 1 lb (= 16 oz) from 12 lb to add to the 4 oz; 12 lb 4 oz becomes 11 lb 20 oz. 9. 18.50 dg

Exercise Set 9.5 1. 32 oz 3. 10,000 lb 5. 9 tons 7. $3\frac{3}{4}$ lb 9. $1\frac{3}{4}$ tons 11. 204 oz 13. 9800 lb 15. 76 oz 17. 1.5 tons

19. $\frac{1}{20}$ lb 21. 92 oz 23. 161 oz 25. 5 lb 9 oz 27. 53 lb 10 oz 29. 8 tons 750 lb 31. 3 tons 175 lb 33. 8 lb 11 oz 35. 31 lb 2 oz

37. 1 ton 700 lb 39. 0.5 kg 41. 4000 mg 43. 25,000 g 45. 0.048 g 47. 0.0063 kg 49. 15,140 mg 51. 6250 g 53. 350,000 cg
55. 13.5 mg 57. 5.815 g or 5815 mg 59. 1850 mg or 1.85 g 61. 1360 g or 1.36 kg 63. 13.52 kg 65. 2.125 kg 67. 200,000; 3,200,000
69. $\frac{269}{400}$ or 0.6725; 21,520 71. 0.5; 0.0005; 50 73. 21,000; 21,000,000; 2,100,000 75. 8.064 kg 77. 30 mg 79. 5 lb 8 oz
81. 35 lb 14 oz 83. 6 lb 15.5 oz 85. 144 mg 87. 6.12 kg 89. 130 lb 91. 211 lb 93. 0.16 95. 0.875 97. no 99. yes
101. no 103. answers may vary; for example, 250 mg or 0.25 g 105. true 107. answers may vary

Section 9.6

Vocabulary, Readiness & Video Check 1. capacity 3. fluid ounces 5. cups 7. quarts 9. We can't subtract 3 qt from 0 qt, so we borrow 1 gal (= 4 qt) from 3 gal to get 2 gal 4 qt. 11. 0.45 dal

Exercise Set 9.6 1. 4 c 3. 16 pt 5. $3\frac{1}{2}$ gal 7. 5 pt 9. 8 c 11. $3\frac{3}{4}$ qt 13. $10\frac{1}{2}$ qt 15. 9 c 17. 23 qt 19. $\frac{1}{4}$ pt 21. 14 gal 2 qt

23. 4 gal 3 qt 1 pt 25. 22 pt 27. 13 gal 2 qt 29. 4 c 4 fl oz 31. 1 gal 1 qt 33. 2 gal 3 qt 1 pt 35. 17 gal 37. 4 gal 3 qt
39. 5000 ml 41. 0.00016 kl 43. 5.6 L 45. 320 cl 47. 0.41 kl 49. 0.064 L 51. 160 L 53. 3600 ml 55. 19.3 L

57. 4.5 L or 4500 ml 59. 8410 ml or 8.41 L 61. 16,600 ml or 16.6 L 63. 3840 ml 65. 162.4 L 67. 336; 84; 168 69. $\frac{1}{4}$; 1; 2

71. 1.59 L 73. 18.954 L 75. 4.3 fl oz 77. yes 79. \$0.316 81. $\frac{4}{5}$ 83. $\frac{3}{5}$ 85. $\frac{9}{10}$ 87. no 89. no 91. less than; answers may vary 93. answers may vary 95. 128 fl oz 97. 1.5 cc 99. 2.7 cc 101. 54 u or 0.54 cc 103. 86 u or 0.86 cc

Section 9.7

Vocabulary, Readiness & Video Check 1. 1 L $\approx$ 0.26 gal or 3.79 L $\approx$ 1 gal 3. $F = 1.8C + 32$; 27

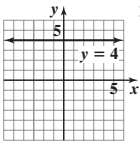
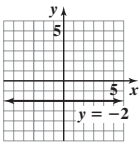
Exercise Set 9.7 1. 25.57 fl oz 3. 218.44 cm 5. 40 oz 7. 57.66 mi 9. 3.77 gal 11. 13.5 kg 13. 1.5; $1\frac{2}{3}$; 150; 60 15. 55; 5500; 180; 2160 17. 3.94 in. 19. 80.5 kph 21. 0.008 oz 23. 229.6 ft 25. 9.92 billion mi 27. yes 29. 2790 mi 31. 90 mm 33. 112.5 g
35. 104 mph 37. 26.24 ft 39. 3 mi 41. 8 fl oz 43. b 45. b 47. c 49. d 51. d 53. 25°C 55. 40°C 57. 122°F 59. 239°F
61. -6.7°C 63. 61.2°C 65. 197.6°F 67. 54.3°F 69. 56.7°C 71. 80.6°F 73. 21.1°C 75. 244.4°F 77. 7232°F 79. 29 81. 36
83. yes 85. no 87. no 89. yes 91. 2.13 sq m 93. 1.19 sq m 95. 1.69 sq m 97. 188.8°C 99. answers may vary

Chapter 9 Vocabulary Check 1. Weight 2. Mass 3. meter 4. unit fractions 5. gram 6. liter 7. line segment
8. complementary 9. line 10. perimeter 11. angle; vertex 12. Area 13. ray 14. transversal 15. straight 16. volume
17. vertical 18. adjacent 19. obtuse 20. right 21. acute 22. supplementary 23. surface area

Chapter 9 Review 1. right 2. straight 3. acute 4. obtuse 5. 65° 6. 75° 7. 58° 8. 98° 9. 90° 10. 25° 11. $\angle a$ and $\angle b$; $\angle b$ and $\angle c$; $\angle c$ and $\angle d$; $\angle d$ and $\angle a$ 12. $\angle x$ and $\angle w$; $\angle y$ and $\angle z$ 13. $m\angle x = 100^\circ$; $m\angle y = 80^\circ$; $m\angle z = 80^\circ$ 14. $m\angle x = 155^\circ$; $m\angle y = 155^\circ$; $m\angle z = 25^\circ$ 15. $m\angle x = 53^\circ$; $m\angle y = 53^\circ$; $m\angle z = 127^\circ$ 16. $m\angle x = 42^\circ$; $m\angle y = 42^\circ$; $m\angle z = 138^\circ$ 17. 69 m 18. 30.6 cm 19. 36 m 20. 90 ft 21. 32 ft 22. 440 ft 23. 5.338 in. 24. 31.4 yd 25. 240 sq ft 26. 189 sq yd 27. 600 sq cm 28. 82.81 sq m 29. 49π sq ft ≈ 153.86 sq ft 30. 9π sq in. ≈ 28.26 sq in. 31. 119 sq in. 32. 140 sq m 33. 144 sq m 34. 1625 sq cm 35. 432 sq ft 36. 130 sq ft 37. $V = 15\frac{5}{8}$ cu in.; $SA = 37\frac{1}{2}$ sq in. 38. $V = 84$ cu ft; $SA = 136$ sq ft 39. $V = 20,000\pi$ cu cm $\approx 62,800$ cu cm 40. $V = \frac{1}{6}\pi$ cu km $\approx \frac{11}{21}$ cu km 41. $2\frac{2}{3}$ cu ft 42. 307.72 cu in. 43. $7\frac{1}{2}$ cu ft 44. 0.5π cu ft or $\frac{1}{2}\pi$ cu ft 45. 9 ft 46. 24 yd 47. 7920 ft 48. 18 in. 49. 17 yd 1 ft 50. 3 ft 10 in. 51. 4200 cm 52. 820 mm 53. 0.01218 m 54. 0.00231 km 55. 21 yd 1 ft 56. 3 ft 8 in. 57. 9.5 cm or 95 mm 58. 2.74 m or 274 cm 59. 169 yd 2 ft 60. 258 ft 4 in. 61. 6175 km 62. 0.24 sq m 63. $4\frac{1}{8}$ lb 64. 4600 lb 65. 3 lb 4 oz 66. 5 tons 300 lb 67. 0.027 g 68. 40,000 g 69. 21 dag 70. 0.0003 dg 71. 3 lb 9 oz 72. 33 lb 8 oz 73. 21.5 mg 74. 0.6 kg or 600 g 75. 4 lb 4 oz 76. 9 tons 1075 lb 77. 14 qt 78. 5 c 79. 7 pt 80. 72 c 81. 4 qt 1 pt 82. 3 gal 3 qt 83. 3800 ml 84. 1.4 kl 85. 3060 cl 86. 0.00245 L 87. 1 gal 1 qt 88. 7 gal 89. 736 ml or 0.736 L 90. 15.5 L or 15,500 ml 91. 2 gal 3 qt 92. 6 fl oz 93. 10.88 L 94. yes 95. 22.96 ft 96. 10.55 m 97. 4.55 gal 98. 8.27 qt 99. 425.25 g 100. 10.35 kg 101. 2.36 in. 102. 180.4 lb 103. 1076°F 104. 320°F 105. 5.2°C 106. 26.7°C 107. 1.7°C 108. 329°F 109. 108° 110. 89° 111. 95° 112. 57° 113. 273 in. 114. 194 ft 115. 1624 sq m 116. 9π sq m ≈ 28.26 sq m 117. $346\frac{1}{2}$ cu in. 118. $V = 140$ cu in.; $SA = 166$ sq in. 119. 75 in. 120. 4 tons 200 lb 121. 500 cm 122. 0.000286 km 123. 1.4 g 124. 27 qt 125. 186.8°F 126. 11°C 127. 9117 m or 9.117 km 128. 35.7 L or 35,700 ml 129. 8 gal 1 qt 130. 12.8 kg

Chapter 9 Getting Ready for the Test 1. D 2. B 3. E 4. A 5. F 6. C 7. B 8. C 9. A 10. B 11. B 12. C 13. A and D 14. B 15. A or D 16. C

Chapter 9 Test 1. 12° 2. 56° 3. 50° 4. $m\angle x = 118^\circ$; $m\angle y = 62^\circ$; $m\angle z = 118^\circ$ 5. $m\angle x = 73^\circ$; $m\angle y = 73^\circ$; $m\angle z = 73^\circ$ 6. 6.2 m 7. 10 in. 8. circumference = 18π in. ≈ 56.52 in.; area = 81π sq in. ≈ 254.34 sq in. 9. perimeter = 24.6 yd; area = 37.1 sq yd 10. perimeter = 68 in.; area = 185 sq in. 11. $62\frac{6}{7}$ cu in. 12. 30 cu ft 13. 16 in. 14. 18 cu ft 15. 62 ft; \$115.94 16. 23 ft 4 in. 17. 10 qt 18. $1\frac{7}{8}$ lb 19. 5600 lb 20. $4\frac{3}{4}$ gal 21. 0.04 g 22. 2400 g 23. 36 mm 24. 0.43 g 25. 830 ml 26. 1 gal 2 qt 27. 3 lb 13 oz 28. 8 ft 3 in. 29. 2 gal 3 qt 30. 66 mm or 6.6 cm 31. 2.256 km or 2256 m 32. 28.9°C 33. 54.7°F 34. 5.6 m 35. 4 gal 3 qt 36. 91.4 m 37. 16 ft 6 in.

Cumulative Review Chapters 1–9 1. 5; Sec. 3.3, Ex. 1 2. 6; Sec. 3.3 3. a. $\frac{16}{625}$ b. $\frac{1}{16}$; Sec. 4.3, Ex. 10 4. a. $-\frac{1}{27}$ b. $\frac{9}{49}$; Sec. 4.3 5. $9\frac{3}{10}$; Sec. 4.7, Ex. 11 6. $9\frac{11}{15}$; Sec. 4.7 7. $20x - 10.9$; Sec. 5.2, Ex. 13 8. $1.2y + 1.8$; Sec. 5.2 9. 28.4405; Sec. 5.5, Ex. 14 10. 2.16; Sec. 5.5 11. =; Sec. 5.5, Ex. 9 12. >; Sec. 5.5 13. -1.3; Sec. 5.6, Ex. 6 14. 30; Sec. 5.6 15. 424 ft; Sec. 6.4, Ex. 12 16. 236 ft; Sec. 6.4 17. a. $\frac{5}{7}$ b. $\frac{7}{24}$; Sec. 6.1, Ex. 6 18. a. $\frac{1}{4}$ b. $\frac{4}{9}$; Sec. 6.1 19. $\frac{180 \text{ dollars}}{1 \text{ week}}$; Sec. 6.1, Ex. 7 20. $\frac{1 \text{ chaperone}}{5 \text{ students}}$; Sec. 6.1 21. 0.44; Sec. 6.2; Ex. 10 22. 1.02; Sec. 6.2 23. 22.4 cc; Sec. 6.3, Ex. 2 24. 7.5 cups; Sec. 6.3 25. 17%; Sec. 7.1, Ex. 1 26. 38%; Sec. 7.1 27. 200; Sec. 7.2, Ex. 10 28. 1200; Sec. 7.2 29. 2.7; Sec. 7.3, Ex. 7 30. 12.6; Sec. 7.3 31. 32%; Sec. 7.4, Ex. 5 32. 27%; Sec. 7.4 33. sales tax: \$6.41; total price: \$91.91; Sec. 7.5, Ex. 1 34. sales tax: \$30; total price: \$405; Sec. 7.5 35. A(-4, 2), B(1, 2), C(0, 1), D(-3, 0), E(5, -4); Sec. 8.3, Ex. 2 36. A(2, -3), B(-5, 0), C(0, 4), D(-3, -2); Sec. 8.3 37. ; Sec. 8.4, Ex. 4 38. ; Sec. 8.4 39. 57; Sec. 5.7, Ex. 4 40. 83; Sec. 5.7 41. $\frac{1}{4}$; Sec. 8.5, Ex. 5

42. $\frac{2}{7}$; Sec. 8.5 43. 42° ; Sec. 9.1, Ex. 4 44. 43° ; Sec. 9.1 45. 96 in.; Sec. 9.4, Ex. 1 46. 21 feet; Sec. 9.4 47. 9π sq ft ≈ 28.26 sq ft; Sec. 9.3, Ex. 4 48. 4π sq mi ≈ 12.56 sq mi; Sec. 9.3 49. 15°C ; Sec. 9.7, Ex. 6 50. 30°C ; Sec. 9.7

Chapter 10 Exponents and Polynomials

Section 10.1

Vocabulary, Readiness & Video Check 1. terms 3. binomial 5. add 7. Terms where everything is the same except for the numerical coefficient. 9. 2; -5

Exercise Set 10.1 1. $-5x - 24$ 3. $-11z^2 - 3z - 1$ 5. $9y^2 + 25y - 40$ 7. $-4.3a^4 - 2a^2 + 9$ 9. $-9x + 16$ 11. $3z^2 - z + 7$
 13. $5a - 13$ 15. $-2x^2 + 4x + 1$ 17. $-20y^3 + 12y^2$ 19. $-9x^2 - x + 16$ 21. $8y^2 - 10y - 8$ 23. $5x - 12$ 25. $y - 4$
 27. $-5x^2 + 3x + 5$ 29. $4x - 4.1$ 31. $4a - 7$ 33. $-15y + 3.6$ 35. $19t^2 - 11t + 3$ 37. $2b^3 + 13b^2 + 4b - 2$
 39. $-5x^2 - 11x + 13$ 41. $\frac{9}{7}$ 43. 5 45. -5 47. -8 49. 20 51. 25 53. 50 55. 576 ft 57. \$3200 59. a. 835 million or 835,000,000 b. 832 million or 832,000,000 c. yes; yes; answers may vary 61. 547.4 million or 547,400,000 63. 611 ft
 65. 909 ft 67. 86.5% cell phone-only U.S. households 69. 81 71. 25 73. x^3 75. 2^2a^4 77. $(8x + 2)$ in. 79. $(4x - 15)$ units
 81. 20; 6; 2 83. 72752 85. 29 ft; -243 ft; answers may vary

Section 10.2

Vocabulary, Readiness & Video Check 1. exponent 3. multiply 5. $x^2 \cdot x^3 = x^5$ 7. 3 and a

Exercise Set 10.2 1. x^{14} 3. a^4 5. $15z^5$ 7. $-40x^2$ 9. $42x^3$ 11. $12a^{17}$ 13. $25x^6y^4$ 15. $28a^5b^6$ 17. x^{15} 19. z^{30} 21. b^{62}
 23. $81a^4$ 25. $a^{33}b^{24}$ 27. $1000x^{15}y^9$ 29. $-24y^{22}$ 31. $256x^9y^{13}$ 33. $7x - 21$ 35. $-6a - 4b$ 37. $9x + 18y - 27$
 39. $16x^{12}$ sq in. 41. $12a^4b^5$ sq m 43. 18,003,384 $a^{45}b^{30}$ 45. 34,867,84401 x^{50} 47. $x^{270}y^{216}$ 49. answers may vary

Integrated Review 1. $2x - 3$ 2. $20y - 11$ 3. $10x + 3$ 4. $-5y - 4$ 5. $-2a^4 + 3a^2 + 9a$ 6. $-4a^3 - 12a^2$
 7. $7.3x^2 - 4.2x - 5.3$ 8. $1.8y^2 - 2.4y - 5.6$ 9. $6x + 7$ 10. $2x^2 + 3x - 12$ 11. -1 12. 26 13. x^{20} 14. x^{10} 15. y^4
 16. a^{11} 17. x^{77} 18. x^{36} 19. x^{42} 20. y^{27} 21. $125x^3$ 22. $32y^5$ 23. $-12x^2y^7$ 24. $12a^3b^4$ 25. $y^{33}z^{39}$ 26. $a^{20}b^{48}$ 27. $300x^4y^3$
 28. $128y^6z^7$ 29. $144a^{38}b^{12}$ 30. $125x^{22}y^{28}$ 31. \$130.3 billion

Section 10.3

Vocabulary, Readiness & Video Check 1. The monomial is multiplied by each term in the trinomial. 3. To make the point that the power rule applies only to products and not to sums, so we cannot apply the power rule to a binomial squared. 5. No; it is a binomial times a trinomial, and FOIL can only be used to multiply a binomial times a binomial.

Exercise Set 10.3 1. $27x^3 - 9x$ 3. $-6a^3 + 9a^2 + 15a$ 5. $42x^4 - 35x^3 + 49x^2$ 7. $x^2 + 13x + 30$ 9. $2x^2 + 2x - 24$
 11. $36a^2 + 48a + 16$ 13. $a^3 - 33a + 18$ 15. $8x^3 + 2x^2 - 55x + 50$ 17. $x^5 + 4x^4 + 6x^3 + 7x^2 + 2x$ 19. $-30r^2 + 20r$
 21. $-6y^3 - 2y^4 + 12y^2$ 23. $x^2 + 14x + 24$ 25. $4a^2 - 9$ 27. $x^2 + 10x + 25$ 29. $b^2 + \frac{7}{5}b + \frac{12}{25}$ 31. $6x^3 + 25x^2 + 10x + 1$
 33. $49x^2 + 70x + 25$ 35. $4x^2 - 4x + 1$ 37. $8x^5 - 8x^3 - 6x^2 - 6x + 9$ 39. $x^5 + 2x^4 + 3x^3 + 2x^2 + x$
 41. $10z^4 - 3z^3 + 3z - 2$ 43. $2 \cdot 5^2$ 45. $2^3 \cdot 3^2$ 47. $2^3 \cdot 5^2$ 49. $(y^3 - 3y^2 - 16y - 12)$ sq ft 51. $(x^4 - 3x^2 + 1)$ sq m
 53. answers may vary

Section 10.4

Vocabulary, Readiness & Video Check 1. factor; product 3. smallest 5. Factor means to write as a product. 7. Find the GCF of the terms of the binomial and then factor it out.

Exercise Set 10.4 1. 3 3. 12 5. 4 7. 4 9. y^2 11. a^5 13. xy^2 15. x 17. $2z^3$ 19. $3y(y + 6)$ 21. $5a^6(2 - a^2)$
 23. $4x(x^2 + 3x + 5)$ 25. $z^5(z^2 - 6)$ 27. $-7(5 - 2y + y^2)$ or $7(-5 + 2y - y^2)$ 29. $12a^5(1 - 3a)$ 31. 36 33. $\frac{4}{5}$
 35. 37.5% 37. a. $x^2 + 2x$ b. answers may vary 39. answers may vary 41. $4y^2 + 28y + 49$ 43. answers may vary

Chapter 10 Vocabulary Check 1. Factoring 2. greatest common factor 3. FOIL 4. trinomial 5. binomial 6. monomial 7. polynomials 8. exponent

Chapter 10 Review 1. $10b - 3$ 2. $21s - 15$ 3. $-x + 2.8$ 4. $-y - 12$ 5. $4z^2 + 11z - 6$ 6. $17a^3 + 25a^2$ 7. $-11y^2 - y + \frac{3}{4}$
 8. $x^2 - 7x + 3$ 9. 45 10. -19 11. $(26x + 28)$ ft 12. $(16x^2 + 4)$ m 13. x^{24} 14. y^7 15. $24z^7$ 16. $-15x^3y^5$ 17. a^{35}
 18. x^{28} 19. $81b^2$ 20. $a^{20}b^{10}c^5$ 21. $56x^{16}$ 22. $108x^{30}y^{25}$ 23. $81a^{14}$ sq mi 24. $27x^5$ sq in. 25. $10a^3 - 12a$
 26. $-3y^4 + 6y^3 - 3y^2$ 27. $x^2 + 8x + 12$ 28. $15x^2 - 32x + 9$ 29. $y^2 - 10y + 25$ 30. $49a^2 + 14a + 1$ 31. $x^3 - x^2 + x + 3$
 32. $8y^4 + 4y^3 - 2y^2 - 3y - 3$ 33. $3z^4 + 5z^3 + 6z^2 + 3z + 1$ 34. $(a^3 + 5a^2 - 5a + 6)$ sq cm 35. 5 36. 4 37. 6 38. 5
 39. x^2 40. y^7 41. xy 42. a^5b^2 43. $5a$ 44. $4y^2z$ 45. $2x(x + 6)$ 46. $6a(a - 2)$ 47. $y^4(6 - y^2)$ 48. $7(x^2 + 2x + 1)$
 49. $a^3(5a^4 - a + 1)$ 50. $10y(y^5 - 1)$ 51. $z^2 + z + 4$ 52. $-4y - 2$ 53. x^{21} 54. y^9 55. $a^{18}b^{30}c^6$ 56. $81x^6$
 57. $12a^4 - 15a$ 58. $x^2 + 9x + 20$ 59. $9x^2 + 24x + 16$ 60. $6z^2 - 7z - 10$ 61. 4 62. z^4 63. $z^7(z^2 + 4)$ 64. $x^5(x^7 - 6)$
 65. $15a^4(1 + 3a)$ 66. $8z^5(2 - 3z^3)$

Chapter 10 Getting Ready for the Test 1. C 2. A 3. A 4. B 5. C 6. B 7. D 8. C 9. D

Chapter 10 Test 1. $15x - 4$ 2. $7x - 2$ 3. $3.4y^2 + 2y - 3$ 4. $-2a^2 + a + 1$ 5. 17 6. y^{14} 7. y^{33} 8. $16x^8$ 9. $-12a^{10}$
 10. p^{54} 11. $72a^{20}b^5$ 12. $10x^3 + 6.5x$ 13. $-2y^4 - 12y^3 + 8y$ 14. $x^2 - x - 6$ 15. $25x^2 + 20x + 4$ 16. $a^3 + 8$
 17. perimeter: $(14x - 4)$ in.; area: $(5x^2 + 33x - 14)$ sq in. 18. 15 19. $3y^3$ 20. $3y(y - 5)$ 21. $2a(5a + 6)$ 22. $6(x^2 - 2x - 5)$
 23. $x^3(7x^3 - 6x + 1)$

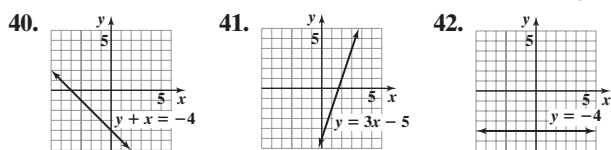
Cumulative Review Chapters 1–10 1. 106,400 sq mi; Sec. 1.5, Ex. 6 2. 147 trees; Sec. 1.5 3. -8 ; Sec. 2.2, Ex. 15 4. -10 ;
 Sec. 2.2 5. -7 ; Sec. 2.3, Ex. 6 6. -3 ; Sec. 2.3 7. 1; Sec. 2.3, Ex. 7 8. 5; Sec. 2.3 9. -46 ; Sec. 2.6, Ex. 4 10. -2 ; Sec. 2.6
 11. -1 ; Sec. 3.3, Ex. 2 12. 1; Sec. 3.3 13. $\frac{13x}{30}$; Sec. 4.5, Ex. 2 14. $\frac{1}{14y}$; Sec. 4.5 15. 736.2; Sec. 5.1, Ex. 15 16. 328.2; Sec. 5.1
 17. 25.454; Sec. 5.2, Ex. 1 18. 17052; Sec. 5.2 19. no; Sec. 5.3, Ex. 13 20. yes; Sec. 5.3 21. 0.7861; Sec. 5.4, Ex. 8 22. 0.818;
 Sec. 5.4 23. 0.012; Sec. 5.4, Ex. 9 24. 0.0503; Sec. 5.4 25. -2.6 ; Sec. 5.5, Ex. 16 26. -13.6 ; Sec. 5.5 27. 3.14; Sec. 5.5, Ex. 4
 28. 1.947; Sec. 5.5 29. $\frac{1}{6}$; Sec. 6.4, Ex. 5 30. $\frac{2}{5}$; Sec. 6.4 31. 46 ft; Sec. 6.5, Ex. 4 32. 3 ft; Sec. 6.5 33. $1.2 = 30\% \cdot x$; Sec. 7.2, Ex. 2
 34. $9 = 45\% \cdot x$; Sec. 7.2 35. 16%; Sec. 7.3, Ex. 10 36. 25%; Sec. 7.3 37. 775 freshmen; Sec. 7.4, Ex. 3 38. 1450 apples; Sec. 7.4
 39. \$160; Sec. 7.6, Ex. 2 40. \$25; Sec. 7.6 41. 55%; Sec. 8.2, Ex. 2 42. 31%; Sec. 8.2 43. 28 in.; Sec. 9.2, Ex. 2 44. 25 ft; Sec. 9.2
 45. 5.1 sq mi; Sec. 9.3, Ex. 2 46. 68 sq in.; Sec. 9.3 47. 4 tons 1650 lb; Sec. 9.5, Ex. 4 48. 16 tons 100 lb; Sec. 9.5 49. 3.21 L;
 Sec. 9.6, Ex. 5 50. 43.21 L; Sec. 9.6 51. $-3x + 1$; Sec. 10.1, Ex. 1 52. $4a + 12$; Sec. 10.1 53. $x^2 + 5x + 6$; Sec. 10.3, Ex. 3
 54. $2x^2 + 19x + 35$; Sec. 10.3

Appendices

Appendix B Exercise Set 1. x^2 3. 81 5. p^6q^5 7. $\frac{xy^3}{2}$ 9. 1 11. 2 13. -1 15. 1 17. $\frac{1}{64}$ 19. $\frac{7}{x^3}$ 21. $\frac{5}{6}$ 23. p^3 25. $\frac{1}{x^3}$
 27. z^3 29. $\frac{4}{9}$ 31. $\frac{y^2}{25}$ 33. p^4 35. a^6 37. x^6y^6 39. $\frac{1}{x^{11}}$ 41. $\frac{15}{x^3}$ 43. $\frac{1}{y^{12}}$ 45. $\frac{56m^7}{n^5}$ 47. x^7 49. a^8b^{13} 51. $\frac{1}{x^6}$ 53. 3
 55. $\frac{1}{125}$ 57. $\frac{8}{x^9}$ 59. $\frac{3}{10}$ 61. $\frac{1}{z^7}$ 63. $\frac{1}{x^9}$ 65. $\frac{a^8}{b^8}$ 67. $\frac{30x^9}{y^6}$

Appendix C Exercise Set 1. 7.8×10^4 3. 1.67×10^{-6} 5. 6.35×10^{-3} 7. 1.16×10^6 9. 4.2×10^3 11. 0.0000000008673
 13. 0.033 15. 20,320 17. 700,000,000 19. 5.3×10^{12} 21. 8,100,000,000,000 23. 4,300,000,000,000; 4.3×10^{12}
 25. 0.000036 27. 0.0000000000000000028 29. 0.0000005 31. 200,000 33. 2.7×10^9 gal

Practice Final Exam 1. 200 2. 21 3. -6 4. -100 5. 7 6. -117 7. 0 8. undefined 9. $-\frac{2}{3y}$ 10. $\frac{3}{4}$ 11. $\frac{1}{a^2}$
 12. $\frac{64}{3}$ or $21\frac{1}{3}$ 13. $16\frac{8}{11}$ 14. -6.2 15. 40.902 16. 0.006 17. 610% 18. 37.5% 19. $\frac{69}{200}$ 20. -0.5 21. 34.9 22. 30
 23. 1 24. $\frac{4}{31}$ 25. $-8z - 20$ 26. $(15x + 15)$ in. 27. -28 28. -2 29. 3 30. -4 31. 3.7 32. $\frac{48}{11}$ or $4\frac{4}{11}$ 33. 60 yd; 200 sq yd
 34. -4 35. -2 36. 24 mi 37. 244 women 38. $53\frac{1}{3}$ g 39. discount: \$18; sale price: \$102



43. $15x - 4$ 44. $-2a^2 + a + 1$ 45. $-12a^{10}$ 46. $72a^{20}b^5$ 47. $x^2 - x - 6$ 48. $3y(y - 5)$ 49. 12°
 50. $m\angle x = 73^\circ$; $m\angle y = 73^\circ$; $m\angle z = 73^\circ$ 51. perimeter = 68 in.; area = 185 sq in. 52. circumference = 18π in. ≈ 56.52 in.;
 area = 81π sq in. ≈ 254.34 sq in. 53. 10 qt 54. 2400 g

Solutions to Selected Exercises

Chapter 1

Exercise Set 1.2

1. The place value of the 5 in 657 is tens.
5. The place value of the 5 in 43,526,000 is hundred-housands.
9. 354 is written as three hundred fifty-four.
13. 26,990 is written as twenty-six thousand, nine hundred ninety.
17. 24,350,185 is written as twenty-four million, three hundred fifty thousand, one hundred eighty-five.
21. 2720 is written as two thousand, seven hundred twenty.
25. 14,433 is written as fourteen thousand, four hundred thirty-three.
29. Six thousand, five hundred eighty-seven in standard form is 6587.
33. Thirteen million, six hundred one thousand, eleven in standard form is 13,601,011.
37. Two hundred sixty thousand, nine hundred ninety-seven in standard form is 260,997.
41. One thousand, twenty-four in standard form is 1024.
45. Five hundred sixty-five in standard form is 565.
49. $3470 = 3000 + 400 + 70$
53. $66,049 = 60,000 + 6000 + 40 + 9$
57. Mount Shasta erupted in 1786.
61. The earliest date in the table is 1750, which is an eruption of Glacier Peak.
65. Labrador retrievers are the most popular breed; 75 is written as seventy-five
69. The largest number is 9861.
73. answers may vary

Exercise Set 1.3

1.
$$\begin{array}{r} 14 \\ + 22 \\ \hline 36 \end{array}$$
5.
$$\begin{array}{r} 12 \\ 13 \\ + 24 \\ \hline 49 \end{array}$$
9.
$$\begin{array}{r} 1 \quad 1 \quad 1 \\ 22,781 \\ + 186,297 \\ \hline 209,078 \end{array}$$
13.
$$\begin{array}{r} 22 \\ 81 \\ 17 \\ 23 \\ 79 \\ + 12 \\ \hline 212 \end{array}$$
17.
$$\begin{array}{r} 1 \quad 1 \quad 1 \\ 6 \, 820 \\ 4 \, 271 \\ + 5 \, 626 \\ \hline 16,717 \end{array}$$

$$\begin{array}{r} 21. \quad \begin{array}{r} 122 \quad 21 \\ 121,742 \\ 57,279 \\ 26,586 \\ + 426,782 \\ \hline 632,389 \end{array} \end{array}$$

$$\begin{array}{r} 25. \quad \begin{array}{r} 62 \\ - 37 \\ \hline 25 \end{array} \end{array}$$

$$\begin{array}{r} 29. \quad \begin{array}{r} 600 \\ - 432 \\ \hline 168 \end{array} \end{array}$$

$$\begin{array}{r} 33. \quad \begin{array}{r} 533 \\ - 29 \\ \hline 504 \end{array} \end{array}$$

$$\begin{array}{r} 37. \quad \begin{array}{r} 50,000 \\ - 17,289 \\ \hline 32,711 \end{array} \end{array}$$

$$\begin{array}{r} 41. \quad \begin{array}{r} 51,111 \\ - 19,898 \\ \hline 31,213 \end{array} \end{array}$$

$$\begin{array}{r} 45. \quad \begin{array}{r} 76 \\ - 67 \\ \hline 9 \end{array} \end{array}$$

$$\begin{array}{r} 49. \quad \begin{array}{r} 11 \quad 1 \\ 10,962 \\ 4 \, 851 \\ + 7 \, 063 \\ \hline 22,876 \end{array} \end{array}$$

53. Opposite sides of a rectangle have the same length.
 $4 + 8 + 4 + 8 = 12 + 12 = 24$
 The perimeter is 24 inches.

57. The unknown vertical side has length
 $12 - 5 = 7$ meters.
 The unknown horizontal side has length
 $10 - 5 = 5$ meters.
 $10 + 12 + 5 + 7 + 5 + 5 = 44$
 The perimeter is 44 meters.

61. "Find the total" indicates addition.

$$\begin{array}{r} 13 \\ 76 \\ 39 \\ 8 \\ 17 \\ + 126 \\ \hline 266 \end{array}$$

The total of 76, 39, 8, 17, and 126 is 266.

$$\begin{array}{r} \text{Check: } \begin{array}{r} 1 \\ 25 \\ + 37 \\ \hline 62 \end{array} \end{array}$$

$$\begin{array}{r} \text{Check: } \begin{array}{r} 11 \\ 168 \\ + 432 \\ \hline 600 \end{array} \end{array}$$

$$\begin{array}{r} \text{Check: } \begin{array}{r} 1 \\ 504 \\ + 29 \\ \hline 533 \end{array} \end{array}$$

$$\begin{array}{r} \text{Check: } \begin{array}{r} 11 \quad 11 \\ 32,711 \\ + 17,289 \\ \hline 50,000 \end{array} \end{array}$$

$$\begin{array}{r} \text{Check: } \begin{array}{r} 11 \quad 11 \\ 31,213 \\ + 19,898 \\ \hline 51,111 \end{array} \end{array}$$

65. "Increased by" indicates addition.

$$\begin{array}{r} 1 \\ 452 \\ + 92 \\ \hline 544 \end{array}$$

452 increased by 92 is 544.

69. "Subtracted from" indicates subtraction.

$$\begin{array}{r} 100 \\ - 12 \\ \hline 88 \end{array}$$

12 subtracted from 100 is 88.

73. Subtract the cost of the DVD player from the amount in her savings account.

$$\begin{array}{r} 914 \\ - 295 \\ \hline 619 \end{array}$$

She will have \$619 left.

77. 530,000

$$\begin{array}{r} 530,000 \\ - 247,000 \\ \hline 283,000 \end{array}$$

The Missouri sub-basin drains 283,000 square miles more than the Arkansas Red-White sub-basin.

81. 503

$$\begin{array}{r} 503 \\ - 239 \\ \hline 264 \end{array}$$

She must read 264 more pages.

85. Live rock music has a decibel level of 100 dB.

89. 2410

$$\begin{array}{r} 2410 \\ + 1034 \\ \hline 3444 \end{array}$$

There were 3444 stores worldwide.

93. California has the most CVS pharmacies.

97. Pennsylvania and New York:

$$\begin{array}{r} 1 \\ 408 \\ + 486 \\ \hline 894 \end{array}$$

Florida and Georgia:

$$\begin{array}{r} 1 \\ 756 \\ + 313 \\ \hline 1069 \end{array}$$

Florida and Georgia have more CVS pharmacies.

101. The minuend is 48 and the subtrahend is 1.

105. answers may vary

109. $\begin{array}{r} 22 \\ 14 \end{array}$

$$\begin{array}{r} 173 \\ 86 \\ + 257 \\ \hline 530 \end{array}$$

The given sum is incorrect, the correct sum is 530.

113. $\begin{array}{r} 1 \\ 141 \\ + 888 \end{array}$

$$\begin{array}{r} 1029 \end{array}$$

The given difference is correct.

117. answers may vary

Exercise Set 1.4

1. To round 423 to the nearest ten, observe that the digit in the ones place is 3. Since this digit is less than 5, we do not add 1 to the digit in the tens place. The number 423 rounded to the nearest ten is 420.

5. To round 2791 to the nearest hundred, observe that the digit in the tens place is 9. Since this digit is 5 or greater, we add 1 to the digit in the hundreds place. The number 2791 rounded to the nearest hundred is 2800.

9. To round 21,094 to the nearest thousand, observe that the digit in the hundreds place is 0. Since this digit is less than 5, we do not add 1 to the digit in the thousands place. The number 21,094 rounded to the nearest thousand is 21,000.

13. To round 328,495 to the nearest hundred, observe that the digit in the tens place is 9. Since this digit is 5 or greater, we add 1 to the digit in the hundreds place. The number 328,495 rounded to the nearest hundred is 328,500.

17. To round 39,994 to the nearest ten, observe that the digit in the ones place is 4. Since this digit is less than 5, we do not add 1 to the digit in the tens place. The number 39,994 rounded to the nearest ten is 39,990.

21. Estimate 5281 to a given place value by rounding it to that place value. 5281 rounded to the tens place is 5280, to the hundreds place is 5300, and to the thousands place is 5000.

25. Estimate 14,876 to a given place value by rounding it to that place value. 14,876 rounded to the tens place is 14,880, to the hundreds place is 14,900, and to the thousands place is 15,000.

29. To round 60,149 to the nearest hundred, observe that the digit in the tens place is 4. Since this digit is less than 5, we do not add 1 to the digit in the hundreds place. Therefore, 60,149 days rounded to the nearest hundred is 60,100 days.

33. To round 4,155,907 to the nearest hundred-thousand, observe that the digit in the ten-thousands place is 5. Since this digit is 5 or greater, we add 1 to the digit in the hundred-thousands place. Therefore, \$4,155,907 rounded to the nearest hundred-thousand is \$4,200,000.

37. 39 rounds to 40

$$45 \text{ rounds to } 50$$

$$22 \text{ rounds to } 20$$

$$\begin{array}{r} + 17 \text{ rounds to } + 20 \\ \hline 130 \end{array}$$

41. 1913 rounds to 1900

$$1886 \text{ rounds to } 1900$$

$$\begin{array}{r} + 1925 \text{ rounds to } + 1900 \\ \hline 5700 \end{array}$$

45. 3995 rounds to 4000

$$2549 \text{ rounds to } 2500$$

$$\begin{array}{r} + 4944 \text{ rounds to } + 4900 \\ \hline 11,400 \end{array}$$

49. $229 + 443 + 606$ is approximately $230 + 440 + 610 = 1280$.

The answer of 1278 is correct.

53. 899 rounds to 900

$$1499 \text{ rounds to } 1500$$

$$\begin{array}{r} + 999 \text{ rounds to } + 1000 \\ \hline 3400 \end{array}$$

The total cost is approximately \$3400.

$$\begin{array}{r} 57. \quad 20,320 \text{ rounds to } 20,000 \\ - 14,410 \text{ rounds to } - 14,000 \\ \hline 6000 \end{array}$$

The difference in elevation is approximately 6000 feet.

$$\begin{array}{r} 61. \quad 1,128,030 \text{ rounds to } 1,128,000 \\ - 946,357 \text{ rounds to } - 946,000 \\ \hline 182,000 \end{array}$$

The decrease in enrollment was approximately 182,000 children.

65. 641 hundred-thousand is 64,100,000 in standard form.
 64,100,000 rounded to the nearest million is 64,000,000.
 64,100,000 rounded to the nearest ten-million is 60,000,000.

69. a. The smallest possible number that rounds to 8600 is 8550.
 b. The largest possible number that rounds to 8600 is 8649.

$$\begin{array}{l} 73. \quad 54 \text{ rounds to } 50 \\ 17 \text{ rounds to } 20 \\ 50 + 20 + 50 + 20 = 140 \end{array}$$

The perimeter is approximately 140 meters.

Exercise Set 1.5

1. $1 \cdot 24 = 24$
 5. $8 \cdot 0 \cdot 9 = 0$
 9. $6(3 + 8) = 6 \cdot 3 + 6 \cdot 8$
 13. $20(14 + 6) = 20 \cdot 14 + 20 \cdot 6$

$$\begin{array}{r} 17. \quad 613 \\ \times 6 \\ \hline 3678 \end{array}$$

$$\begin{array}{r} 21. \quad 1074 \\ \times 6 \\ \hline 6444 \end{array}$$

$$\begin{array}{r} 25. \quad 421 \\ \times 58 \\ \hline 3368 \\ 21050 \\ \hline 24,418 \end{array}$$

$$\begin{array}{r} 29. \quad 780 \\ \times 20 \\ \hline 15,600 \end{array}$$

$$33. (640)(1)(10) = (640)(10) = 6400$$

$$\begin{array}{r} 37. \quad 609 \\ \times 234 \\ \hline 2436 \\ 18270 \\ 121800 \\ \hline 142,506 \end{array}$$

$$\begin{array}{r} 41. \quad 589 \\ \times 110 \\ \hline 5890 \\ 58900 \\ \hline 64,790 \end{array}$$

$$\begin{array}{l} 45. \text{ Area} = (\text{length})(\text{width}) \\ = (9 \text{ meters})(7 \text{ meters}) \\ = 63 \text{ square meters} \end{array}$$

$$\begin{array}{l} \text{Perimeter} = (9 + 7 + 9 + 7) \text{ meters} \\ = 32 \text{ meters} \end{array}$$

$$\begin{array}{r} 49. \quad 576 \text{ rounds to } 600 \\ \times 354 \text{ rounds to } \times 400 \\ \hline 240,000 \end{array}$$

53. 38×42 is approximately 40×40 , which is 1600. The best estimate is c.

$$\begin{array}{l} 57. \quad 80 \times 11 = (8 \times 10) \times 11 \\ = 8 \times (10 \times 11) \\ = 8 \times 110 \\ = 880 \end{array}$$

$$\begin{array}{r} 61. \quad 2240 \\ \times 2 \\ \hline 4480 \end{array}$$

$$\begin{array}{r} 65. \quad 94 \\ \times 35 \\ \hline 470 \\ 2820 \\ \hline 3290 \end{array}$$

The total cost is \$3290.

$$\begin{array}{l} 69. \text{ Area} = (\text{length})(\text{width}) \\ = (110 \text{ feet})(80 \text{ feet}) \\ = 8800 \text{ square feet} \end{array}$$

The area is 8800 square feet.

$$\begin{array}{r} 73. \quad 94 \\ \times 62 \\ \hline 188 \\ 5640 \\ \hline 5828 \end{array}$$

There are 5828 pixels on the screen.

$$\begin{array}{r} 77. \quad 160 \\ \times 8 \\ \hline 1280 \end{array}$$

There are 1280 calories in 8 ounces.

81. There are 60 minutes in one hour, so there are 24×60 minutes in one day.

$$\begin{array}{l} 24 \times 60 \times 1000 = 24 \times 6 \times 10 \times 1000 \\ = 144 \times 10,000 \\ = 1,440,000 \end{array}$$

They produce 1,440,000 tea bags in one day.

$$\begin{array}{r} 85. \quad 134 \\ \times 16 \\ \hline 804 \\ 1340 \\ \hline 2144 \end{array}$$

$$\begin{array}{r} 89. \quad 19 \\ - 4 \\ \hline 15 \end{array}$$

The difference of 19 and 4 is 15.

93. a. $3 \cdot 5 = 5 + 5 + 5 = 3 + 3 + 3 + 3 + 3$
 b. answers may vary

$$\begin{array}{l} 97. \quad 42 \times 3 = 126 \\ 42 \times 9 = 378 \\ \text{The problem is } 42 \\ \times 93 \end{array}$$

101. On a side with 7 windows per row, there are $7 \times 23 = 161$ windows. On a side with 4 windows per row, there are $4 \times 23 = 92$ windows.
 $161 + 161 + 92 + 92 = 506$
 There are 506 windows on the building.

Exercise Set 1.6

1. $54 \div 9 = 6$
 5. $0 \div 8 = 0$
 9. $\frac{18}{18} = 1$
 13. $26 \div 0$ is undefined.
 17. $0 \div 14 = 0$

21.
$$\begin{array}{r} 29 \\ 3 \overline{)87} \\ \underline{-6} \\ 27 \\ \underline{-27} \\ 0 \end{array}$$

 Check: $3 \cdot 29 = 87$

25.
$$\begin{array}{r} 338 \\ 3 \overline{)1014} \\ \underline{-9} \\ 11 \\ \underline{-9} \\ 24 \\ \underline{-24} \\ 0 \end{array}$$

 Check: $3 \cdot 338 = 1014$

29.
$$\begin{array}{r} 9 \\ 7 \overline{)63} \\ \underline{-63} \\ 0 \end{array}$$

 Check: $7 \cdot 9 = 63$

33.
$$\begin{array}{r} 68 \text{ R } 3 \\ 7 \overline{)479} \\ \underline{-42} \\ 59 \\ \underline{-56} \\ 3 \end{array}$$

 Check: $7 \cdot 68 + 3 = 479$

37.
$$\begin{array}{r} 38 \text{ R } 1 \\ 8 \overline{)305} \\ \underline{-24} \\ 65 \\ \underline{-64} \\ 1 \end{array}$$

 Check: $8 \cdot 38 + 1 = 305$

41.
$$\begin{array}{r} 13 \\ 55 \overline{)715} \\ \underline{-55} \\ 165 \\ \underline{-165} \\ 0 \end{array}$$

 Check: $55 \cdot 13 = 715$

45.
$$\begin{array}{r} 97 \text{ R } 8 \\ 97 \overline{)9417} \\ \underline{-873} \\ 687 \\ \underline{-679} \\ 8 \end{array}$$

 Check: $97 \cdot 97 + 8 = 9417$

49.
$$\begin{array}{r} 506 \\ 13 \overline{)6578} \\ \underline{-65} \\ 07 \\ \underline{-0} \\ 78 \\ \underline{-78} \\ 0 \end{array}$$

 Check: $13 \cdot 506 = 6578$

53.
$$\begin{array}{r} 54 \\ 236 \overline{)12744} \\ \underline{-1180} \\ 944 \\ \underline{-944} \\ 0 \end{array}$$

 Check: $236 \cdot 54 = 12,744$

57.
$$\begin{array}{r} 202 \text{ R } 15 \\ 102 \overline{)20619} \\ \underline{-204} \\ 21 \\ \underline{-0} \\ 219 \\ \underline{-204} \\ 15 \end{array}$$

 Check: $102 \cdot 202 + 15 = 20,619$

61.
$$\begin{array}{r} 17 \\ 7 \overline{)119} \\ \underline{-7} \\ 49 \\ \underline{-49} \\ 0 \end{array}$$

65.
$$\begin{array}{r} 2132 \text{ R } 32 \\ 40 \overline{)85312} \\ \underline{-80} \\ 53 \\ \underline{-40} \\ 131 \\ \underline{-120} \\ 112 \\ \underline{-80} \\ 32 \end{array}$$

69.
$$\begin{array}{r} 23 \text{ R } 2 \\ 5 \overline{)117} \\ \underline{-10} \\ 17 \\ \underline{-15} \\ 2 \end{array}$$

 The quotient of 117 and 5 is 23 R 2.

73.
$$\begin{array}{r} 20 \text{ R } 2 \\ 3 \overline{)62} \\ \underline{-6} \\ 02 \\ \underline{-0} \\ 2 \end{array}$$

 The quotient of 62 and 3 is 20 R 2.

$$\begin{array}{r}
 77. \quad \begin{array}{r} 165 \\ 318 \overline{)52470} \\ \underline{-318} \\ 2067 \\ \underline{-1908} \\ 1590 \\ \underline{-1590} \\ 0 \end{array}
 \end{array}$$

The person weighs 165 pounds on Earth.

$$\begin{array}{r}
 81. \quad \begin{array}{r} 88 \text{ R } 1 \\ 3 \overline{)265} \\ \underline{-24} \\ 25 \\ \underline{-24} \\ 1 \end{array}
 \end{array}$$

There are 88 bridges every 3 miles over the 265 miles, plus the first bridge for a total of 89 bridges.

$$\begin{array}{r}
 85. \quad \begin{array}{r} 5 \\ 5280 \overline{)26400} \\ \underline{-26400} \\ 0 \end{array}
 \end{array}$$

Broad Peak is 5 miles tall.

$$\begin{array}{r}
 \begin{array}{r} 2 \\ 10 \\ 24 \\ 35 \end{array} \qquad \begin{array}{r} 20 \\ 6 \overline{)120} \\ \underline{-12} \\ 0 \end{array}
 \end{array}$$

$$\begin{array}{r}
 89. \quad \begin{array}{r} 22 \\ 17 \\ + 12 \\ \hline 120 \end{array}
 \end{array}$$

$$\text{Average} = \frac{120}{6} = 20$$

$$\begin{array}{r}
 \begin{array}{r} 2 \\ 86 \\ 79 \\ 81 \\ 69 \\ + 80 \\ \hline 395 \end{array} \qquad \begin{array}{r} 79 \\ 5 \overline{)395} \\ \underline{-35} \\ 45 \\ \underline{-45} \\ 0 \end{array}
 \end{array}$$

$$\text{Average} = \frac{395}{5} = 79$$

$$\begin{array}{r}
 97. \quad \begin{array}{r} 111 \\ 82 \\ 463 \\ 29 \\ + 8704 \\ \hline 9278 \end{array}
 \end{array}$$

$$\begin{array}{r}
 101. \quad \begin{array}{r} 722 \\ -43 \\ \hline 679 \end{array}
 \end{array}$$

$$\begin{array}{r}
 105. \quad \begin{array}{r} 9 \text{ R } 12 \\ 24 \overline{)228} \\ \underline{-216} \\ 12 \end{array}
 \end{array}$$

109. 200 divided by 20 is $200 \div 20$, which is choice b.

113. The average will increase; answers may vary.

$$117. \text{ Since Area} = \text{length} \cdot \text{width, length} = \frac{\text{Area}}{\text{width}}.$$

$$\text{length} = \frac{60 \text{ square feet}}{5 \text{ feet}} = \frac{60}{5} \text{ feet} = 12 \text{ feet}$$

The length is 12 feet.

$$\begin{array}{r}
 121. \quad \begin{array}{r} 26 \\ -5 \\ \hline 21 \\ -5 \\ \hline 16 \\ -5 \\ \hline 11 \\ -5 \\ \hline 6 \\ -5 \\ \hline 1 \end{array}
 \end{array}$$

$$\text{Thus } 26 \div 5 = 5 \text{ R } 1.$$

Exercise Set 1.7

$$1. 4 \cdot 4 \cdot 4 = 4^3$$

$$5. 12 \cdot 12 \cdot 12 = 12^3$$

$$9. 9 \cdot 8 \cdot 8 = 9 \cdot 8^2$$

$$13. 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 3 \cdot 2^4 \cdot 5^5$$

$$17. 5^3 = 5 \cdot 5 \cdot 5 = 125$$

$$21. 1^{10} = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

$$25. 2^7 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 128$$

$$29. 4^4 = 4 \cdot 4 \cdot 4 \cdot 4 = 256$$

$$33. 12^2 = 12 \cdot 12 = 144$$

$$37. 20^1 = 20$$

$$41. 3 \cdot 2^6 = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 192$$

$$45. 15 + 3 \cdot 2 = 15 + 6 = 21$$

$$49. 32 \div 4 - 3 = 8 - 3 = 5$$

$$53. 6 \cdot 5 + 8 \cdot 2 = 30 + 16 = 46$$

$$\begin{aligned}
 57. (7 + 5^2) \div 4 \cdot 2^3 &= (7 + 25) \div 4 \cdot 2^3 \\
 &= 32 \div 4 \cdot 2^3 \\
 &= 32 \div 4 \cdot 8 \\
 &= 8 \cdot 8 \\
 &= 64
 \end{aligned}$$

$$61. \frac{18 + 6}{2^4 - 2^2} = \frac{24}{16 - 4} = \frac{24}{12} = 2$$

$$65. \frac{7(9 - 6) + 3}{3^2 - 3} = \frac{7(3) + 3}{9 - 3} = \frac{21 + 3}{6} = \frac{24}{6} = 4$$

$$\begin{aligned}
 69. 2^4 \cdot 4 - (25 \div 5) &= 2^4 \cdot 4 - 5 \\
 &= 16 \cdot 4 - 5 \\
 &= 64 - 5 \\
 &= 59
 \end{aligned}$$

$$\begin{aligned}
 73. (7 \cdot 5) + [9 \div (3 \div 3)] &= (7 \cdot 5) + [9 \div (1)] \\
 &= 35 + 9 \\
 &= 44
 \end{aligned}$$

$$\begin{aligned}
 77. \frac{9^2 + 2^2 - 1^2}{8 \div 2 \cdot 3 \cdot 1 \div 3} &= \frac{81 + 4 - 1}{4 \cdot 3 \cdot 1 \div 3} \\
 &= \frac{85 - 1}{12 \cdot 1 \div 3} \\
 &= \frac{84}{12 \div 3} \\
 &= \frac{84}{4} \\
 &= 21
 \end{aligned}$$

$$\begin{aligned}
 81. \quad 9 \div 3 + 5^2 \cdot 2 - 10 &= 9 \div 3 + 25 \cdot 2 - 10 \\
 &= 3 + 50 - 10 \\
 &= 53 - 10 \\
 &= 43
 \end{aligned}$$

$$\begin{aligned}
 85. \quad 7^2 - \{18 - [40 \div (5 \cdot 1) + 2] + 5^2\} \\
 &= 7^2 - \{18 - [40 \div (5) + 2] + 5^2\} \\
 &= 7^2 - \{18 - [8 + 2] + 5^2\} \\
 &= 7^2 - \{18 - [10] + 5^2\} \\
 &= 7^2 - \{18 - 10 + 25\} \\
 &= 7^2 - \{8 + 25\} \\
 &= 7^2 - \{33\} \\
 &= 49 - 33 \\
 &= 16
 \end{aligned}$$

$$\begin{aligned}
 89. \quad \text{Area of a square} &= (\text{side})^2 \\
 &= (23 \text{ miles})^2 \\
 &= 529 \text{ square miles} \\
 \text{Perimeter} &= (23 + 23 + 23 + 23) \text{ meters} \\
 &= 92 \text{ meters}
 \end{aligned}$$

$$93. \quad 2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

The statement is false.

$$97. \quad 24 \div (3 \cdot 2) + 2 \cdot 5 = 24 \div 6 + 2 \cdot 5 = 4 + 10 = 14$$

$$\begin{aligned}
 101. \quad (7 + 2^4)^5 - (3^5 - 2^4)^2 &= (7 + 16)^5 - (243 - 16)^2 \\
 &= 23^5 - 227^2 \\
 &= 6,436,343 - 51,529 \\
 &= 6,384,814
 \end{aligned}$$

Exercise Set 1.8

1.

a	b	$a + b$	$a - b$	$a \cdot b$	$a \div b$
21	7	$21 + 7 = 28$	$21 - 7 = 14$	$21 \cdot 7 = 147$	$21 \div 7 = 3$

5.

a	b	$a + b$	$a - b$	$a \cdot b$	$a \div b$
56	1	$56 + 1 = 57$	$56 - 1 = 55$	$56 \cdot 1 = 56$	$56 \div 1 = 56$

$$9. \quad 3xz - 5x = 3(2)(3) - 5(2) = 18 - 10 = 8$$

$$13. \quad 4x - z = 4(2) - 3 = 8 - 3 = 5$$

$$\begin{aligned}
 17. \quad 2xy^2 - 6 &= 2(2)(5)^2 - 6 \\
 &= 2 \cdot 2 \cdot 25 - 6 \\
 &= 100 - 6 \\
 &= 94
 \end{aligned}$$

$$\begin{aligned}
 21. \quad x^5 + (y - z) &= 2^5 + (5 - 3) \\
 &= 2^5 + 2 \\
 &= 32 + 2 \\
 &= 34
 \end{aligned}$$

$$25. \quad \frac{2y - 2}{x} = \frac{2(5) - 2}{2} = \frac{10 - 2}{2} = \frac{8}{2} = 4$$

$$29. \quad \frac{5x}{y} - \frac{10}{y} = \frac{5(2)}{5} - \frac{10}{5} = \frac{10}{5} - 2 = 2 - 2 = 0$$

$$\begin{aligned}
 33. \quad (4y - 5z)^3 &= (4 \cdot 5 - 5 \cdot 3)^3 \\
 &= (20 - 15)^3 \\
 &= (5)^3 \\
 &= 125
 \end{aligned}$$

$$\begin{aligned}
 37. \quad 2y(4z - x) &= 2 \cdot 5(4 \cdot 3 - 2) \\
 &= 2 \cdot 5(12 - 2) \\
 &= 2 \cdot 5(10) \\
 &= 10(10) \\
 &= 100
 \end{aligned}$$

$$41. \quad \frac{7x + 2y}{3x} = \frac{7(2) + 2(5)}{3(2)} = \frac{14 + 10}{6} = \frac{24}{6} = 4$$

45. Let n be 10.

$$\begin{aligned}
 n - 8 &= 2 \\
 10 - 8 &\stackrel{?}{=} 2 \\
 2 &= 2 \quad \text{True}
 \end{aligned}$$

Yes, 10 is a solution.

49. Let n be 7.

$$\begin{aligned}
 3n - 5 &= 10 \\
 3(7) - 5 &\stackrel{?}{=} 10 \\
 21 - 5 &\stackrel{?}{=} 10 \\
 16 &= 10 \quad \text{False}
 \end{aligned}$$

No, 7 is not a solution.

53. Let x be 0.

$$\begin{aligned}
 5x + 3 &= 4x + 13 \\
 5(0) + 3 &\stackrel{?}{=} 4(0) + 13 \\
 0 + 3 &\stackrel{?}{=} 0 + 13 \\
 3 &= 13 \quad \text{False}
 \end{aligned}$$

No, 0 is not a solution.

57. $n - 2 = 10$

$$\begin{aligned}
 \text{Let } n \text{ be } 10. \\
 10 - 2 &\stackrel{?}{=} 10 \\
 8 &= 10 \quad \text{False}
 \end{aligned}$$

Let n be 12.

$$\begin{aligned}
 12 - 2 &\stackrel{?}{=} 10 \\
 10 &= 10 \quad \text{True}
 \end{aligned}$$

Let n be 14.

$$\begin{aligned}
 14 - 2 &\stackrel{?}{=} 10 \\
 12 &= 10 \quad \text{False}
 \end{aligned}$$

12 is a solution.

61. $6n + 2 = 26$

$$\begin{aligned}
 \text{Let } n \text{ be } 0. \\
 6(0) + 2 &\stackrel{?}{=} 26 \\
 0 + 2 &\stackrel{?}{=} 26
 \end{aligned}$$

$$2 = 26 \quad \text{False}$$

Let n be 2.

$$\begin{aligned}
 6(2) + 2 &\stackrel{?}{=} 26 \\
 12 + 2 &\stackrel{?}{=} 26 \\
 14 &= 26 \quad \text{False}
 \end{aligned}$$

Let n be 4.

$$\begin{aligned}
 6(4) + 2 &\stackrel{?}{=} 26 \\
 24 + 2 &\stackrel{?}{=} 26 \\
 26 &= 26 \quad \text{True}
 \end{aligned}$$

4 is a solution.

65. $7x - 9 = 5x + 13$

$$\begin{aligned}
 \text{Let } x \text{ be } 3. \\
 7(3) - 9 &\stackrel{?}{=} 5(3) + 13 \\
 21 - 9 &\stackrel{?}{=} 15 + 13 \\
 12 &= 28 \quad \text{False}
 \end{aligned}$$

Let x be 7.

$$\begin{aligned}
 7(7) - 9 &\stackrel{?}{=} 5(7) + 13 \\
 49 - 9 &\stackrel{?}{=} 35 + 13 \\
 40 &= 48 \quad \text{False}
 \end{aligned}$$

Let x be 11.

$$\begin{aligned}
 7(11) - 9 &\stackrel{?}{=} 5(11) + 13 \\
 77 - 9 &\stackrel{?}{=} 55 + 13 \\
 68 &= 68 \quad \text{True}
 \end{aligned}$$

11 is a solution.

69. The total of a number and 8 is $x + 8$.

73. The product of 512 and a number is $512x$.

77. The sum of seventeen and a number added to the product of five and the number is $5x + (17 + x)$.

81. A number subtracted from 11 is $11 - x$.

85. 6 divided by a number is $6 \div x$ or $\frac{6}{x}$.

89. The variables were not replaced with the correct values. Replace x with 3 and y with 5.

$$\begin{aligned} 2x + 3y &= 2(3) + 3(2) \\ &= 6 + 5 \\ &= 11 \end{aligned}$$

93. $x^4 - y^2 = 23^4 - 72^2$
 $= 279,841 - 5184$
 $= 274,657$

97. $5x$ is the largest; answers may vary.

Chapter 1 Test

1. 82,426 in words is eighty-two thousand, four hundred twenty-six.

5.
$$\begin{array}{r} 496 \\ \times 30 \\ \hline 14,880 \end{array}$$

9. $0 \div 49 = 0$

13. $6^1 \cdot 2^3 = 6 \cdot 2 \cdot 2 \cdot 2 = 48$

17. To round 52,369 to the nearest thousand, observe that the digit in the hundreds place is 3. Since this digit is less than 5, we do not add 1 to the digit in the thousands place. The number 52,369 rounded to the nearest thousand is 52,000.

21.
$$\begin{array}{r} 15 \\ + 107 \\ \hline 122 \end{array}$$

25.
$$\begin{array}{r} 725 \\ - 599 \\ \hline 126 \end{array}$$

The higher-priced one is \$126 more.

29. Perimeter = $(20 + 10 + 20 + 10)$ yards = 60 yards

$$\begin{aligned} \text{Area} &= (\text{length})(\text{width}) \\ &= (20 \text{ yards})(10 \text{ yards}) \\ &= 200 \text{ square yards} \end{aligned}$$

33. Let n be 6.

$$5n - 11 = 19$$

$$5(6) - 11 \stackrel{?}{=} 19$$

$$30 - 11 \stackrel{?}{=} 19$$

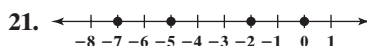
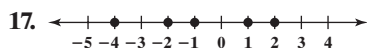
$$19 = 19 \quad \text{True}$$

6 is a solution.

Chapter 2

Exercise Set 2.1

1. If 0 represents ground level, then 1235 feet underground is -1235 .
5. If 0 represents zero degrees Fahrenheit, then 120 degrees above zero is $+120$.
9. If 0 represents a loss of \$0, then a loss of \$1270 million is -1270 million.
13. The coldest temperature on record for the state of Idaho is -60° or -60°F .



25. $-7 < -5$ since -7 is to the left of -5 on a number line.

29. $-26 < 26$ since -26 is to the left of 26 on a number line.

33. $|-8| = 8$ since -8 is 8 units from 0 on a number line.

37. $|-55| = 55$ since -55 is 55 units from 0 on a number line.

41. The opposite of negative 4 is 4. $-(-4) = 4$

45. The opposite of negative 85 is 85. $-(-85) = 85$

49. $-|20| = -20$

53. $-(-43) = 43$

57. $-(-33) = 33$

61. $-|-x| = -|-2| = -2$

65. $-|x| = -|7| = -7$

69. $|-8| = 8$

$$|-11| = 11$$

Since $8 < 11$, $|-8| < |-11|$.

73. $-|-12| = -12$

$$-(-12) = 12$$

Since $-12 < 12$, $-|-12| < -(-12)$.

77. $|0| = 0$

$$|-9| = 9$$

Since $0 < 9$, $|0| < |-9|$.

81. $-(-12) = 12$

$$-(-18) = 18$$

Since $12 < 18$, $-(-12) < -(-18)$.

85. If the opposite of a number is -28 , then the number is 28, and its absolute value is 28.

89. The tallest bar on the graph corresponds to Lake Superior, so Lake Superior has the highest elevation.

93. The number on the graph closest to -200°C is -186°C , which corresponds to the element oxygen.

97.
$$\begin{array}{r} 15 \\ + 20 \\ \hline 35 \end{array}$$

101. $2^2 = 4$, $-|3| = -3$, $-(-5) = 5$, and $-|-8| = -8$, so the numbers in order from least to greatest are $-|-8|$, $-|3|$, 2^2 , $-(-5)$.

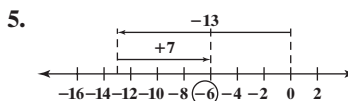
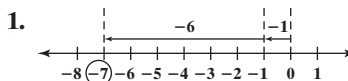
105. $-(-2) = 2$, $5^2 = 25$, $-10 = -10$, $-|-9| = -9$, and $|-12| = 12$, so the numbers in order from least to greatest are -10 , $-|-9|$, $-(-2)$, $|-12|$, 5^2 .

109. $-(-|-8|) = -(-8) = 8$

113. True; a positive number will always be to the right of a negative number on a number line.

117. answers may vary

Exercise Set 2.2



9. $|-8| + |-2| = 8 + 2 = 10$

The common sign is negative, so $-8 + (-2) = -10$.

13. $|6| - |-2| = 6 - 2 = 4$

$6 > 2$, so the answer is positive.

$$6 + (-2) = 4$$

17. $|-5| - |3| = 5 - 3 = 2$

$5 > 3$, so the answer is negative.

$$3 + (-5) = -2$$

21. $|-12| + |-12| = 12 + 12 = 24$

The common sign is negative, so $-12 + (-12) = -24$.

25. $|12| - |-5| = 12 - 5 = 7$
 $12 > 5$, so the answer is positive.
 $12 + (-5) = 7$
29. $|-56| - |26| = 56 - 26 = 30$
 $56 > 26$, so the answer is negative.
 $-56 + 26 = -30$
33. $|-144| - |124| = 144 - 124 = 20$
 $144 > 124$, so the answer is negative.
 $124 + (-144) = -20$
37. $-4 + 2 + (-5) = -2 + (-5) = -7$
41. $12 + (-4) + (-4) + 12 = 8 + (-4) + 12$
 $= 4 + 12$
 $= 16$
45. $-6 + (-15) + (-7) = -21 + (-7) = -28$
49. $5 + (-2) + 17 = 3 + 17 = 20$
53. $3 + 14 + (-18) = 17 + (-18) = -1$
57. $-13 + 8 + (-10) + (-27) = -5 + (-10) + (-27)$
 $= -15 + (-27)$
 $= -42$
61. $3x + y = 3(2) + (-3) = 6 + (-3) = 3$
65. The sum of -6 and 25 is $-6 + 25 = 19$.
69. $0 + (-215) + (-16) = -215 + (-16) = -231$
The diver's present depth is 231 feet below the surface.
73. The bar for 2016 has a height of 45,687, so the net income in 2016 was \$45,687,000,000.
77. $-10 + 12 = 2$
The temperature at 11 p.m. was 2°C .
81. $-61 + 30 = -31$
Oklahoma's all-time record low temperature is -31°F .
85. $44 - 0 = 44$
89. answers may vary
93. $-10 + (-12) = -22$
97. False; for example, $4 + (-2) = 2 > 0$.

Exercise Set 2.3

1. $-8 - (-8) = -8 + 8 = 0$
5. $3 - 8 = 3 + (-8) = -5$
9. $-4 - (-7) = -4 + 7 = 3$
13. $3 - 15 = 3 + (-15) = -12$
17. $478 - (-30) = 478 + 30 = 508$
21. $-7 - (-3) = -7 + 3 = -4$
25. $-25 - 17 = -25 + (-17) = -42$
29. $2 - (-12) = 2 + 12 = 14$
33. $8 - 13 = 8 + (-13) = -5$
37. $30 - 67 = 30 + (-67) = -37$
41. $13 - 5 - 7 = 13 + (-5) + (-7) = 8 + (-7) = 1$
45. $-11 + (-6) - 14 = -11 + (-6) + (-14)$
 $= -17 + (-14)$
 $= -31$
49. $-(-5) - 21 + (-16) = 5 + (-21) + (-16)$
 $= -16 + (-16)$
 $= -32$
53. $-3 + 4 - (-23) - 10 = -3 + 4 + 23 + (-10)$
 $= 1 + 23 + (-10)$
 $= 24 + (-10)$
 $= 14$

57. $x - y = 8 - (-23) = 8 + 23 = 31$
61. $2x - y = 2(1) - (-18) = 2 + 18 = 20$
65. The state with the coldest minimum temperature is Alaska.
The extreme temperatures for Alaska are 100 and -80° .
 $100 - (-80) = 100 + 180 = 180$
The temperature extreme for Alaska is 180°F .
69. $+7 - (-19) = 7 + 19 = 26$
The difference in scores was 26 strokes.
73. $-282 - (-436) = -282 + 436 = 154$
The difference in elevation is 154 feet.
77. $600 - (-52) = 600 + 52 = 652$
The difference in elevation is 652 feet.
81. $867 - (-330) = 867 + 330 = 1197$
The difference in temperatures is 1197°F .
85. The sum of -5 and a number is $-5 + x$.
89. $\frac{100}{20} = 5$
93. answers may vary
97. $10 - 30 = 10 + (-30) = -20$
101. $|-5| - |5| = 5 - 5 = 0$
105. $|-8 - 3| = |-8 + (-3)| = |-11| = 11$
 $8 - 3 = 8 + (-3) = 5$
Since $11 \neq 5$, the statement is false.

Exercise Set 2.4

1. $-6(-2) = 12$
5. $9(-9) = -81$
9. $6(-2)(-4) = -12(-4) = 48$
13. $-4(4)(-5) = -16(-5) = 80$
17. $-5(3)(-1)(-1) = -15(-1)(-1) = 15(-1) = -15$
21. $(-3)^3 = (-3)(-3)(-3) = 9(-3) = -27$
25. $(-4)^3 = (-4)(-4)(-4) = 16(-4) = -64$
29. $\frac{-30}{6} = -5$
33. $\frac{0}{-21} = 0$
37. $\frac{56}{-4} = -14$
41. $-5(3) = -15$
45. $-7(-6) = 42$
49. $(-7)^2 = (-7)(-7) = 49$
53. $-\frac{72}{8} = -9$
57. $4(-10)(-3) = -40(-3) = 120$
61. $\frac{-25}{0}$ is undefined.
65. $280 \div (-40) = \frac{280}{-40} = -7$
69. $-1^4 = -(1 \cdot 1 \cdot 1 \cdot 1) = -1$
73. $-2(3)(5)(-6) = -6(5)(-6) = -30(-6) = 180$
77. $-2(-3)(-5) = 6(-5) = -30$
81. $\frac{35}{\frac{\times 82}{70}}$
 $\frac{2800}{2870}$
 $35 \cdot (-82) = -2870$

$$85. ab = 9(-2) = -18$$

$$89. \frac{x}{y} = \frac{5}{-5} = -1$$

$$93. \frac{x}{y} = \frac{-36}{-6} = 6$$

$$97. xy = 0(-8) = 0$$

$$\frac{x}{y} = \frac{0}{-8} = 0$$

$$101. \begin{array}{r} 42 \\ \times 6 \\ \hline 252 \end{array}$$

$$-42(-6) = 252$$

$$105. \text{ Subtract a number from } -16 \text{ is } -16 - x.$$

$$109. \text{ Divide a number by } -33 \text{ is } \frac{x}{-33} \text{ or } x \div (-33).$$

$$113. \text{ Each move of 20 feet down is represented by } -20.$$

$$5 \cdot (-20) = -100$$

The diver is at a depth of 100 feet.

$$117. -3 \cdot 63 = -189$$

The melting point of argon is -189°C .

$$121. \text{ a. } 1109 - 24,814 = -23,705$$

There were 23,705 fewer analog movie screens in 2015.

This is a change of $-23,705$ movie screens.

$$\text{b. This is a period of 5 years.}$$

$$\frac{-23,705}{5} = -4741$$

The average change was -4741 analog movie screens per year.

$$125. 12 \div 4 - 2 + 7 = 3 - 2 + 7 = 1 + 7 = 8$$

$$129. -8 - 20 = -8 + (-20) = -28$$

$$133. \text{ The product of an odd number of negative numbers is negative, so the product of seven negative numbers is negative.}$$

$$137. \text{ answers may vary}$$

Exercise Set 2.5

$$1. (-5)^3 = (-5)(-5)(-5) = 25(-5) = -125$$

$$5. 8 \cdot 2^2 = 8 \cdot 4 = 32$$

$$9. 7 + 3(-6) = 7 + (-18) = -11$$

$$13. -10 + 4 \div 2 = -10 + 2 = -8$$

$$17. \frac{16 - 13}{-3} = \frac{3}{-3} = -1$$

$$21. 5(-3) - (-12) = -15 - (-12) = -15 + 12 = -3$$

$$\begin{aligned} 25. 8 \cdot 6 - 3 \cdot 5 + (-20) &= 48 - 3 \cdot 5 + (-20) \\ &= 48 - 15 + (-20) \\ &= 33 + (-20) \\ &= 13 \end{aligned}$$

$$29. |7 + 3| \cdot 2^3 = |10| \cdot 2^3 = 10 \cdot 2^3 = 10 \cdot 8 = 80$$

$$\begin{aligned} 33. 7^2 - (4 - 2^3) &= 7^2 - (4 - 8) \\ &= 7^2 - (-4) \\ &= 49 - (-4) \\ &= 49 + 4 \\ &= 53 \end{aligned}$$

$$37. -(-2)^6 = -64$$

$$\begin{aligned} 41. |8 - 24| \cdot (-2) \div (-2) &= |-16| \cdot (-2) \div (-2) \\ &= 16 \cdot (-2) \div (-2) \\ &= -32 \div (-2) \\ &= 16 \end{aligned}$$

$$\begin{aligned} 45. 5(5 - 2) + (-5)^2 - 6 &= 5(3) + (-5)^2 - 6 \\ &= 5(3) + 25 - 6 \\ &= 15 + 25 - 6 \\ &= 40 - 6 \\ &= 34 \end{aligned}$$

$$49. (-36 \div 6) - (4 \div 4) = -6 - 1 = -7$$

$$\begin{aligned} 53. 2(8 - 10)^2 - 5(1 - 6)^2 &= 2(-2)^2 - 5(-5)^2 \\ &= 2(4) - 5(25) \\ &= 8 - 125 \\ &= -117 \end{aligned}$$

$$\begin{aligned} 57. \frac{(-7)(-3) - (4)(3)}{3[7 \div (3 - 10)]} &= \frac{21 - 12}{3[7 \div (-7)]} \\ &= \frac{9}{3(-1)} \\ &= \frac{9}{-3} \\ &= -3 \end{aligned}$$

$$61. x + y + z = -2 + 4 + (-1) = 2 + (-1) = 1$$

$$65. x^2 - y = (-2)^2 - 4 = 4 - 4 = 0$$

$$69. x^2 = (-3)^2 = 9$$

$$73. 2z^3 = 2(-4)^3 = 2(-64) = -128$$

$$\begin{aligned} 77. 2x^3 - z &= 2(-3)^3 - (-4) \\ &= 2(-27) - (-4) \\ &= -54 - (-4) \\ &= -54 + 4 \\ &= -50 \end{aligned}$$

$$81. \text{ average} = \frac{-17 + (-26) + (-20) + (-13)}{4}$$

$$\begin{aligned} &= \frac{-76}{4} \\ &= -19 \end{aligned}$$

$$85. \text{ average} = \frac{-6 + 0 + 2 + 4}{4}$$

$$\begin{aligned} &= \frac{0}{4} \\ &= 0 \end{aligned}$$

The average of the scores is 0.

$$89. 45$$

$$\begin{array}{r} \times 90 \\ 4050 \end{array}$$

$$93. 8 + 8 + 8 + 8 = 32$$

The perimeter is 32 inches.

$$97. 2 \cdot (7 - 5) \cdot 3 = 2 \cdot 2 \cdot 3 = 4 \cdot 3 = 12$$

$$101. \text{ answers may vary}$$

$$105. (-12)^4 = (-12)(-12)(-12)(-12) = 20,736$$

$$\begin{aligned} 109. (xy + z)^x &= [2(-5) + 7]^2 \\ &= [-10 + 7]^2 \\ &= [-3]^2 \\ &= 9 \end{aligned}$$

Exercise Set 2.6

$$1. x - 8 = -2$$

$$6 - 8 \stackrel{?}{=} -2$$

$$-2 = -2 \quad \text{True}$$

Since $-2 = -2$ is true, 6 is a solution of the equation.

$$\begin{aligned}
 5. \quad & -9f = 64 - f \\
 & -9(-8) \stackrel{?}{=} 64 - (-8) \\
 & \quad 72 \stackrel{?}{=} 64 + 8 \\
 & \quad 72 = 72 \quad \text{True} \\
 & \text{Since } 72 = 72 \text{ is true, } -8 \text{ is a solution of the equation.}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & a + 5 = 23 \\
 & a + 5 - 5 = 23 - 5 \\
 & \quad a = 18 \\
 & \text{Check: } a + 5 = 23 \\
 & \quad 18 + 5 \stackrel{?}{=} 23 \\
 & \quad 23 = 23 \quad \text{True} \\
 & \text{The solution is 18.}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & 7 = y - 2 \\
 & 7 + 2 = y - 2 + 2 \\
 & \quad 9 = y \\
 & \text{Check: } 7 = y - 2 \\
 & \quad 7 \stackrel{?}{=} 9 - 2 \\
 & \quad 7 = 7 \quad \text{True} \\
 & \text{The solution is 9.}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & 5x = 20 \\
 & \frac{5x}{5} = \frac{20}{5} \\
 & \frac{5}{5} \cdot x = \frac{20}{5} \\
 & \quad x = 4 \\
 & \text{Check: } 5x = 20 \\
 & \quad 5(4) \stackrel{?}{=} 20 \\
 & \quad 20 = 20 \quad \text{True} \\
 & \text{The solution is 4.}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & \frac{n}{7} = -2 \\
 & 7 \cdot \frac{n}{7} = 7 \cdot (-2) \\
 & \frac{7}{7} \cdot n = 7 \cdot (-2) \\
 & \quad n = -14 \\
 & \text{Check: } \frac{n}{7} = -2 \\
 & \quad \frac{-14}{7} \stackrel{?}{=} -2 \\
 & \quad -2 = -2 \quad \text{True} \\
 & \text{The solution is } -14.
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & -4y = 0 \\
 & \frac{-4y}{-4} = \frac{0}{-4} \\
 & \frac{-4}{-4} \cdot y = \frac{0}{-4} \\
 & \quad y = 0 \\
 & \text{Check: } -4y = 0 \\
 & \quad -4(0) \stackrel{?}{=} 0 \\
 & \quad 0 = 0 \quad \text{True} \\
 & \text{The solution is 0.}
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & 5x = -35 \\
 & \frac{5x}{5} = \frac{-35}{5} \\
 & \frac{5}{5} \cdot x = \frac{-35}{5} \\
 & \quad x = -7 \\
 & \text{The solution is } -7.
 \end{aligned}$$

$$\begin{aligned}
 33. \quad & -15 = y + 10 \\
 & -15 - 10 = y + 10 - 10 \\
 & \quad -25 = y \\
 & \text{The solution is } -25.
 \end{aligned}$$

$$\begin{aligned}
 37. \quad & n = -10 + 31 \\
 & \quad n = 21 \\
 & \text{The solution is 21.}
 \end{aligned}$$

$$\begin{aligned}
 41. \quad & \frac{n}{4} = -20 \\
 & 4 \cdot \frac{n}{4} = 4 \cdot (-20) \\
 & \frac{4}{4} \cdot n = 4 \cdot (-20) \\
 & \quad n = -80 \\
 & \text{The solution is } -80.
 \end{aligned}$$

45. A number decreased by -2 is $x - (-2)$.

49. The sum of -15 and a number is $-15 + x$.

$$\begin{aligned}
 53. \quad & n - 42,860 = -1286 \\
 & n - 42,860 + 42,860 = -1286 + 42,860 \\
 & \quad n = 41,574 \\
 & \text{The solution is 41,574.}
 \end{aligned}$$

57. answers may vary

Chapter 2 Test

$$\begin{aligned}
 1. \quad & -5 + 8 = 3 \\
 5. \quad & -18 + (-12) = -30 \\
 9. \quad & |-25| + (-13) = 25 + (-13) = 12 \\
 13. \quad & -8 + 9 \div (-3) = -8 + (-3) = -11 \\
 17. \quad & -(-7)^2 \div 7 \cdot (-4) = -49 \div 7 \cdot (-4) = -7 \cdot (-4) = 28 \\
 21. \quad & \frac{|25 - 30|^2}{2(-6) + 7} = \frac{|-5|^2}{-12 + 7} = \frac{(5)^2}{-5} = \frac{25}{-5} = -5 \\
 25. \quad & \frac{3z}{2y} = \frac{3(2)}{2(-3)} = \frac{6}{-6} = -1 \\
 29. \quad & \text{Subtract the depth of the lake from the elevation of the surface.} \\
 & 1495 - 5315 = 1495 + (-5315) = -3820 \\
 & \text{The deepest point on the lake is 3820 feet below sea level.}
 \end{aligned}$$

$$\begin{aligned}
 33. \quad & \frac{n}{-7} = 4 \\
 & -7 \cdot \frac{n}{-7} = -7 \cdot 4 \\
 & \frac{-7}{-7} \cdot n = -7 \cdot 4 \\
 & \quad n = -28 \\
 & \text{The solution is } -28.
 \end{aligned}$$

Chapter 3

Exercise Set 3.1

$$\begin{aligned}
 1. \quad & 3x + 5x = (3 + 5)x = 8x \\
 5. \quad & 4c + c - 7c = (4 + 1 - 7)c = -2c \\
 9. \quad & 3a + 2a + 7a - 5 = (3 + 2 + 7)a - 5 = 12a - 5 \\
 13. \quad & -3(11y) = (-3 \cdot 11)y = -33y \\
 17. \quad & 2(y + 3) = 2 \cdot y + 2 \cdot 3 = 2y + 6 \\
 21. \quad & -4(3x + 7) = -4 \cdot 3x + (-4) \cdot 7 = -12x - 28
 \end{aligned}$$

$$\begin{aligned}
 25. \quad 8 + 5(3c - 1) &= 8 + 5 \cdot 3c - 5 \cdot 1 \\
 &= 8 + 15c - 5 \\
 &= 15c + 8 - 5 \\
 &= 15c + 3
 \end{aligned}$$

$$\begin{aligned}
 29. \quad 3 + 6(w + 2) + w &= 3 + 6 \cdot w + 6 \cdot 2 + w \\
 &= 3 + 6w + 12 + w \\
 &= 6w + w + 3 + 12 \\
 &= 7w + 15
 \end{aligned}$$

$$\begin{aligned}
 33. \quad -(2y - 6) + 10 &= -1(2y - 6) + 10 \\
 &= -1 \cdot 2y - (-1) \cdot 6 + 10 \\
 &= -2y + 6 + 10 \\
 &= -2y + 16
 \end{aligned}$$

$$37. \quad z - 8z = (1 - 8)z = -7z$$

$$41. \quad 2y - 6 + 4y - 8 = 2y + 4y - 6 - 8 = 6y - 14$$

$$\begin{aligned}
 45. \quad 2(x + 1) + 20 &= 2 \cdot x + 2 \cdot 1 + 20 \\
 &= 2x + 2 + 20 \\
 &= 2x + 22
 \end{aligned}$$

$$\begin{aligned}
 49. \quad -5(z + 3) + 2z &= -5 \cdot z + (-5) \cdot 3 + 2z \\
 &= -5z - 15 + 2z \\
 &= -5z + 2z - 15 \\
 &= -3z - 15
 \end{aligned}$$

$$\begin{aligned}
 53. \quad -7(x + 5) + 5(2x + 1) \\
 &= -7 \cdot x + (-7) \cdot 5 + 5 \cdot 2x + 5 \cdot 1 \\
 &= -7x - 35 + 10x + 5 \\
 &= -7x + 10x - 35 + 5 \\
 &= 3x - 30
 \end{aligned}$$

$$\begin{aligned}
 57. \quad -3(n - 1) - 4n &= -3 \cdot n - (-3) \cdot 1 - 4n \\
 &= -3n + 3 - 4n \\
 &= -3n - 4n + 3 \\
 &= -7n + 3
 \end{aligned}$$

$$\begin{aligned}
 61. \quad 6(2x - 1) - 12x &= 6 \cdot 2x - 6 \cdot 1 - 12x \\
 &= 12x - 6 - 12x \\
 &= 12x - 12x - 6 \\
 &= -6
 \end{aligned}$$

$$\begin{aligned}
 65. \quad -(4x - 10) + 2(3x + 5) \\
 &= -1(4x - 10) + 2 \cdot 3x + 2 \cdot 5 \\
 &= -1 \cdot 4x - (-1) \cdot 10 + 6x + 10 \\
 &= -4x + 10 + 6x + 10 \\
 &= -4x + 6x + 10 + 10 \\
 &= 2x + 20
 \end{aligned}$$

$$\begin{aligned}
 69. \quad 5y - 2(y - 1) + 3 &= 5y - 2 \cdot y - (-2) \cdot 1 + 3 \\
 &= 5y - 2y + 2 + 3 \\
 &= 3y + 5
 \end{aligned}$$

$$\begin{aligned}
 73. \quad 2a + 2a + 6 + 5a + 6 + 2a \\
 &= 2a + 2a + 5a + 2a + 6 + 6 \\
 &= (2 + 2 + 5 + 2)a + 6 + 6 \\
 &= 11a + 12
 \end{aligned}$$

The perimeter is $(11a + 12)$ feet.

$$\begin{aligned}
 77. \quad \text{Area} &= (\text{length}) \cdot (\text{width}) \\
 &= (4y) \cdot (9) \\
 &= (4 \cdot 9)y \\
 &= 36y
 \end{aligned}$$

The area is $36y$ square inches.

$$\begin{aligned}
 81. \quad \text{Area} &= (\text{length}) \cdot (\text{width}) \\
 &= (3y + 1) \cdot (20) \\
 &= 3y \cdot 20 + 1 \cdot 20 \\
 &= (3 \cdot 20)y + 20 \\
 &= 60y + 20
 \end{aligned}$$

The area is $(60y + 20)$ square miles.

$$\begin{aligned}
 85. \quad \text{Perimeter} &= 2 \cdot (\text{length}) + 2 \cdot (\text{width}) \\
 &= 2 \cdot (18) + 2 \cdot (14) \\
 &= 36 + 28 \\
 &= 64
 \end{aligned}$$

The perimeter is 64 feet.

$$89. \quad -13 + 10 = -3$$

$$93. \quad -4 + 4 = 0$$

$$\begin{aligned}
 97. \quad 2(xy) &= 2 \cdot x \cdot y \\
 &= 2xy
 \end{aligned}$$

The expressions are not equivalent.

$$\begin{aligned}
 101. \quad 4(y - 3) + 11 &= 4 \cdot y - 4 \cdot 3 + 11 \\
 &= 4y - 12 + 11 \\
 &= 4y - 1
 \end{aligned}$$

The expressions are not equivalent.

105. The order of the terms is not changed, but the grouping is. This is the associative property of addition.

$$\begin{aligned}
 109. \quad 9684q - 686 - 4860q + 12,960 \\
 &= 9684q - 4860q - 686 + 12,960 \\
 &= 4824q + 12,274
 \end{aligned}$$

113. answers may vary

Exercise Set 3.2

$$\begin{aligned}
 1. \quad x - 3 &= -1 + 4 \\
 x - 3 &= 3 \\
 x - 3 + 3 &= 3 + 3 \\
 x &= 6
 \end{aligned}$$

$$\begin{aligned}
 5. \quad 2w - 12w &= 40 \\
 -10w &= 40 \\
 \frac{-10w}{-10} &= \frac{40}{-10} \\
 w &= -4
 \end{aligned}$$

$$\begin{aligned}
 9. \quad 2z &= 12 - 14 \\
 2z &= -2 \\
 \frac{2z}{2} &= \frac{-2}{2} \\
 z &= -1
 \end{aligned}$$

$$\begin{aligned}
 13. \quad -3x - 3x &= 50 - 2 \\
 -6x &= 48 \\
 \frac{-6x}{-6} &= \frac{48}{-6} \\
 x &= -8
 \end{aligned}$$

$$\begin{aligned}
 17. \quad 7x + 7 - 6x &= 10 \\
 7x - 6x + 7 &= 10 \\
 x + 7 &= 10 \\
 x + 7 - 7 &= 10 - 7 \\
 x &= 3
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & 2(5x - 3) = 11x \\
 & 2 \cdot 5x - 2 \cdot 3 = 11x \\
 & 10x - 6 = 11x \\
 & 10x - 10x - 6 = 11x - 10x \\
 & -6 = x
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & 21y = 5(4y - 6) \\
 & 21y = 5 \cdot 4y - 5 \cdot 6 \\
 & 21y = 20y - 30 \\
 & 21y - 20y = 20y - 20y - 30 \\
 & y = -30
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & 2x - 8 = 0 \\
 & 2x - 8 + 8 = 0 + 8 \\
 & 2x = 8 \\
 & \frac{2x}{2} = \frac{8}{2} \\
 & x = 4
 \end{aligned}$$

$$\begin{aligned}
 33. \quad & -7 = 2x - 1 \\
 & -7 + 1 = 2x - 1 + 1 \\
 & -6 = 2x \\
 & \frac{-6}{2} = \frac{2x}{2} \\
 & -3 = x
 \end{aligned}$$

$$\begin{aligned}
 37. \quad & 11(x - 6) = -4 - 7 \\
 & 11 \cdot x - 11 \cdot 6 = -11 \\
 & 11x - 66 = -11 \\
 & 11x - 66 + 66 = -11 + 66 \\
 & 11x = 55 \\
 & \frac{11x}{11} = \frac{55}{11} \\
 & x = 5
 \end{aligned}$$

$$\begin{aligned}
 41. \quad & y - 20 = 6y \\
 & y - y - 20 = 6y - y \\
 & -20 = 5y \\
 & \frac{-20}{5} = \frac{5y}{5} \\
 & -4 = y
 \end{aligned}$$

$$\begin{aligned}
 45. \quad & -2 - 3 = -4 + x \\
 & -5 = -4 + x \\
 & -5 + 4 = -4 + 4 + x \\
 & -1 = x
 \end{aligned}$$

$$\begin{aligned}
 49. \quad & 3w - 12w = -27 \\
 & -9w = -27 \\
 & \frac{-9w}{-9} = \frac{-27}{-9} \\
 & w = 3
 \end{aligned}$$

$$\begin{aligned}
 53. \quad & 18 - 11 = \frac{x}{-5} \\
 & 7 = \frac{x}{-5} \\
 & -5 \cdot 7 = -5 \cdot \frac{x}{-5} \\
 & -5 \cdot 7 = \frac{-5}{-5} \cdot x \\
 & -35 = x
 \end{aligned}$$

$$\begin{aligned}
 57. \quad & 10 = 7t - 12t \\
 & 10 = -5t \\
 & \frac{10}{-5} = \frac{-5t}{-5} \\
 & -2 = t
 \end{aligned}$$

$$\begin{aligned}
 61. \quad & 50y = 7(7y + 4) \\
 & 50y = 7 \cdot 7y + 7 \cdot 4 \\
 & 50y = 49y + 28 \\
 & 50y - 49y = 49y - 49y + 28 \\
 & y = 28
 \end{aligned}$$

$$\begin{aligned}
 65. \quad & 7x + 14 - 6x = -4 - 10 \\
 & 7x - 6x + 14 = -14 \\
 & x + 14 = -14 \\
 & x + 14 - 14 = -14 - 14 \\
 & x = -28
 \end{aligned}$$

$$\begin{aligned}
 69. \quad & 23x + 8 - 25x = 7 - 9 \\
 & 23x - 25x + 8 = -2 \\
 & -2x + 8 = -2 \\
 & -2x + 8 - 8 = -2 - 8 \\
 & -2x = -10 \\
 & \frac{-2x}{-2} = \frac{-10}{-2} \\
 & x = 5
 \end{aligned}$$

73. "The sum of -7 and a number" is $-7 + x$.

77. "The product of -13 and a number" is $-13x$.

81. "The product of -11 and a number, increased by 5 " is $-11x + 5$.

85. "Seven added to the product of 4 and a number" is $4x + 7$.

89. "The product of -6 and the sum of a number and 15 " is $-6(x + 15)$.

93. "The quotient of seventeen and a number, increased by -15 " is $\frac{17}{x} + (-15)$ or $\frac{17}{x} - 15$.

97. From the graph the daily cost of business travel is \$554 in London, \$497 in Geneva, and \$484 in Zurich.

$$554 + 497 + 484 = 1535$$

The combined daily cost for a business traveler going to London, Geneva, and Zurich is \$1535.

101. no; answers may vary

$$105. \quad \frac{y}{72} = -86 - (-1029)$$

$$\frac{y}{72} = -86 + 1029$$

$$\frac{y}{72} = 943$$

$$72 \cdot \frac{y}{72} = 72 \cdot 943$$

$$\frac{72}{72} \cdot y = 72 \cdot 943$$

$$y = 67,896$$

$$109. \quad |-13| + 3^2 = 100y - |-20| - 99y$$

$$13 + 3^2 = 100y - 20 - 99y$$

$$13 + 9 = 100y - 99y - 20$$

$$22 = y - 20$$

$$22 + 20 = y - 20 + 20$$

$$42 = y$$

Exercise Set 3.3

1. $3x - 7 = 4x + 5$

$$3x - 3x - 7 = 4x - 3x + 5$$

$$-7 = x + 5$$

$$-7 - 5 = x + 5 - 5$$

$$-12 = x$$

5. $19 - 3x = 14 + 2x$

$$19 - 3x + 3x = 14 + 2x + 3x$$

$$19 = 14 + 5x$$

$$19 - 14 = 14 - 14 + 5x$$

$$5 = 5x$$

$$\frac{5}{5} = \frac{5x}{5}$$

$$1 = x$$

9. $x + 20 + 2x = -10 - 2x - 15$

$$x + 2x + 20 = -10 - 15 - 2x$$

$$3x + 20 = -25 - 2x$$

$$3x + 2x + 20 = -25 - 2x + 2x$$

$$5x + 20 = -25$$

$$5x + 20 - 20 = -25 - 20$$

$$5x = -45$$

$$\frac{5x}{5} = \frac{-45}{5}$$

$$x = -9$$

13. $35 - 17 = 3(x - 2)$

$$18 = 3x - 6$$

$$18 + 6 = 3x - 6 + 6$$

$$24 = 3x$$

$$\frac{24}{3} = \frac{3x}{3}$$

$$8 = x$$

17. $2(y - 3) = y - 6$

$$2y - 6 = y - 6$$

$$2y - y - 6 = y - y - 6$$

$$y - 6 = -6$$

$$y - 6 + 6 = -6 + 6$$

$$y = 0$$

21. $2t - 1 = 3(t + 7)$

$$2t - 1 = 3t + 21$$

$$2t - 2t - 1 = 3t - 2t + 21$$

$$-1 = t + 21$$

$$-1 - 21 = t + 21 - 21$$

$$-22 = t$$

25. $-4x = 44$

$$\frac{-4x}{-4} = \frac{44}{-4}$$

$$x = -11$$

29. $8 - b = 13$

$$8 - 8 - b = 13 - 8$$

$$-b = 5$$

$$\frac{-b}{-1} = \frac{5}{-1}$$

$$b = -5$$

33. $3r + 4 = 19$

$$3r + 4 - 4 = 19 - 4$$

$$3r = 15$$

$$\frac{3r}{3} = \frac{15}{3}$$

$$r = 5$$

37. $8y - 13y = -20 - 25$

$$-5y = -45$$

$$\frac{-5y}{-5} = \frac{-45}{-5}$$

$$y = 9$$

41. $-4 + 12 = 16x - 3 - 15x$

$$8 = x - 3$$

$$8 + 3 = x - 3 + 3$$

$$11 = x$$

45. $4x + 3 = 2x + 11$

$$4x - 2x + 3 = 2x - 2x + 11$$

$$2x + 3 = 11$$

$$2x + 3 - 3 = 11 - 3$$

$$2x = 8$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

49. $-8n + 1 = -6n - 5$

$$-8n + 8n + 1 = -6n + 8n - 5$$

$$1 = 2n - 5$$

$$1 + 5 = 2n - 5 + 5$$

$$6 = 2n$$

$$\frac{6}{2} = \frac{2n}{2}$$

$$3 = n$$

53. $9a + 29 + 7 = 0$

$$9a + 36 = 0$$

$$9a + 36 - 36 = 0 - 36$$

$$9a = -36$$

$$\frac{9a}{9} = \frac{-36}{9}$$

$$a = -4$$

57. $12 + 5t = 6(t + 2)$

$$12 + 5t = 6t + 12$$

$$12 + 5t - 5t = 6t - 5t + 12$$

$$12 = t + 12$$

$$12 - 12 = t + 12 - 12$$

$$0 = t$$

61. $10 + 5(z - 2) = 4z + 1$

$$10 + 5z - 10 = 4z + 1$$

$$5z = 4z + 1$$

$$5z - 4z = 4z - 4z + 1$$

$$z = 1$$

65. "The sum of -42 and 16 is -26 " translates to

$$-42 + 16 = -26.$$

69. "Three times the difference of -14 and 2 amounts to -48 " translates to $3(-14 - 2) = -48$.

73. The 2016 bar is labeled 134. Thus, 134,000,000 returns were filed electronically in 2016.

$$\begin{aligned} 77. \quad x^3 - 2xy &= 3^3 - 2(3)(-1) \\ &= 27 - 2(3)(-1) \\ &= 27 - (-6) \\ &= 27 + 6 \\ &= 33 \end{aligned}$$

81. The first step in solving $2x - 5 = -7$ is to add 5 to both sides, which is choice b.

85. The error is in the second line.

$$\begin{aligned} 2(3x - 5) &= 5x - 7 \\ 6x - 10 &= 5x - 7 \\ 6x - 10 + 10 &= 5x - 7 + 10 \\ 6x &= 5x + 3 \\ 6x - 5x &= 5x + 3 - 5x \\ x &= 3 \end{aligned}$$

$$\begin{aligned} 89. \quad 2^3(x + 4) &= 3^2(x + 4) \\ 8(x + 4) &= 9(x + 4) \\ 8x + 32 &= 9x + 36 \\ 8x + 32 - 36 &= 9x + 36 - 36 \\ 8x - 4 &= 9x \\ 8x - 4 - 8x &= 9x - 8x \\ -4 &= x \end{aligned}$$

Exercise Set 3.4

1. "A number added to -5 is -7 " is $-5 + x = -7$.
5. "A number subtracted from -20 amounts to 104 " is $-20 - x = 104$.
9. "The product of 5 and the sum of -3 and a number is -20 " is $5(-3 + x) = -20$.
13. "The sum of 3, 4, and a number amounts to 16" is

$$\begin{aligned} 3 + 4 + x &= 16 \\ 7 + x &= 16 \\ x + 7 - 7 &= 16 - 7 \\ x &= 9 \end{aligned}$$
17. "Thirty less a number is equal to the product of 3 and the sum of the number and 6" is

$$\begin{aligned} 30 - x &= 3(x + 6) \\ 30 - x &= 3x + 18 \\ 30 - 30 - x &= 3x + 18 - 30 \\ -x &= 3x - 12 \\ -x - 3x &= 3x - 3x - 12 \\ -4x &= -12 \\ \frac{-4x}{-4} &= \frac{-12}{-4} \\ x &= 3 \end{aligned}$$
21. "Three times the difference of some number and 5 amounts to the quotient of 108 and 12" is

$$\begin{aligned} 3(x - 5) &= \frac{108}{12} \\ 3x - 15 &= 9 \\ 3x - 15 + 15 &= 9 + 15 \\ 3x &= 24 \\ \frac{3x}{3} &= \frac{24}{3} \\ x &= 8 \end{aligned}$$

25. Let x be the number of electoral votes that California has. Since Florida has 26 fewer electoral votes, Florida has $x - 26$ electoral votes. Since the two states have a total of 84 electoral votes, the sum of x and $x - 26$ is 84.

$$\begin{aligned} x + x - 26 &= 84 \\ 2x - 26 &= 84 \\ 2x - 26 + 26 &= 84 + 26 \\ 2x &= 110 \\ \frac{2x}{2} &= \frac{110}{2} \\ x &= 55 \end{aligned}$$

Thus, California has 55 electoral votes and Florida has $55 - 26 = 29$ electoral votes.

29. Let x be the average daily circulation, in thousands, of *The Wall Street Journal*. Since the average daily circulation of *USA Today* is 226 thousand more than the average daily circulation of *The Wall Street Journal*, the average daily circulation of *USA Today* is $x + 226$. Since the combined average daily circulation is 4340 thousand, the sum of x and $x + 226$ is 4340.

$$\begin{aligned} x + x + 226 &= 4340 \\ 2x + 226 &= 4340 \\ 2x + 226 - 226 &= 4340 - 226 \\ 2x &= 4114 \\ \frac{2x}{2} &= \frac{4114}{2} \\ x &= 2057 \end{aligned}$$

Thus, the average daily circulation of *The Wall Street Journal* is 2057 thousand and the average daily circulation of *USA Today* is $2057 + 226 = 2283$ thousand.

33. Let x be the cost of the games. Since the cost of the Xbox 360 is 3 times as much as the cost of the games, the cost of the Xbox 360 is $3x$. Since the total cost of the Xbox 360 and the games is \$440, the sum of x and $3x$ is 440.

$$\begin{aligned} x + 3x &= 440 \\ 4x &= 440 \\ \frac{4x}{4} &= \frac{440}{4} \\ x &= 110 \\ 3x &= 3(110) = 330 \end{aligned}$$

The cost of the games is \$110 and the cost of the Xbox 360 is \$330.

37. Let x be the capacity of Michigan Stadium. Since the capacity of Beaver Stadium is 1081 more than the capacity of Michigan Stadium, the capacity of Beaver Stadium is $x + 1081$. Since the combined capacity of the two stadiums is 213,483, the sum of x and $x + 1081$ is 213,483.

$$\begin{aligned} x + x + 1081 &= 213,483 \\ 2x + 1081 &= 213,483 \\ 2x + 1081 - 1081 &= 213,483 - 1081 \\ 2x &= 212,402 \\ \frac{2x}{2} &= \frac{212,402}{2} \\ x &= 106,201 \end{aligned}$$

The capacity of Michigan Stadium is 106,201 and the capacity of Beaver Stadium is $106,201 + 1081 = 107,282$.

41. Let x be the number of motor vehicles manufactured in Germany in 2015. Since the number of motor vehicles manufactured in the United States in 2015 was twice the number of motor vehicles manufactured in Germany, the number of motor vehicles manufactured in the United States in 2015 was $2x$. Since the total number of motor vehicles manufactured in these countries in 2015 was 18,300,000, the sum of x and $2x$ is 18,300,000.

$$x + 2x = 18,300,000$$

$$3x = 18,300,000$$

$$\frac{3x}{3} = \frac{18,300,000}{3}$$

$$x = 6,100,000$$

$$2x = 2(6,100,000) = 12,200,000$$

The number of motor vehicles manufactured in Germany in 2015 was 6,100,000 and the number of motor vehicles manufactured in the United States in 2015 was 12,200,000.

45. Let x be the maximum length, in yards, of a soccer field for play by under-10-year-olds. Since the maximum length of a soccer field for Olympic play is 50 yards longer than the maximum length for play by under-10-year-olds, the maximum length of a soccer field for Olympic play is $x + 50$. Since the total length of these two categories of soccer fields is 190 yards, the sum of x and $x + 50$ is 190.

$$x + x + 50 = 190$$

$$2x + 50 = 190$$

$$2x + 50 - 50 = 190 - 50$$

$$2x = 140$$

$$\frac{2x}{2} = \frac{140}{2}$$

$$x = 70$$

Thus, the maximum length of a soccer field for play by under-10-year-olds is 70 yards and the maximum length of a soccer field for Olympic play is $70 + 50 = 120$ yards.

49. Let x be the estimated amount, in billions of dollars, that will be spent on TV advertising in 2017. Since the estimated amount that will be spent on Internet advertising in 2017 is 0.6 billion dollars more than the amount estimated to be spent on TV advertising, the estimated amount that will be spent on Internet advertising in 2017 is $x + 0.6$. Since the total amount that is estimated to be spent on advertising in these two mediums is \$150 billion, the sum of x and $x + 0.6$ is 150.

$$x + x + 0.6 = 150$$

$$2x + 0.6 = 150$$

$$2x + 0.6 - 0.6 = 150 - 0.6$$

$$2x = 149.4$$

$$\frac{2x}{2} = \frac{149.4}{2}$$

$$x = 74.7$$

$$x + 0.6 = 74.7 + 0.6 = 75.3$$

Thus, it is estimated that \$74.7 billion will be spent on TV advertising in 2017 and \$75.3 billion will be spent on Internet advertising in 2017.

53. 1026 rounded to the nearest hundred is 1000.

57. yes; answers may vary

61. Use $P = C + M$, with $P = 999$ and $M = 450$.

$$P = C + M$$

$$999 = C + 450$$

$$999 - 450 = C + 450 - 450$$

$$549 = C$$

The wholesale cost of the computer is \$549.

Chapter 3 Test

$$\begin{aligned} 1. \quad 7x - 5 - 12x + 10 &= 7x - 12x - 5 + 10 \\ &= (7 - 12)x - 5 + 10 \\ &= -5x + 5 \end{aligned}$$

$$\begin{aligned} 5. \quad \text{Area} &= (\text{length}) \cdot (\text{width}) \\ &= 4 \cdot (3x - 1) \\ &= 4 \cdot 3x - 4 \cdot 1 \\ &= 12x - 4 \end{aligned}$$

The area is $(12x - 4)$ square meters.

$$\begin{aligned} 9. \quad -4x + 7 &= 15 \\ -4x + 7 - 7 &= 15 - 7 \\ -4x &= 8 \\ \frac{-4x}{-4} &= \frac{8}{-4} \\ x &= -2 \end{aligned}$$

$$\begin{aligned} 13. \quad 4(5x + 3) &= 2(7x + 6) \\ 20x + 12 &= 14x + 12 \\ 20x + 12 - 14x &= 14x + 12 - 14x \\ 6x + 12 &= 12 \\ 6x + 12 - 12 &= 12 - 12 \\ 6x &= 0 \\ \frac{6x}{6} &= \frac{0}{6} \\ x &= 0 \end{aligned}$$

17. "The sum of twice 5 and -15 is -5 " translates to $2 \cdot 5 + (-15) = -5$.

21. Let x be the number of women runners entered in the race. Since the number of men entered in the race is 112 more than the number of women, the number of men is $x + 112$. Since the total number of runners in the race is 600, the sum of x and $x + 112$ is 600.

$$x + x + 112 = 600$$

$$2x + 112 = 600$$

$$2x + 112 - 112 = 600 - 112$$

$$2x = 488$$

$$\frac{2x}{2} = \frac{488}{2}$$

$$x = 244$$

244 women entered the race.

Chapter 4

Exercise Set 4.1

1. In the fraction $\frac{1}{2}$, the numerator is 1 and the denominator is 2. Since $1 < 2$, the fraction is proper.
5. In the fraction $\frac{15}{15}$, the numerator is 15 and the denominator is 15. Since $15 \geq 15$, the fraction is improper.

9. Each part is $\frac{1}{4}$ of a whole and there are 11 parts shaded, or 2 wholes and 3 more parts.

a. $\frac{11}{4}$ b. $2\frac{3}{4}$

13. 7 out of 12 equal parts are shaded: $\frac{7}{12}$

17. 4 out of 9 equal parts are shaded: $\frac{4}{9}$

21. Each part is $\frac{1}{2}$ of a whole and there are 11 parts shaded, or 5 wholes and 1 more part.

a. $\frac{11}{2}$ b. $5\frac{1}{2}$

25. 5 of 8 equal parts are shaded: $\frac{5}{8}$

29. 

33. freshmen $\rightarrow 42$
students $\rightarrow 131$

$\frac{42}{131}$ of the students are freshmen.

37. born in Ohio $\rightarrow 7$
U.S. presidents $\rightarrow 44$

$\frac{7}{44}$ of U.S. presidents were born in Ohio.

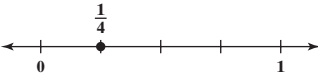
41. 11 of 31 days of March is $\frac{11}{31}$ of the month.

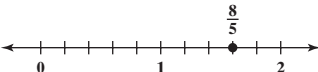
45. There are 50 states total. 33 states contain federal Indian reservations.

- a. $\frac{33}{50}$ of the states contain federal Indian reservations.

- b. $50 - 33 = 17$
17 states do not contain federal Indian reservations.

- c. $\frac{17}{50}$ of the states do not contain federal Indian reservations.

49. 

53. 

57. $\frac{12}{12} = 12 \div 12 = 1$

61. $\frac{0}{-2} = 0 \div (-2) = 0$

65. $\frac{-9}{0} = -9 \div 0$ is undefined

69. $2\frac{1}{3} = \frac{3 \cdot 2 + 1}{3} = \frac{6 + 1}{3} = \frac{7}{3}$

73. $6\frac{5}{8} = \frac{8 \cdot 6 + 5}{8} = \frac{48 + 5}{8} = \frac{53}{8}$

77. $9\frac{7}{20} = \frac{20 \cdot 9 + 7}{20} = \frac{180 + 7}{20} = \frac{187}{20}$

81.
$$\begin{array}{r} 3 \\ 5 \overline{)17} \\ \underline{-15} \\ 2 \end{array}$$

 $\frac{17}{5} = 3\frac{2}{5}$

85.
$$\begin{array}{r} 3 \\ 15 \overline{)47} \\ \underline{-45} \\ 2 \end{array}$$

$\frac{47}{15} = 3\frac{2}{15}$

89.
$$\begin{array}{r} 1 \\ 175 \overline{)182} \\ \underline{-175} \\ 7 \end{array}$$

$\frac{182}{175} = 1\frac{7}{175}$

93. $3^2 = 3 \cdot 3 = 9$

97. $-\frac{11}{2} = \frac{-11}{2} = \frac{11}{-2}$

101. answers may vary

105. 1 is close to 0, so $\frac{1}{8}$ is close to 0. $7\frac{1}{8}$ rounded to the nearest whole number is 7.

109. $1500 + 80 = 1580$

1500 of the 1580 Habitat for Humanity affiliates are located in the United States: $\frac{1500}{1580}$.

Exercise Set 4.2

1. $20 = 2 \cdot 10$

$$\begin{array}{c} \downarrow \quad \swarrow \searrow \\ 2 \cdot 2 \cdot 5 = 2^2 \cdot 5 \end{array}$$

5. $81 = 9 \cdot 9$

$$\begin{array}{c} \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 3 \cdot 3 \cdot 3 \cdot 3 = 3^4 \end{array}$$

9. $110 = 2 \cdot 55$

$$\begin{array}{c} \downarrow \quad \downarrow \\ 2 \cdot 5 \cdot 11 = 2 \cdot 5 \cdot 11 \end{array}$$

13. $240 = 2 \cdot 120$

$$\begin{array}{c} \downarrow \quad \downarrow \quad \downarrow \\ 2 \cdot 2 \cdot 60 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 2 \cdot 2 \cdot 2 \cdot 30 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 2 \cdot 2 \cdot 2 \cdot 2 \cdot 15 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 = 2^4 \cdot 3 \cdot 5 \end{array}$$

In exercises 17 through 49, common factors are divided out to simplify.

17. $\frac{3}{12} = \frac{3 \cdot 1}{3 \cdot 4} = \frac{1}{4}$

21. $\frac{14}{16} = \frac{2 \cdot 7}{2 \cdot 8} = \frac{7}{8}$

25. $\frac{35a}{50a} = \frac{5 \cdot 7 \cdot a}{5 \cdot 10 \cdot a} = \frac{7}{10}$

29. $\frac{30x^2}{36x} = \frac{5 \cdot 6 \cdot x \cdot x}{6 \cdot 6 \cdot x} = \frac{5 \cdot x}{6} = \frac{5x}{6}$

33. $\frac{25xy}{40y} = \frac{5 \cdot 5 \cdot x \cdot y}{5 \cdot 8 \cdot y} = \frac{5 \cdot x}{8} = \frac{5x}{8}$

37. $\frac{36x^3y^2}{24xy} = \frac{3 \cdot 12 \cdot x \cdot x \cdot x \cdot y \cdot y}{2 \cdot 12 \cdot x \cdot y} = \frac{3 \cdot x \cdot x \cdot y}{2} = \frac{3x^2y}{2}$

41. $\frac{40xy}{64xyz} = \frac{5 \cdot 8 \cdot x \cdot y}{8 \cdot 8 \cdot x \cdot y \cdot z} = \frac{5}{8 \cdot z} = \frac{5}{8z}$

45. $-\frac{55}{85y} = -\frac{5 \cdot 11}{5 \cdot 17 \cdot y} = -\frac{11}{17 \cdot y} = -\frac{11}{17y}$

$$49. \frac{224a^3b^4c^2}{16ab^4c^2} = \frac{14 \cdot 16 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot c \cdot c}{1 \cdot 16 \cdot a \cdot b \cdot b \cdot b \cdot b \cdot c \cdot c}$$

$$= \frac{14 \cdot a \cdot a}{1}$$

$$= 14a^2$$

53. Not equivalent, since the cross products are not equal:
 $7 \cdot 8 = 56$ and $5 \cdot 11 = 55$.

57. Equivalent, since the cross products are equal:
 $3 \cdot 18 = 54$ and $9 \cdot 6 = 54$

61. Not equivalent, since the cross products are not equal:
 $8 \cdot 24 = 192$ and $12 \cdot 18 = 216$

$$65. \frac{2640 \text{ feet}}{5280 \text{ feet}} = \frac{2640 \cdot 1}{2640 \cdot 2} = \frac{1}{2}$$

2640 feet represents $\frac{1}{2}$ of a mile.

$$69. \frac{10 \text{ inches}}{24 \text{ inches}} = \frac{2 \cdot 5}{2 \cdot 12} = \frac{5}{12}$$

$\frac{5}{12}$ of the wall is concrete.

$$73. \frac{45 \text{ individuals}}{366 \text{ individuals}} = \frac{3 \cdot 15}{3 \cdot 122} = \frac{15}{122}$$

$\frac{15}{122}$ of U.S. astronauts are currently active.

$$77. 2y = 2(-7) = -14$$

$$81. \frac{3975}{6625} = \frac{3 \cdot 1325}{5 \cdot 1325} = \frac{3}{5}$$

85. $3 + 1 = 4$ blood donors have an AB blood type.

$$\frac{4 \text{ donors}}{100 \text{ donors}} = \frac{4 \cdot 1}{4 \cdot 25} = \frac{1}{25}$$

$\frac{1}{25}$ of blood donors have an AB blood type.

89. answers may vary

93. The piece representing education is labeled $\frac{6}{100}$.

$$\frac{6}{100} = \frac{2 \cdot 3}{2 \cdot 50} = \frac{3}{50}$$

$\frac{3}{50}$ of entering college freshmen plan to major in education.

97. The piece representing historic parks is labeled $\frac{24}{100}$.

$$\frac{24}{100} = \frac{4 \cdot 6}{4 \cdot 25} = \frac{6}{25}$$

$\frac{6}{25}$ of National Park Service area are historic parks.

101. 786, 22, 222, 900, and 1470 are divisible by 2 because each number ends with an even digit. 8691, 786, 2235, 105, 222, 900, and 1470 are divisible by 3 because the sum of each number's digits is divisible by 3.

786, 222, 900, and 1470 are divisible by both 2 and 3.

Exercise Set 4.3

In exercises 1 through 113, common factors are divided out as needed to simplify.

$$1. \frac{6}{11} \cdot \frac{3}{7} = \frac{6 \cdot 3}{11 \cdot 7} = \frac{18}{77}$$

$$5. \frac{1}{2} \cdot \frac{2}{15} = \frac{1 \cdot 2}{2 \cdot 15} = \frac{1}{15}$$

$$9. 3a^2 \cdot \frac{1}{4} = \frac{3a^2}{1} \cdot \frac{1}{4} = \frac{3a^2 \cdot 1}{1 \cdot 4} = \frac{3a^2}{4}$$

$$13. 0 \cdot \frac{8}{9} = 0$$

$$17. \frac{11}{20} \cdot \frac{1}{7} \cdot \frac{5}{22} = \frac{11 \cdot 1 \cdot 5}{20 \cdot 7 \cdot 22}$$

$$= \frac{11 \cdot 1 \cdot 5}{5 \cdot 4 \cdot 7 \cdot 11 \cdot 2}$$

$$= \frac{1}{4 \cdot 7 \cdot 2}$$

$$= \frac{1}{56}$$

$$21. \left(-\frac{2}{3}\right)^2 = -\frac{2}{3} \cdot -\frac{2}{3} = \frac{2 \cdot 2}{3 \cdot 3} = \frac{4}{9}$$

$$25. \frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \cdot \frac{6}{5} = \frac{2 \cdot 6}{3 \cdot 5} = \frac{2 \cdot 2 \cdot 3}{3 \cdot 5} = \frac{2 \cdot 2}{5} = \frac{4}{5}$$

$$29. -\frac{8}{9} \div \frac{x}{2} = -\frac{8}{9} \cdot \frac{2}{x} = -\frac{8 \cdot 2}{9 \cdot x} = -\frac{16}{9x}$$

$$33. -\frac{2}{3} \div 4 = -\frac{2}{3} \div \frac{4}{1}$$

$$= -\frac{2}{3} \cdot \frac{1}{4}$$

$$= -\frac{2 \cdot 1}{3 \cdot 4}$$

$$= -\frac{2 \cdot 1}{3 \cdot 2 \cdot 2}$$

$$= -\frac{1}{3 \cdot 2}$$

$$= -\frac{1}{6}$$

$$37. \frac{2}{3} \cdot \frac{5}{9} = \frac{2 \cdot 5}{3 \cdot 9} = \frac{10}{27}$$

$$41. \frac{16}{27y} \div \frac{8}{15y} = \frac{16}{27y} \cdot \frac{15y}{8}$$

$$= \frac{16 \cdot 15y}{27y \cdot 8}$$

$$= \frac{8 \cdot 2 \cdot 3 \cdot 5 \cdot y}{3 \cdot 9 \cdot y \cdot 8}$$

$$= \frac{2 \cdot 5}{9}$$

$$= \frac{10}{9}$$

$$45. \left(-\frac{3}{4}\right)^2 = -\frac{3}{4} \cdot -\frac{3}{4} = \frac{3 \cdot 3}{4 \cdot 4} = \frac{9}{16}$$

$$49. 7 \div \frac{2}{11} = \frac{7}{1} \div \frac{2}{11} = \frac{7}{1} \cdot \frac{11}{2} = \frac{7 \cdot 11}{1 \cdot 2} = \frac{77}{2}$$

$$53. \left(\frac{2}{7} \div \frac{7}{2}\right) \cdot \frac{3}{4} = \left(\frac{2}{7} \cdot \frac{2}{7}\right) \cdot \frac{3}{4}$$

$$= \frac{4}{49} \cdot \frac{3}{4}$$

$$= \frac{3}{49}$$

$$57. -\frac{2}{3} \cdot -\frac{6}{11} = \frac{2 \cdot 6}{3 \cdot 11} = \frac{2 \cdot 2 \cdot 3}{3 \cdot 11} = \frac{2 \cdot 2}{11} = \frac{4}{11}$$

$$61. \frac{21x^2}{10y} \div \frac{14x}{25y} = \frac{21x^2}{10y} \cdot \frac{25y}{14x}$$

$$= \frac{21x^2 \cdot 25y}{10y \cdot 14x}$$

$$= \frac{3 \cdot 7 \cdot x \cdot x \cdot 5 \cdot 5 \cdot y}{2 \cdot 5 \cdot y \cdot 2 \cdot 7 \cdot x}$$

$$= \frac{3 \cdot x \cdot 5}{2 \cdot 2}$$

$$= \frac{15x}{4}$$

$$\begin{aligned}
 65. \quad \frac{a^3}{2} \div 30a^3 &= \frac{a^3}{2} \div \frac{30a^3}{1} \\
 &= \frac{a^3}{2} \cdot \frac{1}{30a^3} \\
 &= \frac{a^3 \cdot 1}{2 \cdot 30a^3} \\
 &= \frac{a \cdot a \cdot a \cdot 1}{2 \cdot 30 \cdot a \cdot a \cdot a} \\
 &= \frac{1}{2 \cdot 30} \\
 &= \frac{1}{60}
 \end{aligned}$$

$$\begin{aligned}
 69. \quad \left(\frac{1}{2} \cdot \frac{2}{3}\right) \div \frac{5}{6} &= \left(\frac{1 \cdot 2}{2 \cdot 3}\right) \div \frac{5}{6} \\
 &= \frac{1}{3} \div \frac{5}{6} \\
 &= \frac{1}{3} \cdot \frac{6}{5} \\
 &= \frac{1 \cdot 6}{3 \cdot 5} \\
 &= \frac{1 \cdot 2 \cdot 3}{3 \cdot 5} \\
 &= \frac{1 \cdot 2}{5} \\
 &= \frac{2}{5}
 \end{aligned}$$

$$73. \text{ a. } xy = \frac{2}{5} \cdot \frac{5}{6} = \frac{2 \cdot 5}{5 \cdot 6} = \frac{2 \cdot 5}{5 \cdot 2 \cdot 3} = \frac{1}{3}$$

$$\text{ b. } x \div y = \frac{2}{5} \div \frac{5}{6} = \frac{2}{5} \cdot \frac{6}{5} = \frac{2 \cdot 6}{5 \cdot 5} = \frac{12}{25}$$

$$\begin{aligned}
 77. \quad 3x &= -\frac{5}{6} \\
 3\left(-\frac{5}{18}\right) &\stackrel{?}{=} -\frac{5}{6} \\
 \frac{3}{1} \cdot -\frac{5}{18} &\stackrel{?}{=} -\frac{5}{6} \\
 -\frac{3 \cdot 5}{1 \cdot 18} &\stackrel{?}{=} -\frac{5}{6} \\
 -\frac{3 \cdot 5}{3 \cdot 6} &\stackrel{?}{=} -\frac{5}{6} \\
 -\frac{5}{6} &= -\frac{5}{6} \quad \text{True}
 \end{aligned}$$

Yes, $-\frac{5}{18}$ is a solution.

$$\begin{aligned}
 81. \quad \frac{1}{4} \text{ of } 200 &= \frac{1}{4} \cdot 200 \\
 &= \frac{1}{4} \cdot \frac{200}{1} \\
 &= \frac{1 \cdot 200}{4 \cdot 1} \\
 &= \frac{1 \cdot 4 \cdot 50}{4 \cdot 1} \\
 &= \frac{50}{1} \\
 &= 50
 \end{aligned}$$

$$\begin{aligned}
 85. \quad \frac{7}{50} \text{ of } 800 &= \frac{7}{50} \cdot 800 \\
 &= \frac{7}{50} \cdot \frac{800}{1} \\
 &= \frac{7 \cdot 800}{50 \cdot 1} \\
 &= \frac{7 \cdot 50 \cdot 16}{50 \cdot 1} \\
 &= \frac{7 \cdot 16}{1} \\
 &= 112
 \end{aligned}$$

112 of the students would be expected to major in business.

$$\begin{aligned}
 89. \quad \frac{2}{5} \text{ of } 2170 &= \frac{2}{5} \cdot 2170 \\
 &= \frac{2}{5} \cdot \frac{2170}{1} \\
 &= \frac{2 \cdot 2170}{5 \cdot 1} \\
 &= \frac{2 \cdot 5 \cdot 434}{5 \cdot 1} \\
 &= \frac{2 \cdot 434}{1} \\
 &= 868
 \end{aligned}$$

He has hiked 868 miles.

$$\begin{aligned}
 93. \quad \frac{2}{3} \text{ of } 2757 &= \frac{2}{3} \cdot 2757 \\
 &= \frac{2}{3} \cdot \frac{2757}{1} \\
 &= \frac{2 \cdot 2757}{3 \cdot 1} \\
 &= \frac{2 \cdot 3 \cdot 919}{3 \cdot 1} \\
 &= \frac{2 \cdot 919}{1} \\
 &= 1838
 \end{aligned}$$

The sale price is \$1838.

$$97. \text{ Area} = \text{length} \cdot \text{width} = \frac{5}{14} \cdot \frac{1}{5} = \frac{5 \cdot 1}{14 \cdot 5} = \frac{1}{14}$$

The area is $\frac{1}{14}$ square foot.

$$\begin{aligned}
 101. \quad \frac{1}{5} \cdot 12,000 &= \frac{1}{5} \cdot \frac{12,000}{1} \\
 &= \frac{1 \cdot 5 \cdot 2400}{5 \cdot 1} \\
 &= \frac{1 \cdot 2400}{1} \\
 &= 2400
 \end{aligned}$$

The family drove 2400 miles for family business.

$$\begin{aligned}
 105. \quad &968 \\
 &\underline{-772} \\
 &196
 \end{aligned}$$

$$\begin{aligned}
 109. \quad \frac{42}{25} \cdot \frac{125}{36} \div \frac{7}{6} &= \frac{42}{25} \cdot \frac{125}{36} \cdot \frac{6}{7} \\
 &= \frac{42 \cdot 125 \cdot 6}{25 \cdot 36 \cdot 7} \\
 &= \frac{6 \cdot 7 \cdot 5 \cdot 25 \cdot 6}{25 \cdot 6 \cdot 6 \cdot 7} \\
 &= \frac{5}{1} \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 113. \quad \frac{63}{200} \text{ of } 27,000 &= \frac{63}{200} \cdot 27,000 \\
 &= \frac{63}{200} \cdot \frac{27,000}{1} \\
 &= \frac{63 \cdot 200 \cdot 135}{200 \cdot 1} \\
 &= \frac{63 \cdot 135}{1} \\
 &= 8505
 \end{aligned}$$

The National Park Service is charged with maintaining 8505 monuments and statues.

Exercise Set 4.4

In exercises 1 through 93, common factors are divided out as needed to simplify.

$$1. \quad \frac{5}{11} + \frac{2}{11} = \frac{5+2}{11} = \frac{7}{11}$$

$$5. \quad -\frac{6}{20} + \frac{1}{20} = \frac{-6+1}{20} = \frac{-5}{20} = -\frac{1 \cdot 5}{4 \cdot 5} = -\frac{1}{4}$$

$$9. \quad \frac{2}{9x} + \frac{4}{9x} = \frac{2+4}{9x} = \frac{6}{9x} = \frac{2 \cdot 3}{3 \cdot 3 \cdot x} = \frac{2}{3x}$$

$$13. \quad \frac{10}{11} - \frac{4}{11} = \frac{10-4}{11} = \frac{6}{11}$$

$$17. \quad \frac{1}{y} - \frac{4}{y} = \frac{1-4}{y} = \frac{-3}{y} = -\frac{3}{y}$$

$$21. \quad \frac{20}{21} - \frac{10}{21} - \frac{17}{21} = \frac{20-10-17}{21}$$

$$= \frac{-7}{21}$$

$$= -\frac{1 \cdot 7}{3 \cdot 7} = -\frac{1}{3}$$

$$25. \quad -\frac{9}{100} + \frac{99}{100} = \frac{-9+99}{100} = \frac{90}{100} = \frac{9 \cdot 10}{10 \cdot 10} = \frac{9}{10}$$

$$29. \quad \frac{9x}{15} + \frac{1}{15} = \frac{9x+1}{15}$$

$$33. \quad \frac{9}{12} - \frac{7}{12} - \frac{10}{12} = \frac{9-7-10}{12} = \frac{-8}{12} = -\frac{2 \cdot 4}{3 \cdot 4} = -\frac{2}{3}$$

$$37. \quad x + y = \frac{3}{4} + \frac{2}{4} = \frac{3+2}{4} = \frac{5}{4}$$

$$41. \quad \frac{4}{20} + \frac{7}{20} + \frac{9}{20} = \frac{4+7+9}{20} = \frac{20}{20} = 1$$

The perimeter is 1 inch.

45. To find the remaining amount of track to be inspected,

subtract the $\frac{5}{20}$ mile that has already been inspected from the $\frac{19}{20}$ mile total that must be inspected.

$$\frac{19}{20} - \frac{5}{20} = \frac{19-5}{20} = \frac{14}{20} = \frac{2 \cdot 7}{2 \cdot 10} = \frac{7}{10}$$

$\frac{7}{10}$ of a mile of track remains to be inspected.

49. North America takes up $\frac{16}{100}$ of the world's land area,

while South America takes up $\frac{12}{100}$ of the land area.

$$\frac{16}{100} + \frac{12}{100} = \frac{16+12}{100} = \frac{28}{100} = \frac{4 \cdot 7}{4 \cdot 25} = \frac{7}{25}$$

$\frac{7}{25}$ of the world's land area is within North America and South America.

53. Multiples of 15:

$$15 \cdot 1 = 15, \text{ not a multiple of } 9$$

$$15 \cdot 2 = 30, \text{ not a multiple of } 9$$

$$15 \cdot 3 = 45, \text{ a multiple of } 9$$

LCD: 45

$$57. \quad 6 = \boxed{2} \cdot 3$$

$$15 = \boxed{3} \cdot 5$$

$$25 = \boxed{5 \cdot 5}$$

$$\text{LCD} = 2 \cdot 3 \cdot 5 \cdot 5 = 150$$

$$61. \quad 18 = \boxed{2 \cdot 3 \cdot 3}$$

$$21 = 3 \cdot \boxed{7}$$

$$\text{LCD} = 2 \cdot 3 \cdot 3 \cdot 7 = 126$$

$$65. \quad \frac{2}{3} = \frac{2 \cdot 7}{3 \cdot 7} = \frac{14}{21}$$

$$69. \quad \frac{1}{2} = \frac{1 \cdot 25}{2 \cdot 25} = \frac{25}{50}$$

$$73. \quad \frac{2y}{3} = \frac{2y \cdot 4}{3 \cdot 4} = \frac{8y}{12}$$

$$77. \text{ books and magazines: } \frac{27}{50} = \frac{27 \cdot 2}{50 \cdot 2} = \frac{54}{100}$$

$$\text{clothing and accessories: } \frac{1}{2} = \frac{1 \cdot 50}{2 \cdot 50} = \frac{50}{100}$$

$$\text{computer hardware: } \frac{23}{50} = \frac{23 \cdot 2}{50 \cdot 2} = \frac{46}{100}$$

$$\text{computer software: } \frac{1}{2} = \frac{1 \cdot 50}{2 \cdot 50} = \frac{50}{100}$$

$$\text{drugs, health and beauty aids: } \frac{3}{20} = \frac{3 \cdot 5}{20 \cdot 5} = \frac{15}{100}$$

$$\text{electronics and appliances: } \frac{13}{20} = \frac{13 \cdot 5}{20 \cdot 5} = \frac{65}{100}$$

$$\text{food, beer, and wine: } \frac{9}{20} = \frac{9 \cdot 5}{20 \cdot 5} = \frac{45}{100}$$

$$\text{home furnishings: } \frac{13}{25} = \frac{13 \cdot 4}{25 \cdot 4} = \frac{52}{100}$$

$$\text{music and videos: } \frac{3}{5} = \frac{3 \cdot 20}{5 \cdot 20} = \frac{60}{100}$$

$$\text{office equipment and supplies: } \frac{61}{100} = \frac{61}{100}$$

$$\text{sporting goods: } \frac{12}{25} = \frac{12 \cdot 4}{25 \cdot 4} = \frac{48}{100}$$

$$\text{toys, hobbies, and games: } \frac{1}{2} = \frac{1 \cdot 50}{2 \cdot 50} = \frac{50}{100}$$

$$81. \quad 3^2 = 3 \cdot 3 = 9$$

$$85. \quad 7^2 = 7 \cdot 7 = 49$$

$$89. \quad \frac{2}{7} + \frac{9}{7} = \frac{2+9}{7} = \frac{11}{7}$$

$$93. \quad \frac{16}{100} + \frac{12}{100} + \frac{7}{100} + \frac{20}{100} + \frac{30}{100} + \frac{6}{100} + \frac{9}{100}$$

$$= \frac{16+12+7+20+30+6+9}{100}$$

$$= \frac{100}{100}$$

$$= 1$$

answers may vary

97. answers may vary

Exercise Set 4.5

1. The LCD of 3 and 6 is 6.

$$\frac{2}{3} + \frac{1}{6} = \frac{2 \cdot 2}{3 \cdot 2} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{5}{6}$$

5. The LCD of 11 and 33 is 33.

$$-\frac{2}{11} + \frac{2}{33} = -\frac{2 \cdot 3}{11 \cdot 3} + \frac{2}{33} = -\frac{6}{33} + \frac{2}{33} = -\frac{4}{33}$$

9. The LCD of 35 and 7 is 35.

$$\begin{aligned}\frac{11x}{35} + \frac{2x}{7} &= \frac{11x}{35} + \frac{2x \cdot 5}{7 \cdot 5} \\ &= \frac{11x}{35} + \frac{10x}{35} \\ &= \frac{21x}{35} \\ &= \frac{3 \cdot 7 \cdot x}{5 \cdot 7} \\ &= \frac{3x}{5}\end{aligned}$$

13. The LCD of 12 and 9 is 36.

$$\frac{5}{12} - \frac{1}{9} = \frac{5 \cdot 3}{12 \cdot 3} - \frac{1 \cdot 4}{9 \cdot 4} = \frac{15}{36} - \frac{4}{36} = \frac{11}{36}$$

17. The LCD of 11 and 9 is 99.

$$\begin{aligned}\frac{5a}{11} + \frac{4a}{9} &= \frac{5a \cdot 9}{11 \cdot 9} + \frac{4a \cdot 11}{9 \cdot 11} \\ &= \frac{45a}{99} + \frac{44a}{99} \\ &= \frac{89a}{99}\end{aligned}$$

21. The LCD of 2 and
- x
- is
- $2x$
- .

$$\frac{1}{2} + \frac{3}{x} = \frac{1 \cdot x}{2 \cdot x} + \frac{3 \cdot 2}{x \cdot 2} = \frac{x}{2x} + \frac{6}{2x} = \frac{x+6}{2x}$$

25. The LCD of 14 and 7 is 14.

$$\frac{9}{14} - \frac{3}{7} = \frac{9}{14} - \frac{3 \cdot 2}{7 \cdot 2} = \frac{9}{14} - \frac{6}{14} = \frac{3}{14}$$

29. The LCD of 9 and 12 is 36.

$$\frac{1}{9} - \frac{5}{12} = \frac{1 \cdot 4}{9 \cdot 4} - \frac{5 \cdot 3}{12 \cdot 3} = \frac{4}{36} - \frac{15}{36} = -\frac{11}{36}$$

33. The LCD of 7 and 8 is 56.

$$\frac{5}{7} - \frac{1}{8} = \frac{5 \cdot 8}{7 \cdot 8} - \frac{1 \cdot 7}{8 \cdot 7} = \frac{40}{56} - \frac{7}{56} = \frac{33}{56}$$

- 37.
- $\frac{3}{9} - \frac{5}{9} = \frac{3-5}{9} = \frac{-2}{9} = -\frac{2}{9}$

41. The LCD of 11 and 3 is 33.

$$\begin{aligned}\frac{5}{11} + \frac{y}{3} &= \frac{5 \cdot 3}{11 \cdot 3} + \frac{y \cdot 11}{3 \cdot 11} \\ &= \frac{15}{33} + \frac{11y}{33} \\ &= \frac{15 + 11y}{33}\end{aligned}$$

45. The LCD of 2, 4, and 16 is 16.

$$\begin{aligned}\frac{x}{2} + \frac{x}{4} + \frac{2x}{16} &= \frac{x \cdot 8}{2 \cdot 8} + \frac{x \cdot 4}{4 \cdot 4} + \frac{2x}{16} \\ &= \frac{8x}{16} + \frac{4x}{16} + \frac{2x}{16} \\ &= \frac{14x}{16} \\ &= \frac{2 \cdot 7x}{2 \cdot 8} \\ &= \frac{7x}{8}\end{aligned}$$

49. The LCD of 3 and 13 is 39.

$$\begin{aligned}\frac{2a}{3} + \frac{6a}{13} &= \frac{2a \cdot 13}{3 \cdot 13} + \frac{6a \cdot 3}{13 \cdot 3} \\ &= \frac{26a}{39} + \frac{18a}{39} \\ &= \frac{44a}{39}\end{aligned}$$

53. The LCD of 9 and
- y
- is
- $9y$
- .

$$\begin{aligned}\frac{5}{9} + \frac{1}{y} &= \frac{5 \cdot y}{9 \cdot y} + \frac{1 \cdot 9}{y \cdot 9} \\ &= \frac{5y}{9y} + \frac{9}{9y} \\ &= \frac{5y+9}{9y}\end{aligned}$$

57. The LCD of 5 and 9 is 45.

$$\frac{4}{5} + \frac{4}{9} = \frac{4 \cdot 9}{5 \cdot 9} + \frac{4 \cdot 5}{9 \cdot 5} = \frac{36}{45} + \frac{20}{45} = \frac{56}{45}$$

61. The LCD of 12, 24, and 6 is 24.

$$\begin{aligned}-\frac{9}{12} + \frac{17}{24} - \frac{1}{6} &= -\frac{9 \cdot 2}{12 \cdot 2} + \frac{17}{24} - \frac{1 \cdot 4}{6 \cdot 4} \\ &= -\frac{18}{24} + \frac{17}{24} - \frac{4}{24} \\ &= -\frac{5}{24}\end{aligned}$$

65. The LCD of 7 and 10 is 70. Write each fraction as an equivalent fraction with a denominator of 70.

$$\frac{2}{7} = \frac{2 \cdot 10}{7 \cdot 10} = \frac{20}{70}$$

$$\frac{3}{10} = \frac{3 \cdot 7}{10 \cdot 7} = \frac{21}{70}$$

$$\text{Since } 20 < 21, \frac{20}{70} < \frac{21}{70}, \text{ so } \frac{2}{7} < \frac{3}{10}.$$

69. The LCD of 4 and 14 is 28. Write each fraction as an equivalent fraction with a denominator of 28.

$$-\frac{3}{4} = -\frac{3 \cdot 7}{4 \cdot 7} = -\frac{21}{28}$$

$$-\frac{11}{14} = -\frac{11 \cdot 2}{14 \cdot 2} = -\frac{22}{28}$$

$$\text{Since } -21 > -22, -\frac{21}{28} > -\frac{22}{28}, \text{ so } -\frac{3}{4} > -\frac{11}{14}.$$

- 73.
- $xy = \frac{1}{3} \cdot \frac{3}{4} = \frac{1 \cdot 3}{3 \cdot 4} = \frac{1}{4}$

77. The LCD of 3 and 5 is 15.

$$\begin{aligned}\frac{4}{5} + \frac{1}{3} + \frac{4}{5} + \frac{1}{3} &= \frac{4 \cdot 3}{5 \cdot 3} + \frac{1 \cdot 5}{3 \cdot 5} + \frac{4 \cdot 3}{5 \cdot 3} + \frac{1 \cdot 5}{3 \cdot 5} \\ &= \frac{12}{15} + \frac{5}{15} + \frac{12}{15} + \frac{5}{15} \\ &= \frac{34}{15}\end{aligned}$$

$$\text{The perimeter is } \frac{34}{15} \text{ centimeters or } 2\frac{4}{15} \text{ centimeters.}$$

81. "The sum of a number and
- $\frac{1}{2}$
- " translates as
- $x + \frac{1}{2}$
- .

85. The LCD of 10 and 100 is 100.

$$\frac{17}{100} - \frac{1}{10} = \frac{17}{100} - \frac{1 \cdot 10}{10 \cdot 10} = \frac{17}{100} - \frac{10}{100} = \frac{7}{100}$$

$$\text{A sloth can travel } \frac{7}{100} \text{ mph faster in the trees.}$$

89. The LCD of 20 and 25 is 100.

$$\frac{13}{20} - \frac{4}{25} = \frac{13}{20} \cdot \frac{5}{5} - \frac{4}{25} \cdot \frac{4}{4} = \frac{65}{100} - \frac{16}{100} = \frac{49}{100}$$

Math or science is the favorite subject for $\frac{49}{100}$ of these students.

93. The LCD of 50 and 2 is 50.

$$\begin{aligned}\frac{13}{50} + \frac{1}{2} &= \frac{13}{50} + \frac{1}{2} \cdot \frac{25}{25} \\ &= \frac{13}{50} + \frac{25}{50} \\ &= \frac{38}{50} \\ &= \frac{2 \cdot 19}{2 \cdot 25} \\ &= \frac{19}{25}\end{aligned}$$

The Pacific and Atlantic Oceans account for $\frac{19}{25}$ of the world's water surface area.

$$97. 1 - \frac{37}{100} = \frac{100}{100} - \frac{37}{100} = \frac{63}{100}$$

$\frac{63}{100}$ of the areas maintained by the National Park Service are not national monuments.

$$101. (8 - 6) \cdot (4 - 7) = 2 \cdot (-3) = -6$$

105. The LCD of 3, 4, and 540 is 540.

$$\begin{aligned}\frac{2}{3} - \frac{1}{4} - \frac{2}{540} &= \frac{2}{3} \cdot \frac{180}{180} - \frac{1}{4} \cdot \frac{135}{135} - \frac{2}{540} \\ &= \frac{360}{540} - \frac{135}{540} - \frac{2}{540} \\ &= \frac{225}{540} - \frac{2}{540} \\ &= \frac{223}{540}\end{aligned}$$

109. answers may vary

Exercise Set 4.6

In exercises 1 through 53, common factors are divided out as needed to simplify.

$$1. \frac{\frac{1}{8}}{\frac{3}{4}} = \frac{1}{8} \div \frac{3}{4} = \frac{1}{8} \cdot \frac{4}{3} = \frac{1 \cdot 4}{8 \cdot 3} = \frac{1 \cdot 4}{2 \cdot 4 \cdot 3} = \frac{1}{6}$$

$$5. \frac{\frac{2x}{27}}{\frac{4}{9}} = \frac{2x}{27} \div \frac{4}{9} = \frac{2x}{27} \cdot \frac{9}{4} = \frac{2x \cdot 9}{27 \cdot 4} = \frac{2 \cdot x \cdot 9}{3 \cdot 9 \cdot 2 \cdot 2} = \frac{x}{6}$$

$$\begin{aligned}9. \frac{\frac{3x}{4}}{5 - \frac{1}{8}} &= \frac{8 \cdot \left(\frac{3x}{4}\right)}{8 \cdot \left(5 - \frac{1}{8}\right)} \\ &= \frac{\frac{8}{1} \cdot \frac{3x}{4}}{8 \cdot 5 - 8 \cdot \frac{1}{8}} \\ &= \frac{\frac{2 \cdot 4 \cdot 3x}{1 \cdot 4}}{40 - 1} \\ &= \frac{6x}{39} \\ &= \frac{2 \cdot 3 \cdot x}{3 \cdot 13} \\ &= \frac{2x}{13}\end{aligned}$$

$$\begin{aligned}13. \frac{5}{6} \div \frac{1}{3} \cdot \frac{1}{4} &= \frac{5}{6} \cdot \frac{3}{1} \cdot \frac{1}{4} \\ &= \frac{5 \cdot 3 \cdot 1}{2 \cdot 3 \cdot 1 \cdot 4} \\ &= \frac{5}{2 \cdot 4} \\ &= \frac{5}{8}\end{aligned}$$

$$\begin{aligned}17. \left(\frac{2}{9} + \frac{4}{9}\right)\left(\frac{1}{3} - \frac{9}{10}\right) &= \left(\frac{6}{9}\right)\left(\frac{1}{3} \cdot \frac{10}{10} - \frac{9}{10} \cdot \frac{3}{3}\right) \\ &= \left(\frac{6}{9}\right)\left(\frac{10}{30} - \frac{27}{30}\right) \\ &= \left(\frac{6}{9}\right)\left(\frac{-17}{30}\right) \\ &= -\frac{6 \cdot 17}{9 \cdot 30} \\ &= -\frac{3 \cdot 2 \cdot 17}{3 \cdot 3 \cdot 2 \cdot 15} \\ &= -\frac{17}{3 \cdot 15} \\ &= -\frac{17}{45}\end{aligned}$$

$$\begin{aligned}21. 2 \cdot \left(\frac{1}{4} + \frac{1}{5}\right) + 2 &= 2 \cdot \left(\frac{1}{4} \cdot \frac{5}{5} + \frac{1}{5} \cdot \frac{4}{4}\right) + 2 \\ &= 2 \cdot \left(\frac{5}{20} + \frac{4}{20}\right) + 2 \\ &= 2 \cdot \left(\frac{9}{20}\right) + 2 \\ &= \left(\frac{2}{1}\right)\left(\frac{9}{20}\right) + 2 \\ &= \frac{2 \cdot 9}{1 \cdot 20} + 2 \\ &= \frac{2 \cdot 9}{1 \cdot 2 \cdot 10} + 2 \\ &= \frac{9}{10} + 2 \\ &= \frac{9}{10} + \frac{2}{1} \cdot \frac{10}{10} \\ &= \frac{9}{10} + \frac{20}{10} \\ &= \frac{29}{10}\end{aligned}$$

$$\begin{aligned}25. \left(\frac{2}{5} - \frac{3}{10}\right)^2 &= \left(\frac{2}{5} \cdot \frac{2}{2} - \frac{3}{10}\right)^2 \\ &= \left(\frac{4}{10} - \frac{3}{10}\right)^2 \\ &= \left(\frac{1}{10}\right)^2 \\ &= \frac{1}{10} \cdot \frac{1}{10} \\ &= \frac{1}{100}\end{aligned}$$

$$\begin{aligned}
 29. \quad 5y - z &= 5\left(\frac{2}{5}\right) - \frac{5}{6} \\
 &= 2 - \frac{5}{6} \\
 &= \frac{2}{1} \cdot \frac{6}{6} - \frac{5}{6} \\
 &= \frac{12}{6} - \frac{5}{6} \\
 &= \frac{7}{6}
 \end{aligned}$$

$$\begin{aligned}
 33. \quad x^2 - yz &= \left(-\frac{1}{3}\right)^2 - \left(\frac{2}{5}\right)\left(\frac{5}{6}\right) \\
 &= \left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right) - \left(\frac{2}{5}\right)\left(\frac{5}{6}\right) \\
 &= \frac{1}{9} - \frac{2 \cdot 5}{5 \cdot 2 \cdot 3} \\
 &= \frac{1}{9} - \frac{1}{3} \\
 &= \frac{1}{9} - \frac{1}{3} \cdot \frac{3}{3} \\
 &= \frac{1}{9} - \frac{3}{9} \\
 &= -\frac{2}{9}
 \end{aligned}$$

$$37. \quad \frac{\frac{5a}{24}}{\frac{1}{12}} = \frac{5a}{24} \div \frac{1}{12} = \frac{5a}{24} \cdot \frac{12}{1} = \frac{5 \cdot a \cdot 12}{12 \cdot 2 \cdot 1} = \frac{5a}{2}$$

$$41. \quad \left(-\frac{1}{2}\right)^2 + \frac{1}{5} = \frac{1}{4} + \frac{1}{5} = \frac{1 \cdot 5}{4 \cdot 5} + \frac{1 \cdot 4}{5 \cdot 4} = \frac{5}{20} + \frac{4}{20} = \frac{9}{20}$$

$$45. \quad \left(1 - \frac{2}{5}\right)^2 = \left(\frac{5}{5} - \frac{2}{5}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}$$

$$\begin{aligned}
 49. \quad \left(-\frac{2}{9} - \frac{7}{9}\right)^4 &= \left(-\frac{9}{9}\right)^4 \\
 &= (-1)^4 \\
 &= (-1)(-1)(-1)(-1) \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 53. \quad \left(\frac{3}{4} \div \frac{6}{5}\right) - \left(\frac{3}{4} \cdot \frac{6}{5}\right) &= \left(\frac{3}{4} \cdot \frac{5}{6}\right) - \left(\frac{3}{4} \cdot \frac{6}{5}\right) \\
 &= \frac{3 \cdot 5}{4 \cdot 2 \cdot 3} - \frac{3 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 5} \\
 &= \frac{5}{4 \cdot 2} - \frac{3 \cdot 3}{2 \cdot 5} \\
 &= \frac{5}{8} - \frac{9}{10} \\
 &= \frac{5 \cdot 5}{8 \cdot 5} - \frac{9 \cdot 4}{10 \cdot 4} \\
 &= \frac{25}{40} - \frac{36}{40} \\
 &= -\frac{11}{40}
 \end{aligned}$$

$$57. \quad 3 + \frac{1}{2} = \frac{3}{1} + \frac{1}{2} = \frac{6}{2} + \frac{1}{2} = \frac{7}{2} \text{ or } 3\frac{1}{2}$$

61. no; answers may vary

$$\begin{aligned}
 65. \quad \frac{\frac{1}{4} + \frac{2}{14}}{2} &= \frac{28\left(\frac{1}{4} + \frac{2}{14}\right)}{28(2)} \\
 &= \frac{28 \cdot \frac{1}{4} + 28 \cdot \frac{2}{14}}{28 \cdot 2} \\
 &= \frac{7 + 4}{56} \\
 &= \frac{11}{56}
 \end{aligned}$$

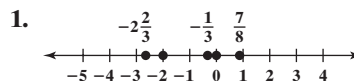
69. False; the average cannot be greater than the greatest number.

73. true

77. subtraction, multiplication, addition, division

$$\begin{aligned}
 81. \quad \frac{2+x}{y} &= \frac{2 + \frac{3}{4}}{-\frac{4}{7}} \\
 &= \frac{28\left(2 + \frac{3}{4}\right)}{28\left(-\frac{4}{7}\right)} \\
 &= \frac{28 \cdot 2 + 28 \cdot \frac{3}{4}}{28 \cdot \left(-\frac{4}{7}\right)} \\
 &= \frac{56 + 21}{-16} \\
 &= -\frac{77}{16}
 \end{aligned}$$

Exercise Set 4.7



5. $2\frac{11}{12}$ rounds to 3.

$1\frac{1}{4}$ rounds to 1.

$$3 \cdot 1 = 3$$

The best estimate is **b**.

$$9. \quad 2\frac{2}{3} \cdot \frac{1}{7} = \frac{8}{3} \cdot \frac{1}{7} = \frac{8}{21}$$

$$13. \text{ Exact: } 2\frac{1}{5} \cdot 3\frac{1}{2} = \frac{11}{5} \cdot \frac{7}{2} = \frac{77}{10} \text{ or } 7\frac{7}{10}$$

Estimate: $2\frac{1}{5}$ rounds to 2, $3\frac{1}{2}$ rounds to 4.

$$2 \cdot 4 = 8 \text{ so the answer is reasonable.}$$

$$17. \quad 5 \cdot 2\frac{1}{2} = \frac{5}{1} \cdot \frac{5}{2} = \frac{25}{2} \text{ or } 12\frac{1}{2}$$

$$21. \quad 2\frac{2}{3} \div \frac{1}{7} = \frac{8}{3} \cdot \frac{7}{1} = \frac{56}{3} \text{ or } 18\frac{2}{3}$$

25. $8\frac{1}{3}$ rounds to 8.

$1\frac{1}{2}$ rounds to 2.

$$8 + 2 = 10$$

The best estimate is **b**.

$$29. \text{ Exact: } 10\frac{3}{14} + 3\frac{4}{7} = 10\frac{3}{14} + 3\frac{8}{14} = 13\frac{11}{14}$$

Estimate: $10\frac{3}{14}$ rounds to 10, $3\frac{4}{7}$ rounds to 4.

$$10 + 4 = 14 \text{ so the answer is reasonable.}$$

$$\begin{array}{r} 33. \quad 12\frac{3}{14} \quad 12\frac{18}{84} \\ 10 \quad 10 \\ + 25\frac{5}{12} \quad + 25\frac{35}{84} \\ \hline \quad \quad 47\frac{53}{84} \end{array}$$

$$\begin{array}{r} 37. \quad 3\frac{5}{8} \quad 3\frac{15}{24} \\ 2\frac{1}{6} \quad 2\frac{4}{24} \\ + 7\frac{3}{4} \quad + 7\frac{18}{24} \\ \hline 12\frac{37}{24} = 12 + 1\frac{13}{24} = 13\frac{13}{24} \end{array}$$

$$\begin{array}{r} 41. \quad 10\frac{13}{14} \quad 10\frac{13}{14} \\ - 3\frac{4}{7} \quad - 3\frac{8}{14} \\ \hline \text{Exact: } 7\frac{5}{14} \\ \text{Estimate: } 10\frac{13}{14} \text{ rounds to } 11, 3\frac{4}{7} \text{ rounds to } 4. \\ 11 - 4 = 7 \text{ so the answer is reasonable.} \end{array}$$

$$\begin{array}{r} 45. \quad 6 \quad 5\frac{9}{9} \\ - 2\frac{4}{9} \quad - 2\frac{4}{9} \\ \hline \quad \quad 3\frac{5}{9} \end{array}$$

$$\begin{array}{r} 49. \quad 2\frac{3}{4} \\ + 1\frac{1}{4} \\ \hline 3\frac{4}{4} = 3 + 1 = 4 \end{array}$$

$$53. \quad 3\frac{1}{9} \cdot 2 = \frac{28}{9} \cdot \frac{2}{1} = \frac{56}{9} \text{ or } 6\frac{2}{9}$$

$$57. \quad 22\frac{4}{9} + 13\frac{5}{18} = 22\frac{8}{18} + 13\frac{5}{18} = 35\frac{13}{18}$$

$$\begin{array}{r} 61. \quad 15\frac{1}{5} \quad 15\frac{6}{30} \\ 20\frac{3}{10} \quad 20\frac{9}{30} \\ + 37\frac{2}{15} \quad + 37\frac{4}{30} \\ \hline \quad \quad 72\frac{19}{30} \end{array}$$

$$\begin{aligned} 65. \quad 4\frac{2}{7} \cdot 1\frac{3}{10} &= \frac{30}{7} \cdot \frac{13}{10} \\ &= \frac{30 \cdot 13}{7 \cdot 10} \\ &= \frac{3 \cdot 10 \cdot 13}{7 \cdot 10} \\ &= \frac{39}{7} \text{ or } 5\frac{4}{7} \end{aligned}$$

$$69. \quad \text{"}-5\frac{2}{7} \text{ decreased by a number"} \text{ translates as } -5\frac{2}{7} - x.$$

$$73. \quad 12\frac{3}{4} \div 4 = \frac{51}{4} \div \frac{4}{1} = \frac{51}{4} \cdot \frac{1}{4} = \frac{51}{16} \text{ or } 3\frac{3}{16}$$

The patient walked $3\frac{3}{16}$ miles per day.

$$\begin{array}{r} 77. \quad 11\frac{1}{4} \quad 11\frac{5}{20} \quad 10\frac{25}{20} \\ - 3\frac{3}{5} \quad - 3\frac{12}{20} \quad - 3\frac{12}{30} \\ \hline \quad \quad 7\frac{13}{20} \end{array}$$

Tucson gets an average of $7\frac{13}{20}$ inches more rain than Yuma.

$$81. \quad \frac{3}{4} \cdot 1\frac{1}{4} = \frac{3}{4} \cdot \frac{5}{4} = \frac{3 \cdot 5}{4 \cdot 4} = \frac{15}{16}$$

The area is $\frac{15}{16}$ square inch.

$$\begin{array}{r} 85. \quad 15\frac{2}{3} - \left(3\frac{1}{4} + 2\frac{1}{2}\right) = 15\frac{2}{3} - \left(3\frac{1}{4} + 2\frac{2}{4}\right) \\ = 15\frac{2}{3} - 5\frac{3}{4} \\ 15\frac{2}{3} \quad 15\frac{8}{12} \quad 14\frac{20}{12} \\ - 5\frac{3}{4} \quad - 5\frac{9}{12} \quad - 5\frac{9}{12} \\ \hline \quad \quad 9\frac{11}{12} \end{array}$$

No; the remaining pipe is $9\frac{11}{12}$ feet, which is $\frac{1}{12}$ foot short.

$$\begin{array}{r} 89. \quad 2\frac{2}{3} \quad 2\frac{40}{60} \\ 4\frac{7}{15} \quad 4\frac{28}{60} \\ + 2\frac{37}{60} \quad + 2\frac{37}{60} \\ \hline 8\frac{105}{60} = 8\frac{7}{4} = 8 + 1\frac{3}{4} = 9\frac{3}{4} \end{array}$$

The total duration of the eclipses is $9\frac{3}{4}$ minutes.

$$\begin{aligned} 93. \quad -4\frac{2}{5} \cdot 2\frac{3}{10} &= -\frac{22}{5} \cdot \frac{23}{10} \\ &= -\frac{2 \cdot 11 \cdot 23}{5 \cdot 2 \cdot 5} \\ &= -\frac{253}{25} \text{ or } -10\frac{3}{25} \end{aligned}$$

$$\begin{aligned} 97. \quad -31\frac{2}{15} + 17\frac{3}{20} &= -31\frac{8}{60} + 17\frac{9}{60} \\ &= -30\frac{68}{60} + 17\frac{9}{60} \\ &= -13\frac{59}{60} \end{aligned}$$

$$\begin{aligned} 101. \quad 11\frac{7}{8} - 13\frac{5}{6} &= 11\frac{21}{24} - 13\frac{20}{24} \\ &= -\left(13\frac{20}{24} - 11\frac{21}{24}\right) \\ &= -\left(12\frac{44}{24} - 11\frac{21}{24}\right) \\ &= -1\frac{23}{24} \end{aligned}$$

$$105. \frac{1}{3}(3x) = \left(\frac{1}{3} \cdot 3\right)x = 1 \cdot x = x$$

$$109. \text{a. } 9\frac{5}{5} = 9 + 1 = 10$$

$$\text{b. } 9\frac{100}{100} = 9 + 1 = 10$$

$$\text{c. } 6\frac{44}{11} = 6 + 4 = 10$$

$$\text{d. } 8\frac{13}{13} = 8 + 1 = 9$$

a, b, and c are equivalent to 10.

113. answers may vary

117. answers may vary

Exercise Set 4.8

$$1. \quad x + \frac{1}{3} = -\frac{1}{3}$$

$$x + \frac{1}{3} - \frac{1}{3} = -\frac{1}{3} - \frac{1}{3}$$

$$x = -\frac{2}{3}$$

$$\text{Check: } x + \frac{1}{3} = -\frac{1}{3}$$

$$-\frac{2}{3} + \frac{1}{3} \stackrel{?}{=} -\frac{1}{3}$$

$$-\frac{1}{3} = -\frac{1}{3} \quad \text{True}$$

The solution is $-\frac{2}{3}$.

$$5. \quad 3x - \frac{1}{5} - 2x = \frac{1}{5} + \frac{2}{5}$$

$$x - \frac{1}{5} = \frac{3}{5}$$

$$x - \frac{1}{5} + \frac{1}{5} = \frac{3}{5} + \frac{1}{5}$$

$$x = \frac{4}{5}$$

$$\text{Check: } 3x - \frac{1}{5} - 2x = \frac{1}{5} + \frac{2}{5}$$

$$3 \cdot \frac{4}{5} - \frac{1}{5} - 2 \cdot \frac{4}{5} \stackrel{?}{=} \frac{1}{5} + \frac{2}{5}$$

$$\frac{12}{5} - \frac{1}{5} - \frac{8}{5} \stackrel{?}{=} \frac{1}{5} + \frac{2}{5}$$

$$\frac{3}{5} = \frac{3}{5} \quad \text{True}$$

The solution is $\frac{4}{5}$.

$$9. \quad \frac{2}{5} + y = -\frac{3}{10}$$

$$\frac{2}{5} + y - \frac{2}{5} = -\frac{3}{10} - \frac{2}{5}$$

$$y = -\frac{3}{10} - \frac{2}{5} \cdot \frac{2}{2}$$

$$y = -\frac{3}{10} - \frac{4}{10}$$

$$y = -\frac{7}{10}$$

$$\text{Check: } \frac{2}{5} + y = -\frac{3}{10}$$

$$\frac{2}{5} + \left(-\frac{7}{10}\right) \stackrel{?}{=} -\frac{3}{10}$$

$$\frac{2}{5} \cdot \frac{2}{2} + \left(-\frac{7}{10}\right) \stackrel{?}{=} -\frac{3}{10}$$

$$\frac{4}{10} + \left(-\frac{7}{10}\right) \stackrel{?}{=} -\frac{3}{10}$$

$$-\frac{3}{10} = -\frac{3}{10} \quad \text{True}$$

The solution is $-\frac{7}{10}$.

$$13. \quad -\frac{2}{9} = x - \frac{5}{6}$$

$$-\frac{2}{9} + \frac{5}{6} = x - \frac{5}{6} + \frac{5}{6}$$

$$-\frac{2}{9} \cdot \frac{2}{2} + \frac{5}{6} \cdot \frac{3}{3} = x$$

$$-\frac{4}{18} + \frac{15}{18} = x$$

$$\frac{11}{18} = x$$

$$\text{Check: } -\frac{2}{9} = x - \frac{5}{6}$$

$$-\frac{2}{9} \stackrel{?}{=} \frac{11}{18} - \frac{5}{6}$$

$$-\frac{2}{9} \cdot \frac{2}{2} \stackrel{?}{=} \frac{11}{18} - \frac{5}{6} \cdot \frac{3}{3}$$

$$-\frac{4}{18} \stackrel{?}{=} \frac{11}{18} - \frac{15}{18}$$

$$-\frac{4}{18} = -\frac{4}{18} \quad \text{True}$$

The solution is $\frac{11}{18}$.

$$17. \quad \frac{1}{4}x = 3$$

$$4 \cdot \frac{1}{4}x = 4 \cdot 3$$

$$x = 12$$

$$21. \quad -\frac{4}{9}z = -\frac{3}{2}$$

$$-\frac{9}{4} \cdot \frac{4}{9}z = -\frac{9}{4} \cdot -\frac{3}{2}$$

$$z = \frac{27}{8}$$

$$25. \quad -3x = -\frac{6}{11}$$

$$-\frac{1}{3} \cdot -3x = -\frac{1}{3} \cdot -\frac{6}{11}$$

$$x = \frac{6}{3 \cdot 11}$$

$$x = \frac{2}{11}$$

$$29. \quad \frac{x}{3} + 2 = \frac{7}{3}$$

$$3\left(\frac{x}{3} + 2\right) = 3 \cdot \frac{7}{3}$$

$$3 \cdot \frac{x}{3} + 3 \cdot 2 = 7$$

$$x + 6 = 7$$

$$x + 6 - 6 = 7 - 6$$

$$x = 1$$

$$33. \quad \frac{1}{2} - \frac{3}{5} = \frac{x}{10}$$

$$10\left(\frac{1}{2} - \frac{3}{5}\right) = 10 \cdot \frac{x}{10}$$

$$10 \cdot \frac{1}{2} - 10 \cdot \frac{3}{5} = x$$

$$5 - 6 = x$$

$$-1 = x$$

$$37. \quad \frac{x}{7} - \frac{4}{3} = \frac{x}{7} \cdot \frac{3}{3} - \frac{4}{3} \cdot \frac{7}{7}$$

$$= \frac{3x}{21} - \frac{28}{21}$$

$$= \frac{3x - 28}{21}$$

$$41. \quad \frac{3x}{10} + \frac{x}{6} = \frac{3x}{10} \cdot \frac{3}{3} + \frac{x}{6} \cdot \frac{5}{5}$$

$$= \frac{9x}{30} + \frac{5x}{30}$$

$$= \frac{14x}{30}$$

$$= \frac{2 \cdot 7 \cdot x}{2 \cdot 15}$$

$$= \frac{7x}{15}$$

$$45. \quad \frac{2}{3} - \frac{x}{5} = \frac{4}{15}$$

$$15\left(\frac{2}{3} - \frac{x}{5}\right) = 15 \cdot \frac{4}{15}$$

$$15 \cdot \frac{2}{3} - 15 \cdot \frac{x}{5} = 4$$

$$10 - 3x = 4$$

$$10 - 3x - 10 = 4 - 10$$

$$-3x = -6$$

$$\frac{-3x}{-3} = \frac{-6}{-3}$$

$$x = 2$$

$$49. \quad -3m - 5m = \frac{4}{7}$$

$$-8m = \frac{4}{7}$$

$$-\frac{1}{8} \cdot -8m = -\frac{1}{8} \cdot \frac{4}{7}$$

$$m = -\frac{1 \cdot 4}{2 \cdot 4 \cdot 7}$$

$$m = -\frac{1}{14}$$

$$53. \quad \frac{5}{9} - \frac{2}{3} = \frac{5}{9} - \frac{2}{3} \cdot \frac{3}{3} = \frac{5}{9} - \frac{6}{9} = -\frac{1}{9}$$

$$57. \quad \frac{5}{7}y = -\frac{15}{49}$$

$$\frac{7}{5} \cdot \frac{5}{7}y = \frac{7}{5} \cdot -\frac{15}{49}$$

$$y = -\frac{7 \cdot 15}{5 \cdot 49}$$

$$y = -\frac{7 \cdot 3 \cdot 5}{5 \cdot 7 \cdot 7}$$

$$y = -\frac{3}{7}$$

$$61. \quad -\frac{5}{8}y = \frac{3}{16} - \frac{9}{16}$$

$$16\left(-\frac{5}{8}y\right) = 16\left(\frac{3}{16} - \frac{9}{16}\right)$$

$$-10y = 16 \cdot \frac{3}{16} - 16 \cdot \frac{9}{16}$$

$$-10y = 3 - 9$$

$$-10y = -6$$

$$\frac{-10y}{-10} = \frac{-6}{-10}$$

$$y = \frac{3}{5}$$

$$65. \quad \frac{7}{6}x = \frac{1}{4} - \frac{2}{3}$$

$$12 \cdot \frac{7}{6}x = 12\left(\frac{1}{4} - \frac{2}{3}\right)$$

$$14x = 12 \cdot \frac{1}{4} - 12 \cdot \frac{2}{3}$$

$$14x = 3 - 8$$

$$14x = -5$$

$$\frac{14x}{14} = \frac{-5}{14}$$

$$x = -\frac{5}{14}$$

$$69. \quad \frac{x}{3} + 2 = \frac{x}{2} + 8$$

$$6\left(\frac{x}{3} + 2\right) = 6\left(\frac{x}{2} + 8\right)$$

$$6 \cdot \frac{x}{3} + 6 \cdot 2 = 6 \cdot \frac{x}{2} + 6 \cdot 8$$

$$2x + 12 = 3x + 48$$

$$2x + 12 - 2x = 3x + 48 - 2x$$

$$12 = x + 48$$

$$12 - 48 = x + 48 - 48$$

$$-36 = x$$

73. 327 rounded to the nearest ten is 330.

$$77. \quad \frac{14}{11} + \frac{3x}{8} = \frac{x}{2}$$

$$88\left(\frac{14}{11} + \frac{3x}{8}\right) = 88 \cdot \frac{x}{2}$$

$$88 \cdot \frac{14}{11} + 88 \cdot \frac{3x}{8} = 44x$$

$$112 + 33x = 44x$$

$$112 + 33x - 33x = 44x - 33x$$

$$112 = 11x$$

$$\frac{112}{11} = \frac{11x}{11}$$

$$\frac{112}{11} = x$$

Chapter 4 Test

1. 7 of the 16 equal parts are shaded: $\frac{7}{16}$
5. $-\frac{42x}{70} = -\frac{3 \cdot 14 \cdot x}{5 \cdot 14} = -\frac{3x}{5}$
9. $495 = 3 \cdot 165 = 3 \cdot 3 \cdot 55 = 3 \cdot 3 \cdot 5 \cdot 11 = 3^2 \cdot 5 \cdot 11$
13. The LCD of 7 and x is $7x$.
 $\frac{1}{7} - \frac{3}{x} = \frac{1}{7} \cdot \frac{x}{x} - \frac{3}{x} \cdot \frac{7}{7} = \frac{x}{7x} - \frac{21}{7x} = \frac{x-21}{7x}$
17. $-\frac{8}{15y} - \frac{2}{15y} = \frac{-8-2}{15y} = \frac{-10}{15y} = -\frac{2 \cdot 5}{3 \cdot 5 \cdot y} = -\frac{2}{3y}$
21.
$$\begin{array}{r} 19 \\ -2\frac{3}{11} \\ \hline \end{array} \quad \begin{array}{r} 18\frac{11}{11} \\ -2\frac{3}{11} \\ \hline \end{array}$$

$$16\frac{8}{11}$$
25. $\frac{1}{2} \div \frac{2}{3} \cdot \frac{3}{4} = \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{3}{4} = \frac{1 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 4} = \frac{9}{16}$
29. $\frac{5 + \frac{3}{7}}{2 - \frac{1}{2}} = \frac{14(5 + \frac{3}{7})}{14(2 - \frac{1}{2})} = \frac{14 \cdot 5 + 14 \cdot \frac{3}{7}}{14 \cdot 2 - 14 \cdot \frac{1}{2}} = \frac{70 + 6}{28 - 7} = \frac{76}{21} \text{ or } 3\frac{13}{21}$
33. $-5x = -5\left(-\frac{1}{2}\right) = \frac{5}{1} \cdot \frac{1}{2} = \frac{5 \cdot 1}{1 \cdot 2} = \frac{5}{2} \text{ or } 2\frac{1}{2}$
37. Education: $\frac{1}{50}$
 Transportation: $\frac{1}{5}$
 Clothing: $\frac{1}{25}$
- $$\begin{aligned} \frac{1}{50} + \frac{1}{5} + \frac{1}{25} &= \frac{1}{50} + \frac{1}{5} \cdot \frac{10}{10} + \frac{1}{25} \cdot \frac{2}{2} \\ &= \frac{1}{50} + \frac{10}{50} + \frac{2}{50} \\ &= \frac{13}{50} \end{aligned}$$
- $\frac{13}{50}$ of spending goes for education, transportation, and clothing.

Chapter 5**Exercise Set 5.1**

1. 5.62 in words is five and sixty-two hundredths.
5. -0.205 in words is negative two hundred five thousandths.
9. 3000.04 in words is three thousand and four hundredths.
13. 2.43 in words is two and forty-three hundredths.
17. The check should be paid to "Verizon," for the amount of "91.68," which is written in words as "Ninety-one and $\frac{68}{100}$."
21. Nine and eight hundredths is 9.08.
25. Forty-six ten-thousandths is 0.0046.
29. $0.27 = \frac{27}{100}$
33. $5.4 = 5\frac{4}{10} = 5\frac{2}{5}$
37. $7.008 = 7\frac{8}{1000} = 7\frac{1}{125}$
41. $0.3005 = \frac{3005}{10,000} = \frac{601}{2000}$
45. In words, 0.077 is seventy-seven thousandths. As a fraction,
 $0.077 = \frac{77}{1000}$
49.
$$\begin{array}{cc} 0.57 & 0.54 \\ \uparrow & \uparrow \\ 7 & > 4 \end{array}$$
 so $0.57 > 0.54$
 Thus $-0.57 < -0.54$.
53.
$$\begin{array}{cc} 0.54900 & 0.549 \\ \uparrow & \uparrow \\ 9 & = 9 \end{array}$$
 so $0.54900 = 0.549$
57.
$$\begin{array}{cc} 1.062 & 1.07 \\ \uparrow & \uparrow \\ 6 & < 7 \end{array}$$
 so $1.062 < 1.07$
 Thus, $-1.062 > -1.07$.
61.
$$\begin{array}{cc} 0.023 & 0.024 \\ \uparrow & \uparrow \\ 3 & < 4 \end{array}$$
 so $0.023 < 0.024$
 Thus, $-0.023 > -0.024$.
65. To round 98,207.23 to the nearest ten, observe that the digit in the ones place is 7. Since this digit is 5 or greater, we add 1 to the digit in the tens place. The number 98,207.23 rounded to the nearest ten is 98,210.
69. To round 0.5942 to the nearest thousandth, observe that the digit in the ten-thousandths place is 2. Since this digit is less than 5, we do not add 1 to the digit in the thousandths place. The number 0.5942 rounded to the nearest thousandth is 0.594.
73. To round $\pi \approx 3.14159265$ to the nearest thousandth, observe that the digit in the ten-thousandths place is 5. Since this digit is 5 or greater, we add 1 to the digit in the thousandths place. The number $\pi \approx 3.14159265$ rounded to the nearest thousandth is 3.142.
77. To round 0.1992 to the nearest hundredth, observe that the digit in the thousandths place is 9. Since this digit is 5 or greater, we add 1 to the digit in the hundredths place. The number 0.1992 rounded to the nearest hundredth is 0.20. The amount is \$0.20.
81. To round 1.918 to the nearest hundredth, observe that the digit in the thousandths place is 8. Since this digit is 5 or greater, we add 1 to the digit in the hundredths place. The number 1.918 rounded to the nearest hundredth is 1.92. The time is 1.92 minutes.
85. To round 224.695 to the nearest one, observe that the digit in the tenths place is 6. Since this digit is 5 or greater, we add 1 to the digit in the ones place. The number 224.695 rounded to the nearest one is 225. This is 225 days.
89.
$$\begin{array}{r} 82 \\ -47 \\ \hline 35 \end{array}$$
93. To round 2849.1738 to the nearest hundredth, observe that the digit in the thousandths place is 3. Since this digit is less than 5, we do not add 1 to the digit in the hundredths place. 2849.1738 rounded to the nearest hundredth is 2849.17, which is choice **a**.
97. $7\frac{12}{100} = 7.12$

101. answers may vary

105. 0.26499 and 0.25786 rounded to the nearest hundredth are 0.26. 0.26559 rounds to 0.27 and 0.25186 rounds to 0.25.

109. Round each amount to the nearest hundred million, then add.

936.7	rounds to	900
760.5	rounds to	800
658.7	rounds to	700
652.5	rounds to	700
623.2	rounds to	600
+ 533.3	rounds to	+ 500
		4200

The total amount of money is estimated as \$4200 million.

Exercise Set 5.2

1. 5.6

$$\begin{array}{r} + 2.1 \\ 5.6 \\ \hline 7.7 \end{array}$$

5. $\begin{array}{r} 24.6000 \\ 2.3900 \end{array}$

$$\begin{array}{r} + 0.0678 \\ 24.6000 \\ 2.3900 \\ \hline 27.0578 \end{array}$$

9. $18.56 + (-8.23)$

Subtract the absolute values.

$$\begin{array}{r} 18.56 \\ - 8.23 \\ \hline 10.33 \end{array}$$

Attach the sign of the larger absolute value.

$$18.56 + (-8.23) = 10.33$$

13. Exact: $\begin{array}{r} 100.009 \\ 6.080 \\ + 9.034 \\ \hline 115.123 \end{array}$ Estimate: $\begin{array}{r} 100 \\ 6 \\ + 9 \\ \hline 115 \end{array}$

17. $\begin{array}{r} 12.6 \\ - 8.2 \\ \hline 4.4 \end{array}$ Check: $\begin{array}{r} 4.4 \\ + 8.2 \\ \hline 12.6 \end{array}$

21. $\begin{array}{r} 654.90 \\ - 56.67 \\ \hline 598.23 \end{array}$ Check: $\begin{array}{r} 598.23 \\ + 56.67 \\ \hline 654.90 \end{array}$

25. Exact: $\begin{array}{r} 1000.0 \\ - 123.4 \\ \hline 876.6 \end{array}$ Check: $\begin{array}{r} 876.6 \\ + 123.4 \\ \hline 1000.0 \end{array}$

Estimate: $\begin{array}{r} 1000 \\ - 100 \\ \hline 900 \end{array}$

29. $-1.12 - 5.2 = -1.12 + (-5.2)$

Add the absolute values.

$$\begin{array}{r} 1.12 \\ + 5.20 \\ \hline 6.32 \end{array}$$

Attach the common sign.

$$-1.12 - 5.2 = -6.32$$

33. $-2.6 - (-5.7) = -2.6 + 5.7$

Subtract the absolute values.

$$\begin{array}{r} 5.7 \\ - 2.6 \\ \hline 3.1 \end{array}$$

Attach the sign of the larger absolute value.

$$-2.6 - (-5.7) = 3.1$$

37. $\begin{array}{r} 23.0 \\ - 6.7 \\ \hline 16.3 \end{array}$ Check: $\begin{array}{r} 16.3 \\ + 6.7 \\ \hline 23.0 \end{array}$

41. $-6.06 + 0.44$

Subtract the absolute values.

$$\begin{array}{r} 6.06 \\ - 0.44 \\ \hline 5.62 \end{array}$$

Attach the sign of the larger absolute value.

$$-6.06 + 0.44 = -5.62$$

45. $50.2 - 600 = 50.2 + (-600)$

Subtract the absolute values.

$$\begin{array}{r} 600.0 \\ - 50.2 \\ \hline 549.8 \end{array}$$

Attach the sign of the larger absolute value.

$$50.2 - 600 = -549.8$$

49. $\begin{array}{r} 200.008 \\ 7.060 \end{array}$

$$\begin{array}{r} + 9.046 \\ 200.008 \\ 7.060 \\ \hline 216.114 \end{array}$$

53. $-102.4 - 78.04 = -102.4 + (-78.04)$

Add the absolute values.

$$\begin{array}{r} 102.40 \\ + 78.04 \\ \hline 180.44 \end{array}$$

Attach the common sign.

$$-102.4 - 78.04 = -180.44$$

57. $x + z = 3.6 + 0.21 = 3.81$

61. $y - x + z = 5 - 3.6 + 0.21$
 $= 5.00 - 3.60 + 0.21$
 $= 1.40 + 0.21$
 $= 1.61$

65. $27.4 + y = 16$

$$27.4 + (-11.4) \stackrel{?}{=} 16$$

$$16 = 16 \quad \text{True}$$

Yes, -11.4 is a solution.

69. $30.7x + 17.6 - 23.8x - 10.7$
 $= 30.7x - 23.8x + 17.6 - 10.7$
 $= 6.9x + 6.9$

73. $\begin{array}{r} 40.00 \\ - 32.48 \\ \hline 7.52 \end{array}$

Her change was \$7.52.

77. Perimeter = $7.14 + 7.14 + 7.14 + 7.14 = 28.56$ meters

81. The phrase "How much faster" indicates that we should subtract the average wind speed from the record speed.

$$\begin{array}{r} 231.0 \\ - 35.2 \\ \hline 195.8 \end{array}$$

The highest wind speed is 195.8 miles per hour faster than the average wind speed.

85. To find the total, we add.

$$\begin{array}{r}
 2111 \\
 936.7 \\
 760.5 \\
 + 658.7 \\
 \hline
 2355.9
 \end{array}$$

The total ticket sales rounded were \$2356 million.

89. Add the lengths of the sides to get the perimeter.

$$\begin{array}{r}
 11 \\
 12.40 \\
 29.34 \\
 + 25.70 \\
 \hline
 67.44
 \end{array}$$

67.44 feet of border material is needed.

93. The tallest bar indicates the greatest chocolate consumption per person, so Switzerland has the greatest chocolate consumption per person.

Country	Pounds of Chocolate per Person
Switzerland	19.8
Germany	17.4
UK	16.3
Ireland	16.2
Norway	14.6
Sweden	11.9

$$101. \left(\frac{2}{3}\right)^2 = \frac{2}{3} \cdot \frac{2}{3} = \frac{2 \cdot 2}{3 \cdot 3} = \frac{4}{9}$$

$$105. 10.68 - (2.3 + 2.3) = 10.68 - 4.60 = 6.08$$

The unknown length is 6.08 inches

109. 1 nickel, 1 dime, and 2 pennies:

$$0.05 + 0.10 + 0.01 + 0.01 = 0.17$$

3 nickels and 2 pennies:

$$0.05 + 0.05 + 0.05 + 0.01 + 0.01 = 0.17$$

1 dime and 7 pennies:

$$0.10 + 0.01 + 0.01 + 0.01 + 0.01 + 0.01 + 0.01 + 0.01 = 0.17$$

2 nickels and 7 pennies:

$$0.05 + 0.05 + 0.01 + 0.01 + 0.01 + 0.01 + 0.01 + 0.01 = 0.17$$

113. answers may vary

Exercise Set 5.3

1. 0.17 2 decimal places

$$\begin{array}{r} \times 8 \\ 1.36 \end{array}$$

$$1.36 \quad 2 + 0 = 2 \text{ decimal places}$$

5. The product
- $(-2.3)(7.65)$
- is negative.

$$\begin{array}{r} 7.65 \\ \times 2.3 \\ \hline \end{array}$$

$$2295$$

$$15300$$

$$-17.595$$

2 + 1 = 3 decimal places and include the negative sign

9. Exact: 6.8

$$\begin{array}{r}
 \times 4.2 \\
 136 \\
 2720 \\
 \hline
 28.56
 \end{array}$$

- Estimate: 7

$$\begin{array}{r}
 \times 4 \\
 28
 \end{array}$$

13. Exact: 1.0047

$$\begin{array}{r}
 \times 8.2 \\
 20094 \\
 803760 \\
 \hline
 8.23854
 \end{array}$$

- Estimate: 1

$$\begin{array}{r}
 \times 8 \\
 8
 \end{array}$$

- 17.
- $6.5 \times 10 = 65$

$$21. (-7.093)(1000) = -7093$$

$$25. (-9.83)(-0.01) = 0.0983$$

29. 0.123

$$\begin{array}{r}
 \times 0.4 \\
 0.0492
 \end{array}$$

33. 8.6

$$\begin{array}{r}
 \times 0.15 \\
 430 \\
 860 \\
 \hline
 1.290
 \end{array}$$

or 1.29

$$37. 562.3 \times 0.001 = 0.5623$$

$$\begin{aligned}
 41. 1.5 \text{ billion} &= 1.5 \times 1 \text{ billion} \\
 &= 1.5 \times 1,000,000,000 \\
 &= 1,500,000,000
 \end{aligned}$$

The cost at launch was \$1,500,000,000.

$$45. xy = 3(-0.2) = -0.6$$

$$49. 0.6x = 4.92$$

$$0.6(14.2) \stackrel{?}{=} 4.92$$

$$8.52 \stackrel{?}{=} 4.92 \quad \text{False}$$

No, 14.2 is not a solution.

$$53. C = \pi d \text{ is } \pi(10 \text{ cm}) = 10\pi \text{ cm}$$

$$C \approx 10(3.14) \text{ cm} = 31.4 \text{ cm}$$

57. Multiply his hourly wage by the number of hours worked.

$$17.88$$

$$\begin{array}{r}
 \times 40 \\
 715.20
 \end{array}$$

His pay for last week was \$715.20.

61. Area = length
- $\cdot$
- width

$$5.44$$

$$\begin{array}{r}
 \times 2.6 \\
 3264
 \end{array}$$

$$10880$$

$$14.144$$

The face is 14.144 square inches.

$$65. C = \pi \cdot d$$

$$C = \pi \cdot 135 = 135\pi$$

$$135$$

$$\begin{array}{r}
 \times 3.14 \\
 540
 \end{array}$$

$$1350$$

$$40500$$

$$423.90$$

He travels 135π meters or approximately 423.9 meters.

69. a. Circumference = $2 \cdot \pi \cdot \text{radius}$

Smaller circle:

$$C = 2 \cdot \pi \cdot 10 = 20\pi$$

$$C \approx 20(3.14) = 62.8$$

The circumference of the smaller circle is approximately 62.8 meters.

Larger circle:

$$C = 2 \cdot \pi \cdot 20 = 40\pi$$

$$C \approx 40(3.14) = 125.6$$

The circumference of the larger circle is approximately 125.6 meters.

- b. Yes, the circumference gets doubled when the radius is doubled.

73. 1.3032

$$\begin{array}{r} \times 750 \\ 651600 \\ 9122400 \\ 9774000 \end{array}$$

750 U.S. dollars is equivalent to 977.4 Canadian dollars.

$$\begin{array}{r} 486 \\ 77. \quad 6 \overline{)2916} \\ \underline{-24} \\ 51 \\ \underline{-48} \\ 36 \\ \underline{-36} \\ 0 \end{array}$$

81. 3.60

$$\begin{array}{r} + 0.04 \\ 3.64 \end{array}$$

85. The product of a negative number and a positive number is a negative number.

$$0.221$$

$$\times 0.5$$

$$0.1105$$

The product is -0.1105 .

89. answers may vary

Exercise Set 5.4

$$\begin{array}{r} 4.6 \\ 1. \quad 6 \overline{)27.6} \\ \underline{-24} \\ 36 \\ \underline{-36} \\ 0 \end{array}$$

$$\begin{array}{r} 300 \\ 5. \quad 0.06 \overline{)18} \text{ becomes } 6 \overline{)1800} \\ \underline{-18} \\ 0 \end{array}$$

$$\begin{array}{r} 6.6 \\ 9. \quad \text{Exact: } 5.5 \overline{)36.3} \text{ becomes } 55 \overline{)363.0} \\ \underline{-330} \\ 330 \\ \underline{-330} \\ 0 \end{array}$$

$$\text{Estimate: } \frac{6}{6 \overline{)36}}$$

13. A positive number divided by a negative number is a negative number.

$$\begin{array}{r} 600 \\ 0.06 \overline{)36} \text{ becomes } 6 \overline{)3600} \\ \underline{-36} \\ 0 \end{array}$$

$$36 \div (-0.06) = -600$$

$$\begin{array}{r} 4.8 \\ 17. \quad 0.27 \overline{)1.296} \text{ becomes } 27 \overline{)129.6} \\ \underline{-108} \\ 216 \\ \underline{-216} \\ 0 \end{array}$$

$$\begin{array}{r} 5.8 \\ 21. \quad 0.82 \overline{)4.756} \text{ because } 82 \overline{)475.6} \\ \underline{-410} \\ 656 \\ \underline{-656} \\ 0 \end{array}$$

$$\begin{array}{r} 9.8 \\ 25. \quad \text{Exact: } 7.2 \overline{)70.56} \text{ becomes } 72 \overline{)705.6} \\ \underline{-648} \\ 576 \\ \underline{-576} \\ 0 \end{array}$$

$$\text{Estimate: } \frac{10}{7 \overline{)70}}$$

$$\begin{array}{r} 45 \\ 29. \quad 0.027 \overline{)1.215} \text{ becomes } 27 \overline{)1215} \\ \underline{-108} \\ 135 \\ \underline{-135} \\ 0 \end{array}$$

$$\begin{array}{r} 0.0055 \\ 33. \quad 3.78 \overline{)0.02079} \text{ becomes } 378 \overline{)2.0790} \\ \underline{-1890} \\ 1890 \\ \underline{-1890} \\ 0 \end{array}$$

$$\begin{array}{r} 113.98 \approx 114.0 \\ 37. \quad 0.6 \overline{)68.39} \text{ becomes } 6 \overline{)683.90} \\ \underline{-6} \\ 08 \\ \underline{-6} \\ 23 \\ \underline{-18} \\ 59 \\ \underline{-54} \\ 50 \\ \underline{-48} \\ 2 \end{array}$$

$$41. \quad \frac{26.87}{10} = 2.687$$

$$\begin{array}{r}
 12.6 \\
 45. \quad 7 \overline{) 88.2} \\
 \underline{-7} \\
 18 \\
 \underline{-14} \\
 42 \\
 \underline{-42} \\
 0
 \end{array}$$

$$49. \frac{456.25}{10,000} = 0.045625$$

$$\begin{array}{r}
 8 \\
 53. \quad 0.6 \overline{) 4.8} \text{ becomes } 6 \overline{) 48} \\
 \underline{-48} \\
 0
 \end{array}$$

$$4.8 \div (-0.6) = -8$$

$$\begin{array}{r}
 1400 \\
 57. \quad 0.03 \overline{) 42} \text{ becomes } 3 \overline{) 4200} \\
 \underline{-3} \\
 12 \\
 \underline{-12} \\
 0
 \end{array}$$

$$\begin{array}{r}
 58,000 \\
 61. \quad 0.0015 \overline{) 87} \text{ becomes } 15 \overline{) 870,000} \\
 \underline{-75} \\
 120 \\
 \underline{-120} \\
 0
 \end{array}$$

$$87 \div (-0.0015) = -58,000$$

$$65. -2.4 \div (-100) = \frac{-2.4}{-100} = 0.024$$

$$69. z \div y = 4.52 \div (-0.8)$$

$$\begin{array}{r}
 5.65 \\
 0.8 \overline{) 4.52} \text{ becomes } 8 \overline{) 45.20} \\
 \underline{-40} \\
 52 \\
 \underline{-48} \\
 40 \\
 \underline{-40} \\
 0
 \end{array}$$

$$z \div y = 4.52 \div (-0.8) = -5.65$$

$$73. \quad \frac{x}{4} = 3.04$$

$$\frac{12.16}{4} \stackrel{?}{=} 3.04$$

$$3.04 = 3.04 \quad \text{True}$$

Yes, 12.16 is a solution.

77. Divide the square feet by the square feet per quart.

$$\begin{array}{r}
 10.5 \approx 11 \\
 52 \overline{) 546.0} \\
 \underline{-52} \\
 26 \\
 \underline{-0} \\
 260 \\
 \underline{-260} \\
 0
 \end{array}$$

Since only whole quarts are sold, 11 quarts are needed.

81. Divide the number of crayons by 64.

11.40 rounded to the nearest tenth is 11.4 boxes.

$$\begin{array}{r}
 64 \overline{) 730.00} \\
 \underline{-64} \\
 90 \\
 \underline{-64} \\
 260 \\
 \underline{-256} \\
 40
 \end{array}$$

85. From Exercise 83, we know that there are 24 teaspoons in 4 fluid ounces. Thus, there are 48 half teaspoons (0.5 tsp) or doses in 4 fluid ounces. To see how long the medicine will last, if a dose is taken every 4 hours, there are $24 \div 4 = 6$ doses taken per day. $48 \text{ (doses)} \div 6 \text{ (per day)} = 8 \text{ days}$. The medicine will last 8 days.

89. Divide the number of books sold by the number of hours.

$$\begin{array}{r}
 345.5 \\
 24 \overline{) 8292.0} \\
 \underline{-72} \\
 109 \\
 \underline{-96} \\
 132 \\
 \underline{-120} \\
 120 \\
 \underline{-120} \\
 0
 \end{array}$$

There were 345.5 thousand books sold per hour.

$$93. \quad \frac{3}{5} - \frac{7}{10} = \frac{3}{5} \cdot \frac{2}{2} - \frac{7}{10} = \frac{6}{10} - \frac{7}{10} = -\frac{1}{10}$$

$$97. \quad 1.278$$

$$\begin{array}{r}
 +0.300 \\
 1.578
 \end{array}$$

$$101. \quad 1000.00$$

$$\begin{array}{r}
 -95.71 \\
 904.29
 \end{array}$$

105. $78.6 \div 97$ is approximately $78.6 \div 100 = 0.786$, which is choice b.

109. Area = (length)(width)

$$\begin{array}{r}
 8.6 \\
 4.5 \overline{) 38.7} \text{ becomes } 45 \overline{) 387.0} \\
 \underline{-360} \\
 270 \\
 \underline{-270} \\
 0
 \end{array}$$

The length is 8.6 feet.

$$113. \quad 1.15 \overline{) 75} \text{ becomes } 115 \overline{) 7500.00} \approx 65.2$$

$$\begin{array}{r}
 65.21 \\
 \underline{-690} \\
 600 \\
 \underline{-575} \\
 250 \\
 \underline{-230} \\
 200 \\
 \underline{-115} \\
 85
 \end{array}$$

$$1.15\overline{)95} \text{ becomes } 115\overline{)9500.00} \approx 82.6$$

$$\begin{array}{r} -920 \\ 300 \\ -230 \\ 700 \\ -690 \\ 100 \\ -0 \\ 100 \end{array}$$

The range of wind speeds is 65.2–82.6 knots.

Exercise Set 5.5

$$1. \quad 5\overline{)1.0} \quad \begin{array}{r} 0.2 \\ -1.0 \\ 0 \end{array}$$

$$\frac{1}{5} = 0.2$$

$$5. \quad 4\overline{)3.00} \quad \begin{array}{r} 0.75 \\ -28 \\ 20 \\ -20 \\ 0 \end{array}$$

$$\frac{3}{4} = 0.75$$

$$9. \quad 4\overline{)9.00} \quad \begin{array}{r} 2.25 \\ -8 \\ 10 \\ -8 \\ 20 \\ -20 \\ 0 \end{array}$$

$$\frac{9}{4} = 2.25$$

$$13. \quad 40\overline{)17.000} \quad \begin{array}{r} 0.425 \\ -160 \\ 100 \\ -80 \\ 200 \\ -200 \\ 0 \end{array}$$

$$\frac{17}{40} = 0.425$$

$$17. \quad 3\overline{)1.000} \quad \begin{array}{r} 0.333... \\ -9 \\ 10 \\ -9 \\ 10 \\ -9 \\ 1 \end{array}$$

$$-\frac{1}{3} = -0.\overline{3}$$

$$21. \quad 11\overline{)7.000000} \quad \begin{array}{r} 0.636363... \\ -66 \\ 40 \\ -33 \\ 70 \\ -66 \\ 40 \\ -33 \\ 70 \\ -66 \\ 40 \\ -33 \\ 7 \end{array}$$

$$\frac{7}{11} = 0.\overline{63}$$

$$25. \quad 125\overline{)78.000} \quad \begin{array}{r} 0.624 \\ -750 \\ 300 \\ -250 \\ 500 \\ -500 \\ 0 \end{array}$$

$$\frac{78}{125} = 0.624$$

$$29. \quad \frac{7}{16} = 0.4375 \approx 0.44$$

$$33. \quad 91\overline{)56.000} \quad \begin{array}{r} 0.615 \approx 0.62 \\ -546 \\ 140 \\ -91 \\ 490 \\ -455 \\ 35 \end{array}$$

$$37. \quad 50\overline{)1.00} \quad \begin{array}{r} 0.02 \\ -100 \\ 0 \end{array}$$

$$41. \quad 200\overline{)43.000} \quad \begin{array}{r} 0.215 \\ -400 \\ 300 \\ -200 \\ 1000 \\ -1000 \\ 0 \end{array}$$

$$0.215 = \frac{43}{200}$$

$$45. \quad 6\overline{)5.000} \quad \begin{array}{r} 0.833... \\ -48 \\ 20 \\ -18 \\ 20 \\ -18 \\ 2 \end{array}$$

$$\frac{5}{6} = 0.8\overline{3} \text{ and } 0.\overline{6} < 0.8\overline{3}, \text{ so } 0.\overline{6} < \frac{5}{6}$$

$$49. \quad 7\overline{)4.000} \quad \begin{array}{r} 0.571 \\ -35 \\ 50 \\ -49 \\ 10 \\ -7 \\ 3 \end{array}$$

$$\frac{4}{7} \approx 0.57 \text{ and } 0.57 > 0.14, \text{ so } \frac{4}{7} > 0.14.$$

$$53. \quad 64\overline{)456.000} \quad \begin{array}{r} 7.125 \\ -448 \\ 80 \\ -64 \\ 160 \\ -128 \\ 320 \\ -320 \\ 0 \end{array}$$

$$\frac{456}{64} = 7.125 \text{ and } 7.123 < 7.125, \text{ so } 7.123 < \frac{456}{64}.$$

57. $0.49 = 0.490$
 $0.49, 0.491, 0.498$

61. $\frac{5}{8} = 0.625$
 $0.612 = 0.612$
 $0.649 = 0.649$
 $0.612, \frac{5}{8}, 0.649$

65. $\frac{1 + 0.8}{-0.6} = \frac{1.8}{-0.6} = \frac{18}{-6} = -3$

69. $(5.6 - 2.3)(2.4 + 0.4) = (3.3)(2.8) = 9.24$

73. $\frac{7 + 0.74}{-6} = \frac{7.74}{-6} = -1.29$

77. $\frac{1}{4}(-9.6 - 5.2) = \frac{1}{4}(-14.8) = -3.7$

81. $\text{Area} = l \cdot w = (0.62)\left(\frac{2}{5}\right) = (0.62)(0.4) = 0.248$

The area is 0.248 square yard.

85. $x - y = 6 - 0.3 = 5.7$

89. $\frac{9}{10} + \frac{16}{25} = \frac{9}{10} \cdot \frac{5}{5} + \frac{16}{25} \cdot \frac{2}{2} = \frac{45}{50} + \frac{32}{50} = \frac{77}{50}$

93. $1.0 = 1$

97. $\frac{99}{100} = 0.99$

$\frac{99}{100} < 1$

101.	2291	rounds to	2300
	1915	rounds to	1900
	1309	rounds to	1300
	1186	rounds to	1200
	924	rounds to	900
	<u>+ 582</u>	rounds to	<u>+ 600</u>
			8200

There are approximately 8200 stations with the top six formats.

Exercise Set 5.6

1. $x + 1.2 = 7.1$

$x + 1.2 - 1.2 = 7.1 - 1.2$

$x = 5.9$

5. $6.2 = y - 4$

$6.2 + 4 = y - 4 + 4$

$10.2 = y$

9. $-3.5x + 2.8 = -11.2$

$-3.5x + 2.8 - 2.8 = -11.2 - 2.8$

$-3.5x = -14$

$\frac{-3.5x}{-3.5} = \frac{-14}{-3.5}$

$x = 4$

13. $2(x - 1.3) = 5.8$

$2x - 2.6 = 5.8$

$2x - 2.6 + 2.6 = 5.8 + 2.6$

$2x = 8.4$

$\frac{2x}{2} = \frac{8.4}{2}$

$x = 4.2$

17. Multiply both sides of the equation by 10.

$7x - 10.8 = x$

$10(7x - 10.8) = 10(x)$

$10(7x) - 10(10.8) = 10x$

$70x - 108 = 10x$

$70x - 108 - 70x = 10x - 70x$

$-108 = -60x$

$\frac{-108}{-60} = \frac{-60x}{-60}$

$1.8 = x$

21. $y - 3.6 = 4$

$y - 3.6 + 3.6 = 4 + 3.6$

$y = 7.6$

25. $6.5 = 10x + 7.2$

$6.5 - 7.2 = 10x + 7.2 - 7.2$

$-0.7 = 10x$

$\frac{-0.7}{10} = \frac{10x}{10}$

$-0.07 = x$

29. $200x - 0.67 = 100x + 0.81$

$200x - 0.67 + 0.67 = 100x + 0.81 + 0.67$

$200x = 100x + 1.48$

$200x - 100x = 100x - 100x + 1.48$

$100x = 1.48$

$\frac{100x}{100} = \frac{1.48}{100}$

$x = 0.0148$

33. $8x - 5 = 10x - 8$

$8x - 5 + 8 = 10x - 8 + 8$

$8x + 3 = 10x$

$8x + 3 - 8x = 10x - 8x$

$3 = 2x$

$\frac{3}{2} = \frac{2x}{2}$

$1.5 = x$

37. $-0.9x + 2.65 = -0.5x + 5.45$

Multiply each term by 100.

$-90x + 265 = -50x + 545$

$-90x + 265 + 90x = -50x + 545 + 90x$

$265 = 40x + 545$

$265 - 545 = 40x + 545 - 545$

$-280 = 40x$

$\frac{-280}{40} = \frac{40x}{40}$

$-7 = x$

41. $0.7x + 13.8 = x - 2.16$

Multiply each term by 100.

$70x + 1380 = 100x - 216$

$70x + 1380 + 216 = 100x - 216 + 216$

$70x + 1596 = 100x$

$70x + 1596 - 70x = 100x - 70x$

$1596 = 30x$

$\frac{1596}{30} = \frac{30x}{30}$

$53.2 = x$

$$45. \frac{6x}{5} \cdot \frac{1}{2x^2} = \frac{6x \cdot 1}{5 \cdot 2x^2} = \frac{2 \cdot 3 \cdot x}{5 \cdot 2 \cdot x \cdot x} = \frac{3}{5x}$$

$$49. \quad b + 4.6 = 8.3 \\ b + 4.6 - 4.6 = 8.3 - 4.6 \\ b = 3.7$$

$$53. \quad 5y - 1.2 - 7y + 8 = 5y - 7y - 1.2 + 8 \\ = -2y + 6.8$$

$$57. \quad 4.7x + 8.3 = -5.8 \\ 4.7x + 8.3 - 8.3 = -5.8 - 8.3 \\ 4.7x = -14.1 \\ \frac{4.7x}{4.7} = \frac{-14.1}{4.7} \\ x = -3$$

$$61. \quad 5(x - 3.14) = 4x \\ 5 \cdot x - 5 \cdot 3.14 = 4x \\ 5x - 15.7 = 4x \\ 5x - 5x - 15.7 = 4x - 5x \\ -15.7 = -x \\ 15.7 = x$$

$$65. \quad 9.6z - 3.2 - 11.7z - 6.9 = 9.6z - 11.7z - 3.2 - 6.9 \\ = -2.1z - 10.1$$

69. answers may vary

$$73. \quad 1.95y + 6.834 = 7.65y - 19.8591 \\ 1.95y + 6.834 - 6.834 = 7.65y - 19.8591 - 6.834 \\ 1.95y = 7.65y - 26.6931 \\ 1.95y - 7.65y = 7.65y - 7.65y - 26.6931 \\ -5.7y = -26.6931 \\ \frac{-5.7y}{-5.7} = \frac{-26.6931}{-5.7} \\ y = 4.683$$

Exercise Set 5.7

$$1. \text{ Mean: } \frac{15 + 23 + 24 + 18 + 25}{5} = \frac{105}{5} = 21$$

Median: Write the numbers in order: 15, 18, 23, 24, 25

The middle number is 23.

Mode: There is no mode, since each number occurs once.

$$5. \text{ Mean: } \frac{0.5 + 0.2 + 0.2 + 0.6 + 0.3 + 1.3 + 0.8 + 0.1 + 0.5}{9} \\ = \frac{4.5}{9} \\ = 0.5$$

Median: Write the numbers in order:

0.1, 0.2, 0.2, 0.3, 0.5, 0.5, 0.6, 0.8, 1.3

The middle number is 0.5.

Mode: Since 0.2 and 0.5 occur twice, there are two modes, 0.2 and 0.5.

$$9. \text{ Mean: } \frac{2717 + 2073 + 1972 + 1965 + 1819}{5} = \frac{10,546}{5} = 2109.2$$

The mean height of the five tallest buildings is 2109.2 feet.

13. answers may vary

$$17. \text{ GPA} = \frac{4 \cdot 3 + 4 \cdot 3 + 4 \cdot 4 + 3 \cdot 3 + 2 \cdot 1}{3 + 3 + 4 + 3 + 1} \\ = \frac{51}{14} \\ \approx 3.64$$

21. Mode: 6.9 since this number appears twice.

$$25. \text{ Mean: } \frac{\text{sum of 15 pulse rates}}{15} = \frac{1095}{15} = 73$$

29. There are 9 rates lower than the mean. They are 66, 68, 71, 64, 71, 70, 65, 70, and 72.

$$33. \quad \frac{18}{30y} = \frac{3 \cdot 6}{5 \cdot 6 \cdot y} = \frac{3}{5y}$$

37. Since the mode is 35, 35 must occur at least twice in the set. Since there is an odd number of numbers in the set, the median, 37, is in the set.

Let n be the remaining unknown number.

$$\text{Mean: } \frac{35 + 35 + 37 + 40 + n}{5} = 38$$

$$\frac{147 + n}{5} = 38$$

$$5 \cdot \frac{147 + n}{5} = 5 \cdot 38$$

$$147 + n = 190$$

$$147 - 147 + n = 190 - 147$$

$$n = 43$$

The missing numbers are 35, 35, 37, and 43.

Chapter 5 Test

1. 45.092 in words is forty-five and ninety-two thousandths.

5. Subtract the absolute values.

$$30.25$$

$$- 9.83$$

$$20.42$$

Attach the sign of the larger absolute value.

$$9.83 - 30.25 = -20.42$$

9. 0.8623 rounded to the nearest thousandth is 0.862.

$$13. \quad -24.73 = -24\frac{73}{100}$$

$$17. \quad \frac{0.23 + 1.63}{-0.3} = \frac{1.86}{-0.3} = -6.2$$

$$21. \text{ Mean: } \frac{26 + 32 + 42 + 43 + 49}{5} = \frac{192}{5} = 38.4$$

Median: The numbers are listed in order. The middle number is 42.

Mode: There is no mode since each number occurs only once.

$$25. \text{ Area} = \frac{1}{2}(4.2 \text{ miles})(1.1 \text{ miles}) \\ = 0.5(4.2)(1.1) \text{ square miles} \\ = 2.31 \text{ square miles}$$

Chapter 6

Exercise Set 6.1

1. The ratio of 16 to 24 is $\frac{16}{24} = \frac{8 \cdot 2}{8 \cdot 3} = \frac{2}{3}$.
5. The ratio of 4.63 to 8.21 is $\frac{4.63}{8.21} = \frac{4.63 \cdot 100}{8.21 \cdot 100} = \frac{463}{821}$.
9. The ratio of \$32 to \$100 is $\frac{32}{100} = \frac{4 \cdot 8}{4 \cdot 25} = \frac{8}{25}$.
13. The ratio of $3\frac{1}{2}$ to $12\frac{1}{4}$ is

$$\begin{aligned}\frac{3\frac{1}{2}}{12\frac{1}{4}} &= 3\frac{1}{2} \div 12\frac{1}{4} \\ &= \frac{7}{2} \div \frac{49}{4} \\ &= \frac{7}{2} \cdot \frac{4}{49} \\ &= \frac{7 \cdot 2 \cdot 2}{2 \cdot 7 \cdot 7} \\ &= \frac{2}{7}\end{aligned}$$

17. The ratio of the average Fin Whale weight to the average Blue Whale weight is

$$\frac{50 \text{ tons}}{145 \text{ tons}} = \frac{5 \cdot 10}{5 \cdot 29} = \frac{10}{29}.$$

21. The ratio of women to men is

$$\frac{125}{100} = \frac{5 \cdot 25}{4 \cdot 25} = \frac{5}{4}.$$

25. The ratio of the miniDVD diameter to the standard DVD diameter is

$$\frac{8 \text{ centimeters}}{12 \text{ centimeters}} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{2}{3}.$$

29. Perimeter = $8 + 15 + 17 = 40$

The ratio of the longest side to the perimeter is

$$\frac{17 \text{ feet}}{40 \text{ feet}} = \frac{17}{40}.$$

33. The rate of 5 shrubs every 15 feet is

$$\frac{5 \text{ shrubs}}{15 \text{ feet}} = \frac{1 \text{ shrub}}{3 \text{ feet}}.$$

37. The rate of 6 laser printers for 28 computers is

$$\frac{6 \text{ laser printers}}{28 \text{ computers}} = \frac{3 \text{ laser printers}}{14 \text{ computers}}.$$

41. $3\overline{)330}$

$$\begin{array}{r} 110 \\ 3\overline{)330} \\ \underline{-3} \\ 03 \\ \underline{-3} \\ 0 \end{array}$$

330 calories in a 3-ounce serving is $\frac{110 \text{ calories}}{1 \text{ ounce}}$ or 110 calories/ounce.

$$\begin{array}{r} 90 \\ 60\overline{)5400} \\ \underline{-540} \\ 0 \end{array}$$

5400 wingbeats per 60 seconds is $\frac{90 \text{ wingbeats}}{1 \text{ second}}$ or 90 wingbeats/second.

$$\begin{array}{r} 1,655,975 \\ 2\overline{)3,311,950} \\ \underline{-2} \\ 13 \\ \underline{-12} \\ 11 \\ \underline{-10} \\ 11 \\ \underline{-10} \\ 19 \\ \underline{-18} \\ 15 \\ \underline{-14} \\ 10 \\ \underline{-10} \\ 0 \end{array}$$

3,311,950 voters for 2 senators is $\frac{1,655,975 \text{ voters}}{1 \text{ senator}}$ or 1,655,975 voters/senator.

$$\begin{array}{r} 4,517,558.7 \\ 15\overline{)67,763,381.0} \\ \underline{-60} \\ 77 \\ \underline{-75} \\ 26 \\ \underline{-15} \\ 113 \\ \underline{-105} \\ 83 \\ \underline{-75} \\ 88 \\ \underline{-75} \\ 131 \\ \underline{-120} \\ 110 \\ \underline{-105} \\ 5 \end{array}$$

\$67,763,281 for 15 players is about $\frac{\$4,517,559}{1 \text{ player}}$ or \$4,517,559/player.

57. a. $14.5\overline{)400}$ becomes $145\overline{)4000.00}$

$$\begin{array}{r} 27.58 \approx 27.6 \\ 145\overline{)4000.00} \\ \underline{-290} \\ 1100 \\ \underline{-1015} \\ 850 \\ \underline{-725} \\ 1250 \\ \underline{-1160} \\ 90 \end{array}$$

The unit rate for the car is ≈ 27.6 miles/gallon.

$$\text{b. } 9.25 \overline{)270} \text{ becomes } 925 \overline{)27000.00} \quad \frac{29.18}{9.25} \approx 29.2$$

$$\begin{array}{r} -1850 \\ 8500 \\ -8325 \\ \hline 1750 \\ -925 \\ \hline 8250 \\ -7400 \\ \hline 850 \end{array}$$

The unit rate for the truck is ≈ 29.2 miles/gallon.

c. From parts **a** and **b**, we see that the truck gets better gas mileage.

$$\text{61. } 7 \overline{)1.19} \quad \frac{0.17}{7 \overline{)1.19}}$$

$$\begin{array}{r} -7 \\ \hline 49 \\ -49 \\ \hline 0 \end{array}$$

The unit price is \$0.17 per banana.

$$\text{65. } 16 \overline{)1.8900} \quad \frac{0.1181}{16 \overline{)1.8900}} \approx 0.118$$

$$\begin{array}{r} -16 \\ 29 \\ -16 \\ \hline 130 \\ -128 \\ \hline 20 \\ -16 \\ \hline 4 \end{array}$$

The 16-ounce size costs \$0.118 per ounce.

$$\frac{0.115}{6 \overline{)0.690}}$$

$$\begin{array}{r} -6 \\ 9 \\ -6 \\ \hline 30 \\ -30 \\ \hline 0 \end{array}$$

The 6-ounce size costs \$0.115 per ounce.

The 6-ounce size is the better buy.

$$\text{69. } 100 \overline{)0.5900} \quad \frac{0.0059}{100 \overline{)0.5900}} \approx 0.006$$

$$\begin{array}{r} -500 \\ 900 \\ -900 \\ \hline 0 \end{array}$$

The 100-count size costs \$0.006 per napkin.

$$\frac{0.0051}{180 \overline{)0.9300}} \approx 0.005$$

$$\begin{array}{r} -900 \\ 300 \\ -180 \\ \hline 120 \end{array}$$

The 180-count size costs \$0.005 per napkin.

The 180-count size is the better buy.

$$\text{73. } 3.7 \overline{)0.555} \text{ becomes } 37 \overline{)5.55} \quad \frac{0.15}{3.7 \overline{)0.555}}$$

$$\begin{array}{r} -37 \\ 185 \\ -185 \\ \hline 0 \end{array}$$

$$\text{77. no; } \frac{6 \text{ inches}}{15 \text{ inches}} = \frac{2 \cdot 3}{5 \cdot 3} = \frac{2}{5}$$

$$\text{81. } 29,543 - 29,286 = 257$$

$$13.4 \overline{)257} \text{ becomes } 134 \overline{)2570.00} \quad \frac{19.17}{13.4 \overline{)257}} \approx 19.2$$

$$\begin{array}{r} -134 \\ 1230 \\ -1206 \\ \hline 240 \\ -134 \\ \hline 1060 \\ -938 \\ \hline 122 \end{array}$$

There were 257 miles driven and the miles per gallon was 19.2.

$$\text{85. } 7759 \overline{)11,674.00} \quad \frac{1.50}{7759 \overline{)11,674.00}} \approx 1.5$$

$$\begin{array}{r} -7759 \\ 39150 \\ -38795 \\ \hline 3550 \end{array}$$

The unit rate is 1.5 steps/foot.

89. answers may vary

Exercise Set 6.2

$$\text{1. } \frac{10 \text{ diamonds}}{6 \text{ opals}} = \frac{5 \text{ diamonds}}{3 \text{ opals}}$$

$$\text{5. } \frac{6 \text{ eagles}}{58 \text{ sparrows}} = \frac{3 \text{ eagles}}{29 \text{ sparrows}}$$

$$\text{9. } \frac{22 \text{ vanilla wafers}}{1 \text{ cup cookie crumbs}} = \frac{55 \text{ vanilla wafers}}{2.5 \text{ cups cookie crumbs}}$$

$$\text{13. } \frac{5}{8} \stackrel{?}{=} \frac{4}{7}$$

$$5 \cdot 7 \stackrel{?}{=} 8 \cdot 4$$

$$35 \neq 32$$

The proportion is false.

$$\text{17. } \frac{5}{8} \stackrel{?}{=} \frac{625}{1000}$$

$$5 \cdot 1000 \stackrel{?}{=} 8 \cdot 625$$

$$5000 = 5000$$

The proportion is true.

$$\text{21. } \frac{4.2}{8.4} \stackrel{?}{=} \frac{5}{10}$$

$$4.2 \cdot 10 \stackrel{?}{=} 8.4 \cdot 5$$

$$42 = 42$$

The proportion is true.

$$\begin{aligned}
 25. \quad & \frac{2\frac{2}{5}}{\frac{2}{3}} \stackrel{?}{=} \frac{\frac{10}{9}}{\frac{1}{4}} \\
 & 2\frac{2}{5} \cdot \frac{1}{4} \stackrel{?}{=} \frac{2}{3} \cdot \frac{10}{9} \\
 & \frac{12}{5} \cdot \frac{1}{4} \stackrel{?}{=} \frac{20}{27} \\
 & \frac{12}{20} \neq \frac{20}{27}
 \end{aligned}$$

The proportion is false.

$$\begin{aligned}
 29. \quad & \frac{10}{15} \stackrel{?}{=} \frac{4}{6} \\
 & 10 \cdot 6 \stackrel{?}{=} 15 \cdot 4 \\
 & 60 = 60
 \end{aligned}$$

The proportion $\frac{10}{15} = \frac{4}{6}$ is true.

$$\begin{aligned}
 33. \quad & \frac{0.15}{3} \stackrel{?}{=} \frac{0.35}{7} \\
 & 0.15 \cdot 7 \stackrel{?}{=} 3 \cdot 0.35 \\
 & 1.05 = 1.05
 \end{aligned}$$

The proportion $\frac{0.15}{3} = \frac{0.35}{7}$ is true.

$$\begin{aligned}
 37. \quad & \frac{x}{5} = \frac{6}{10} \\
 & x \cdot 10 = 5 \cdot 6 \\
 & 10x = 30 \\
 & \frac{10x}{10} = \frac{30}{10} \\
 & x = 3
 \end{aligned}$$

$$\begin{aligned}
 41. \quad & \frac{30}{10} = \frac{15}{y} \\
 & 30 \cdot y = 10 \cdot 15 \\
 & 30y = 150 \\
 & \frac{30y}{30} = \frac{150}{30} \\
 & y = 5
 \end{aligned}$$

$$\begin{aligned}
 45. \quad & \frac{24}{x} = \frac{60}{96} \\
 & 24 \cdot 96 = x \cdot 60 \\
 & 2304 = 60x \\
 & \frac{2304}{60} = \frac{60x}{60} \\
 & 38.4 = x
 \end{aligned}$$

$$\begin{aligned}
 49. \quad & \frac{n}{0.6} = \frac{0.05}{12} \\
 & n \cdot 12 = 0.6 \cdot 0.05 \\
 & 12n = 0.030 \\
 & \frac{12n}{12} = \frac{0.03}{12} \\
 & n = 0.0025
 \end{aligned}$$

$$\begin{aligned}
 53. \quad & \frac{\frac{1}{3}}{\frac{3}{8}} = \frac{\frac{2}{5}}{n} \\
 & \frac{1}{3} \cdot n = \frac{3}{8} \cdot \frac{2}{5} \\
 & \frac{n}{3} = \frac{3}{20} \\
 & 3 \cdot \frac{n}{3} = 3 \cdot \frac{3}{20} \\
 & n = \frac{9}{20}
 \end{aligned}$$

$$\begin{aligned}
 57. \quad & \frac{n}{1\frac{1}{5}} = \frac{4\frac{1}{6}}{6\frac{2}{3}} \\
 & n \cdot 6\frac{2}{3} = 1\frac{1}{5} \cdot 4\frac{1}{6} \\
 & n \cdot \frac{20}{3} = \frac{6}{5} \cdot \frac{25}{6} \\
 & \frac{20}{3}n = 5 \\
 & \frac{3}{20} \cdot \frac{20}{3}n = \frac{3}{20} \cdot 5 \\
 & n = \frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 61. \quad & \frac{3.2}{0.3} = \frac{x}{1.4} \\
 & 3.2 \cdot 1.4 = 0.3 \cdot x \\
 & 4.48 = 0.3x \\
 & \frac{4.48}{0.3} = \frac{0.3x}{0.3} \\
 & 14.9 \approx x
 \end{aligned}$$

$$\begin{aligned}
 65. \quad & \frac{7}{18} = \frac{x}{5} \\
 & 7 \cdot 5 = 18 \cdot x \\
 & 35 = 18x \\
 & \frac{35}{18} = \frac{18x}{18} \\
 & 1.9 \approx x
 \end{aligned}$$

$$\begin{aligned}
 69. \quad & \begin{array}{cc} 8.01 & 8.1 \\ \uparrow & \uparrow \\ 0 & < & 1 \end{array} \\
 & 8.01 < 8.1
 \end{aligned}$$

$$73. \quad \frac{75}{125} = \frac{3 \cdot 25}{5 \cdot 25} = \frac{3}{5}$$

$$\begin{aligned}
 77. \quad & \frac{9}{15} = \frac{3}{5} \\
 & \frac{9}{3} = \frac{15}{5} \\
 & \frac{5}{15} = \frac{3}{9} \\
 & \frac{15}{9} = \frac{5}{3}
 \end{aligned}$$

$$81. \frac{a}{b} = \frac{c}{d}$$

Possible answers include:

$$\frac{d}{b} = \frac{c}{a}$$

$$\frac{a}{c} = \frac{b}{d}$$

$$\frac{b}{a} = \frac{d}{c}$$

$$85. \frac{x}{7} = \frac{0}{8}$$

$$x \cdot 8 = 7 \cdot 0$$

$$8x = 0$$

$$\frac{8x}{8} = \frac{0}{8}$$

$$x = 0$$

$$89. \frac{222}{1515} = \frac{37}{y}$$

$$222 \cdot y = 1515 \cdot 37$$

$$222y = 56,055$$

$$\frac{222y}{222} = \frac{56,055}{222}$$

$$y = 252.5$$

Exercise Set 6.3

1. Let x be the number of field goals (baskets) made.

$$\text{baskets} \rightarrow \frac{45}{100} = \frac{x}{800} \leftarrow \text{baskets}$$

$$\text{attempts} \rightarrow \frac{45}{100} = \frac{x}{800} \leftarrow \text{attempts}$$

$$45 \cdot 800 = 100 \cdot x$$

$$36,000 = 100x$$

$$\frac{36,000}{100} = \frac{100x}{100}$$

$$360 = x$$

He made 360 baskets.

5. Let x be the number of applications received.

$$\text{accepted} \rightarrow \frac{2}{7} = \frac{180}{x} \leftarrow \text{accepted}$$

$$\text{applied} \rightarrow \frac{2}{7} = \frac{180}{x} \leftarrow \text{applied}$$

$$2 \cdot x = 7 \cdot 180$$

$$2x = 1260$$

$$\frac{2x}{2} = \frac{1260}{2}$$

$$x = 630$$

The school received 630 applications.

9. Let x be the number of square feet required.

$$\text{floor space} \rightarrow \frac{9}{1} = \frac{x}{30} \leftarrow \text{floor space}$$

$$\text{students} \rightarrow \frac{9}{1} = \frac{x}{30} \leftarrow \text{students}$$

$$9 \cdot 30 = 1 \cdot x$$

$$270 = x$$

30 students require 270 square feet of floor space.

13. Let x be the distance between Milan and Rome.

$$\text{kilometers} \rightarrow \frac{30}{1} = \frac{x}{15} \leftarrow \text{kilometers}$$

$$\text{cm on map} \rightarrow \frac{30}{1} = \frac{x}{15} \leftarrow \text{cm on map}$$

$$30 \cdot 15 = 1 \cdot x$$

$$450 = x$$

Milan and Rome are 450 kilometers apart.

17. Let x be the number of hits the player is expected to get.

$$\text{hits} \rightarrow \frac{3}{8} = \frac{x}{40} \leftarrow \text{hits}$$

$$\text{at bats} \rightarrow \frac{3}{8} = \frac{x}{40} \leftarrow \text{at bats}$$

$$3 \cdot 40 = 8 \cdot x$$

$$120 = 8x$$

$$\frac{120}{8} = \frac{8x}{8}$$

$$15 = x$$

The player would be expected to get 15 hits.

21. Let x be the number of applications she should expect.

$$\text{applications} \rightarrow \frac{4}{3} = \frac{x}{14} \leftarrow \text{applications}$$

$$\text{ounces} \rightarrow \frac{4}{3} = \frac{x}{14} \leftarrow \text{ounces}$$

$$4 \cdot 14 = 3 \cdot x$$

$$56 = 3x$$

$$\frac{56}{3} = \frac{3x}{3}$$

$$18\frac{2}{3} = x$$

She should expect 18 applications from the 14-ounce bottle.

25. Let x be the number of servings he can make.

$$\text{milk} \rightarrow \frac{1\frac{1}{2}}{4} = \frac{4}{x} \leftarrow \text{milk}$$

$$\text{servings} \rightarrow \frac{1\frac{1}{2}}{4} = \frac{4}{x} \leftarrow \text{servings}$$

$$1\frac{1}{2} \cdot x = 4 \cdot 4$$

$$\frac{3}{2}x = 16$$

$$\frac{2}{3} \cdot \frac{3}{2}x = \frac{2}{3} \cdot 16$$

$$x = \frac{32}{3} = 10\frac{2}{3}$$

He can make $10\frac{2}{3}$ servings.

29. a. Let x be the number of teaspoons of granules needed.

$$\text{water} \rightarrow \frac{25}{1} = \frac{450}{x} \leftarrow \text{water}$$

$$\text{granules} \rightarrow \frac{25}{1} = \frac{450}{x} \leftarrow \text{granules}$$

$$25 \cdot x = 1 \cdot 450$$

$$25x = 450$$

$$\frac{25x}{25} = \frac{450}{25}$$

$$x = 18$$

18 teaspoons of granules are needed.

- b. Let x be the number of tablespoons of granules needed.

$$\text{tsp} \rightarrow \frac{3}{1} = \frac{18}{x} \leftarrow \text{tsp}$$

$$\text{tbsp} \rightarrow \frac{3}{1} = \frac{18}{x} \leftarrow \text{tbsp}$$

$$3 \cdot x = 1 \cdot 18$$

$$3x = 18$$

$$\frac{3x}{3} = \frac{18}{3}$$

$$x = 6$$

6 tablespoons of granules are needed.

33. Let x be the estimated head-to-toe height of the Statue of Liberty.

$$\begin{array}{lcl} \text{height} & \rightarrow \frac{x}{42} = \frac{5\frac{1}{3}}{2} \leftarrow & \text{height} \\ \text{arm length} & \rightarrow & \text{arm length} \end{array}$$

$$x \cdot 2 = 42 \cdot 5\frac{1}{3}$$

$$2x = 42 \cdot \frac{16}{3}$$

$$2x = 224$$

$$\frac{2x}{2} = \frac{224}{2}$$

$$x = 112$$

The estimated height is 112 feet.

$$112 - 111\frac{1}{12} = \frac{11}{12}$$

The difference is $\frac{11}{12}$ foot or 11 inches.

37. Let x be the estimated height of the Empire State Building.

$$\begin{array}{lcl} \text{height} & \rightarrow \frac{x}{102} = \frac{850}{69} \leftarrow & \text{height} \\ \text{stories} & \rightarrow & \text{stories} \end{array}$$

$$x \cdot 69 = 102 \cdot 850$$

$$69x = 86,700$$

$$\frac{69x}{69} = \frac{86,700}{69}$$

$$x \approx 1257$$

The height of the Empire State Building is approximately 1257 feet.

41. Let x be the number expected to have gotten their first job in the restaurant industry.

$$\begin{array}{lcl} \text{restaurant} & \rightarrow \frac{x}{84} = \frac{1}{3} \leftarrow & \text{restaurant} \\ \text{workers} & \rightarrow & \text{workers} \end{array}$$

$$x \cdot 3 = 84 \cdot 1$$

$$3x = 84$$

$$\frac{3x}{3} = \frac{84}{3}$$

$$x = 28$$

You would expect 28 of the workers to have gotten their first job in the restaurant industry.

45. a. Let x be the number of gallons of oil needed.

$$\begin{array}{lcl} \text{oil} & \rightarrow \frac{x}{5} = \frac{1}{50} \leftarrow & \text{oil} \\ \text{gas} & \rightarrow & \text{gas} \end{array}$$

$$x \cdot 50 = 5 \cdot 1$$

$$50x = 5$$

$$\frac{50x}{50} = \frac{5}{50}$$

$$x = \frac{1}{10} = 0.1$$

0.1 gallon of oil is needed.

- b. 0.1 gallon = 0.1(1 gallon)

$$= 0.1(128 \text{ fluid ounces})$$

$$= 12.8 \text{ fluid ounces}$$

0.1 gallon is approximately 13 fluid ounces.

$$49. 200 = 2 \cdot 100$$

$$\begin{array}{c} \downarrow \quad \downarrow \quad \downarrow \\ = 2 \cdot 4 \cdot 25 \\ \downarrow \quad \downarrow \quad \downarrow \\ = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \\ = 2^3 \cdot 5^2 \end{array}$$

53. Let x be the number of ml.

$$\text{mg} \rightarrow \frac{15}{1} = \frac{12}{x} \leftarrow \text{mg}$$

$$\text{ml} \rightarrow \frac{1}{15} = \frac{x}{12} \leftarrow \text{ml}$$

$$15 \cdot x = 1 \cdot 12$$

$$15x = 12$$

$$\frac{15x}{15} = \frac{12}{15}$$

$$x = \frac{4}{5} = 0.8$$

0.8 ml of the medicine should be administered.

57. 11 muffins are approximately 1 dozen (12) muffins.

$$1.5 \cdot 8 = 12$$

Approximately 12 cups of milk will be needed.

61. answers may vary

Exercise Set 6.4

1. $\sqrt{4} = 2$ because $2^2 = 4$.

5. $\sqrt{\frac{1}{81}} = \frac{1}{9}$ because $\left(\frac{1}{9}\right)^2 = \frac{1}{9} \cdot \frac{1}{9} = \frac{1}{81}$.

9. $\sqrt{3} \approx 1.732$

13. $\sqrt{31} \approx 5.568$

17. Since 38 is between $36 = 6 \cdot 6$ and $49 = 7 \cdot 7$, $\sqrt{38}$ is between 6 and 7; $\sqrt{38} \approx 6.16$.

21. $\sqrt{256} = 16$ because $16^2 = 256$.

25. $\sqrt{\frac{49}{144}} = \frac{7}{12}$ because $\left(\frac{7}{12}\right)^2 = \frac{7}{12} \cdot \frac{7}{12} = \frac{49}{144}$.

29. Let $a = 5$ and $b = 12$.

$$a^2 + b^2 = c^2$$

$$5^2 + 12^2 = c^2$$

$$25 + 144 = c^2$$

$$169 = c^2$$

$$13 = c$$

The missing length is 13 inches.

33. Let $a = 22$ and $b = 48$.

$$c^2 = a^2 + b^2$$

$$c^2 = 22^2 + 48^2$$

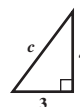
$$c^2 = 484 + 2304$$

$$c^2 = 2788$$

$$c = \sqrt{2788} \approx 52.802$$

The missing length is approximately 52.802 meters.

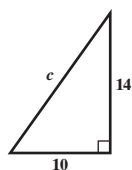
- 37.



$$\begin{aligned} \text{hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{other leg})^2} \\ &= \sqrt{(3)^2 + (4)^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

The hypotenuse has length 5 units.

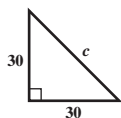
41.



$$\begin{aligned}
 \text{hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{other leg})^2} \\
 &= \sqrt{(10)^2 + (14)^2} \\
 &= \sqrt{100 + 196} \\
 &= \sqrt{296} \\
 &\approx 17.205
 \end{aligned}$$

The hypotenuse is about 17.205 units.

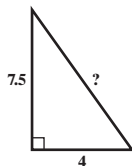
45.



$$\begin{aligned}
 \text{hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{other leg})^2} \\
 &= \sqrt{(30)^2 + (30)^2} \\
 &= \sqrt{900 + 900} \\
 &= \sqrt{1800} \\
 &\approx 42.426
 \end{aligned}$$

The hypotenuse is about 42.426 units.

49.



$$\begin{aligned}
 \text{hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{other leg})^2} \\
 &= \sqrt{(7.5)^2 + (4)^2} \\
 &= \sqrt{56.25 + 16} \\
 &= \sqrt{72.25} \\
 &= 8.5
 \end{aligned}$$

The hypotenuse has length 8.5 units.

$$\begin{aligned}
 53. \text{ leg} &= \sqrt{(\text{hypotenuse})^2 - (\text{other leg})^2} \\
 &= \sqrt{(32)^2 - (20)^2} \\
 &= \sqrt{1024 - 400} \\
 &= \sqrt{624} \\
 &\approx 25.0
 \end{aligned}$$

The tree is about 25 feet tall.

$$57. \frac{10}{12} = \frac{2 \cdot 5}{2 \cdot 6} = \frac{5}{6}$$

$$61. \frac{9}{13y} + \frac{12}{13y} = \frac{9+12}{13y} = \frac{21}{13y}$$

65. From exercise 17, $\sqrt{38}$ is between 6 and 7. Since 38 is closer to 36 than it is to 49, $\sqrt{38}$ is approximately 6. $\sqrt{38} \approx 6.16$.

69. answers may vary

73. x is the difference between the vertical (unmarked) legs of the two triangles. In the larger triangle, $a = 8$ in. and $c = 12$ in.

$$a^2 + b^2 = c^2$$

$$8^2 + b^2 = 12^2$$

$$64 + b^2 = 144$$

$$b^2 = 80$$

$$b = \sqrt{80}$$

In the smaller triangle, $a = 8$ in. and $c = 10$ in.

$$a^2 + b^2 = c^2$$

$$8^2 + b^2 = 10^2$$

$$64 + b^2 = 100$$

$$b^2 = 36$$

$$b = 6$$

Thus $x = (\sqrt{80} - 6)$ inches ≈ 2.94 inches.

Exercise Set 6.5

1. The triangles are congruent by Side-Side-Side.

5. The triangles are congruent by Angle-Side-Angle.

$$9. \frac{22}{11} = \frac{14}{7} = \frac{12}{6} = \frac{2}{1}$$

The ratio of corresponding sides is $\frac{2}{1}$.

$$13. \frac{x}{3} = \frac{9}{6}$$

$$6 \cdot x = 3 \cdot 9$$

$$6x = 27$$

$$\frac{6x}{6} = \frac{27}{6}$$

$$x = 4.5$$

$$17. \frac{y}{3.75} = \frac{12}{9}$$

$$9 \cdot y = 12 \cdot 3.75$$

$$9y = 45$$

$$\frac{9y}{9} = \frac{45}{9}$$

$$y = 5$$

$$21. \frac{x}{3.25} = \frac{17.5}{3.25}$$

$$3.25 \cdot x = 17.5 \cdot 3.25$$

$$3.25x = 56.875$$

$$\frac{3.25x}{3.25} = \frac{56.875}{3.25}$$

$$x = 17.5$$

$$25. \frac{z}{60} = \frac{15}{32}$$

$$32 \cdot z = 60 \cdot 15$$

$$32z = 900$$

$$\frac{32z}{32} = \frac{900}{32}$$

$$z = 28.125$$

$$29. \frac{x}{13} = \frac{80}{2}$$

$$2 \cdot x = 80 \cdot 13$$

$$2x = 1040$$

$$\frac{2x}{2} = \frac{1040}{2}$$

$$x = 520$$

The observation deck is 520 feet high.

$$\begin{aligned}
 33. \quad \frac{x}{18} &= \frac{24}{30} \\
 30 \cdot x &= 24 \cdot 18 \\
 30x &= 432 \\
 \frac{30x}{30} &= \frac{432}{30} \\
 x &= 14.4
 \end{aligned}$$

The shadow of the tree is 14.4 feet long.

$$\begin{aligned}
 37. \quad \text{Let } a &= 200 \text{ and } c = 430. \\
 a^2 + b^2 &= c^2 \\
 200^2 + b^2 &= 430^2 \\
 40,000 + b^2 &= 184,900 \\
 40,000 - 40,000 + b^2 &= 184,900 - 40,000 \\
 b^2 &= 144,900 \\
 b &= \sqrt{144,900} \approx 381
 \end{aligned}$$

The gantry is approximately 381 feet tall.

$$41. (0.41)(-3) = -1.23$$

$$\begin{aligned}
 45. \quad \frac{n}{5.2} &= \frac{12.6}{7.8} \\
 7.8 \cdot n &= 12.6 \cdot 5.2 \\
 7.8n &= 65.52 \\
 n &= \frac{65.52}{7.8} \\
 n &= 8.4
 \end{aligned}$$

$$\begin{aligned}
 49. \quad \frac{x}{5} &= \frac{10}{1} \\
 x &= 50 \\
 x \cdot \frac{1}{4} &= 50 \cdot \frac{1}{4} \\
 x &= 125 \\
 \frac{y}{7} &= \frac{1}{2} \\
 y &= 3.5 \\
 y \cdot \frac{1}{4} &= 3.5 \cdot \frac{1}{4} \\
 y &= 0.875 \\
 \frac{z}{10} &= \frac{1}{8} \\
 z &= 12.5 \\
 z \cdot \frac{1}{4} &= 12.5 \cdot \frac{1}{4} \\
 z &= 3.125
 \end{aligned}$$

The actual proposed dimensions are 200 feet by 300 feet by 425 feet.

Chapter 6 Test

1. The ratio 4500 trees to 6500 trees is

$$\frac{4500 \text{ trees}}{6500 \text{ trees}} = \frac{9 \cdot 500}{13 \cdot 500} = \frac{9}{13}$$

5. The ratio 590 feet to 186 feet is $\frac{590 \text{ feet}}{186 \text{ feet}} = \frac{2 \cdot 295}{2 \cdot 93} = \frac{295}{93}$.

$$\begin{aligned}
 9. \quad \frac{0.1487}{8} &\approx 0.149 \\
 8 \overline{)1.1900} \\
 \underline{-8} & \\
 39 & \\
 \underline{-32} & \\
 70 & \\
 \underline{-64} & \\
 60 & \\
 \underline{-56} & \\
 4 &
 \end{aligned}$$

The 8-ounce size costs \$0.149/ounce.

$$\begin{aligned}
 \frac{0.1575}{12} &\approx 0.158 \\
 12 \overline{)1.8900} \\
 \underline{-12} & \\
 69 & \\
 \underline{-60} & \\
 90 & \\
 \underline{-84} & \\
 60 & \\
 \underline{-60} & \\
 0 &
 \end{aligned}$$

The 12-ounce size costs \$0.158/ounce.

The 8-ounce size is the better buy.

$$\begin{aligned}
 13. \quad \frac{n}{3} &= \frac{15}{9} \\
 n \cdot 9 &= 3 \cdot 15 \\
 9n &= 45 \\
 \frac{9n}{9} &= \frac{45}{9} \\
 n &= 5
 \end{aligned}$$

17. Let x be the length of the home in feet.

$$\begin{aligned}
 \text{feet} &\rightarrow \frac{x}{11} = \frac{9}{2} \leftarrow \text{feet} \\
 \text{inches} &\rightarrow \frac{x}{11} = \frac{9}{2} \leftarrow \text{inches} \\
 x \cdot 2 &= 11 \cdot 9 \\
 2x &= 99 \\
 \frac{2x}{2} &= \frac{99}{2} \\
 x &= 49\frac{1}{2}
 \end{aligned}$$

The home is $49\frac{1}{2}$ feet long.

$$21. \sqrt{157} \approx 12.530$$

25. Let x be the height of the tower.

$$\begin{aligned}
 \frac{x}{5} &= \frac{48}{3} \\
 x &= 80 \\
 x \cdot 4 &= 80 \cdot 4 \\
 4x &= 320 \\
 \frac{4x}{4} &= \frac{320}{4} \\
 x &= 80
 \end{aligned}$$

The tower is approximately 80 feet tall.

Chapter 7

Exercise Set 7.1

1. $\frac{96}{100} = 96\%$
5. 37 of the adults preferred football, while 13 preferred soccer. Thus, $37 + 13 = 50$ preferred football or soccer.
 $\frac{50}{100} = 50\%$
9. $6\% = 6(0.01) = 0.06$
13. $73.6\% = 73.6(0.01) = 0.736$
17. $0.6\% = 0.6(0.01) = 0.006$
21. $32.58\% = 32.58(0.01) = 0.3258$
25. $4\% = 4 \cdot \frac{1}{100} = \frac{4}{100} = \frac{1 \cdot 4}{4 \cdot 25} = \frac{1}{25}$
29. $175\% = 175 \cdot \frac{1}{100} = \frac{175}{100} = \frac{7 \cdot 25}{4 \cdot 25} = \frac{7}{4}$ or $1\frac{3}{4}$
33. $10\frac{1}{3}\% = 10\frac{1}{3} \cdot \frac{1}{100} = \frac{31}{3} \cdot \frac{1}{100} = \frac{31}{300}$
37. $0.22 = 0.22(100\%) = 22\%$
41. $5.3 = 5.3(100\%) = 530\%$
45. $0.2228 = 0.2228(100\%) = 22.28\%$
49. $0.7 = 0.7(100\%) = 70\%$
53. $\frac{4}{5} = \frac{4}{5} \cdot 100\% = \frac{400}{5}\% = 80\%$
57. $\frac{3}{8} = \frac{3}{8} \cdot 100\% = \frac{300}{8}\% = \frac{75}{2}\% = 37\frac{1}{2}\%$
61. $4\frac{1}{2} = 4\frac{1}{2} \cdot 100\% = \frac{9}{2} \cdot 100\% = \frac{900}{2}\% = 450\%$
65. $\frac{9}{11} = \frac{9}{11} \cdot 100\% = \frac{900}{11}\%$

$$\frac{81.818}{11} \approx 81.82$$

$$11 \overline{)900.000}$$

$$\begin{array}{r} -88 \\ 20 \\ -11 \\ 90 \\ -88 \\ 20 \\ -11 \\ 90 \\ -88 \\ 2 \end{array}$$

$\frac{9}{11}$ is approximately 81.82%.

69.

Percent	Decimal	Fraction
60%	0.6	$\frac{3}{5}$
$23\frac{1}{2}\%$	0.235	$\frac{47}{200}$
80%	0.8	$\frac{4}{5}$
$33\frac{1}{3}\%$	0.3333	$\frac{1}{3}$
87.5%	0.875	$\frac{7}{8}$
7.5%	0.075	$\frac{3}{40}$

73. $38\% = 38(0.01) = 0.38$
 $38\% = 38 \cdot \frac{1}{100} = \frac{38}{100} = \frac{19}{50}$
77. $49.2\% = 49.2(0.01) = 0.492$
 $49.2\% = 49.2 \cdot \frac{1}{100} = \frac{49.2}{100} \cdot \frac{10}{10} = \frac{492}{1000} = \frac{123}{250}$
81. $16.2\% = 16.2(0.01) = 0.162$
 $16.2\% = 16.2 \cdot \frac{1}{100}$
 $= \frac{16.2}{100}$
 $= \frac{16.2}{100} \cdot \frac{10}{10}$
 $= \frac{162}{1000}$
 $= \frac{81}{500}$
85. $\frac{7}{1000} = \frac{7}{1000} (100\%) = \frac{700}{1000}\% = \frac{7}{10}\% = 0.7\%$
89. $\frac{3}{4} - \frac{1}{2} \cdot \frac{8}{9} = \frac{3}{4} - \frac{1 \cdot 8}{2 \cdot 9}$
 $= \frac{3}{4} - \frac{1 \cdot 2 \cdot 4}{2 \cdot 9}$
 $= \frac{3}{4} - \frac{4}{9}$
 $= \frac{3}{4} \cdot \frac{9}{9} - \frac{4}{9} \cdot \frac{4}{4}$
 $= \frac{27}{36} - \frac{16}{36}$
 $= \frac{27 - 16}{36}$
 $= \frac{11}{36}$

93. a. 52.8647% rounded to the nearest tenth percent is 52.9%.
 b. 52.8647% rounded to the nearest hundredth percent is 52.86%.
97. $45\% + 40\% + 11\% = 96\%$
 $100\% - 96\% = 4\%$
 4% of the U.S. population have AB blood type.
101. A fraction written as a percent is greater than 100% when the numerator is greater than the denominator.
105. The longest bar corresponds to occupational therapy assistants, so that is predicted to be the fastest-growing occupation.
109. answers may vary

Exercise Set 7.2

1. 18% of 81 is what number?

$$\begin{array}{c} \downarrow \downarrow \downarrow \downarrow \downarrow \\ 18\% \cdot 81 = \quad x \end{array}$$

5. 0.6 is 40% of what number?

$$\begin{array}{c} \downarrow \downarrow \downarrow \downarrow \downarrow \\ 0.6 = 40\% \cdot \quad x \end{array}$$

9. what number is 9% of 43?

$$\begin{array}{c} \downarrow \downarrow \downarrow \downarrow \downarrow \\ x = 9\% \cdot 43 \end{array}$$

$$\begin{aligned}
 13. \quad & 10\% \cdot 35 = x \\
 & 0.10 \cdot 35 = x \\
 & 3.5 = x \\
 & 10\% \text{ of } 35 \text{ is } 3.5.
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & 1.2 = 12\% \cdot x \\
 & 1.2 = 0.12 \cdot x \\
 & \frac{1.2}{0.12} = \frac{0.12x}{0.12} \\
 & 10 = x \\
 & 1.2 \text{ is } 12\% \text{ of } 10.
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & x \cdot 80 = 88 \\
 & \frac{x \cdot 80}{80} = \frac{88}{80} \\
 & x = 1.1 \\
 & x = 110\% \\
 & 88 \text{ is } 110\% \text{ of } 80.
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & 0.1 = 10\% \cdot x \\
 & 0.1 = 0.10 \cdot x \\
 & \frac{0.1}{0.1} = \frac{0.1x}{0.1} \\
 & 1 = x \\
 & 0.1 \text{ is } 10\% \text{ of } 1.
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & 82.5 = 16\frac{1}{2}\% \cdot x \\
 & 82.5 = 0.165 \cdot x \\
 & \frac{82.5}{0.165} = \frac{0.165x}{0.165} \\
 & 500 = x \\
 & 82.5 \text{ is } 16\frac{1}{2}\% \text{ of } 500.
 \end{aligned}$$

$$\begin{aligned}
 33. \quad & x = 42\% \cdot 60 \\
 & x = 0.42 \cdot 60 \\
 & x = 25.2 \\
 & 25.2 \text{ is } 42\% \text{ of } 60.
 \end{aligned}$$

$$\begin{aligned}
 37. \quad & 120\% \cdot x = 42 \\
 & 1.20 \cdot x = 42 \\
 & \frac{1.2x}{1.2} = \frac{42}{1.2} \\
 & x = 35 \\
 & 120\% \text{ of } 35 \text{ is } 42.
 \end{aligned}$$

$$\begin{aligned}
 41. \quad & x \cdot 600 = 3 \\
 & \frac{x \cdot 600}{600} = \frac{3}{600} \\
 & x = 0.005 \\
 & x = 0.5\% \\
 & 0.5\% \text{ of } 600 \text{ is } 3.
 \end{aligned}$$

$$\begin{aligned}
 45. \quad & 1575 = x \cdot 2500 \\
 & \frac{1575}{2500} = \frac{x \cdot 2500}{2500} \\
 & 0.63 = x \\
 & 63\% = x \\
 & 1575 \text{ is } 63\% \text{ of } 2500.
 \end{aligned}$$

$$\begin{aligned}
 49. \quad & \frac{27}{x} = \frac{9}{10} \\
 & 27 \cdot 10 = x \cdot 9 \\
 & 270 = 9x \\
 & \frac{270}{9} = \frac{9x}{9} \\
 & 30 = x
 \end{aligned}$$

$$53. \quad \frac{17}{12} = \frac{x}{20}$$

$$\begin{aligned}
 57. \quad & 5 \cdot n = 32 \\
 & \frac{5 \cdot n}{5} = \frac{32}{5} \\
 & n = \frac{32}{5}
 \end{aligned}$$

Choice **c** is correct.

61. answers may vary

65. Since 85 is less than 120, the percent is less than 100%; **c**.69. Since 100% is 1, 100% of 45 is equal to 45; **a**.

73. answers may vary

$$\begin{aligned}
 77. \quad & 22,113 = 180\% \cdot x \\
 & 22,113 = 1.80 \cdot x \\
 & \frac{22,113}{1.8} = \frac{1.8 \cdot x}{1.8} \\
 & 12,285 = x \\
 & 22,113 \text{ is } 180\% \text{ of } 12,285.
 \end{aligned}$$

Exercise Set 7.31. 98% of 45 is what number?

$$\begin{array}{ccc}
 \downarrow & \downarrow & \downarrow \\
 \text{percent} & \text{base} & \text{amount} = a \\
 \frac{a}{45} = \frac{98}{100}
 \end{array}$$

5. 14.3 is 26% of what number?

$$\begin{array}{ccc}
 \downarrow & \downarrow & \downarrow \\
 \text{amount} & \text{percent} & \text{base} = b \\
 \frac{14.3}{b} = \frac{26}{100}
 \end{array}$$

9. what percent of 400 is 70?

$$\begin{array}{ccc}
 \downarrow & \downarrow & \downarrow \\
 \text{percent} = p & \text{base} & \text{amount} \\
 \frac{70}{400} = \frac{p}{100}
 \end{array}$$

$$13. \quad \frac{a}{65} = \frac{40}{100} \quad \text{or} \quad \frac{a}{65} = \frac{2}{5}$$

$$a \cdot 5 = 65 \cdot 2$$

$$5a = 130$$

$$\frac{5a}{5} = \frac{130}{5}$$

$$a = 26$$

40% of 65 is 26.

$$17. \quad \frac{90}{b} = \frac{15}{100} \quad \text{or} \quad \frac{90}{b} = \frac{3}{20}$$

$$90 \cdot 20 = b \cdot 3$$

$$1800 = 3b$$

$$\frac{1800}{3} = \frac{3b}{3}$$

$$600 = b$$

15% of 600 is 90.

$$21. \quad \frac{42}{35} = \frac{p}{100} \quad \text{or} \quad \frac{6}{5} = \frac{p}{100}$$

$$6 \cdot 100 = 5 \cdot p$$

$$600 = 5p$$

$$\frac{600}{5} = \frac{5p}{5}$$

$$120 = p$$

42 is 120% of 35.

$$25. \quad \frac{3.7}{b} = \frac{10}{100} \quad \text{or} \quad \frac{3.7}{b} = \frac{1}{10}$$

$$3.7 \cdot 10 = b \cdot 1$$

$$37 = b$$

3.7 is 10% of 37.

$$29. \quad \frac{160}{b} = \frac{16}{100} \quad \text{or} \quad \frac{160}{b} = \frac{4}{25}$$

$$160 \cdot 25 = b \cdot 4$$

$$4000 = 4b$$

$$\frac{4000}{4} = \frac{4b}{4}$$

$$1000 = b$$

160 is 16% of 1000.

$$33. \quad \frac{a}{62} = \frac{89}{100}$$

$$a \cdot 100 = 62 \cdot 89$$

$$100a = 5518$$

$$\frac{100a}{100} = \frac{5518}{100}$$

$$a = 55.18$$

55.18 is 89% of 62.

$$37. \quad \frac{105}{b} = \frac{140}{100} \quad \text{or} \quad \frac{105}{b} = \frac{7}{5}$$

$$105 \cdot 5 = b \cdot 7$$

$$525 = 7b$$

$$\frac{525}{7} = \frac{7b}{7}$$

$$75 = b$$

140% of 75 is 105.

$$41. \quad \frac{4}{800} = \frac{p}{100} \quad \text{or} \quad \frac{1}{200} = \frac{p}{100}$$

$$1 \cdot 100 = 200 \cdot p$$

$$100 = 200p$$

$$\frac{100}{200} = \frac{200p}{200}$$

$$0.5 = p$$

0.5% of 800 is 4.

$$45. \quad \frac{a}{48} = \frac{20}{100} \quad \text{or} \quad \frac{a}{48} = \frac{1}{5}$$

$$a \cdot 5 = 48 \cdot 1$$

$$5a = 48$$

$$\frac{5a}{5} = \frac{48}{5}$$

$$a = 9.6$$

20% of 48 is 9.6.

$$49. \quad -\frac{11}{16} + \left(-\frac{3}{16}\right) = \frac{-11-3}{16} = \frac{-14}{16} = -\frac{2 \cdot 7}{2 \cdot 8} = -\frac{7}{8}$$

$$53. \quad \begin{array}{r} 0.41 \\ +0.29 \\ \hline 0.70 \end{array}$$

57. answers may vary

$$61. \quad \frac{p}{100} = \frac{13}{52}$$

$$\frac{25}{100} \stackrel{?}{=} \frac{13}{52}$$

$$\frac{1}{4} \stackrel{?}{=} \frac{1}{4}$$

True

Yes, the percent is 25.

$$65. \quad \frac{a}{53,862} = \frac{22.3}{100}$$

$$a \cdot 100 = 22.3 \cdot 53,862$$

$$100a = 1,201,122.6$$

$$\frac{100a}{100} = \frac{1,201,122.6}{100}$$

$$a \approx 12,011.2$$

Exercise Set 7.4

1. 24 is 1.5% of what number?

Method 1:

$$24 = 1.5\% \cdot x$$

$$24 = 0.015x$$

$$\frac{24}{0.015} = \frac{0.015x}{0.015}$$

$$1600 = x$$

1600 bolts were inspected.

Method 2:

$$\frac{24}{b} = \frac{1.5}{100}$$

$$24 \cdot 100 = 1.5 \cdot b$$

$$2400 = 1.5b$$

$$\frac{2400}{1.5} = \frac{1.5b}{1.5}$$

$$1600 = b$$

1600 bolts were inspected.

5. 378 is what percent of 2700?

Method 1:

$$378 = x \cdot 2700$$

$$\frac{378}{2700} = \frac{x \cdot 2700}{2700}$$

$$0.14 = x$$

$$14\% = x$$

The student spent 14% of last semester's college costs on books.

Method 2:

$$\frac{378}{2700} = \frac{p}{100}$$

$$378 \cdot 100 = 2700 \cdot p$$

$$37,800 = 2700p$$

$$\frac{37,800}{2700} = \frac{2700p}{2700}$$

$$14 = p$$

$$14 = p$$

$$14 = p$$

The student spent 14% of last semester's college costs on books.

9. 14,155 is what percent of 36,899?

Method 1:

$$14,155 = x \cdot 36,899$$

$$\frac{14,155}{36,899} = \frac{x \cdot 36,899}{36,899}$$

$$0.38 \approx x$$

$$38\% \approx x$$

38% of McDonald's restaurants in 2016 were in the United States.

Method 2:

$$\frac{14,155}{36,899} = \frac{p}{100}$$

$$14,155 \cdot 100 = 36,899 \cdot p$$

$$1,415,500 = 36,899p$$

$$\frac{1,415,500}{36,899} = \frac{36,899p}{36,899}$$

$$38 \approx p$$

38% of McDonald's restaurants in 2016 were in the United States.

13. What number is 43% of 41,900?

Method 1:

$$x = 43\% \cdot 41,900$$

$$x = 0.43 \cdot 41,900$$

$$x = 18,017$$

The number of people employed as occupational therapy assistants is expected to be $41,900 + 18,017 = 59,917$.

Method 2:

$$\frac{a}{41,900} = \frac{43}{100}$$

$$a \cdot 100 = 41,900 \cdot 43$$

$$100a = 1,801,700$$

$$\frac{100a}{100} = \frac{1,801,700}{100}$$

$$a = 18,017$$

The number of people employed as occupational therapy assistants is expected to be $41,900 + 18,017 = 59,917$.

17. 41 is what percent of 135?

Method 1:

$$41 = x \cdot 135$$

$$\frac{41}{135} = \frac{x \cdot 135}{135}$$

$$0.30 \approx x$$

$$30\% \approx x$$

30% of the ski runs at Keystone ski area are rated intermediate.

Method 2:

$$\frac{41}{135} = \frac{p}{100}$$

$$41 \cdot 100 = 135 \cdot p$$

$$4100 = 135p$$

$$\frac{4100}{135} = \frac{135p}{135}$$

$$0.30 \approx p$$

30% of the ski runs at Keystone ski area are rated intermediate.

21. 10 is what percent of 80?

Method 1:

$$10 = x \cdot 80$$

$$\frac{10}{80} = \frac{x \cdot 80}{80}$$

$$0.125 = x$$

$$12.5\% = x$$

12.5% of the total calories come from fat.

Method 2:

$$\frac{10}{80} = \frac{p}{100}$$

$$10 \cdot 100 = 80 \cdot p$$

$$1000 = 80p$$

$$\frac{1000}{80} = \frac{80p}{80}$$

$$12.5 = p$$

12.5% of the total calories come from fat.

25. 26,250 is 15% of what number?

Method 1:

$$26,250 = 15\% \cdot x$$

$$26,250 = 0.15 \cdot x$$

$$\frac{26,250}{0.15} = \frac{0.15 \cdot x}{0.15}$$

$$175,000 = x$$

The price of the home was \$175,000.

Method 2:

$$\frac{26,250}{b} = \frac{15}{100}$$

$$26,250 \cdot 100 = b \cdot 15$$

$$2,625,000 = 15b$$

$$\frac{2,625,000}{15} = \frac{15b}{15}$$

$$175,000 = b$$

The price of the home was \$175,000.

29. What number is 4.5% of 19,286?

Method 1:

$$x = 4.5\% \cdot 19,286$$

$$x = 0.045 \cdot 19,286$$

$$x = 867.87$$

The price of the car will increase by \$867.87. The new price of that model will be $\$19,286 + \$867.87 = \$20,153.87$.

Method 2:

$$\frac{a}{19,286} = \frac{4.5}{100}$$

$$a \cdot 100 = 4.5 \cdot 19,286$$

$$100a = 86,787$$

$$a = \frac{86,787}{100}$$

$$a = 867.87$$

The price of the car will increase by \$867.87. The new price of that model will be $\$19,286 + \$867.87 = \$20,153.87$.

33. 11.6% of \$8994 is what number?

Method 1:

$$11.6\% (8994) = x$$

$$0.116(8994) = x$$

$$1043.304 = x$$

$$1043 \approx x$$

The increase in tuition is \$1043. The tuition for the 2016–2017 school year is $\$1043 + \$8994 = \$10,037$.

Method 2:

$$\begin{aligned}\frac{a}{8994} &= \frac{11.6}{100} \\ a \cdot 100 &= 11.6 \cdot 8994 \\ 100a &= 104,330.4 \\ \frac{100a}{100} &= \frac{104,330.4}{100} \\ a &= 1043.304 \\ a &\approx 1043\end{aligned}$$

The increase in tuition is \$1043. The tuition for the 2016–2017 school year is \$1043 + \$8994 = \$10,037.

37.

Original Amount	New Amount	Amount of Increase	Percent of Increase
50	80	$80 - 50 = 30$	$\frac{30}{50} = 0.6 = 60\%$

41.

Original Amount	New Amount	Amount of Decrease	Percent of Decrease
8	6	$8 - 6 = 2$	$\frac{2}{8} = 0.25 = 25\%$

45. percent of decrease = $\frac{\text{amount of decrease}}{\text{original amount}}$

$$\begin{aligned}&= \frac{150 - 84}{150} \\&= \frac{66}{150} \\&= 0.44\end{aligned}$$

The decrease in calories is 44%.

49. percent of increase = $\frac{\text{amount of increase}}{\text{original amount}}$

$$\begin{aligned}&= \frac{337 - 71}{71} \\&= \frac{266}{71} \\&\approx 3.746\end{aligned}$$

The projected increase in virtual reality devices is 374.6%.

53. percent of decrease = $\frac{\text{amount of decrease}}{\text{original amount}}$

$$\begin{aligned}&= \frac{1199 - 1021}{1199} \\&= \frac{178}{1199} \\&\approx 0.148\end{aligned}$$

The decrease in permanent digital downloads of singles was 14.8%.

57. percent of increase = $\frac{\text{amount of increase}}{\text{original amount}}$

$$\begin{aligned}&= \frac{420 - 174}{174} \\&= \frac{246}{174} \\&\approx 1.414\end{aligned}$$

The increase in the size of farms in the United States was 141.4%.

61. 0.12

$$\begin{aligned}&\times \frac{38}{96} \\&\hline 4.56\end{aligned}$$

65. $-\frac{3}{8} + \frac{5}{12} = -\frac{3}{8} \cdot \frac{3}{3} + \frac{5}{12} \cdot \frac{2}{2}$

$$\begin{aligned}&= -\frac{9}{24} + \frac{10}{24} \\&= \frac{1}{24}\end{aligned}$$

69. The increased number is double the original number.

73. 35,196 is what percent of 37,863?

Method 1:

$$\begin{aligned}35,196 &= x \cdot 37,863 \\ \frac{35,196}{37,863} &= \frac{x \cdot 37,863}{37,863}\end{aligned}$$

$$0.9296 \approx x$$

93% of the population of Monaco are Internet users.

Method 2:

$$\begin{aligned}\frac{35,196}{37,863} &= \frac{p}{100} \\ 35,196 \cdot 100 &= 37,863 \cdot p \\ 3,519,600 &= 37,863p \\ \frac{3,519,600}{37,863} &= \frac{37,863p}{37,863} \\ 92.96 &\approx p\end{aligned}$$

93% of the population of Monaco are Internet users.

77. answers may vary

81. False. The percents are different. The percent of increase was 20% while the percent of decrease was $16\frac{2}{3}\%$.

Exercise Set 7.5

1. sales tax = $5\% \cdot \$150 = 0.05 \cdot \$150 = \$7.50$

The sales tax is \$7.50.

5. $\$335.30 = r \cdot \$4790$

$$\begin{aligned}\frac{335.30}{4790} &= r \\ 0.07 &= r\end{aligned}$$

The sales tax rate is 7%.

9. sales tax = $6.5\% \cdot \$1800 = 0.065 \cdot \$1800 = \$117$

$$\text{total price} = \$1800 + \$117 = \$1917$$

The sales tax is \$117 and the total price of the bracelet is \$1917.

13. $\$98.70 = r \cdot \$1645$

$$\begin{aligned}\frac{98.70}{1645} &= r \\ 0.06 &= r\end{aligned}$$

The sales tax rate is 6%.

17. commission = $4\% \cdot \$1,329,401 = 0.04 \cdot \$1,329,401$

$$= \$53,176.04$$

Her commission was \$53,176.04.

21. commission = $1.5\% \cdot \$325,900 = 0.015 \cdot \$325,900$

$$= \$4888.50$$

His commission will be \$4888.50.

	Original Price	Discount Rate	Amount of Discount	Sale Price
25.	\$89	10%	$10\% \cdot \$89 = \$8.90$	$\$89 - \$8.90 = \$80.10$
29.	\$410	35%	$35\% \cdot \$410 = \$143.50$	$\$410 - \$143.50 = \$266.50$

33. discount = $15\% \cdot \$300 = 0.15 \cdot \$300 = \$45$
 sale price = $\$300 - \$45 = \$255$
 The discount is \$45 and the sale price is \$255.

Purchase Price	Tax Rate	Sales Tax	Total Price
\$56	5.5%	$5.5\% \cdot \$56 = \$3.08$	$\$56 + \$3.08 = \$59.08$

Sale	Commission Rate	Commission
\$17,900	$\frac{\$1432}{\$17,900} = 0.08 = 8\%$	\$1432

45. $400 \cdot \frac{3}{100} \cdot 11 = 12 \cdot 11 = 132$

49. Round \$68 to \$70 and 9.5% to 10%.
 $10\% \cdot \$70 = 0.10 \cdot \$70 = \$7$
 $\$70 + \$7 = \$77$
 The best estimate of the total price is \$77; **d**.

Bill Amount	10%	15%	20%
$\$72.17 \approx \$72.00$	\$7.20	$\$7.20 + \frac{1}{2}(\$7.20) = \$7.20 + \$3.60 = \$10.80$	$2(\$7.20) = \$14.40$

57. $7.5\% \cdot \$24,966 = 0.075 \cdot \$24,966 = \$1872.45$
 $\$24,966 + \$1872.45 = \$26,838.45$
 The total price of the necklace is \$26,838.45.

Exercise Set 7.6

1. simple interest = principal $\cdot$ rate $\cdot$ time
 $= (\$200)(8\%)(2)$
 $= (\$200)(0.08)(2) = \$32$
5. simple interest = principal $\cdot$ rate $\cdot$ time
 $= (\$5000)(10\%)\left(1\frac{1}{2}\right)$
 $= (\$5000)(0.10)(1.5) = \$750$
9. simple interest = principal $\cdot$ rate $\cdot$ time
 $= (\$2500)(16\%)\left(\frac{21}{12}\right)$
 $= (\$2500)(0.16)(1.75) = \$700$
13. simple interest = principal $\cdot$ rate $\cdot$ time
 $= \$5000(9\%)\left(\frac{15}{12}\right)$
 $= \$5000(0.09)(1.25) = \$562.50$
 Total = $\$5000 + \$562.50 = \$5562.50$
17. $A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$
 $= 6150\left(1 + \frac{0.14}{2}\right)^{2 \cdot 15}$
 $= 6150(1.07)^{30}$
 $\approx 46,815.37$
 The total amount is \$46,815.37.

21. $A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$
 $= 10,000\left(1 + \frac{0.09}{2}\right)^{2 \cdot 20}$
 $= 10,000(1.045)^{40}$
 $\approx 58,163.65$
 The total amount is \$58,163.65.

25. $A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$
 $= 2000\left(1 + \frac{0.08}{1}\right)^{1 \cdot 5}$
 $= 2000(1.08)^5$
 ≈ 2938.66

The total amount is \$2938.66

29. perimeter = $10 + 6 + 10 + 6 = 32$
 The perimeter is 32 yards.

33. $\frac{x}{4} + \frac{x}{5} = \frac{x}{4} \cdot \frac{5}{5} + \frac{x}{5} \cdot \frac{4}{4} = \frac{5x}{20} + \frac{4x}{20} = \frac{9x}{20}$

37. answers may vary

Chapter 7 Test

1. $85\% = 85(0.01) = 0.85$
5. $6.1 = 6.1(100\%) = 610\%$
9. $0.2\% = 0.2 \cdot \frac{1}{100} = \frac{0.2}{100} = \frac{2}{1000} = \frac{1}{500}$
13. $\frac{1}{4} = \frac{1}{4} \cdot \frac{100\%}{1} = \frac{100}{4}\% = 25\%$
17. Method 1:
 $567 = x \cdot 756$
 $\frac{567}{756} = \frac{x \cdot 756}{756}$
 $0.75 = x$
 $75\% = x$
 567 is 75% of 756.
 Method 2:
 $\frac{567}{756} = \frac{p}{100}$
 $567 \cdot 100 = 756 \cdot p$
 $56,700 = 756p$
 $\frac{56,700}{756} = \frac{756p}{756}$
 $75 = p$
 567 is 75% of 756.

21. percent of increase = $\frac{\text{amount of increase}}{\text{original amount}}$
 $= \frac{26,460 - 25,200}{25,200}$
 $= \frac{1260}{25,200}$
 $= 0.05$

The increase in population was 5%.

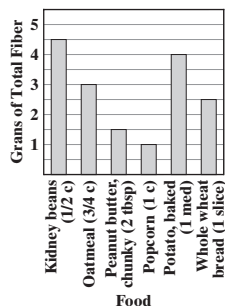
25. simple interest = principal $\cdot$ rate $\cdot$ time
 $= (\$2000)(9.25\%)\left(3\frac{1}{2}\right)$
 $= (\$2000)(0.0925)(3.5)$
 $= \$647.50$

Chapter 8

Exercise Set 8.1

1. Kansas has the greatest number of wheat icons, so the greatest acreage of wheat was planted in the state of Kansas.
5. The states with fewer than 3 wheat icons are South Dakota, Colorado, and Washington, so the states of South Dakota, Colorado, and Washington plant less than 3 million acres in wheat.
9. The year 2013 has 4 flames and each flames represents 12,000 wildfires, so there were approximately $4(12,000) = 48,000$ wildfires in 2013.
13. 2012 has 5.5 flames and 2013 has 4 flames, which is 1.5 less. Thus, the decrease in the number of wildfires from 2012 to 2013 was about $1.5(12,000) = 18,000$.
17. The longest bar corresponds to September, so the month in which most hurricanes made landfall is September.
21. Two of the 78 hurricanes that made landfall in August did so in 2008. The fraction is $\frac{2}{78} = \frac{1}{39}$.
25. The only bar corresponding to a city in the United States is the bar for New York City. The population is approximately 20.6 million or 20,600,000.

29. Fiber Content of Selected Foods



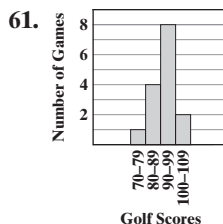
33. The height of the bar for 100–149 miles per week is 15, so 15 of the adults drive 100–149 miles per week.
37. 15 of the adults drive 100–149 miles per week and 9 of the adults drive 150–199 miles per week, so $15 + 9 = 24$ of the adults drive 100–199 miles per week.
41. 9 of the 100 adults surveyed drive 150–199 miles per week, so the ratio is $\frac{9}{100}$.
45. According to the bar graph, approximately 24 million householders were 55–64 years old.
49. According to the bar graph, 23 million householders were 45–54 years old and 21 million householders were 35–44 years old. $23 - 21 = 2$ million more householders were 45–54 than were 35–44 years old.

53.

Class Interval (Scores)	Tally	Class Frequency (Number of Games)
90–99		8

57.

Class Interval (Account Balances)	Tally	Class Frequency (Number of People)
\$200–\$299		6



- 61.
65. The highest point on the graph corresponds to 2003, so the average number of goals per game was the greatest in 2003.
69. The dots for 2001, 2007, and 2013 are below the 8-level, so the average number of goals per game was less than 8 in 2001, 2007, and 2013.
73. $\text{range} = 556 - 206 = 350$
77. $\text{range} = 5.00 - 3.30 = 1.70$ or 1.7
81. a.
$$\text{mean} = \frac{5 \cdot 2 + 7 \cdot 3 + 4 \cdot 4 + 7 \cdot 5 + 8 \cdot 6 + 8 \cdot 7 + 8 \cdot 8 + 6 \cdot 9 + 5 \cdot 10}{58} = \frac{354}{58} \approx 6.1$$

b. median: $\frac{n+1}{2} = \frac{58+1}{2} = 29.5$. Thus the median is the average of the data item in position 29 and position 30.

$$\text{median} = \frac{6+6}{2} = 6$$

c. mode: Three data items occurred 8 times. The modes are 6, 7, and 8.
85. a.
$$\text{mean} = \frac{3 \cdot 2 + 6 \cdot 4 + 6 \cdot 6 + 5 \cdot 8 + 4 \cdot 10}{24} = \frac{146}{24} \approx 6.1$$

b. median: $\frac{n+1}{2} = \frac{24+1}{2} = 12.5$ The median is the average of the data item in position 12 and position 13.

$$\text{median} = \frac{6+6}{2} = 6$$

c. mode: 4 and 6 both occur most often
89. 10% of 62 is $0.10 \cdot 62 = 6.2$
93. $\frac{17}{50} = \frac{17}{50} \cdot 100\% = \frac{17 \cdot 2 \cdot 50}{50}\% = 34\%$
97. The lowest point on the graph of low temperatures corresponds to Sunday. The low temperature on Sunday was 68°F .
101. answers may vary

Exercise Set 8.2

1. The largest sector corresponds to the category “parent or guardian’s home,” so most of the students live in a parent or guardian’s home.
 5. 180 of the students live in campus housing while 320 live in a parent or guardian’s home.
- $$\frac{180}{320} = \frac{9}{16}$$
- The ratio is $\frac{9}{16}$.

9. $30\% + 7\% = 37\%$

37% of the land on Earth is accounted for by Europe and Asia.

13. Australia accounts for 5% of the land on Earth.

$$5\% \text{ of } 57,000,000 = 0.05 \cdot 57,000,000 \\ = 2,850,000$$

Australia is 2,850,000 square miles.

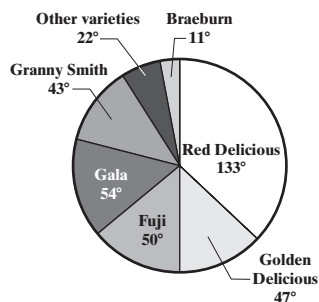
17. The second-largest sector corresponds to nonfiction, so the second-largest category of books is nonfiction.

21. Children's fiction accounts for 22% of the books.

$$22\% \text{ of } 125,600 = 0.22 \cdot 125,600 \\ = 27,632$$

The library has 27,632 children's fiction books.

Type of Apple	Percent	Degrees in Sector
Red Delicious	37%	$37\% \text{ of } 360^\circ = 0.37(360^\circ) \approx 133^\circ$
Golden Delicious	13%	$13\% \text{ of } 360^\circ = 0.13(360^\circ) \approx 47^\circ$
Fuji	14%	$14\% \text{ of } 360^\circ = 0.14(360^\circ) \approx 50^\circ$
Gala	15%	$15\% \text{ of } 360^\circ = 0.15(360^\circ) \approx 54^\circ$
Granny Smith	12%	$12\% \text{ of } 360^\circ = 0.12(360^\circ) \approx 43^\circ$
Other varieties	6%	$6\% \text{ of } 360^\circ = 0.06(360^\circ) \approx 22^\circ$
Braeburn	3%	$3\% \text{ of } 360^\circ = 0.03(360^\circ) \approx 11^\circ$



29. $20 = 2 \times 10 = 2 \times 2 \times 5 = 2^2 \times 5$

33. $85 = 5 \times 17$

37. Pacific Ocean:

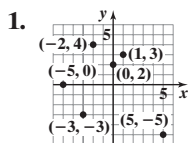
$$49\% \cdot 264,489,800 = 0.49 \cdot 264,489,800 \\ = 129,600,002 \text{ square kilometers}$$

41. $24\% \cdot 2800 = 0.24 \cdot 2800 = 672$

672 respondents said that they spend \$0 online each month.

$$45. \frac{\text{number of respondents who spend \$0}}{\text{number of respondents who spend \$1–\$100}} = \frac{672}{1736} \\ = \frac{12 \cdot 56}{31 \cdot 56} \\ = \frac{12}{31}$$

Exercise Set 8.3



5. Point A has coordinates (0, 0).

Point B has coordinates $(\frac{1}{3}, 0)$.

Point C has coordinates (3, 2).

Point D has coordinates (-1, 3).

Point E has coordinates (-2, -2).

Point F has coordinates (0, -1).

Point G has coordinates (2, -1).

9. $x - y = 3$

$$1 - 2 \stackrel{?}{=} 3$$

$$-1 \stackrel{?}{=} 3 \quad \text{False}$$

No, (1, 2) is not a solution of $x - y = 3$.

13. $x = -3y$

$$6 \stackrel{?}{=} -3(-2)$$

$$6 = 6 \quad \text{True}$$

Yes, (6, -2) is a solution of $x = -3y$.

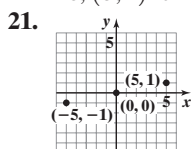
17. $x - 5y = -1$

$$3 - 5(1) \stackrel{?}{=} -1$$

$$3 - 5 \stackrel{?}{=} -1$$

$$-2 \stackrel{?}{=} -1 \quad \text{False}$$

No, (3, 1) is not a solution of $x - 5y = -1$.



25. $y = -9x$

$$y = -9(1)$$

$$y = -9$$

The solution is (1, -9).

$$y = -9x$$

$$y = -9(0)$$

$$y = 0$$

The solution is (0, 0).

$$y = -9x$$

$$-18 = -9x$$

$$-18 = -9x$$

$$\frac{-18}{-9} = \frac{-9x}{-9}$$

$$2 = x$$

The solution is (2, -18).

29. $x + y = -2$

$$-2 + y = -2$$

$$-2 + 2 + y = -2 + 2$$

$$y = 0$$

The solution is (-2, 0).

$$x + y = -2$$

$$1 + y = -2$$

$$1 - 1 + y = -2 - 1$$

$$y = -3$$

The solution is (1, -3).

$$x + y = -2$$

$$x + 5 = -2$$

$$x + 5 - 5 = -2 - 5$$

$$x = -7$$

The solution is (-7, 5).

33. $y = 3x - 5$

$$y = 3 \cdot 1 - 5$$

$$y = 3 - 5$$

$$y = -2$$

The solution is (1, -2).

$$y = 3x - 5$$

$$y = 3 \cdot 2 - 5$$

$$y = 6 - 5$$

$$y = 1$$

The solution is (2, 1).

$$y = 3x - 5$$

$$4 = 3x - 5$$

$$\begin{aligned}
 4 + 5 &= 3x - 5 + 5 \\
 9 &= 3x \\
 \frac{9}{3} &= \frac{3x}{3} \\
 3 &= x
 \end{aligned}$$

The solution is $(3, 4)$.

37. $x + 2y = -8$
 $4 + 2y = -8$
 $4 - 4 + 2y = -8 - 4$
 $2y = -12$
 $\frac{2y}{2} = \frac{-12}{2}$
 $y = -6$

The solution is $(4, -6)$.

$$\begin{aligned}
 x + 2y &= -8 \\
 x + 2(-3) &= -8 \\
 x - 6 &= -8 \\
 x - 6 + 6 &= -8 + 6 \\
 x &= -2
 \end{aligned}$$

The solution is $(-2, -3)$.

$$\begin{aligned}
 x + 2y &= -8 \\
 0 + 2y &= -8 \\
 2y &= -8 \\
 \frac{2y}{2} &= \frac{-8}{2} \\
 y &= -4
 \end{aligned}$$

The solution is $(0, -4)$.

41. $5.6 \cdot 3.9 = 21.84$

45. To plot (a, b) , start at the origin and move a units to the right and b units up. The point will be in quadrant I. Thus, (a, b) is in quadrant I is a true statement.

49. To plot $(0, -b)$, start at the origin and move 0 units to the right and b units down. The point will be on the y -axis. Thus, $(0, -b)$ lies on the x -axis is a false statement.

53. To plot $(4, -3)$, start at the origin and move 4 units to the right, so the point $(4, -3)$ is plotted to the right of the y -axis.

57. $P = 2l + 2w$

Let $l = 6$ and $w = 4$.

$$P = 2 \cdot 6 + 2 \cdot 4 = 12 + 8 = 20$$

The perimeter is 20 units.

Exercise Set 8.4

1. $x + y = 4$

Find any 3 ordered pair solutions.

Let $x = 0$.

$$\begin{aligned}
 x + y &= 4 \\
 0 + y &= 4 \\
 y &= 4
 \end{aligned}$$

$(0, 4)$

Let $x = 2$.

$$\begin{aligned}
 x + y &= 4 \\
 2 + y &= 4 \\
 2 - 2 + y &= 4 - 2 \\
 y &= 2
 \end{aligned}$$

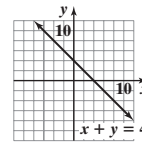
$(2, 2)$

Let $x = 4$.

$$\begin{aligned}
 x + y &= 4 \\
 4 + y &= 4 \\
 4 - 4 + y &= 4 - 4 \\
 y &= 0
 \end{aligned}$$

$(4, 0)$

Plot $(0, 4)$, $(2, 2)$, and $(4, 0)$. Then draw the line through them.



5. $y = 4x$

Find any 3 ordered-pair solutions.

Let $x = 0$.

$$\begin{aligned}
 y &= 4x \\
 y &= 4(0) \\
 y &= 0 \\
 (0, 0)
 \end{aligned}$$

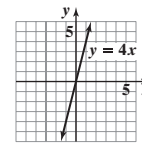
Let $x = 1$.

$$\begin{aligned}
 y &= 4x \\
 y &= 4(1) \\
 y &= 4 \\
 (1, 4)
 \end{aligned}$$

Let $x = -1$.

$$\begin{aligned}
 y &= 4x \\
 y &= 4(-1) \\
 y &= -4 \\
 (-1, -4)
 \end{aligned}$$

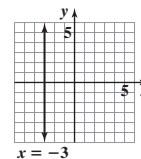
Plot $(0, 0)$, $(1, 4)$, and $(-1, -4)$. Then draw the line through them.



9. $x = -3$

No matter what y -value we choose, x is always -3 .

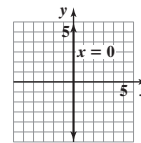
x	y
-3	-4
-3	0
-3	4



13. $x = 0$

No matter what y -value we choose, x is always 0.

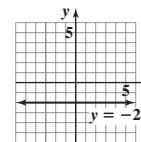
x	y
0	-3
0	0
0	3



17. $y = -2$

No matter what x -value we choose, y is always -2 .

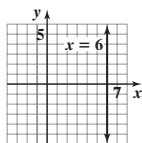
x	y
-4	-2
0	-2
4	-2



21. $x = 6$

No matter what y -value we choose, x is always 6.

x	y
6	-5
6	0
6	5



25. $x = y - 4$

Find any 3 ordered pair solutions.

Let $y = 0$.

$x = y - 4$

$x = 0 - 4$

$x = -4$

$(-4, 0)$

Let $y = 4$.

$x = y - 4$

$x = 4 - 4$

$x = 0$

$(0, 4)$

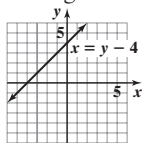
Let $y = 6$.

$x = y - 4$

$x = 6 - 4$

$x = 2$

$(2, 6)$

Plot $(-4, 0)$, $(0, 4)$, and $(2, 6)$. Then draw the line through them.

29. $y = -\frac{1}{4}x$

Find any 3 ordered pair solutions.

Let $x = -4$.

$y = -\frac{1}{4}x$

$y = -\frac{1}{4}(-4)$

$y = 1$

$(-4, 1)$

Let $x = 0$.

$y = -\frac{1}{4}x$

$y = -\frac{1}{4} \cdot 0$

$y = 0$

$(0, 0)$

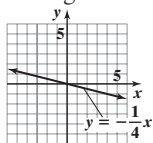
Let $x = 4$.

$y = -\frac{1}{4}x$

$y = -\frac{1}{4} \cdot 4$

$y = -1$

$(4, -1)$

Plot $(-4, 1)$, $(0, 0)$, and $(4, -1)$. Then draw the line through them.

33. $y = 4x + 2$

Find any 3 ordered pair solutions.

Let $x = -1$.

$y = 4x + 2$

$y = 4(-1) + 2$

$y = -4 + 2$

$y = -2$

$(-1, -2)$

Let $x = 0$.

$y = 4x + 2$

$y = 4(0) + 2$

$y = 0 + 2$

$y = 2$

$(0, 2)$

Let $x = 1$.

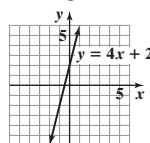
$y = 4x + 2$

$y = 4(1) + 2$

$y = 4 + 2$

$y = 6$

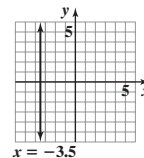
$(1, 6)$

Plot $(-1, -2)$, $(0, 2)$, and $(1, 6)$. Then draw the line through them.

37. $x = -3.5$

No matter what y -value we choose, x is always -3.5 .

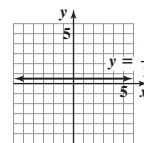
x	y
-3.5	-3
-3.5	0
-3.5	3



41. $y = \frac{1}{2}$

No matter what x -value we choose, y is always $\frac{1}{2}$.

x	y
-4	$\frac{1}{2}$
-0	$\frac{1}{2}$
4	$\frac{1}{2}$



45. $(0.5)\left(-\frac{1}{8}\right) = \left(\frac{1}{2}\right)\left(-\frac{1}{8}\right) = -\frac{1 \cdot 1}{2 \cdot 8} = -\frac{1}{16}$

49. $\frac{2}{11} - \frac{x}{11} = \frac{2-x}{11}$

53. answers may vary

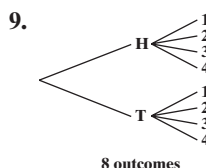
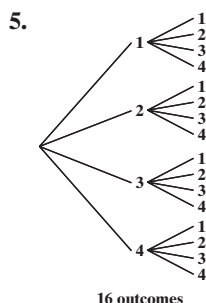
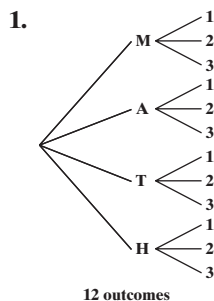
57. The HEV line appears to be at 30,000 above May 2016.

Approximately 30,000 HEVs were sold during May 2016.

61. Because the line corresponding to passenger cars from 2006 to 2009 is decreasing from left to right, passenger car sales were decreasing from 2006 to 2009.

65. answers may vary

Exercise Set 8.5



13. A 1 or a 6 are two of the six possible outcomes. The probability is $\frac{2}{6} = \frac{1}{3}$.

17. Four of the six possible outcomes are numbers greater than 2. The probability is $\frac{4}{6} = \frac{2}{3}$.

21. A 1, a 2, or a 3 are three of three possible outcomes. The probability is $\frac{3}{3} = 1$.

25. One of the seven marbles is red. The probability is $\frac{1}{7}$.

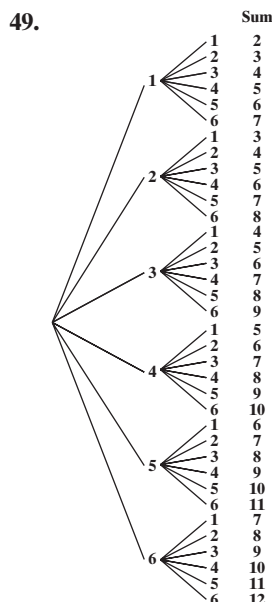
29. Four of the seven marbles are either green or red. The probability is $\frac{4}{7}$.

33. The blood pressure did not change for 10 of the 200 people. The probability is $\frac{10}{200} = \frac{1}{20}$.

37. $\frac{1}{2} \cdot \frac{1}{3} = \frac{1 \cdot 1}{2 \cdot 3} = \frac{1}{6}$

41. One of the 52 cards is the king of hearts. The probability is $\frac{1}{52}$.

45. Thirteen of the 52 cards are hearts. The probability is $\frac{13}{52} = \frac{1}{4}$.



Five of the 36 sums are 6. The probability is $\frac{5}{36}$.

53. answers may vary

Chapter 8 Test

1. There are $4\frac{1}{2}$ dollar symbols for the second week. Each dollar symbol corresponds to \$50.

$$4\frac{1}{2} \cdot \$50 = \frac{9}{2} \cdot \$50 = \frac{\$450}{2} = \$225$$

\$225 was collected during the second week.

5. The shortest bar corresponds to February. The normal monthly precipitation in February in Chicago is 3 centimeters.

9. The line graph is above the 3 level for 2008 and 2011. Thus, the inflation rate was greater than 3% in 2008 and 2011.

13. People under 18 are expected to account for 22% of the U.S. population in 2020.

$$22\% \text{ of } 335 \text{ million} = 0.22 \times 335 \text{ million} = 73.7 \text{ million}$$

74 million people are projected to be under 18 in 2020.

17.

Class Interval (Scores)	Tally	Class Frequency (Number of Students)
40–49		1
50–59		3
60–69		4
70–79		5
80–89		8
90–99		4

$$21. \text{ a. mean} = \frac{90 \cdot 3 + 91 + 92 \cdot 2 + 93 \cdot 8 + 94 \cdot 8}{22} = \frac{2041}{22} \approx 92.8$$

b. median: $\frac{n+1}{2} = \frac{22+1}{2} = 11.5$ The median is the average of the numbers 11th and 12th position.

$$\text{median} = \frac{93 + 93}{2} = 93$$

c. mode: The data items that occur most often are 93 and 94.

$$\text{d. range} = 94 - 90 = 4$$

25. Point C has coordinates $(-3, 4)$

29. $y + x = -4$

Find any 3 ordered pair solutions.

Let $x = 0$

$$y + x = -4$$

$$y + 0 = -4$$

$$y = -4$$

$$(0, -4)$$

Let $y = 0$.

$$y + x = -4$$

$$0 + x = -4$$

$$x = -4$$

$$(-4, 0)$$

Let $x = -2$.

$$y + x = -4$$

$$y + (-2) = -4$$

$$y - 2 = -4$$

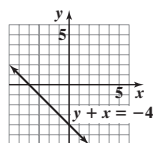
$$y - 2 + 2 = -4 + 2$$

$$y = -2$$

$$(-2, -2)$$

Plot $(0, -4)$, $(-4, 0)$, and $(-2, -2)$.

Then draw the line through them.



33. $y = -\frac{1}{2}x$

Find any 3 ordered pair solutions.

Let $x = 0$

$$y = -\frac{1}{2}x$$

$$y = -\frac{1}{2} \cdot 0$$

$$y = 0$$

$$(0, 0)$$

Let $x = -2$.

$$y = -\frac{1}{2}x$$

$$y = -\frac{1}{2} \cdot -2$$

$$y = 1$$

$$(-2, 1)$$

Let $x = 2$.

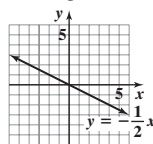
$$y = -\frac{1}{2}x$$

$$y = -\frac{1}{2} \cdot 2$$

$$y = -1$$

$$(2, -1)$$

Plot $(0, 0)$, $(-2, 1)$, and $(2, -1)$. Then draw the line through them.



37. One of the ten possible outcomes is a 6. The probability is $\frac{1}{10}$.

Chapter 9

Exercise Set 9.1

1. The figure extends indefinitely in two directions. It is line CD , line l , or $\overleftrightarrow{CD}$.
5. The figure has two rays with a common endpoint. It is an angle, which can be named $\angle GHI$, $\angle IHG$, or $\angle H$.
9. Two other ways to name $\angle x$ are $\angle CPR$ and $\angle RPC$.
13. $\angle S$ is a straight angle.
17. $\angle Q$ measures between 90° and 180° . It is an obtuse angle.
21. The complement of an angle that measures 23° is an angle that measures $90^\circ - 23^\circ = 67^\circ$.
25. The complement of an angle that measures 58° is an angle that measures $90^\circ - 58^\circ = 32^\circ$.
29. $52^\circ + 38^\circ = 90^\circ$, so $\angle PNQ$ and $\angle QNR$ are complementary. $60^\circ + 30^\circ = 90^\circ$, so $\angle MNP$ and $\angle RNO$ are complementary.
33. $m\angle x = 74^\circ - 47^\circ = 27^\circ$
37. $\angle x$ and the angle marked 150° are supplementary, so $m\angle x = 180^\circ - 150^\circ = 30^\circ$. $\angle y$ and the angle marked 150° are vertical angles, so $m\angle y = 150^\circ$. $\angle z$ and $\angle x$ are vertical angles so $m\angle z = m\angle x = 30^\circ$.
41. $\angle x$ and the angle marked 80° are supplementary, so $m\angle x = 180^\circ - 80^\circ = 100^\circ$. $\angle y$ and the angle marked 80° are alternate interior angles, so $m\angle y = 80^\circ$. $\angle x$ and $\angle z$ are corresponding angles, so $m\angle z = m\angle x = 100^\circ$.
45. $\angle x$ can also be named $\angle ABC$ or $\angle CBA$.
49. $m\angle ABC = 15^\circ$
53. $m\angle DBA = m\angle DBC + m\angle CBA$
 $= 50^\circ + 15^\circ$
 $= 65^\circ$
57. $\frac{7}{8} + \frac{1}{4} = \frac{7}{8} + \frac{2}{8} = \frac{9}{8}$ or $1\frac{1}{8}$
61. $3\frac{1}{3} - 2\frac{1}{2} = \frac{10}{3} - \frac{5}{2}$
 $= \frac{20}{6} - \frac{15}{6}$
 $= \frac{5}{6}$
65. Since there are 360° in a full revolution, there are 360° around the earth at the equator.
69. The supplement of an angle that measures 125.2° is an angle that measures $180^\circ - 125.2^\circ = 54.8^\circ$.
73. True; it is possible to find the supplement of any angle that measures less than 180° .
77. no; answers may vary

Exercise Set 9.2

1. $P = 2 \cdot l + 2 \cdot w$
 $= 2 \cdot 17 \text{ ft} + 2 \cdot 15 \text{ ft}$
 $= 34 \text{ ft} + 30 \text{ ft}$
 $= 64 \text{ ft}$
 The perimeter is 64 feet.
5. $P = a + b + c$
 $= 5 \text{ in.} + 7 \text{ in.} + 9 \text{ in.}$
 $= 21 \text{ in.}$
 The perimeter is 21 inches.
9. All sides of a regular triangle have the same length.
 $P = a + b + c = 14 \text{ in.} + 14 \text{ in.} + 14 \text{ in.} = 42 \text{ in.}$
 The perimeter is 42 inches.

13. Sum the lengths of the sides.

$$P = 5 \text{ ft} + 3 \text{ ft} + 2 \text{ ft} + 7 \text{ ft} + 4 \text{ ft} \\ = 21 \text{ ft}$$

The perimeter is 21 feet.

17. $P = 2 \cdot l + 2 \cdot w$
 $= 2 \cdot 120 \text{ yd} + 2 \cdot 53 \text{ yd}$
 $= 240 \text{ yd} + 106 \text{ yd}$
 $= 346 \text{ yd}$

The perimeter of the football field is 346 yards.

21. The amount of stripping needed is 22 feet.

$$22 \text{ feet} \cdot \$2.50 \text{ per foot} = \$55$$

The total cost of the stripping is \$55.

25. $P = 4 \cdot s = 4 \cdot 7 \text{ in.} = 28 \text{ in.}$

The perimeter is 28 inches.

29. The unmarked vertical side must have length
 $28 \text{ m} - 20 \text{ m} = 8 \text{ m.}$

The unmarked horizontal side must have length
 $20 \text{ m} - 17 \text{ m} = 3 \text{ m.}$

Sum the lengths of the sides.

$$P = 17 \text{ m} + 8 \text{ m} + 3 \text{ m} + 20 \text{ m} + 20 \text{ m} + 28 \text{ m} \\ = 96 \text{ m}$$

The perimeter is 96 meters.

33. The unmarked vertical side must have length
 $5 \text{ cm} + 14 \text{ cm} = 19 \text{ cm.}$

The unmarked horizontal side must have length
 $18 \text{ cm} - 9 \text{ cm} = 9 \text{ cm.}$

Sum the lengths of the sides.

$$P = 18 \text{ cm} + 19 \text{ cm} + 9 \text{ cm} + 14 \text{ cm} + 9 \text{ cm} + 5 \text{ cm} \\ = 74 \text{ cm}$$

The perimeter is 74 centimeters.

37. $C = 2 \cdot \pi \cdot r$
 $= 2 \cdot \pi \cdot 8 \text{ mi}$
 $= 16\pi \text{ mi}$
 $\approx 50.24 \text{ mi}$

The circumference is exactly 16π miles, which is approximately 50.24 miles.

41. $\pi \cdot d = \pi \cdot 15 \text{ ft} = 15\pi \text{ ft} \approx 47.1 \text{ ft}$

He needs 15π feet of netting or 47.1 feet.

45. Sum the lengths of the sides.

$$P = 9 \text{ mi} + 6 \text{ mi} + 11 \text{ mi} + 4.7 \text{ mi} \\ = 30.7 \text{ mi}$$

The perimeter is 30.7 miles.

49. The sides of a regular pentagon all have the same length.
 Sum the lengths of the sides.

$$P = 8 \text{ mm} + 8 \text{ mm} + 8 \text{ mm} + 8 \text{ mm} + 8 \text{ mm} \\ = 40 \text{ mm}$$

The perimeter is 40 millimeters.

53. $5 + 6 \cdot 3 = 5 + 18 = 23$

57. $72 \div (2 \cdot 6) = 72 \div 12 = 6$

61. a. The first age category that 8-year-old children fit into is "Under 9." Thus the minimum width is 30 yards, and the minimum length is 40 yards.

b. $P = 2 \cdot l + 2 \cdot w$
 $= 2 \cdot 40 \text{ yd} + 2 \cdot 30 \text{ yd}$
 $= 80 \text{ yd} + 60 \text{ yd}$
 $= 140 \text{ yd}$

The perimeter of the field is 140 yards.

65. a. Smaller circle:

$$C = 2 \cdot \pi \cdot r \\ = 2 \cdot \pi \cdot 10 \text{ m} \\ = 20\pi \text{ m} \\ \approx 62.8 \text{ m}$$

Larger circle:

$$C = 2 \cdot \pi \cdot r \\ = 2 \cdot \pi \cdot 20 \text{ m} \\ = 40\pi \text{ m} \\ \approx 125.6 \text{ m}$$

- b. Yes, when the radius of a circle is doubled, the circumference is also doubled.

69. The three linear sides each have length 6 meters.

The length of the curved side is half of the circumference of a circle with diameter 6 meters, or

$$\frac{1}{2} \pi \cdot d = \frac{1}{2} \pi \cdot 6 = 3\pi \approx 9.4 \text{ meters}$$

$$6 \text{ m} + 6 \text{ m} + 6 \text{ m} + 9.4 \text{ m} = 27.4 \text{ m}$$

The perimeter is 27.4 meters.

Exercise Set 9.3

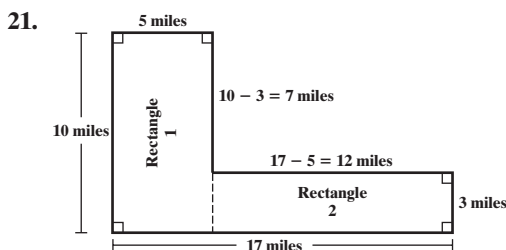
1. $A = l \cdot w = 3.5 \text{ m} \cdot 2 \text{ m} = 7 \text{ sq m}$

5. $A = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 6 \text{ yd} \cdot 5 \text{ yd} = 15 \text{ sq yd}$

9. $A = b \cdot h = 7 \text{ ft} \cdot 5.25 \text{ ft} = 36.75 \text{ sq ft}$

13. $A = \frac{1}{2}(b + B) \cdot h$
 $= \frac{1}{2}(7 \text{ yd} + 4 \text{ yd}) \cdot 4 \text{ yd}$
 $= \frac{1}{2}(11 \text{ yd}) \cdot 4 \text{ yd}$
 $= 22 \text{ sq yd}$

17. $A = b \cdot h$
 $= 5 \text{ in.} \cdot 4\frac{1}{2} \text{ in.}$
 $= 5 \text{ in.} \cdot \frac{9}{2} \text{ in.}$
 $= \frac{45}{2} \text{ sq in.}$
 $= 22\frac{1}{2} \text{ sq in.}$



Rectangle 1:

$$A = l \cdot w = 10 \text{ mi} \cdot 5 \text{ mi} = 50 \text{ sq mi}$$

Rectangle 2:

$$A = l \cdot w = 12 \text{ mi} \cdot 3 \text{ mi} = 36 \text{ sq mi}$$

The area of the figure is

$$50 \text{ sq mi} + 36 \text{ sq mi} = 86 \text{ sq mi.}$$

25. $A = \pi r^2 = \pi (6 \text{ in.})^2 = 36\pi \text{ sq in.} \approx 113\frac{1}{7} \text{ sq in.}$

$$29. V = s^3 = (8 \text{ cm})^3 = 512 \text{ cu cm}$$

$$SA = 6s^2 = 6(8 \text{ cm})^2 = 6 \cdot 64 \text{ sq cm} = 384 \text{ sq cm}$$

$$33. r = \frac{1}{2} \cdot d = \frac{1}{2} \cdot 10 \text{ in.} = 5 \text{ in.}$$

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi (5 \text{ in.})^3$$

$$= \frac{4}{3}\pi \cdot 125 \text{ cu in.}$$

$$= \frac{500}{3}\pi \text{ cu in.}$$

$$\approx \frac{500}{3} \cdot \frac{22}{7} \text{ cu in.} = 523\frac{17}{21} \text{ cu in.}$$

$$SA = 4\pi r^2$$

$$= 4\pi (5 \text{ in.})^2$$

$$= 4\pi \cdot 25 \text{ sq in.}$$

$$= 100\pi \text{ sq in.}$$

$$\approx 100 \cdot \frac{22}{7} \text{ sq in.} = 314\frac{2}{7} \text{ sq in.}$$

$$37. V = \frac{1}{3}s^2h$$

$$= \frac{1}{3}(5 \text{ cm})^2(9 \text{ cm})$$

$$= \frac{1}{3} \cdot 25 \text{ sq cm} \cdot 9 \text{ cm}$$

$$= 75 \text{ cu cm}$$

$$41. V = lwh = (2 \text{ ft})(1.4 \text{ ft})(3 \text{ ft}) = 8.4 \text{ cu ft}$$

$$SA = 2lh + 2wh + 2lw$$

$$= 2(2 \text{ ft})(3 \text{ ft}) + 2(1.4 \text{ ft})(3 \text{ ft}) + 2(2 \text{ ft})(1.4 \text{ ft})$$

$$= 12 \text{ sq ft} + 8.4 \text{ sq ft} + 5.6 \text{ sq ft}$$

$$= 26 \text{ sq ft}$$

$$45. A = l \cdot w = 7 \text{ ft} \cdot 6 \text{ ft} = 42 \text{ sq ft}$$

$$4 \cdot 42 \text{ sq ft} = 168 \text{ sq ft}$$

Four panels have an area of 168 square feet.

$$49. A = \frac{1}{2}(b + B) \cdot h$$

$$= \frac{1}{2}(140 \text{ ft} + 90 \text{ ft}) \cdot 80 \text{ ft}$$

$$= \frac{1}{2} \cdot 230 \text{ ft} \cdot 80 \text{ ft}$$

$$= 9200 \text{ sq ft}$$

$$53. \text{ a. } A = \frac{1}{2}(b + B) \cdot h$$

$$= \frac{1}{2}(25 \text{ ft} + 36 \text{ ft}) \cdot 12\frac{1}{2} \text{ ft}$$

$$= \frac{1}{2} \cdot 61 \text{ ft} \cdot 12\frac{1}{2} \text{ ft}$$

$$= 381\frac{1}{4} \text{ sq ft}$$

To the nearest square foot, the area is 381 square feet.

b. Since each square covers 100 square feet, 4 squares of shingles need to be purchased.

$$57. r = \frac{1}{2} \cdot d = \frac{1}{2} \cdot 4 \text{ ft} = 2 \text{ ft}$$

$$A = \pi r^2 = \pi (2 \text{ ft})^2 = \pi \cdot 4 \text{ sq ft}$$

The area of the pizza is 4π square feet, or approximately $4 \cdot 3.14 = 12.56$ square feet.

$$61. A = l \cdot w = 16 \text{ ft} \cdot 10\frac{1}{2} \text{ ft}$$

$$= 16 \text{ ft} \cdot \frac{21}{2} \text{ ft}$$

$$= 168 \text{ sq ft}$$

The area of the wall is 168 square feet.

$$65. V = lwh = (2 \text{ in.})(2 \text{ in.})(2.2 \text{ in.}) = 8.8 \text{ cu in.}$$

The volume of the Space Cube is 8.8 cubic inches.

$$69. 3^2 = 3 \cdot 3 = 9$$

$$73. 4^2 + 2^2 = 4 \cdot 4 + 2 \cdot 2 = 16 + 4 = 20$$

77. Carpet covers the entire floor of a room, so the situation involves area.

81. A wallpaper border goes around the edge of a room, so the situation involves perimeter.

$$85. r = \frac{1}{2} \cdot d = \frac{1}{2} \cdot 20 \text{ m} = 10 \text{ m}$$

$$V = \frac{1}{2} \cdot \frac{4}{3}\pi r^3$$

$$= \frac{2}{3}\pi (10 \text{ m})^3$$

$$= \frac{2}{3}\pi \cdot 1000 \text{ cu m}$$

$$= \frac{2000}{3}\pi \text{ cu m}$$

$$\approx \frac{2000}{3} \cdot 3.14 \text{ cu m}$$

$$\approx 2093.33 \text{ cu m}$$

The volume of the dome is about 2093.33 cubic meters.

89. The area of the shaded region is the area of the square minus the area of the circle.

$$\text{Square: } A = s^2 = (6 \text{ in.})^2 = 36 \text{ sq in.}$$

Circle:

$$r = \frac{1}{2} \cdot d = \frac{1}{2}(6 \text{ in.}) = 3 \text{ in.}$$

$$A = \pi \cdot r^2 = \pi (3 \text{ in.})^2 = 9\pi \text{ sq in.} \approx 28.26 \text{ sq in.}$$

$$36 \text{ sq in.} - 28.26 \text{ sq in.} = 7.74 \text{ sq in.}$$

The shaded region has an area of approximately 7.74 square inches.

93. no; answers may vary

Exercise Set 9.4

$$1. 60 \text{ in.} = \frac{60 \text{ in.}}{1} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} = \frac{60}{12} \text{ ft} = 5 \text{ ft}$$

$$5. 42,240 \text{ ft} = \frac{42,240 \text{ ft}}{1} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}}$$

$$= \frac{42,240}{5280} \text{ mi}$$

$$= 8 \text{ mi}$$

$$9. 10 \text{ ft} = \frac{10 \text{ ft}}{1} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} = \frac{10}{3} \text{ yd} = 3\frac{1}{3} \text{ yd}$$

$$13. 162 \text{ in.} = \frac{162 \text{ in.}}{1} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} \cdot \frac{1 \text{ yd}}{3 \text{ ft}}$$

$$= \frac{162}{36} \text{ yd}$$

$$= 4.5 \text{ yd}$$

$$17. 40 \text{ ft} = \frac{40 \text{ ft}}{1} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} = \frac{40}{3} \text{ yd}$$

$$13 \text{ yd } 1 \text{ ft}$$

$$\begin{array}{r} 3 \overline{)40} \\ -3 \\ \hline 10 \end{array}$$

$$\begin{array}{r} -3 \\ \hline 10 \end{array}$$

$$\begin{array}{r} -9 \\ \hline 1 \end{array}$$

$$\begin{array}{r} -9 \\ \hline 1 \end{array}$$

$$40 \text{ ft} = 13 \text{ yd } 1 \text{ ft}$$

$$21. 10,000 \text{ ft} = \frac{10,000 \text{ ft}}{1} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} = \frac{10,000}{5280} \text{ mi}$$

$$1 \text{ mi } 4720 \text{ ft}$$

$$\begin{array}{r} 5280 \overline{)10,000} \\ -5280 \\ \hline 4720 \end{array}$$

$$\begin{array}{r} -5280 \\ \hline 4720 \end{array}$$

$$10,000 \text{ ft} = 1 \text{ mi } 4720 \text{ ft}$$

$$25. 8 \text{ yd } 2 \text{ ft} = \frac{8 \text{ yd}}{1} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} + 2 \text{ ft}$$

$$= 24 \text{ ft} + 2 \text{ ft}$$

$$= 26 \text{ ft}$$

$$29. 3 \text{ ft } 10 \text{ in.} + 7 \text{ ft } 4 \text{ in.} = 10 \text{ ft } 14 \text{ in.}$$

$$= 10 \text{ ft} + 1 \text{ ft } 2 \text{ in.}$$

$$= 11 \text{ ft } 2 \text{ in.}$$

$$33. 22 \text{ ft } 8 \text{ in.}$$

$$\begin{array}{r} -16 \text{ ft } 3 \text{ in.} \\ \hline 6 \text{ ft } 5 \text{ in.} \end{array}$$

$$6 \text{ ft } 5 \text{ in.}$$

$$37. 28 \text{ ft } 8 \text{ in.} \div 2 = 14 \text{ ft } 4 \text{ in.}$$

$$41. 60 \text{ m} = \frac{60 \text{ m}}{1} \cdot \frac{100 \text{ cm}}{1 \text{ m}} = 6000 \text{ cm}$$

$$45. 500 \text{ m} = \frac{500 \text{ m}}{1} \cdot \frac{1 \text{ km}}{1000 \text{ m}} = \frac{500}{1000} \text{ km} = 0.5 \text{ km}$$

$$49. 1500 \text{ cm} = \frac{1500 \text{ cm}}{1} \cdot \frac{1 \text{ m}}{100 \text{ cm}} = \frac{1500}{100} \text{ m} = 15 \text{ m}$$

$$53. 7 \text{ km} = \frac{7 \text{ km}}{1} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 7000 \text{ m}$$

$$57. 20.1 \text{ mm} = \frac{20.1 \text{ mm}}{1} \cdot \frac{1 \text{ dm}}{100 \text{ mm}}$$

$$= \frac{20.1}{100} \text{ dm}$$

$$= 0.201 \text{ dm}$$

$$61. 8.6 \text{ m}$$

$$\begin{array}{r} +0.34 \text{ m} \\ \hline 8.94 \text{ m} \end{array}$$

$$8.94 \text{ m}$$

$$65. 24.8 \text{ mm} \quad 24.8 \text{ mm} \quad 2.48 \text{ cm}$$

$$\begin{array}{r} -1.19 \text{ cm} \\ \hline 12.9 \text{ mm} \end{array} \quad \begin{array}{r} -11.9 \text{ mm} \\ \hline 12.9 \text{ mm} \end{array} \quad \text{or} \quad \begin{array}{r} -1.19 \text{ cm} \\ \hline 1.29 \text{ cm} \end{array}$$

$$12.9 \text{ mm} \quad 1.29 \text{ cm}$$

$$69. 18.3 \text{ m} \times 3 = 54.9 \text{ m}$$

	Yards	Feet	Inches
Chrysler Building in New York City	$348\frac{2}{3}$	1046	12,552

	Meters	Millimeters	Kilometers	Centimeters
77. Length of elephant	5	5000	0.005	500
81. Distance from London to Paris	342,000	342,000,000	342	34,200,000

$$85. 6000 \text{ ft}$$

$$\begin{array}{r} -900 \text{ ft} \\ \hline 5100 \text{ ft} \end{array}$$

$$5100 \text{ ft}$$

The Grand Canyon of the Colorado River is 5100 feet deeper than the Grand Canyon of the Yellowstone River.

$$89. 80 \text{ mm} \quad 80.0 \text{ mm}$$

$$\begin{array}{r} -5.33 \text{ cm} \\ \hline -53.3 \text{ mm} \end{array}$$

$$26.7 \text{ mm}$$

The ice must be 26.7 mm thicker before skating is allowed.

$$3.35$$

$$93. 20 \overline{)67.00}$$

$$\begin{array}{r} -60 \\ \hline 70 \end{array}$$

$$\begin{array}{r} -60 \\ \hline 100 \end{array}$$

$$\begin{array}{r} -100 \\ \hline 0 \end{array}$$

$$0$$

$$0$$

$$0$$

Each piece will be 3.35 meters long.

$$97. 0.21 = \frac{21}{100}$$

$$101. \frac{1}{4} = \frac{1}{4} \cdot \frac{25}{25} = \frac{25}{100} = 0.25$$

105. Yes, glass for a drinking glass being 2 millimeters thick is reasonable.

109. 5 yd 2 in. is close to 5 yd. 7 yd 30 in. is close to 7 yd 36 in. = 8 yd. Estimate: 5 yd + 8 yd = 13 yd.

113. answers may vary

Exercise Set 9.5

$$1. 2 \text{ lb} = \frac{2 \text{ lb}}{1} \cdot \frac{16 \text{ oz}}{1 \text{ lb}} = 2 \cdot 16 \text{ oz} = 32 \text{ oz}$$

$$5. 18,000 \text{ lb} = \frac{18,000 \text{ lb}}{1} \cdot \frac{1 \text{ ton}}{2000 \text{ lb}}$$

$$= \frac{18,000}{2000} \text{ tons}$$

$$= 9 \text{ tons}$$

$$9. 3500 \text{ lb} = \frac{3500 \text{ lb}}{1} \cdot \frac{1 \text{ ton}}{2000 \text{ lb}}$$

$$= \frac{3500}{2000} \text{ tons}$$

$$= \frac{7}{4} \text{ tons}$$

$$= 1\frac{3}{4} \text{ tons}$$

$$13. 4.9 \text{ tons} = \frac{4.9 \text{ tons}}{1} \cdot \frac{2000 \text{ lb}}{1 \text{ ton}}$$

$$= 4.9 \cdot 2000 \text{ lb}$$

$$= 9800 \text{ lb}$$

$$17. 2950 \text{ lb} = \frac{2950 \text{ lb}}{1} \cdot \frac{1 \text{ ton}}{2000 \text{ lb}}$$

$$= \frac{2950}{2000} \text{ tons}$$

$$= \frac{59}{40} \text{ tons}$$

$$\approx 1.5 \text{ tons}$$

$$\begin{aligned}
 21. \quad 5\frac{3}{4} \text{ lb} &= \frac{23}{4} \text{ lb} \\
 &= \frac{23}{4} \text{ lb} \cdot \frac{16 \text{ oz}}{1 \text{ lb}} \\
 &= \frac{23}{4} \cdot 16 \text{ oz} \\
 &= 23 \cdot 4 \text{ oz} \\
 &= 92 \text{ oz}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad 89 \text{ oz} &= \frac{89 \text{ oz}}{1} \cdot \frac{1 \text{ lb}}{16 \text{ oz}} = \frac{89}{16} \text{ lb} \\
 &= 5 \text{ lb } 9 \text{ oz} \\
 16 \overline{)89} \\
 \underline{-80} & \\
 9 &
 \end{aligned}$$

$$89 \text{ oz} = 5 \text{ lb } 9 \text{ oz}$$

$$\begin{aligned}
 29. \quad 3 \text{ tons } 1820 \text{ lb} + 4 \text{ tons } 930 \text{ lb} \\
 &= 7 \text{ tons } 2750 \text{ lb} \\
 &= 7 \text{ tons} + 1 \text{ ton } 750 \text{ lb} \\
 &= 8 \text{ tons } 750 \text{ lb}
 \end{aligned}$$

$$\begin{array}{r}
 33. \quad 12 \text{ lb } 4 \text{ oz} \quad 11 \text{ lb } 20 \text{ oz} \\
 \underline{- 3 \text{ lb } 9 \text{ oz}} \quad \underline{- 3 \text{ lb } 9 \text{ oz}} \\
 8 \text{ lb } 11 \text{ oz}
 \end{array}$$

$$\begin{aligned}
 37. \quad 6 \text{ tons } 1500 \text{ lb} \div 5 &= \frac{6}{5} \text{ tons } 300 \text{ lb} \\
 &= 1\frac{1}{5} \text{ tons } 300 \text{ lb} \\
 &= 1 \text{ ton} + \frac{2000 \text{ lb}}{5} + 300 \text{ lb} \\
 &= 1 \text{ ton} + 400 \text{ lb} + 300 \text{ lb} \\
 &= 1 \text{ ton } 700 \text{ lb}
 \end{aligned}$$

$$41. \quad 4 \text{ g} = \frac{4 \text{ g}}{1} \cdot \frac{1000 \text{ mg}}{1 \text{ g}} = 4 \cdot 1000 \text{ mg} = 4000 \text{ mg}$$

$$45. \quad 48 \text{ mg} = \frac{48 \text{ mg}}{1} \cdot \frac{1 \text{ g}}{1000 \text{ mg}} = \frac{48}{1000} \text{ g} = 0.048 \text{ g}$$

$$\begin{aligned}
 49. \quad 15.14 \text{ g} &= \frac{15.14 \text{ g}}{1} \cdot \frac{1000 \text{ mg}}{1 \text{ g}} \\
 &= 15.14 \cdot 1000 \text{ mg} \\
 &= 15,140 \text{ mg}
 \end{aligned}$$

$$\begin{aligned}
 53. \quad 35 \text{ hg} &= \frac{35 \text{ hg}}{1} \cdot \frac{10,000 \text{ cg}}{1 \text{ hg}} \\
 &= 35 \cdot 10,000 \text{ cg} \\
 &= 350,000 \text{ cg}
 \end{aligned}$$

$$\begin{aligned}
 57. \quad 205 \text{ mg} + 5.61 \text{ g} &= 0.205 \text{ g} + 5.61 \text{ g} = 5.815 \text{ g} \\
 \text{or} \\
 205 \text{ mg} + 5.61 \text{ g} &= 205 \text{ mg} + 5610 \text{ mg} \\
 &= 5815 \text{ mg}
 \end{aligned}$$

$$\begin{aligned}
 61. \quad 1.61 \text{ kg} - 250 \text{ g} &= 1.61 \text{ kg} - 0.250 \text{ kg} = 1.36 \text{ kg} \\
 \text{or} \\
 1.61 \text{ kg} - 250 \text{ g} &= 1610 \text{ g} - 250 \text{ g} = 1360 \text{ g}
 \end{aligned}$$

$$65. \quad 17 \text{ kg} \div 8 = \frac{17}{8} \text{ kg}$$

$$\begin{array}{r}
 2.125 \\
 8 \overline{)17.000} \\
 \underline{-16} \\
 10 \\
 \underline{-8} \\
 20 \\
 \underline{-16} \\
 40 \\
 \underline{-40} \\
 0
 \end{array}$$

$$17 \text{ kg} \div 8 = 2.125 \text{ kg}$$

Object	Tons	Pounds	Ounces
A 12-inch cube of osmium	$\frac{269}{400}$ or 0.6725	1345	21,520

Object	Grams	Kilograms	Milligrams	Centigrams
A six-year-old boy	21,000	21	21,000,000	2,100,000

$$77. \quad 0.09 \text{ g} = \frac{0.09 \text{ g}}{1} \cdot \frac{1000 \text{ mg}}{1 \text{ g}} = 0.09 \cdot 1000 \text{ mg} = 90 \text{ mg}$$

$$90 \text{ mg} - 60 \text{ mg} = 30 \text{ mg}$$

The extra-strength tablet contains 30 mg more medication.

$$\begin{array}{r}
 81. \quad 64 \text{ lb } 8 \text{ oz} \quad 63 \text{ lb } 24 \text{ oz} \\
 \underline{-28 \text{ lb } 10 \text{ oz}} \quad \underline{-28 \text{ lb } 10 \text{ oz}} \\
 35 \text{ lb } 14 \text{ oz}
 \end{array}$$

Carla's zucchini was 35 lb 14 oz lighter than the record weight.

$$85. \quad 3 \times 16 = 48$$

3 cartons contain 48 boxes of fruit.

$$3 \text{ mg} \times 48 = 144 \text{ mg}$$

3 cartons contain 144 mg of preservatives.

$$\begin{aligned}
 89. \quad 3 \text{ lb } 4 \text{ oz} \times 10 &= 30 \text{ lb } 40 \text{ oz} \\
 &= 30 \text{ lb} + 2 \text{ lb } 8 \text{ oz} \\
 &= 32 \text{ lb } 8 \text{ oz}
 \end{aligned}$$

Each box weighs 32 lb 8 oz.

$$\begin{aligned}
 32 \text{ lb } 8 \text{ oz} \times 4 &= 128 \text{ lb } 32 \text{ oz} \\
 &= 128 \text{ lb} + 2 \text{ lb} \\
 &= 130 \text{ lb}
 \end{aligned}$$

4 boxes of meat weigh 130 lb.

$$93. \quad \frac{4}{25} = \frac{4}{25} \cdot \frac{4}{4} = \frac{16}{100} = 0.16$$

97. No, a pill containing 2 kg of medication is not reasonable.

101. No, a professor weighing less than 150 g is not reasonable.

105. True, a kilogram is 1000 grams.

Exercise Set 9.6

$$1. \quad 32 \text{ fl oz} = \frac{32 \text{ fl oz}}{1} \cdot \frac{1 \text{ c}}{8 \text{ fl oz}} = \frac{32}{8} \text{ c} = 4 \text{ c}$$

$$5. \quad 14 \text{ qt} = \frac{14 \text{ qt}}{1} \cdot \frac{1 \text{ gal}}{4 \text{ qt}} = \frac{14}{4} \text{ gal} = 3\frac{1}{2} \text{ gal}$$

$$9. \quad 2 \text{ qt} = \frac{2 \text{ qt}}{1} \cdot \frac{2 \text{ pt}}{1 \text{ qt}} \cdot \frac{2 \text{ c}}{1 \text{ pt}} = 2 \cdot 2 \cdot 2 \text{ c} = 8 \text{ c}$$

$$13. 42 \text{ c} = \frac{42 \text{ c}}{1} \cdot \frac{1 \text{ qt}}{4 \text{ c}} = \frac{42}{4} \text{ qt} = 10\frac{1}{2} \text{ qt}$$

$$17. 5 \text{ gal } 3 \text{ qt} = \frac{5 \text{ gal}}{1} \cdot \frac{4 \text{ qt}}{1 \text{ gal}} + 3 \text{ qt} \\ = 5 \cdot 4 \text{ qt} + 3 \text{ qt} \\ = 20 \text{ qt} + 3 \text{ qt} \\ = 23 \text{ qt}$$

$$21. 58 \text{ qt} = 56 \text{ qt} + 2 \text{ qt} \\ = \frac{56 \text{ qt}}{1} \cdot \frac{1 \text{ gal}}{4 \text{ qt}} + 2 \text{ qt} \\ = \frac{56}{4} \text{ gal} + 2 \text{ qt} \\ = 14 \text{ gal } 2 \text{ qt}$$

$$25. 2\frac{3}{4} \text{ gal} = \frac{11}{4} \text{ gal} \\ = \frac{11}{4} \text{ gal} \cdot \frac{4 \text{ qt}}{1 \text{ gal}} \cdot \frac{2 \text{ pt}}{1 \text{ qt}} \\ = \frac{11}{4} \cdot 4 \cdot 2 \text{ pt} \\ = 22 \text{ pt}$$

$$29. 1 \text{ c } 5 \text{ fl oz} + 2 \text{ c } 7 \text{ fl oz} = 3 \text{ c } 12 \text{ fl oz} \\ = 3 \text{ c} + 1 \text{ c } 4 \text{ fl oz} \\ = 4 \text{ c } 4 \text{ fl oz}$$

$$33. \begin{array}{r} 3 \text{ gal } 1 \text{ qt} \qquad 2 \text{ gal } 5 \text{ qt} \qquad 2 \text{ gal } 4 \text{ qt } 2 \text{ pt} \\ \underline{\qquad 1 \text{ qt } 1 \text{ pt} \qquad \underline{\qquad 1 \text{ qt } 1 \text{ pt} \qquad \underline{\qquad 1 \text{ qt } 1 \text{ pt} \qquad} \\ \qquad \qquad \qquad 2 \text{ gal } 3 \text{ qt } 1 \text{ pt} \end{array}$$

$$37. 9 \text{ gal } 2 \text{ qt} \div 2 = (8 \text{ gal } 4 \text{ qt} + 2 \text{ qt}) \div 2 \\ = 8 \text{ gal } 6 \text{ qt} \div 2 \\ = 4 \text{ gal } 3 \text{ qt}$$

$$41. 0.16 \text{ L} = \frac{0.16 \text{ L}}{1} \cdot \frac{1 \text{ kl}}{1000 \text{ L}} = \frac{0.16}{1000} \text{ kl} = 0.00016 \text{ kl}$$

$$45. 3.2 \text{ L} = \frac{3.2 \text{ L}}{1} \cdot \frac{100 \text{ cl}}{1 \text{ L}} = 3.2 \cdot 100 \text{ cl} = 320 \text{ cl}$$

$$49. 64 \text{ ml} = \frac{64 \text{ ml}}{1} \cdot \frac{1 \text{ L}}{1000 \text{ ml}} = \frac{64}{1000} \text{ L} = 0.064 \text{ L}$$

$$53. 3.6 \text{ L} = \frac{3.6 \text{ L}}{1} \cdot \frac{1000 \text{ ml}}{1 \text{ L}} = 3.6 \cdot 1000 \text{ ml} = 3600 \text{ ml}$$

$$57. 2700 \text{ ml} + 1.8 \text{ L} = 2.7 \text{ L} + 1.8 \text{ L} = 4.5 \text{ L} \\ \text{or} \\ 2700 \text{ ml} + 1.8 \text{ L} = 2700 \text{ ml} + 1800 \text{ ml} = 4500 \text{ ml}$$

$$61. 17,500 \text{ ml} - 0.9 \text{ L} = 17,500 \text{ ml} - 900 \text{ ml} \\ = 16,600 \text{ ml}$$

$$\text{or} \\ 17,500 \text{ ml} - 0.9 \text{ L} = 17.5 \text{ L} - 0.9 \text{ L} = 16.6 \text{ L}$$

$$65. 81.2 \text{ L} \div 0.5 = 81.2 \text{ L} \div \frac{1}{2} \\ = 81.2 \text{ L} \cdot 2 \\ = 162.4 \text{ L}$$

Capacity	Cups	Gallons	Quarts	Pints
Your kidneys filter about this amount of blood every minute	4	$\frac{1}{4}$	1	2

$$73. 354 \text{ ml} + 18.6 \text{ L} = 0.354 \text{ L} + 18.6 \text{ L} = 18.954 \text{ L}$$

There were 18.954 liters of gasoline in her tank.

$$77. 5 \text{ pt } 1 \text{ c} + 2 \text{ pt } 1 \text{ c} = 7 \text{ pt } 2 \text{ c} \\ = 7 \text{ pt} + 1 \text{ pt} \\ = 8 \text{ pt} \\ = \frac{8 \text{ pt}}{1} \cdot \frac{1 \text{ qt}}{2 \text{ pt}} \\ = \frac{8}{2} \text{ qt} \\ = 4 \text{ qt} \\ = \frac{4 \text{ qt}}{1} \cdot \frac{1 \text{ gal}}{4 \text{ qt}} \\ = \frac{4}{4} \text{ gal} \\ = 1 \text{ gal}$$

Yes, the liquid can be poured into the container without causing it to overflow.

$$81. \frac{20}{25} = \frac{4 \cdot 5}{5 \cdot 5} = \frac{4}{5}$$

$$85. \frac{72}{80} = \frac{8 \cdot 9}{8 \cdot 10} = \frac{9}{10}$$

89. No, a tub filled with 3000 ml of hot water is not reasonable.

93. answers may vary

97. B indicates 1.5 cc.

101. B indicates 54 u or 0.54 cc.

Exercise Set 9.7

$$1. 756 \text{ ml} \approx \frac{756 \text{ ml}}{1} \cdot \frac{1 \text{ fl oz}}{29.57 \text{ ml}} \approx 25.57 \text{ fl oz}$$

$$5. 1000 \text{ g} \approx \frac{1000 \text{ g}}{1} \cdot \frac{0.04 \text{ oz}}{1 \text{ g}} \approx 40 \text{ oz}$$

$$9. 14.5 \text{ L} \approx \frac{14.5 \text{ L}}{1} \cdot \frac{0.26 \text{ gal}}{1 \text{ L}} \approx 3.77 \text{ gal}$$

	Meters	Yards	Centimeters	Feet	Inches
The height of a woman	1.5	$1\frac{2}{3}$	150	5	60

$$17. 10 \text{ cm} = \frac{10 \text{ cm}}{1} \cdot \frac{1 \text{ in.}}{2.54 \text{ cm}} \approx 3.94 \text{ in.}$$

The balance beam is approximately 3.94 inches wide.

$$21. 200 \text{ mg} = 0.2 \text{ g} \approx \frac{0.2 \text{ g}}{1} \cdot \frac{0.04 \text{ oz}}{1 \text{ g}} \approx 0.008 \text{ oz}$$

$$25. 16 \text{ billion km} \approx \frac{16 \text{ billion km}}{1} \cdot \frac{0.62 \text{ mi}}{1 \text{ km}} \approx 9.92 \text{ billion mi}$$

The antenna can track a spacecraft that is 9.92 billion miles from Earth.

$$29. 4500 \text{ km} \approx \frac{4500 \text{ km}}{1} \cdot \frac{0.62 \text{ mi}}{1 \text{ km}} \approx 2790 \text{ mi}$$

The trip is about 2790 miles.

$$33. 1.5 \text{ lb} - 1.25 \text{ lb} = 0.25 \text{ lb} \\ 0.25 \text{ lb} \approx \frac{0.25 \text{ lb}}{1} \cdot \frac{0.45 \text{ kg}}{1 \text{ lb}} \cdot \frac{1000 \text{ g}}{1 \text{ kg}} \approx 112.5 \text{ g}$$

The difference is approximately 112.5 grams.

$$37. 8 \text{ m} \approx \frac{8 \text{ m}}{1} \cdot \frac{3.28 \text{ ft}}{1 \text{ m}} \approx 26.24 \text{ ft}$$

The base diameter is approximately 26.24 feet.

41. One dose every 4 hours results in $\frac{24}{4} = 6$ doses per day and $6 \times 7 = 42$ doses per week.

$$5 \text{ ml} \times 42 = 210 \text{ ml}$$

$$210 \text{ ml} \approx \frac{210 \text{ ml}}{1} \cdot \frac{1 \text{ fl oz}}{29.57 \text{ ml}} \approx 7.1 \text{ fl oz}$$

8 fluid ounces of medicine should be purchased.

45. A liter has greater capacity than a quart; **b**.

49. An $8\frac{1}{2}$ -ounce glass of water has a capacity of about

$$250 \text{ ml} \left(\frac{1}{4} \text{ L} \right); \text{ d.}$$

$$\begin{aligned} 53. \quad C &= \frac{5}{9} (F - 32) \\ &= \frac{5}{9} (77 - 32) \end{aligned}$$

$$= \frac{5}{9} (45)$$

$$= 25$$

77°F is 25°C.

$$\begin{aligned} 57. \quad F &= \frac{9}{5} C + 32 \\ &= \frac{9}{5} (50) + 32 \\ &= 90 + 32 \\ &= 122 \end{aligned}$$

50°C is 122°F.

$$\begin{aligned} 61. \quad C &= \frac{5}{9} (F - 32) \\ &= \frac{5}{9} (20 - 32) \\ &= \frac{5}{9} (-12) \\ &\approx -6.7 \end{aligned}$$

20°F is -6.7°C.

$$\begin{aligned} 65. \quad F &= 1.8C + 32 \\ &= 1.8(92) + 32 \\ &= 165.6 + 32 \\ &= 197.6 \end{aligned}$$

92°C is 197.6°F.

$$\begin{aligned} 69. \quad C &= \frac{5}{9} (F - 32) \\ &= \frac{5}{9} (134 - 32) \\ &= \frac{5}{9} (102) \\ &\approx 56.7 \end{aligned}$$

134°F is 56.7°C.

$$\begin{aligned} 73. \quad C &= \frac{5}{9} (F - 32) \\ &= \frac{5}{9} (70 - 32) \\ &= \frac{5}{9} (38) \\ &\approx 21.1 \end{aligned}$$

70°F is 21.1°C.

$$\begin{aligned} 77. \quad F &= 1.8C + 32 \\ &= 1.8(4000) + 32 \\ &= 7200 + 32 \\ &= 7232 \end{aligned}$$

4000°C is 7232°F.

$$81. \quad 3[(1 + 5) \cdot (8 - 6)] = 3(6 \cdot 2) = 3(12) = 36$$

85. No, a fever of 40°F is not reasonable.

89. Yes, a fever of 40°C is reasonable.

$$93. \quad 40 \text{ in.} = \frac{40 \text{ in.}}{1} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} = 101.6 \text{ cm}$$

$$BSA = \sqrt{\frac{50 \times 101.6}{3600}} \approx 1.19$$

The BSA is approximately 1.19 sq m.

$$\begin{aligned} 97. \quad C &= \frac{5}{9} (F - 32) \\ &= \frac{5}{9} (371.9 - 32) \\ &= \frac{5}{9} (339.9) \\ &\approx 188.8 \end{aligned}$$

371.9°F is approximately 188.8°C.

Chapter 9 Test

1. The complement of an angle that measures 78° is an angle that measures $90^\circ - 78^\circ = 12^\circ$.

5. $\angle x$ and the angle marked 73° are vertical angles, so $m\angle x = 73^\circ$. $\angle x$ and $\angle y$ are alternate interior angles, so $m\angle y = m\angle x = 73^\circ$. $\angle x$ and $\angle z$ are corresponding angles, so $m\angle z = m\angle x = 73^\circ$.

$$\begin{aligned} 9. \quad P &= 2 \cdot l + 2 \cdot w \\ &= 2(7 \text{ yd}) + 2(5.3 \text{ yd}) \\ &= 14 \text{ yd} + 10.6 \text{ yd} \\ &= 24.6 \text{ yd} \end{aligned}$$

$$A = l \cdot w = 7 \text{ yd} \cdot 5.3 \text{ yd} = 37.1 \text{ sq yd}$$

$$13. \quad P = 4 \cdot s = 4 \cdot 4 \text{ in.} = 16 \text{ in.}$$

The perimeter of the photo is 16 inches.

$$17. \quad 2\frac{1}{2} \text{ gal} = \frac{2\frac{1}{2} \text{ gal}}{1} \cdot \frac{4 \text{ qt}}{1 \text{ gal}} = 10 \text{ qt}$$

$$21. \quad 40 \text{ mg} = \frac{40 \text{ mg}}{1} \cdot \frac{1 \text{ g}}{1000 \text{ mg}} = 0.04 \text{ g}$$

$$25. \quad 0.83 \text{ L} = \frac{0.83 \text{ L}}{1} \cdot \frac{1000 \text{ ml}}{1 \text{ L}} = 830 \text{ ml}$$

$$\begin{aligned} 29. \quad 5 \text{ gal } 2 \text{ qt} \div 2 &= 4 \text{ gal } 6 \text{ qt} \div 2 \\ &= \frac{4}{2} \text{ gal } \frac{6}{2} \text{ qt} \\ &= 2 \text{ gal } 3 \text{ qt} \end{aligned}$$

$$33. \quad F = 1.8C + 32 = 1.8(12.6) + 32 = 22.68 + 32 \approx 54.7$$

12.6°C is 54.7°F

$$\begin{aligned} 37. \quad & \begin{array}{r} 2 \text{ ft } 9 \text{ in.} \\ \times \quad 6 \\ \hline 12 \text{ ft } 54 \text{ in.} \end{array} = 12 \text{ ft } + 4 \text{ ft } 6 \text{ in.} = 16 \text{ ft } 6 \text{ in.} \\ & \text{Thus, 16 ft 6 in. of material is needed.} \end{aligned}$$

Chapter 10

Exercise Set 10.1

1. $(2x + 3) + (-7x - 27) = (2x - 7x) + (3 - 27)$
 $= -5x + (-24)$
 $= -5x - 24$
5. $(12y - 20) + (9y^2 + 13y - 20)$
 $= 9y^2 + (12y + 13y) + (-20 - 20)$
 $= 9y^2 + 25y - 40$
9. $-(9x - 16) = -1(9x - 16)$
 $= -1(9x) + (-1)(-16)$
 $= -9x + 16$
13. $(8a - 5) - (3a + 8) = (8a - 5) + (-3a - 8)$
 $= 8a - 3a - 5 - 8$
 $= 5a - 13$
17. $(10y^2 - 7) - (20y^3 - 2y^2 - 7)$
 $= (10y^2 - 7) + (-20y^3 + 2y^2 + 7)$
 $= (-20y^3) + (10y^2 + 2y^2) + (-7 + 7)$
 $= -20y^3 + 12y^2$
21. $\frac{13y^2 - 6y - 14}{-(5y^2 + 4y - 6)} + \frac{13y^2 - 6y - 14}{-5y^2 - 4y + 6}$
 $\frac{8y^2 - 10y - 8}{8}$
25. $(4y + 4) - (3y + 8) = (4y + 4) + (-3y - 8)$
 $= (4y - 3y) + (4 - 8)$
 $= y - 4$
29. $(5x + 4.5) + (-x - 8.6) = (5x - x) + (4.5 - 8.6)$
 $= 4x - 4.1$
33. $(21y - 4.6) - (36y - 8.2)$
 $= (21y - 4.6) + (-36y + 8.2)$
 $= (21y - 36y) + (-4.6 + 8.2)$
 $= -15y + 3.6$
37. $(2b^3 + 5b^2 - 5b - 8) + (8b^2 + 9b + 6)$
 $= 2b^3 + (5b^2 + 8b^2) + (-5b + 9b) + (-8 + 6)$
 $= 2b^3 + 13b^2 + 4b - 2$
41. $\left(3z + \frac{6}{7}\right) - \left(3z - \frac{3}{7}\right) = \left(3z + \frac{6}{7}\right) + \left(-3z + \frac{3}{7}\right)$
 $= (3z - 3z) + \left(\frac{6}{7} + \frac{3}{7}\right)$
 $= \frac{9}{7}$
45. $x^2 - 6x + 3 = 2^2 - 6(2) + 3$
 $= 4 - 6(2) + 3$
 $= 4 - 12 + 3$
 $= -5$
49. $2x + 10 = 2(5) + 10 = 10 + 10 = 20$
53. $2x^2 + 4x - 20 = 2(5)^2 + 4(5) - 20$
 $= 2(25) + 4(5) - 20$
 $= 50 + 20 - 20$
 $= 50$
57. Let $x = 10$.
 $3000 + 20x = 3000 + 20(10) = 3000 + 200 = 3200$
It costs \$3200 to manufacture 10 file cabinets.

61. Since $2022 - 2014 = 8$, let $x = 8$.
 $8.7x^2 - 2.1x + 7.4 = 8.7(8)^2 - 2.1(8) + 7.4$
 $= 8.7(64) - 2.1(8) + 7.4$
 $= 556.8 - 16.8 + 7.4$
 $= 547.4$

Smartwatch sales are predicted to be 547.4 million or 547,400,000 in 2022.

65. Let $t = 3$.
 $1053 - 16t^2 = 1053 - 16(3)^2$
 $= 1053 - 16(9)$
 $= 1053 - 144$
 $= 909$

After 3 seconds, the object is 909 feet above the river.

69. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$
73. $x \cdot x \cdot x = x^3$
77. $P = (5x - 10) + (2x + 1) + (x + 11)$
 $= 5x + 2x + x - 10 + 1 + 11$
 $= 8x + 2$

The perimeter is $(8x + 2)$ inches.

81. $\frac{3x^2 + \underline{\hspace{1cm}}x - \underline{\hspace{1cm}}}{+ \underline{\hspace{1cm}}x^2 - \underline{\hspace{1cm}}6x + \underline{\hspace{1cm}}2}$
 $\frac{5x^2 + 14x - 4}{5x^2 + 14x - 4}$

Since $3x^2 + 2x^2 = 5x^2$, $20x - 6x = 14x$ and $-6 + 2 = -4$, the missing numbers are 20, 6, and 2.

$$(3x^2 + \underline{20x} - \underline{6}) + (\underline{2}x^2 - 6x + 2) = 5x^2 + 14x - 4$$

85. Let $t = 8$.
 $1053 - 16t^2 = 1053 - 16(8)^2$
 $= 1053 - 16(64)$
 $= 1053 - 1024$
 $= 29$

The height after 8 seconds is 29 feet.

- Let $t = 9$.
 $1053 - 16t^2 = 1053 - 16(9)^2$
 $= 1053 - 16(81)$
 $= 1053 - 1296$
 $= -243$

The height after 9 seconds is -243 feet. answers may vary

Exercise Set 10.2

1. $x^5 \cdot x^9 = x^{5+9} = x^{14}$
5. $3z^3 \cdot 5z^2 = (3 \cdot 5)(z^3 \cdot z^2) = 15z^5$
9. $2x \cdot 3x \cdot 7x = (2 \cdot 3 \cdot 7)(x \cdot x \cdot x) = 42x^3$
13. $(-5x^2y^3)(-5x^4y) = (-5)(-5)(x^2 \cdot x^4)(y^3 \cdot y^1)$
 $= 25x^6y^4$
17. $(x^5)^3 = x^{5 \cdot 3} = x^{15}$
21. $(b^7)^6 \cdot (b^2)^{10} = b^{7 \cdot 6} \cdot b^{2 \cdot 10}$
 $= b^{42} \cdot b^{20}$
 $= b^{42+20}$
 $= b^{62}$
25. $(a^{11}b^8)^3 = a^{11 \cdot 3}b^{8 \cdot 3} = a^{33}b^{24}$
29. $(-3y)(2y^7)^3 = (-3y) \cdot 2^3(y^7)^3$
 $= (-3y) \cdot 8y^{21}$
 $= (-3)(8)(y^1 \cdot y^{21})$
 $= -24y^{22}$

$$33. 7(x - 3) = 7x - 21$$

$$37. 9(x + 2y - 3) = 9x + 18y - 27$$

$$\begin{aligned} 41. \text{Area} &= \frac{1}{2}bh \\ &= \frac{1}{2} \cdot (6a^3b^4) \cdot (4ab) \\ &= \left(\frac{1}{2} \cdot 6 \cdot 4\right)(a^3 \cdot a)(b^4 \cdot b) \\ &= 12a^4b^5 \end{aligned}$$

The area is $12a^4b^5$ square meters.

$$45. (8.1x^{10})^5 = 8.1^5(x^{10})^5 = 34,867.84401x^{50}$$

49. answers may vary

Exercise Set 10.3

1. $3x(9x^2 - 3) = 3x \cdot 9x^2 + 3x \cdot (-3)$
 $= (3 \cdot 9)(x \cdot x^2) + (3)(-3)(x)$
 $= 27x^3 + (-9x)$
 $= 27x^3 - 9x$
5. $7x^2(6x^2 - 5x + 7)$
 $= (7x^2)(6x^2) + (7x^2)(-5x) + (7x^2)(7)$
 $= (7 \cdot 6)(x^2 \cdot x^2) + (7)(-5)(x^2 \cdot x) + (7 \cdot 7)x^2$
 $= 42x^4 - 35x^3 + 49x^2$
9. $(2x - 6)(x + 4)$
 $= 2x(x + 4) - 6(x + 4)$
 $= 2x \cdot x + 2x \cdot 4 - 6 \cdot x - 6 \cdot 4$
 $= 2x^2 + 8x - 6x - 24$
 $= 2x^2 + 2x - 24$
13. $(a + 6)(a^2 - 6a + 3)$
 $= a(a^2 - 6a + 3) + 6(a^2 - 6a + 3)$
 $= a \cdot a^2 + a(-6a) + a \cdot 3 + 6 \cdot a^2 + 6(-6a) + 6 \cdot 3$
 $= a^3 - 6a^2 + 3a + 6a^2 - 36a + 18$
 $= a^3 - 33a + 18$
17. $(x^3 + 2x + x^2)(3x + 1 + x^2)$
 $= x^3(3x + 1 + x^2) + 2x(3x + 1 + x^2) + x^2(3x + 1 + x^2)$
 $= x^3 \cdot 3x + x^3 \cdot 1 + x^3 \cdot x^2 + 2x \cdot 3x + 2x \cdot 1 + 2x \cdot x^2$
 $+ x^2 \cdot 3x + x^2 \cdot 1 + x^2 \cdot x^2$
 $= 3x^4 + x^3 + x^5 + 6x^2 + 2x + 2x^3 + 3x^3 + x^2 + x^4$
 $= x^5 + 4x^4 + 6x^3 + 7x^2 + 2x$
21. $-2y^2(3y + y^2 - 6)$
 $= -2y^2 \cdot 3y + (-2y^2) \cdot y^2 + (-2y^2)(-6)$
 $= -6y^3 - 2y^4 + 12y^2$
25. $(2a + 3)(2a - 3) = 2a(2a - 3) + 3(2a - 3)$
 $= 2a \cdot 2a + 2a(-3) + 3 \cdot 2a + 3(-3)$
 $= 4a^2 - 6a + 6a - 9$
 $= 4a^2 - 9$
29. $\left(b + \frac{3}{5}\right)\left(b + \frac{4}{5}\right) = b\left(b + \frac{4}{5}\right) + \frac{3}{5}\left(b + \frac{4}{5}\right)$
 $= b^2 + \frac{4}{5}b + \frac{3}{5}b + \frac{3}{5} \cdot \frac{4}{5}$
 $= b^2 + \frac{7}{5}b + \frac{12}{25}$
33. $(7x + 5)^2 = (7x + 5)(7x + 5)$
 $= 7x(7x + 5) + 5(7x + 5)$
 $= 49x^2 + 35x + 35x + 25$
 $= 49x^2 + 70x + 25$

$$\begin{aligned} 37. (2x^2 - 3)(4x^3 + 2x - 3) \\ &= 2x^2(4x^3 + 2x - 3) - 3(4x^3 + 2x - 3) \\ &= 8x^5 + 4x^3 - 6x^2 - 12x^3 - 6x + 9 \\ &= 8x^5 - 8x^3 - 6x^2 - 6x + 9 \end{aligned}$$

$$\begin{aligned} 41. \quad & \frac{2z^2 - z + 1}{\times \frac{5z^2 + z - 2}{-4z^2 + 2z - 2}} \\ & \frac{2z^3 - z^2 + z}{10z^4 - 5z^3 + 5z^2} \\ & \frac{10z^4 - 3z^3}{10z^4 - 3z^3} + 3z - 2 \end{aligned}$$

$$45. 72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2$$

$$\begin{aligned} 49. (y - 6)(y^2 + 3y + 2) \\ &= y(y^2 + 3y + 2) - 6(y^2 + 3y + 2) \\ &= y^3 + 3y^2 + 2y - 6y^2 - 18y - 12 \\ &= y^3 - 3y^2 - 16y - 12 \end{aligned}$$

The area is $(y^3 - 3y^2 - 16y - 12)$ square feet.

53. answers may vary

Exercise Set 10.4

1. $48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$
 $15 = 3 \cdot 5$
GCF = 3
5. $12 = 2 \cdot 2 \cdot 3$
 $20 = 2 \cdot 2 \cdot 5$
 $36 = 2 \cdot 2 \cdot 3 \cdot 3$
GCF = $2 \cdot 2 = 4$
9. $y^7 = y^2 \cdot y^5$
 $y^2 = y^2$
 $y^{10} = y^2 \cdot y^8$
GCF = y^2
13. $x^3y^2 = x \cdot x^2 \cdot y^2$
 $xy^2 = x \cdot y^2$
 $x^4y^2 = x \cdot x^3 \cdot y^2$
GCF = $x \cdot y^2 = xy^2$
17. $2 = 2$
 $14 = 2 \cdot 7$
 $18 = 2 \cdot 3 \cdot 3$
GCF = 2
 $z^3 = z^3$
 $z^5 = z^3 \cdot z^2$
 $z^3 = z^3$
GCF = z^3
GCF = $2z^3$
21. $10a^6 = 5a^6 \cdot 2$
 $5a^8 = 5a^6 \cdot a^2$
GCF = $5a^6$
 $10a^6 - 5a^8 = 5a^6 \cdot 2 - 5a^6 \cdot a^2$
 $= 5a^6(2 - a^2)$
25. $z^7 = z^5 \cdot z^2$
 $6z^5 = z^5 \cdot 6$
GCF = z^5
 $z^7 - 6z^5 = z^5 \cdot z^2 - z^5 \cdot 6 = z^5(z^2 - 6)$

29. $12a^5 = 12a^5$

$36a^6 = 12a^5 \cdot 3a$

GCF = $12a^5$

$$12a^5 - 36a^6 = 12a^5 \cdot 1 - 12a^5 \cdot 3a$$
$$= 12a^5(1 - 3a)$$

33. $80\% = \frac{80}{100} = \frac{4}{5}$

37. a. area on the left: $x \cdot x = x^2$

area on the right: $2 \cdot x = 2x$

total area: $x^2 + 2x$

b. answers may vary; notice that $x^2 + 2x = x(x + 2)$

41. Let $x = 2$ and $z = 7$.

$(xy + z)^x = (2y + 7)^2$

$= (2y + 7)(2y + 7)$

$= (2y)(2y) + 2y(7) + 7(2y) + 7(7)$

$= 4y^2 + 14y + 14y + 49$

$= 4y^2 + 28y + 49$

Chapter 10 Test

1. $(11x - 3) + (4x - 1) = (11x + 4x) + (-3 - 1)$
$$= 15x + (-4)$$
$$= 15x - 4$$

5. Let $x = 8$.

$x^2 - 6x + 1 = 8^2 - 6(8) + 1$

$= 64 - 6(8) + 1$

$= 64 - 48 + 1$

$= 17$

9. $(6a^3)(-2a^7) = (6)(-2)(a^3 \cdot a^7) = -12a^{10}$

13. $-2y(y^3 + 6y^2 - 4)$

$= -2y \cdot y^3 - 2y \cdot 6y^2 - 2y \cdot (-4)$

$= -2y^4 - 12y^3 + 8y$

17. Area:

$(x + 7)(5x - 2) = x(5x - 2) + 7(5x - 2)$

$= 5x^2 - 2x + 35x - 14$

$= 5x^2 + 33x - 14$

The area is $(5x^2 + 33x - 14)$ square inches.

Perimeter:

$2(2x) + 2(5x - 2) = 4x + 10x - 4 = 14x - 4$

The perimeter is $(14x - 4)$ inches.

21. $10a^2 = 2a \cdot 5a$

$12a = 2a \cdot 6$

GCF = $2a$

$10a^2 + 12a = 2a \cdot 5a + 2a \cdot 6 = 2a(5a + 6)$

Appendices**Appendix B Exercise Set**

1. $\frac{x^3}{x} = \frac{x^3}{x^1} = x^{3-1} = x^2$

5. $\frac{p^7 q^{20}}{p q^{15}} = \frac{p^7}{p^1} \cdot \frac{q^{20}}{q^{15}} = p^{7-1} \cdot q^{20-15} = p^6 q^5$

9. $7^0 = 1$

13. $-7^0 = -(7^0) = -1$

17. $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

21. $3^{-1} + 2^{-1} = \frac{1}{3} + \frac{1}{2} = \frac{1}{3} \cdot \frac{2}{2} + \frac{1}{2} \cdot \frac{3}{3} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$

25. $\frac{x^{-2}}{x} = \frac{x^{-2}}{x^1} = x^{-2-1} = x^{-3} = \frac{1}{x^3}$

29. $3^{-2} + 3^{-1} = \frac{1}{3^2} + \frac{1}{3^1} = \frac{1}{9} + \frac{1}{3} = \frac{1}{9} + \frac{3}{9} = \frac{4}{9}$

33. $\frac{1}{p^{-4}} = p^4$

37. $(x^8 y^{-6})(x^{-2} y^{12}) = x^{8+(-2)} \cdot y^{-6+12}$
$$= x^6 y^6$$

41. $(5x^{-7})(3x^4) = 5 \cdot 3 \cdot x^{-7+4}$

$= 15 \cdot x^{-3}$

$= 15 \cdot \frac{1}{x^3}$

$= \frac{15}{x^3}$

45. $(8m^5 n^{-1})(7m^2 n^{-4}) = 8 \cdot 7 \cdot m^{5+2} n^{-1+(-4)}$

$= 56 \cdot m^7 \cdot n^{-5}$

$= 56 \cdot m^7 \cdot \frac{1}{n^5}$

$= \frac{56m^7}{n^5}$

49. $\frac{a^9 b^{14}}{ab} = \frac{a^9}{a^1} \cdot \frac{b^{14}}{b^1} = a^{9-1} \cdot b^{14-1} = a^8 b^{13}$

53. $3z^0 = 3 \cdot z^0 = 3 \cdot 1 = 3$

57. $8x^{-9} = 8 \cdot x^{-9} = 8 \cdot \frac{1}{x^9} = \frac{8}{x^9}$

61. $\frac{z^{-8}}{z^{-1}} = z^{-8-(-1)} = z^{-8+1} = z^{-7} = \frac{1}{z^7}$

65. $(a^{-2} b^3)(a^{10} b^{-11}) = a^{-2+10} \cdot b^{3+(-11)}$

$= a^8 \cdot b^{-8}$

$= a^8 \cdot \frac{1}{b^8}$

$= \frac{a^8}{b^8}$

Appendix C Exercise Set

1. $78,000 = 7.8 \times 10^4$

5. $0.00635 = 6.35 \times 10^{-3}$

9. $4200 = 4.2 \times 10^3$

13. $3.3 \times 10^{-2} = 0.033$

17. $7.0 \times 10^8 = 700,000,000$

21. $8.1 \times 10^{12} = 8,100,000,000,000$

25. $(1.2 \times 10^{-3})(3 \times 10^{-2}) = 1.2 \cdot 3 \times 10^{-3+(-2)}$
$$= 3.6 \times 10^{-5}$$
$$= 0.000036$$

29. $\frac{8 \times 10^{-1}}{16 \times 10^5} = \frac{8}{16} \cdot \frac{10^{-1}}{10^5}$
$$= 0.5 \times 10^{-1-5}$$
$$= 0.5 \times 10^{-6}$$
$$= 0.0000005$$

33. $(7.5 \times 10^5)(3600) = (7.5 \times 10^5)(3.6 \times 10^3)$
$$= 7.5 \cdot 3.6 \times 10^{5+3}$$
$$= 27 \times 10^8$$
$$= 2.7 \times 10^1 \times 10^8$$
$$= 2.7 \times 10^9$$

On average, 2.7×10^9 gallons of water flow over Niagara Falls each hour.

Practice Final Exam

1. $2^3 \cdot 5^2 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 = 200$

5. $\sqrt{49} = 7$ because $7^2 = 49$

9. $-\frac{8}{15y} - \frac{2}{15y} = \frac{-8-2}{15y} = \frac{-10}{15y} = -\frac{2 \cdot 5}{3 \cdot 5 \cdot y} = -\frac{2}{3y}$

13.
$$\begin{array}{r} 19 \quad 18 \frac{11}{11} \\ -2 \frac{3}{11} \\ \hline 16 \frac{8}{11} \end{array}$$

17. $6.1 = 6.1(100\%) = 610\%$

21. 34.8923 rounded to the nearest tenth is 34.9.

25.
$$\begin{aligned} -(3z + 2) - 5z - 18 &= -1(3z + 2) - 5z - 18 \\ &= -1 \cdot 3z + (-1) \cdot 2 - 5z - 18 \\ &= -3z - 2 - 5z - 18 \\ &= -3z - 5z - 2 - 18 \\ &= -8z - 20 \end{aligned}$$

29.
$$\begin{aligned} -4(x - 11) - 34 &= 10 - 12 \\ -4x + 44 - 34 &= 10 - 12 \\ -4x + 10 &= -2 \\ -4x + 10 - 10 &= -2 - 10 \\ -4x &= -12 \\ \frac{-4x}{-4} &= \frac{-12}{-4} \\ x &= 3 \end{aligned}$$

33.
$$\begin{aligned} \text{Perimeter} &= (20 + 10 + 20 + 10) \text{ yards} = 60 \text{ yards} \\ \text{Area} &= (\text{length})(\text{width}) \\ &= (20 \text{ yards})(10 \text{ yards}) \\ &= 200 \text{ square yards} \end{aligned}$$

37. Let x be the number of women runners entered in the race. Since the number of men entered in the race is 112 more than the number of women, the number of men is $x + 112$. Since the total number of runners in the race is 600, the sum of x and $x + 112$ is 600.

$$x + x + 112 = 600$$

$$2x + 112 = 600$$

$$2x + 112 - 112 = 600 - 112$$

$$2x = 488$$

$$\frac{2x}{2} = \frac{488}{2}$$

$$x = 244.$$

244 women entered the race.

41. $y = 3x - 5$

Find 3 ordered pair solutions.

Let $x = 0$.

$$y = 3x - 5$$

$$y = 3 \cdot 0 - 5$$

$$y = 0 - 5$$

$$y = -5$$

$$(0, -5)$$

Let $x = 1$.

$$y = 3x - 5$$

$$y = 3 \cdot 1 - 5$$

$$y = 3 - 5$$

$$y = -2$$

$$(1, -2)$$

Let $x = 2$.

$$y = 3x - 5$$

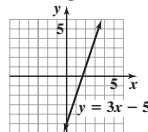
$$y = 3 \cdot 2 - 5$$

$$y = 6 - 5$$

$$y = 1$$

$$(2, 1)$$

Plot $(0, -5)$, $(1, -2)$, and $(2, 1)$. Then draw the line through them.



45. $(6a^3)(-2a^7) = (6)(-2)(a^3 \cdot a^7) = -12a^{10}$

49. The complement of an angle that measures 78° is an angle that measures $90^\circ - 78^\circ = 12^\circ$.

53. $2\frac{1}{2} \text{ gal} = \frac{2\frac{1}{2} \text{ gal}}{1} \cdot \frac{4 \text{ qt}}{1 \text{ gal}} = 10 \text{ qt}$

This page intentionally left blank

Subject Index

A

Absolute value
 explanation of, 152
 method to find, 101
Acute angles, 620
Addends, 17, 19, 203, 714
Addition
 associative property of, 18–19, 167
 on calculators, 25
 by carrying, 17–18
 commutative property of, 18, 19, 167
 of decimals, 344–350, 402
 explanation of, 17
 of fractions, 254–257, 268–274, 312, 319
 given fractional replacement values, 256
 of integers, 108–112, 118, 153
 of mixed numbers, 292–294, 320
 of polynomials, 714–715, 742
 problem solving using, 23–25
 repeated, 40–41
 of signed numbers, 109–110
 of whole numbers, 17–19, 86
 words and phrases for, 79–80
Addition property
 of 0, 18
 of equality, 143–145, 154, 175–178, 186–187, 203, 307
Additive inverse, 117. *See also* Opposites
Adjacent angles, 622
Algebra goals. *See* Study skills
Algebraic expressions
 evaluation of, 75–78, 111, 285
 explanation of, 76, 88
 like terms in, 165–166
 method to simplify, 165–169, 203
 multiplication of, 168–169
Alternate interior angles, 623
Angles
 acute, 620
 adjacent, 622
 alternate interior, 623
 complementary, 621–622
 corresponding, 623
 explanation of, 618, 698
 measurement of, 622–623
 names of, 619
 obtuse, 620
 right, 620
 sides of, 618
 straight, 620
 supplementary, 621–622
 vertical, 622
Angle-side-angle (ASA) triangles, 453
Approximation, of square roots, 446–447
Area
 of circle, 642–643
 explanation of, 639
 formulas for, 639–640, 698

 of parallelograms, 640
 of rectangles, 44–45, 87, 170, 639
 of squares, 71, 88
 surface, 644, 698
 of triangles, 383, 640
Associative property
 of addition, 18–19
 of multiplication, 42
Average. *See also* Mean
 explanation of, 61, 87, 392, 404
 grade point average, 394–395
 method to find, 61, 138

B

Bar graphs
 explanation of, 24, 541, 596–597
 integers in, 103
 method to read and construct, 24–25, 541–543
Base
 explanation of, 68
 of exponential expression, 244
 of percents, 504–506
 of proportions, 493
Binomials. *See also* Polynomials
 explanation of, 714, 742
 multiplication of, 731, 732
 squaring, 731
Borrowing, subtraction by, 21–22, 294
Braces, 79
Brackets, 69, 135
Business analysis, 741

C

Calculators
 addition on, 25
 checking equations on, 189
 compound interest on, 524
 decimals on, 350
 division on, 61
 estimation on, 371
 exponents on, 72
 fractions on, 237, 274, 299
 mixed numbers on, 299
 multiplication on, 47
 negative numbers on, 112
 simplifying expression containing
 fraction bar on, 138
 square roots on, 449
 subtraction on, 25
Capacity
 explanation of, 679
 metric units of, 681–683, 700
 U. S. units of, 679–681, 700
Carrying, adding by, 17–18
Cartesian coordinate system. *See*
 Rectangular coordinate system
Celsius, Anders, 689
Celsius scale

 conversions between Fahrenheit and,
 690–691, 700
 explanation of, 689
Centimeters, 660
Circle graphs
 applications involving, 503
 explanation of, 556, 597
 method to draw, 557–559
 use of, 556–557
 writing ratios from, 418
Circles
 area of, 642–643
 circumference of, 338, 360–361, 402–403,
 632–633, 698
 explanation of, 360, 402–403
Circular cylinders
 explanation of, 644
 volume and surface area of, 644
Circumference
 explanation of, 338, 360–361, 698
 formula for, 632–633
 method to find, 360–361, 402–403
Class frequency, 543, 597
Class interval, 543, 597
Coefficients, numerical, 165–166, 203
Commission problems, 516–517, 530
Common denominator, 254, 258. *See also*
 Least common denominator (LCD)
Commutative property
 of addition, 18, 19, 167
 of multiplication, 42, 167
Complementary angles, 621–622
Complex fractions
 explanation of, 282, 320
 method to simplify, 282–284
Composite numbers, 229, 318
Compound interest, 523–524, 531
Compound interest formula, 524, 531
Cones, volume and surface area of, 644
Congruent triangles
 explanation of, 453, 464
 types of, 453–454
Constant terms, 165
Constants, 165
Convex solids, 643
Corresponding angles, 623
Cross products
 to determine if proportion is true,
 427–428, 463
 explanation of, 235–236
Cubes
 explanation of, 752
 volume and surface area of, 644
Cubic units, 643

D

Data set
 position of the median, 547
 range of, 546–548, 597

- Decimal point, 332, 337, 345, 402
- Decimals
- addition of, 344–350, 402
 - applications with, 392–396
 - on calculators, 350
 - comparing fractions and, 380–381
 - comparison of, 335–337
 - division of, 366–371, 403
 - in equations, 387–389
 - estimating with, 347, 358, 369
 - explanation of, 332
 - method to round, 337–339, 402
 - multiplication of, 357–358, 402–403
 - on number line, 336–338
 - place value for, 332, 337, 401
 - problem solving using, 348–349, 361, 370–371, 382–383
 - as replacement values, 347–348, 360, 370, 383
 - simplifying expressions containing, 348, 381–382
 - in standard form, 334
 - subtraction of, 344–350, 402
 - table of, 753
 - used as replacement values, 347–348, 360
 - in words, 332–334
 - writing fractions as, 378–380, 403
 - writing percents as, 476–479, 528
 - written as fractions, 334–335
 - written in words, 332–334, 402
- Decimeters, 660
- Degrees, 620
- Dekameters, 660
- Denominators
- common, 254
 - explanation of, 215, 417
 - least common, 258–260
 - least common multiple of, 258–259
 - prime factorizations of, 233
- Descartes, René, 565
- Differences
- estimation of, 33–34
 - explanation of, 19–20
- Digits, 8
- Discount problems, 517–518, 530
- Dispersion, of data sets, 546–548
- Distributive property
- explanation of, 166
 - of multiplication, 42–43, 168–169, 204
- Dividends, 53
- Divisibility tests, 230–231
- Division
- on calculators, 61
 - of decimals, 366–371, 403
 - explanation of, 52, 220–221
 - with fractional replacement values, 247
 - of fractions, 244–246, 318
 - of integers, 126–127, 152
 - long, 54–59, 87
 - of mixed numbers, 290–292, 320
 - multiplication to check, 53, 54
 - problem solving using, 59–60
 - properties of, 54, 87, 220–221
 - of whole numbers, 52–61, 87
 - words and phrases for, 79–80
- Division property
- of 0, 54
 - of 1, 54
- Divisor, 53, 57, 366
- E**
- Edge, of polyhedron, 643
- Equality
- addition property of, 143–145, 175–178, 203, 307
 - of fractions, 235–236
 - multiplication property of, 146–148, 175–178, 204, 307
 - words or phrases for, 193
- Equations. *See also* Linear equations in one variable; Linear equations in two variables
- addition property of equality to solve, 154
 - on calculators, 189
 - decimals in, 387–389
 - explanation of, 78, 88
 - expressions vs., 142–143
 - finding unknown numbers in, 486–489
 - fractions in, 307–312, 320
 - multiplication property of equality to solve, 154
 - parentheses in, 187–188
 - percent, 486–489, 529
 - problem-solving steps for, 193–197, 202, 404
 - solutions of, 78–79, 142–148
 - steps to solve, 311, 320, 387
 - writing sentences as, 188–189
- Equilateral triangles, 751
- Equivalent fractions
- explanation of, 231–232, 318
 - method to determine if fractions are, 235–236
 - method to write, 231–235, 260–262
- Estimation. *See also* Rounding
- on calculators, 371
 - to check answers, 290–292
 - with decimals, 347, 358, 369
 - problem solving using, 34–35
 - sums and differences, 33–34
- Evaluating expressions
- explanation of, 68–69, 88, 137–138
 - given replacement values, 118, 127
- Event, probability of, 589–590, 599
- Exams, 5–6, 768–770, 775. *See also* Study skills
- Expanded form, 11
- Experiments, 588, 599
- Exponential expressions
- base of, 244
 - evaluation of, 68–69
 - method to simplify, 759
- Exponential notation, 68
- Exponents
- on calculators, 72
 - explanation of, 68, 88
 - power of product property for, 726–727, 742
 - power property for, 726–727, 759
 - product property for, 724–725, 759
 - quotient rule for, 756, 759
 - simplifying expressions containing, 135–137
 - zero, 756–757
- Expressions. *See also* Algebraic expressions; Exponential expressions
- equations vs., 142–143
 - evaluation of, 76–77
 - explanation of, 76
 - terms of, 165
 - translating word phrases into, 79–80, 179–180
- F**
- Face, of polyhedron, 643
- Factoring, 736–738
- explanation of, 229
- Factors
- explanation of, 41, 229
 - greatest common, 736–738
- Fahrenheit, Gabriel, 689
- Fahrenheit scale
- conversions between Celsius and, 690–691, 700
 - explanation of, 689
- First-degree equations in one variable.
- See* Linear equations in one variable
- First-degree equations in two variables.
- See* Linear equations in two variables
- FOIL order, 732
- Formulas
- area, 639–640, 698
 - circumference, 632–633
 - compound interest, 524, 531
 - geometric, 765
 - perimeter, 629–632, 698
 - volume and surface area, 644, 698
- Fraction bar
- on calculators, 138
 - with complex fractions, 282
 - explanation of, 53, 135, 215
- Fractional replacement values
- addition and subtraction with, 256
 - evaluating expressions given, 272
 - multiplication and division with, 247
- Fractions. *See also* Ratios
- addition of, 254–257, 268–274, 312, 319
 - on calculators, 237, 274, 299
 - comparing decimals and, 380–381
 - complex, 282–286, 320
 - divisibility tests, 230–231
 - division of, 244–246, 318
 - equations containing, 307–312, 320
 - equivalent, 231–232, 235–236, 260–262, 318
 - explanation of, 215, 317
 - graphs of, 219–220, 290
 - improper, 217, 221–222, 317, 318
 - like, 254–257, 319
 - mixed numbers written as improper, 221–222

multiplication of, 242–244, 318
 on number line, 219–220, 290
 problem solving using, 247–248, 382–383
 proper, 217, 317
 rates written as, 419
 ratios written as, 417–421
 from real-life data, 215–218
 reciprocal of, 244, 245, 308, 309, 318
 to represent parts of figures, 215–218
 simplest form of, 232–235, 236, 318
 simplifying expressions containing, 242–243, 381–382
 subtraction of, 254–257, 268–274, 312, 319
 table of, 753
 unit, 656, 669, 682, 699
 unlike, 254
 writing decimals as, 334–335
 writing percents as, 476–479, 528
 written as decimals, 378–380, 403
 Frequency distribution table, 543

G

Geometric figures. *See also specific figures*
 plane, 751–752
 solid, 752
 Geometric formulas, 765
 Grade point average, 394–395
 Grams, 672
 Graphs
 bar, 24, 103, 541–543
 circle, 418
 of fractions, 219–220, 290
 histograms, 543–545
 of integers, 100
 line, 545–546
 of linear equations in two variables, 568, 598
 paired data and, 564–566
 pictographs, 540
 rectangular coordinate system and, 564–566
 review of, 596–599
 scatter diagram, 595
 tree diagram, 588–589
 of whole numbers on number line, 32
 Greatest common factor (GCF)
 explanation of, 736
 factoring out, 737–738
 of list of integers, 736–737
 of list of terms, 737
 Grouping
 of addends, 19
 in addition and multiplication, 167
 of factors, 42
 Grouping symbols, 69, 79, 135, 282

H

Hectometers, 660
 Histograms
 explanation of, 543, 597
 method to read and construct, 543–545

Homework assignments, 772–774.
See also Study skills
 Horizontal lines, 581, 598
 Hypotenuse, 447

I

Improper fractions
 explanation of, 217, 221–222, 317
 as mixed numbers or whole numbers, 221–222, 317, 318
 writing mixed numbers as, 222, 317
 Inches, 656
 Inequality symbols, 100
 Integers. *See also Signed numbers*
 addition of, 108–112, 118, 153
 in bar graphs, 103
 comparison of, 100–101
 division of, 126–127, 153
 explanation of, 99, 152
 graphs of, 100
 greatest common factor of, 736–737
 multiplication of, 123–126, 153
 on number lines, 99, 108, 109, 152–153
 problem solving uses, 127
 problem solving using, 111–112, 119
 subtraction of, 116–120, 153
 Interest problems, 521–524, 529
 Intersecting lines, 622
 Irrational numbers, 338
 Isosceles trapezoids, 751
 Isosceles triangles, 751

K

Kilograms, 672
 Kilometers, 660

L

Least common denominator (LCD)
 complex fractions and, 283–284
 equations solved by multiplying by, 310–312
 explanation of, 258–260, 268, 319
 methods to find, 258–260, 268–274
 to order or compare fractions, 271–272
 Least common multiple (LCM), 258–260
 Legs, of right triangles, 447–448
 Length
 explanation of, 656
 metric units of, 660–663, 699
 U.S. units of, 656–660
 Like fractions
 addition and subtraction of, 254–257, 319
 explanation of, 254
 Like terms
 explanation of, 165–168, 203, 714
 method to combine, 165–166
 Line graphs, 545–546, 564, 596–597
 Line segments, 618, 697
 Linear equations in one variable. *See also Equations*
 addition and multiplication properties to solve, 186–187

on calculators, 189
 explanation of, 186
 parentheses in, 187–188
 problem-solving steps for, 193–197, 202, 204–205
 writing numerical sentences as, 188–189
 Linear equations in two variables. *See also Equations*
 explanation of, 568, 598
 graphs of, 568, 577–581, 598
 Linear measurement, 656
 Lines
 explanation of, 618, 697–698
 horizontal, 581, 598
 intersecting, 622
 parallel, 622, 623
 perpendicular, 622, 623
 vertical, 581, 598
 Liters, 681, 682
 Long division
 examples of, 55–59
 explanation of, 54, 87
 Lowest terms, of fractions, 232–235, 318

M

Magic squares, 151
 Maps, 696
 Mass
 explanation of, 672
 metric units of, 672–674, 699
 U.S. units of, 699
 weight vs., 672
 Mathematics class. *See Study skills*
 Mean
 of data set, 548
 explanation of, 392, 404
 method to find, 392–395
 weighted, 394
 Measurement capacity, 679–683
 linear, 656, 699
 mass, 672–674
 temperature, 687–691
 weight, 669–671
 Measures of central tendency, 392. *See also Mean; Median; Mode*
 Median
 explanation of, 392, 395, 404
 method to find, 395
 Meters, 660
 Metric system. *See also Measurement capacity*
 conversions between U. S. and, 656–657, 659–662
 temperature conversions between U. S. and, 687–691, 700
 units of capacity in, 681–683, 700
 units of length in, 660–662, 699
 units of mass in, 672–674, 699
 units of weight in, 672–674, 699
 use of, 662–663, 674, 683
 Miles, 656
 Milligrams, 672
 Milliliters, 668, 682

- Millimeters, 660
 Minuend, 19–20
 Mixed numbers
 addition and subtraction of, 292–294, 320
 on calculators, 299
 division of, 290–292, 320
 explanation of, 218
 as improper fractions, 221–222
 multiplication of, 290–292, 320
 negative, 296–298
 problem solving with, 295–296
 writing improper fractions as, 222
 Mode
 of data set, 548
 explanation of, 392, 395–396, 404
 method to find, 395–396
 Monomials. *See also* Polynomials
 explanation of, 714, 742
 multiplication of, 730
 Multiplication. *See also* Products
 of algebraic expressions, 168–169
 associative property of, 42
 on calculators, 47
 to check division, 53, 54
 commutative property of, 42
 of decimals, 357–358, 402–403
 distributive property of, 42–43, 168–169, 204
 explanation of, 40
 with fractional replacement values, 247
 of fractions, 242–244, 318
 of integers, 123–126, 153
 of mixed numbers, 290–292, 320
 of polynomials, 730–733, 743
 problem solving using, 46–47, 127, 247–248
 as repeated addition, 40–41
 symbols for, 41, 486
 of whole numbers, 40–47, 87
 words and phrases for, 79–80, 247
 Multiplication property
 of 0, 41
 of 1, 41
 of equality, 146–148, 154, 175–178, 186–187, 204, 307
 of exponents, 724–727, 742
 Multiplication sign, 41

N
 Natural numbers, 8
 Negative exponents
 explanation of, 757–759
 simplifying expressions containing, 757–759
 Negative numbers
 explanation of, 99
 mixed, 296–298
 Notation/symbols. *See also* Words/phrases
 equality, 486
 exponential, 68
 fractional, 417
 grouping, 69, 79, 135, 282
 inequality, 100
 multiplication, 41, 486
 percent, 475, 493
 π (pi), 338, 379, 632, 633, 642
 radical sign, 446
 scientific, 761–763
 Number lines
 decimals on, 336–338
 explanation of, 11
 fractions on, 219–220, 290
 graphs of, 32
 integers on, 99, 108, 109, 152–153
 opposites on, 102
 square roots on, 447
 whole numbers on, 11
 Numbers. *See also* Integers; Signed numbers; Whole numbers
 composite, 229, 318
 irrational, 338
 mixed, 218
 natural, 8
 negative, 99
 positive, 99
 prime, 229–231, 318
 signed, 99
 Numerators, 215, 217, 233, 417
 Numerical coefficients, 165–166, 203

O
 Obtuse angles, 620
 One
 division property of, 54, 220–221
 multiplication property of, 41
 Opposites, 102, 153. *See also* Additive inverse
 Order of operations
 with decimals, 381
 examples of, 70–71
 explanation of, 69, 88, 135, 154, 403
 with fractions, 284–285, 320
 Ordered pairs
 explanation of, 565, 598
 as solutions, 567–569, 578–579, 598
 Organization skills, 2–3, 771–772, 774.
 See also Study skills
 Origin, of rectangular coordinate system, 564, 565
 Ounces, 669
 Outcomes, 588, 589, 599

P
 Paired data, 564–566, 595
 Parallel lines, 622, 623
 Parallelograms
 area of, 640, 641
 explanation of, 751
 Parentheses, 135, 187–188
 Partial products, 44
 Percent decrease, 507, 529
 Percent equations, 486–489, 529
 Percent increase, 506–507, 529
 Percent problems
 applications involving, 503–507, 515–518
 as equations, 486–489
 involving interest, 521–524, 529, 531
 method to solve, 487–489
 proportions to solve, 493–497
 Percent proportions, 493–497, 529
 Percents
 applications with, 479–480
 explanation of, 475, 528
 increase/decrease of, 506–507, 529
 interest problems with, 521–524, 529, 531
 summary for converting, 479–480
 symbol for, 475, 493
 table of, 753, 754
 words and phrases for, 475
 writing decimals or fractions as, 476–479, 528
 written as decimals or fractions, 476–479, 528
 Perfect squares, 446, 447
 Perimeter
 explanation of, 629
 formulas for, 629–632, 698
 of polygons, 22–23, 86, 631
 of rectangles, 629
 of squares, 630
 of triangles, 170, 630–631
 Periods, 9
 Perpendicular lines, 622, 623
 Pictographs, 540
 Place value
 for decimals, 332, 337, 401
 explanation of, 8–9, 85
 rounding whole numbers to, 32–33, 86
 Plane, 564, 618, 639
 Plane figures. *See also specific figures*
 area of, 639–640
 description of, 751
 Plotting the point, 565
 Points
 explanation of, 618
 on graphs of linear equations in two variables, 568, 577–581
 plotting, 564–566
 Polygons
 explanation of, 22, 751
 perimeter of, 22–23, 86, 631
 Polyhedrons, 643–644
 Polynomials
 addition of, 714–715, 742
 evaluation of, 717–718
 explanation of, 714, 742
 methods to factor, 736–738, 743
 multiplication of, 730–733, 743
 subtraction of, 716–717, 742
 types of, 714–715
 Positive numbers, 99
 Positive square roots, 446
 Pounds, 669
 Power of a product property for exponents, 726–727, 742
 Power property for exponents, 726–727, 742, 759

Powers of 10
 division of decimals by, 369
 multiplication of decimals by, 358–360

Prime factorization
 explanation of, 229–231, 318
 to find least common denominators of
 list of denominators, 233, 259–260

Prime numbers, 229–231, 318

Principal, 521

Probability
 of event, 589–590, 599
 explanation of, 588
 tree diagrams and, 588–589

Problem solving. *See also* Notation/symbols;
 the Applications Index; Words/phrases
 with addition, 23–25
 with decimals, 348–349, 361, 370–371
 with division, 59–60
 with equations, 193–197, 202, 204–205, 404
 with estimation, 34–35
 with fractions, 247–248, 272–274
 with integers, 111–112, 119, 127
 with mixed numbers, 295–296
 with multiplication, 46–47
 with percents, 479–480, 487–489,
 515–518, 531
 with proportions, 437–440, 463, 529
 with subtraction, 23–25

Product property for exponents, 724–725,
 742, 759

Products. *See also* Multiplication
 cross, 235–236, 427–428, 463
 explanation of, 41
 partial, 44

Proper fractions, 217, 317

Proportions. *See also* Ratios
 explanation of, 427, 463
 finding unknown numbers in, 428–431
 percent, 493–497, 529
 problem solving with, 437–440, 463, 529
 to solve percent problems, 493–497
 true, 427–428

Protractors, 558–559

Pyramid, square-based, 644

Pythagorean theorem, 445–450, 464

Q

Quadrants, 564

Quadrilaterals, 751

Quotient rule for exponents, 756–757, 759

Quotients
 explanation of, 53
 of like signs, 124
 parts of, 56
 of unlike signs, 124

R

Radical sign, 446

Radius, 24

Rates
 explanation of, 419, 462
 unit, 420, 462
 written as fractions, 419

Ratios. *See also* Fractions; Proportions
 of corresponding sides in similar
 triangles, 454–455
 explanation of, 417, 462
 written as fractions, 417–421

Rays, 618, 698

Reciprocal, of fractions, 244, 245, 308,
 309, 318

Rectangles
 area of, 44–45, 87, 639, 641–642
 explanation of, 751
 perimeter of, 629, 698

Rectangular coordinate system
 explanation of, 564, 598
 method to plot points on, 564–566

Rectangular solids
 explanation of, 752
 volume and surface area of, 644

Remainders, 56

Repeated addition, 40–41

Repeated subtraction, 58–59

Replacement values, use of decimals as,
 347–348, 360, 370, 383

Rhombus, 751

Right angles, 620

Right circular cones, 752

Right circular cylinders, 752

Right triangles, 447–448, 751

Rounding. *See also* Estimation
 of decimals, 337–339, 402
 of whole numbers, 32–33, 86

S

Sales tax problems, 515–516, 530

Scale drawings, 461

Scalene triangles, 751

Scatter diagrams, 595

Scientific notation, 761–763

Sets, 79

Side-angle-side (SAS) triangles, 453

Sides, of angles, 618

Side-side-side (SSS) triangles, 453

Signed numbers. *See also* Integers
 addition of, 109–110
 explanation of, 99, 152

Similar triangles
 explanation of, 454, 464
 finding ratios of corresponding sides in,
 454–455
 finding unknown lengths of sides in,
 455–456

Simple interest, 521–523, 531

Simplest form. *See also* Lowest terms, of
 fractions
 of fractions, 232–235, 318
 problem solving by writing fractions
 in, 236

Single point, 565

Solids
 convex, 643
 description of, 752
 volume and surface area
 of, 643–647

Solutions
 of equations, 78–79, 88, 142–148
 ordered pairs as, 567–569, 578–579, 598

Space, 618

Spheres
 explanation of, 752
 volume and surface area of, 644

Square roots
 approximation of, 446–447
 explanation of, 445, 446, 464
 method to find, 445–446
 on number line, 447
 positive, 446
 Pythagorean theorem and, 447–450
 table of, 755

Square units, 45, 71, 639

Square-based pyramid, 644

Squares
 area of, 71, 88, 639
 explanation of, 445
 perfect, 446, 447
 table of, 755

Squares (geometric)
 area of, 71, 88, 639, 698
 perimeter of, 630, 698

Squaring a binomial, 731

Standard form
 decimals in, 334
 scientific notation numbers written in, 761
 whole numbers in, 11, 86

Statistics
 data set, 546–548, 597
 position of the median, 547

Straight angles, 620

Study guide outline, 778–779

Study skills
 attitude and, 2, 767
 for exams, 5–6, 768–770, 775
 homework assignments and, 772–774
 for learning new terms, 770
 notebooks and, 4–5, 772
 organization skills and, 2–3, 772, 774
 practice final exam and, 780–782
 seeking help and, 5, 776
 self-assessment of, 769
 study guide outline and, 778–779
 textbooks and, 3–4, 776
 time management and, 6
 video resources, 4–5

Subtraction
 by borrowing, 21–22
 on calculators, 25
 of decimals, 344–350, 402
 explanation of, 19
 of fractions, 254–257, 268–274, 312, 319
 given fractional replacement values, 256
 of integers, 116–120, 153
 of mixed numbers, 292–294, 320
 of polynomials, 716–717, 742
 problem solving using, 23–25
 repeated, 58–59
 of whole numbers, 19–22, 86
 words and phrases for, 79–80

Subtraction properties, of 0, 20–21

Subtrahend, 20

Sums

estimation of, 33–34

explanation of, 17

Supplementary angles, 621–622

Surface area

explanation of, 644

formulas for, 644, 698

method to find, 644

Symbols. *See* Notation/symbols

T

Tables, 11–12. *See also* Graphs

Temperature

converting degrees Celsius to

Fahrenheit, 689–690, 700

converting degrees Fahrenheit to

Celsius, 690–691, 700

explanation of, 687

Terms

explanation of, 165

of expression, 203

like, 165–168, 203, 714

variable, 165, 203

Tons, 669

Transversal, 623

Trapezoids, 640, 751

Tree diagrams, 588–589, 599

Triangles

area of, 640, 698

congruent, 453–454, 464

equilateral, 751

explanation of, 751

isosceles, 751

perimeter of, 170, 630–631

right, 447–448, 751

scalene, 751

similar, 454–455, 464

Trinomials. *See also* Polynomials

explanation of, 714, 742

multiplication of, 732–733

U

Undefined, 54

Unit fractions, 656, 669, 682

Unit price, 420–421, 462

Unit rates, 420, 462

Unlike fractions

addition and subtraction of, 268–274, 319

explanation of, 254

problem solving with, 272–274

U.S. measurement system. *See also* Mea-

surement capacity

conversions between metric and,

656–657, 659–662

temperature conversions between met-

ric and, 687–691, 700

units of capacity in, 679–681, 700

units of length in, 656–657, 699

units of mass in, 699

units of weight in, 669–671, 699

use of, 658–660, 671, 680–681

V

Variable terms, 165, 203

Variables, 75, 88

Vertex

explanation of, 618, 698

names of angles, 619

of polyhedron, 643

Vertical lines, 581, 598

Volume

explanation of, 643

formulas for, 644, 698

method to find, 644

W

Weight

explanation of, 669

mass vs., 672

U.S. units of, 669–671, 699

Weighted mean, 394

Whole numbers

addition of, 17–19, 86

division of, 52–61, 87, 290–292

in expanded form, 11

explanation of, 8, 85

graphs of, 32

multiplication of, 40–47, 87, 290–292

rounding of, 32–33, 86

set of, 79

in standard form, 9–11, 86

subtraction of, 19–22, 86

in word form, 9–10

writing improper fractions as, 222

Words/phrases. *See also* Notation/

symbols

for addition, 79–80, 193

for decimals, 332–334, 402

for division, 79–80, 193

for expressions, 79–80, 179–180

for multiplication, 79–80, 193

for multiplication of fractions, 247

for percent, 475

for subtraction, 79–80, 193

translated into variable expressions,

79–80

for whole numbers, 86

X

x-axis, 564, 598

x-coordinate, 565

Y

Yards, 656

y-axis, 564, 598

y-coordinate, 565

Z

Zero

addition property of, 18

division property of, 54, 220–221

multiplication property of, 41

subtraction property of, 20–21

Zero exponents, 756–757

Photo Credits

Chapter 1 p. 6 Bim/E + Getty Images; p. 15 Jeanne Provost/Fotolia; p. 17 Vicki Beaver/Alamy Stock Photo; p. 24 Dirk Ercken/Shutterstock; p. 35 Steve Allen/Allen Creative/Alamy Stock Photo; p. 37 Ken Felepchuk/Shutterstock; p. 46 Sean Pavone/Shutterstock; p. 50 (top) Photographee.eu/Fotolia; (bottom) Pearson Education, Inc.

Chapter 2 p. 104 (left) Neil Lockhart/Shutterstock; (right) Brian Lasenby/Shutterstock; p. 112 Littleny/Shutterstock; p. 122 Pearson Education, Inc.; p. 127 Happy Together/Shutterstock; p. 131 (left) Fuse/Corbis/Getty Images; (right) Koji Hirano/Shutterstock; p. 151 Akg-images/SuperStock; p. 154 (top) Chris Turner/Shutterstock; (bottom) Tusharkoley/Fotolia; p. 156 Itsallgood/Fotolia.

Chapter 3 p. 164 (left) Fotokostic/Shutterstock; (right) Maurizio Borsari/Aflo Co. Ltd/Alamy Stock Photo; p. 196 Maskot/Getty Images; p. 198 (top) Hao Baojing/Fotolia; (bottom) David Bneton/Shutterstock; p. 200 (left) Gretchen Owen/Fotolia; (right) Hellem/Fotolia; p. 211 Wave Break Media/Shutterstock.

Chapter 4 p. 214 Zack Frank/Shutterstock; p. 240 Keith Brofsky/Photodisc/Getty Images; p. 248 The Ohio Collection/Alamy Stock Photo; p. 267 Wave Break Media/Shutterstock; p. 303, 324 Pearson Education, Inc.; p. 305 Maksim Nikalayenka/Shutterstock.

Chapter 5 p. 331 Monkey Business Images/Shutterstock; p. 333 Neal Hamberg/AP Images; p. 342 Scot McKeirnan/Zuma Press, Inc./Alamy Stock Photo; p. 353 (left) Valentin Valkov/Shutterstock; (right) Zeynep Demir/Shutterstock; p. 356, 409 Pearson Education, Inc.; p. 364 Gimmephoto/Fotolia; p. 374 Wave Break Media Micro/Fotolia.

Chapter 6 p. 416, 443, 451 Pearson Education, Inc.; p. 426 (left) Stefan Huwiler/Image Broker/Alamy Stock Photo; (right) ESB Professional/Shutterstock; p. 421, 465 Rachel Youdelman/Pearson Education, Inc.

Chapter 7 p. 475 Pressmaster/Fotolia; p. 480 Ed Aldridge/Shutterstock; p. 504 Michael Jung/Shutterstock; p. 505 Remik44992/Fotolia; p. 509 (right) Elayn Martin-Gay; (left) Burger/Phanie/Alamy Stock Photo; p. 511 Kubais/Fotolia; p. 512 Radostin Todorov/Shutterstock; p. 513 (left) Stockr/Shutterstock; (right) Beneda Miroslav/Shutterstock; p. 515 Narvf/Fotolia; p. 519 (top, left) Pearson Education, Inc.; (top, right) Sashkin/Fotolia; (bottom, left) Terex/Fotolia; (bottom, right) LuckyPhoto/Fotolia.

Chapter 8 p. 539 (top) Ilze Filipova/Shutterstock; (bottom, left) Grzegorz Czapski/Shutterstock; (bottom, right) Jim West/Alamy Stock Photo; p. 574, 588, 594, 600, 611 Pearson Education, Inc.; p. 600 (bottom) Franco's Photos/Shutterstock.

Chapter 9 p. 628 (left) Expose/Shutterstock; (right) Ira Block/National Geographic Creative/Alamy Stock Photo; p. 617, 633, 639, 661 Pearson Education, Inc.; p. 630 Julie/Shutterstock; p. 637 (right) Frankix/Fotolia; p. 652 (left) Jorg Hackemann/Fotolia; Shuttoz/Fotolia; p. 657 Daphot75/Fotolia; p. 672 Michael Simons/Shutterstock; p. 677 (top) Bill/Fotolia; (bottom) Vuktopua/Fotolia; p. 693 Dave/Fotolia; p. 692 (left) Jiang Dao Hua/Shutterstock; (right) Taranova/Shutterstock; p. 693 Dave/Fotolia; p. 709 Worker/Shutterstock.

Chapter 10 p. 713 (top, right) Andrey Popov/Shutterstock; (bottom) Ruslan Kostiv/Shutterstock; (left, top) Adventtr/Stock/Getty Images; p. 722 Bridget Calip/Shutterstock.

Cover NK Natali/Shutterstock

Text Credits

Appendix A Elayn Martin-Gay

Appendix C p. 763 Students for the Exploration and Development of Space; p. 763 National Institute of Standards and Technology; p. 764 CIA World Factbook; p. 764 Niagara-fallslive.com.

This page intentionally left blank

This page intentionally left blank

This page intentionally left blank

This page intentionally left blank

This page intentionally left blank

This page intentionally left blank

This page intentionally left blank



Pearson
MyLab

Get the Most Out of MyLab Math

MyLab™ Math is the world's leading online tutorial and assessment program designed to help you learn and succeed in your mathematics course. MyLab Math courses provide you with:

- ✓ Personalized learning experiences
- ✓ Interactive practice with immediate feedback and just-in-time learning aids
- ✓ Multimedia learning resources
- ✓ Complete eText, accessible anywhere with the Pearson eText app

**MyLab Math is available for this textbook.
To learn more, visit pearson.com/mylab/math.**



Pearson

ISBN-13: 978-0-13-470764-8
ISBN-10: 0-13-470764-8



9 780134 707648



9 0 0 0 0